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CS 2100: Discrete Structures

Homework 1: Logic

Spring 2023

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1. **Exercise 10.d on page 38**
   Suppose x and y indicate particular real numbers. Write conditions that express the following statement: **"Neither x nor y is positive."**, by using comparisons (such as $x > 0$) and the basic operations of logic.

   Solution:

   $x \leq 0 \wedge y \leq 0$

2. **Exercise 17 on page 39**
   Use truth tables to verify the following statement:

$$p \vee (p \wedge q) \equiv p$$

   establishing the second absorption property in Theorem 2.

   Solution:

p	q	$p \wedge q$	$p \vee (p \wedge q)$
f	f	f	f
f	t	f	f
t	f	f	t
t	t	t	t

3. **Exercise 21.e on page 39**
Use truth tables to check if the following statement is true:

$$(p \vee q) \wedge (q \vee r) \equiv (p \wedge r) \vee q$$

Solution:

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \wedge (q \vee r)$	$p \wedge r$	$(p \wedge r) \vee q$
f	f	f	f	f	f	f	f
f	t	f	t	t	t	f	t
t	f	f	t	f	f	f	f
t	t	f	t	t	t	f	t
f	f	t	f	t	f	f	f
f	t	t	t	t	t	f	t
t	f	t	t	t	t	t	t
t	t	t	t	t	t	t	t

4. **Exercise 24.c on page 40**
   By quoting parts of Theorem 2, verify the following logical equivalence:

$$q \wedge (p \vee r) \equiv (p \wedge q) \vee (q \wedge r)$$

   Start with the left side and use parts of Theorem 2 to change the problem, ending with the right side.

   Solution:

$$q \wedge (p \vee r)$$

$$\Big\downarrow \text{ distributive}$$

$$(q \wedge p) \vee (q \wedge r)$$

$$\Big\downarrow \text{ commutative}$$

$$(p \wedge q) \vee (q \wedge r)$$

5. **Exercises 14.d on page 52**
For the following statement:

$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, \frac{x}{y} = 2$$

Write the negation of the statement using Proposition 1 and the rules for negating to simplify each to the point that no $\neg$ symbol occurs to the left of a quantifier. (Recall that $\mathbb{Z}$ denotes the set of all integers, and $\mathbb{R}$ denotes the set of all real numbers.)

Solution:

$$\exists x \in \mathbb{Z}, \forall y \in \mathbb{R}, \frac{x}{y} \neq 2$$

6. **Exercise 15.d on page 52**
For the following statement:
$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, \frac{x}{y} = 2$$
Decide whether the original statement or its negation is true.

Solution:

negation:

$$\exists x \in \mathbb{Z}, \forall y \in \mathbb{R}, \frac{x}{y} \neq 2$$

the original is true.

7. **Exercise 17.c on page 52**
   Write the negation of the following statement as an English sentence:

   "For every positive integer x, there is a positive integer y such that y is smaller than x and y is a factor of x."

   Solution:

original: $\forall x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+, y < x \wedge y | x$

negation: $\exists x \in \mathbb{Z}^+, \forall y \in \mathbb{Z}^+, y \geq x \vee y \nmid x$

Sentence:

There is a positive integer x, for all positive integers y, y is greater or equal to x or y is not a factor of x.

8. **Exercise 20.b on page 53**
For the statement:

> "For every two real numbers x and y, there is an integer n such that $x < n < y$."

Decide whether the original statement or its negation is true.

Solution:

original: $\forall (x, y) \in \mathbb{R}, \exists n \in \mathbb{Z}, x < n < y$

negation: $\exists (x, y) \in \mathbb{R}, \forall n \in \mathbb{Z}, x \geq n \geq y$

the negation is true.

9. **Exercise 11.e on page 66**
Let D be the set $\{1, 3, 5, 7, 8, 10, 11, 12\}$. Decide whether the following statement is true for all the elements of D. If it is not, give a counterexample.

"If $x > 5$ and $x < 7$, then x is negative."

Solution:

false

$x = 6$

10. **Exercise 14.d on page 67**
Express the following statement using predicates and the quantifier $\forall$

"For every positive real number x, if $x < \sqrt{2}$, then $\frac{2}{x} > \sqrt{2}$."

Solution:

$$\forall x \in \mathbb{R}^+, \; x < \sqrt{2} \rightarrow \frac{2}{x} > \sqrt{2}$$

11. **Exercise 17.d on page 67**
Express the following statement using predicates and the quantifiers $\forall$ and $\exists$

"For every real number y, if $y \geqslant 0$, then there exists $x \in \mathbb{R}$ such that $x^2 = y$."

Solution:

$$\forall y \in \mathbb{R}, y \geq 0 \Rightarrow \exists x \in \mathbb{R}, x^2 = y$$

12. **Exercise 25.e on page 67**
Form the contrapositive of the following statement:

"If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle."

Solution:

if a triangle is not an isoscles, then it does not have two equal sides nor angles.

13. **Exercise 26.e on page 67**
Form the converse of the following statement:

"If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle."

Solution:

if a triangle is isosceles, then it has either two equal sides or angles.

14. **Exercise 27.e on page 67**
Form the inverse of the following statement:

"If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle."

Solution:

if a triangle does not have two equal sides nor angles, then it is not isosceles.