Reinforcement Learning

15s1: COMP9417 Machine Learning and Data Mining

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Acknowledgements

Material derived from slides for the book
"Machine Learning" by T. Mitchell
McGraw-Hill (1997)
http://www-2.cs.cmu.edu/~tom/mlbook.html

Aims

This lecture will introduce you to reinforcement learning. Following it you should be able to:

- outline the framework of control learning
- describe control policies that choose optimal actions
- apply the method of Q learning
- outline the convergence properties of Q learning

Control Learning

Consider learning to choose actions, e.g.,

- Learning to control pole-and-cart
- Robot learning to dock on battery charger
- Learning to choose actions to optimize factory output
- Learning to play Backgammon

Control Learning

Note several problem characteristics:

- Delayed reward
- Opportunity for active exploration
- Possibility that state only partially observable
- Possible need to learn multiple tasks with same sensors/effectors

Example: TD-Gammon

Tesauro (1995)

Used Reinforcement Learning to play Backgammon

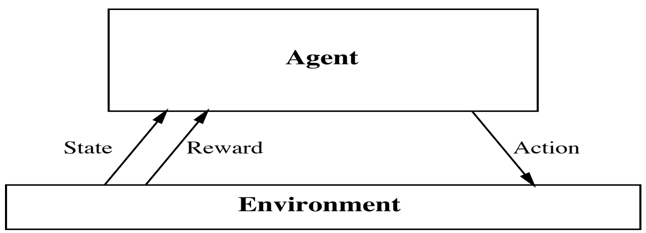
Immediate reward

- +100 if win
- -100 if lose
- 0 for all other states

Trained by playing 1.5 million games against itself

Now approximately equal to best human player

Reinforcement Learning Problem



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \dots$$

Goal: Learn to choose actions that maximize

$$r_0 + \gamma r_1 + \gamma^2 r_2 + \dots \text{, where } 0 \leq \gamma < 1$$

Markov Decision Processes

Assume

- finite set of states S
- set of actions A
- at each discrete time step t agent observes state $s_t \in S$ and chooses action $a_t \in A$
- then receives immediate reward r_t
- and state changes to s_{t+1}

Markov Decision Processes

Markov assumption: $s_{t+1} = \delta(s_t, a_t)$ and $r_t = r(s_t, a_t)$

- r_t and s_{t+1} depend only on *current* state and action
- functions δ and r may be nondeterministic
- functions δ and r not necessarily known to agent

Agent's Learning Task

Execute actions in environment, observe results, and

- learn action policy $\pi : S \to A$ that maximizes

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

  from any starting state in S
- where $0 \le \gamma < 1$ is the discount factor for future rewards

Note something new:

- Target function is $\pi : S \to A$
- but we have no training examples of the form $\langle s, a \rangle$
- in fact training examples are of the form $\langle \langle s, a \rangle, r \rangle$

Value Function

To begin, consider deterministic worlds ...

For each possible policy π the agent might adopt, we can define an evaluation function over states

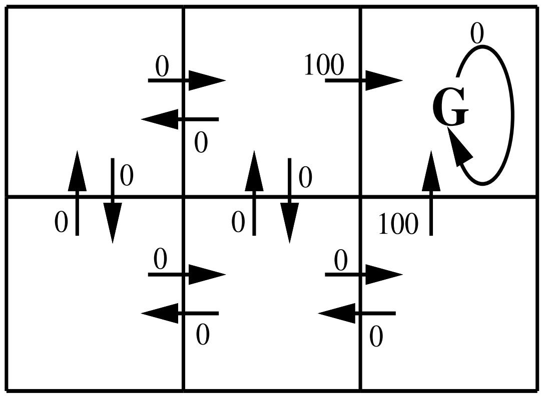
$$\begin{aligned} V^{\pi}(s) &\equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots \\ &\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i} \end{aligned}$$

where $r_t, r_{t+1}, \ldots$ are generated by following policy π starting at state s

Restated, the task is to learn the optimal policy π^*

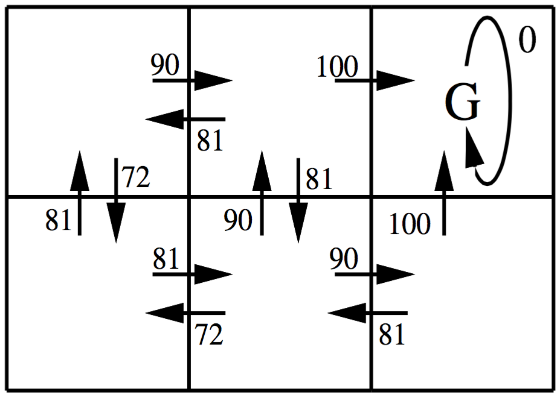
$$\pi^* \equiv \underset{\pi}{\operatorname{argmax}}\, V^{\pi}(s), (\forall s)$$

Q-learning in a simple deterministic grid world – main concepts



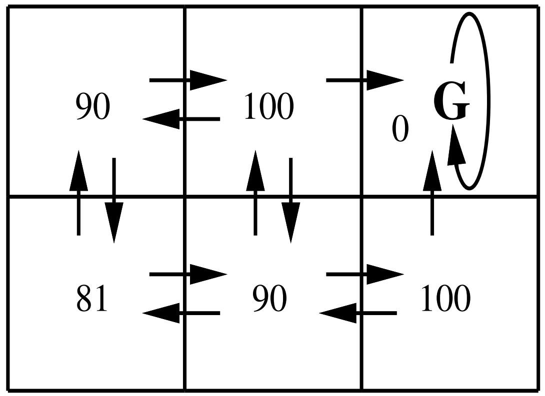
$r(s, a)$ (immediate reward) values

Q-learning in a simple deterministic grid world – main concepts



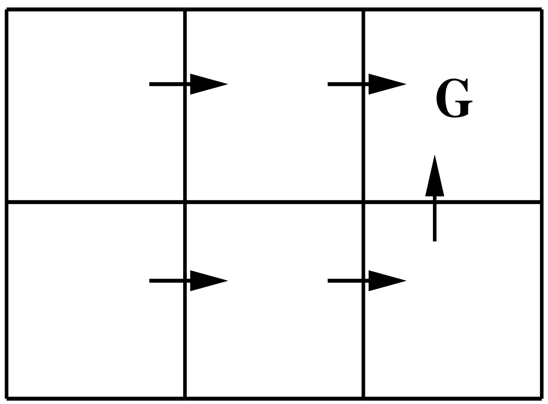
$Q(s, a)$ values

Q-learning in a simple deterministic grid world – main concepts



$V^*(s)$ values

Q-learning in a simple deterministic grid world – main concepts



One optimal policy

What to Learn

Could try to have agent learn the evaluation function V^{π^*} (which we write as V^*)

It could then do a lookahead search to choose best action from any state s because

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}[r(s, a) + \gamma V^*(\delta(s, a))]$$

A problem:

- This works well if agent knows $\delta : S \times A \rightarrow S$, and $r : S \times A \rightarrow \Re$
- But when it doesn't, it can't choose actions this way

Q Function

Define new function very similar to V^*

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

If agent learns Q, it can choose optimal action even without knowing δ!

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}[r(s,a) + \gamma V^*(\delta(s,a))]$$

$$\pi^*(s) = \underset{a}{\operatorname{argmax}} Q(s,a)$$

Q is the evaluation function the agent will learn

Training Rule to Learn Q

Note that Q and V^* are closely related:

$$V^*(s) = \max_{a'} Q(s,a')$$

Which allows us to write Q recursively as

$$\begin{aligned} Q(s_t,a_t) &= r(s_t,a_t) + \gamma V^*(\delta(s_t,a_t))) \\ &= r(s_t,a_t) + \gamma \max_{a'} Q(s_{t+1},a') \end{aligned}$$

Nice! Let $\hat{Q}$ denote learner's current approximation to Q. Consider training rule

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

where s' is the state resulting from applying action a in state s

Q Learning for Deterministic Worlds

For each s,a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

Observe current state s

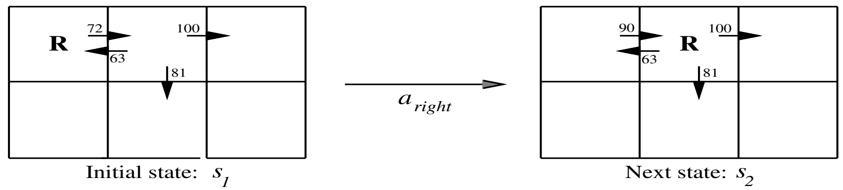
Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s,a)$ as follows:

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

- $s \leftarrow s'$

Updating $\hat{Q}$



$$\begin{aligned} \hat{Q}(s_1,a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2,a') \\ &\leftarrow 0 + 0.9\ \max\{63,81,100\} \\ &\leftarrow 90 \end{aligned}$$

Note: if rewards non-negative, then

$(\forall s,a,n)\ \ \hat{Q}_{n+1}(s,a) \geq \hat{Q}_n(s,a)$ and $(\forall s,a,n)\ \ 0 \leq \hat{Q}_n(s,a) \leq Q(s,a)$

Convergence of Q learning

$\hat{Q}$ converges to Q. Consider case of deterministic world where see each $\langle s, a \rangle$ visited infinitely often.

Proof: Define a full interval to be an interval during which each $\langle s, a \rangle$ is visited. During each full interval the largest error in $\hat{Q}$ table is reduced by factor of γ

Let $\hat{Q}_n$ be table after n updates, and Δ_n be the maximum error in $\hat{Q}_n$; that is

$$\Delta_n = \max_{s,a} |\hat{Q}_n(s,a) - Q(s,a)|$$

Convergence of Q learning

For any table entry $\hat{Q}_n(s,a)$ updated on iteration $n+1$, the error in the revised estimate $\hat{Q}_{n+1}(s,a)$ is

$$\begin{aligned}
|\hat{Q}_{n+1}(s,a) - Q(s,a)| &= |(r + \gamma \max_{a'} \hat{Q}_n(s',a')) \\
&\quad - (r + \gamma \max_{a'} Q(s',a'))| \\
&= \gamma |\max_{a'} \hat{Q}_n(s',a') - \max_{a'} Q(s',a')| \\
&\leq \gamma \max_{a'} |\hat{Q}_n(s',a') - Q(s',a')| \\
&\leq \gamma \max_{s'',a'} |\hat{Q}_n(s'',a') - Q(s'',a')| \\
|\hat{Q}_{n+1}(s,a) - Q(s,a)| &\leq \gamma \Delta_n
\end{aligned}$$

Note we used general fact that

$$|\max_a f_1(a) - \max_a f_2(a)| \leq \max_a |f_1(a) - f_2(a)|$$

Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine V, Q by taking expected values

$$\begin{aligned}
V^\pi(s) &\equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \\
&\equiv E[\sum_{i=0}^{\infty} \gamma^i r_{t+i}]
\end{aligned}$$

$$Q(s,a) \equiv E[r(s,a) + \gamma V^*(\delta(s,a))]$$

Nondeterministic Case

Q learning generalizes to nondeterministic worlds

Alter training rule to

$$\hat{Q}_n(s,a) \leftarrow (1 - \alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n[r + \max_{a'} \hat{Q}_{n-1}(s',a')]$$

where

$$\alpha_n = \frac{1}{1 + visits_n(s,a)}$$

Can still prove convergence of $\hat{Q}$ to Q [Watkins and Dayan, 1992]

Temporal Difference Learning

Q learning: reduce discrepancy between successive Q estimates

One step time difference:

$$Q^{(1)}(s_t, a_t) \equiv r_t + \gamma \max_a \hat{Q}(s_{t+1}, a)$$

Why not two steps?

$$Q^{(2)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 \max_a \hat{Q}(s_{t+2}, a)$$

Or n?

$$Q^{(n)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \cdots + \gamma^{(n-1)} r_{t+n-1} + \gamma^n \max_a \hat{Q}(s_{t+n}, a)$$

Blend all of these:

$$Q^\lambda(s_t, a_t) \equiv (1-\lambda)\left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \cdots\right]$$

Temporal Difference Learning

$$Q^\lambda(s_t, a_t) \equiv (1-\lambda)\left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \cdots\right]$$

Equivalent expression:

$$\begin{aligned} Q^\lambda(s_t, a_t) &= r_t + \gamma[\ (1-\lambda) \max_a \hat{Q}(s_t, a_t) \\ &\quad + \lambda\ Q^\lambda(s_{t+1}, a_{t+1})] \end{aligned}$$

TD(λ) algorithm uses above training rule

- sometimes converges faster than Q learning
- converges for learning V^* for any $0 \le \lambda \le 1$ (Dayan, 1992)
- Tesauro's TD-Gammon uses this algorithm

Further issues

- Replace $\hat{Q}$ table with neural net or other generalizer
- Handle case where state only partially observable
- Design optimal exploration strategies
- Extend to continuous action, state
- Learn and use $\hat{\delta} : S \times A \to S$
- Relationship to dynamic programming

Summary

- Reinforcement Learning enables learning to control (an agent)
- One of the oldest ideas in machine learning, but still useful
- Q learning is a standard approach
- Many further refinements
- Succesfully applied, e.g., in game playing
- Can take a lot of training to learn a good policy
- Exploration – exploitation trade-off (bandit learning)
- Function approximation to capture “state”
- Extended to hierarchical, relational learning