Assignment 6

doesn't matter $= \phi$

3.1

1) $$\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix} = 3(3\cdot -1 - 2\cdot 5) - 0(\quad \phi \quad) + 4(2\cdot 5 - 3\cdot 0)$$
$$= 3(-3-10) + 4(10) = 3(-13) + 40 = -39 + 40 = \boxed{1}$$

$$\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix} = -0(\phi) + 3(3\cdot -1 - 4\cdot 0) - 5(3\cdot 2 - 2\cdot 4)$$
$$= 3(-3) - 5(6-8) = -9 - 5(-2) = -9 + 10 = \boxed{1}$$

3) $$\begin{vmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{vmatrix} = 2(1\cdot -1 - 2\cdot 3) - -2(3\cdot -1 - 2\cdot 1) + 3(3\cdot 3 - 1\cdot 1)$$
$$= 2(-1-6) + 2(-3-2) + 3(9-1)$$
$$= 2(-7) + 2(-5) + 3(8) = -14 - 10 + 24 = \boxed{0}$$

$$\begin{vmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{vmatrix} = --2(3\cdot -1 - 2\cdot 1) + 1(2\cdot -1 - 3\cdot 1) - 3(2\cdot 2 - 3\cdot 3)$$
$$= 2(-3-2) + (-2-3) - 3(4-9)$$
$$= 2(-5) - 5 - 3(-5) = -10 - 5 + 15 = \boxed{0}$$

9) $$\begin{vmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{vmatrix} = 3 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 7 \end{vmatrix} = 3\left(5 \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix}\right) = 15(7\cdot 1 - 2\cdot 3) = 15(7-6) = \boxed{15}$$

11) $$\begin{vmatrix} 3 & 5 & -6 & 4 \\ 0 & -2 & 3 & -3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 3 \end{vmatrix} = 3\cdot -2\cdot 1\cdot 3 = \boxed{-18}$$

15) $$\left|\begin{array}{ccc|cc} 1 & 0 & 4 & 1 & 0 \\ 2 & 3 & 2 & 2 & 3 \\ 0 & 5 & -2 & 0 & 5 \end{array}\right| = 1\cdot 3\cdot -2 + 0 + 4\cdot 2\cdot 5 - (0 + 5\cdot 2\cdot 1 + 0)$$
$$= -6 + 40 - 10 = 40 - 16 = \boxed{24}$$

19) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ swapping rows negated det.

$\begin{vmatrix} c & d \\ a & b \end{vmatrix} = bc - ad$

21) $\begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 3 \cdot 4 - 2 \cdot 5 = 12 - 10 = 2$

$\begin{vmatrix} 3 & 2 \\ 5+3k & 4+2k \end{vmatrix} = 3(4+2k) - 2(5+3k) = 12 + \cancel{6k} - 10 - \cancel{6k} = 12 - 10 = 2$

adding a multiple of another row did nothing to the det.

25) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 0 \\ k & 1 \end{vmatrix} = 1(1 \cdot 1 - 0 \cdot k) = 1(1) = 1$

3.2

1) If 2 rows are interchanged to produce B, then det B = - det A

3) If a multiple of one row of A is added to another row to produce B, then det B = det A

5) $\begin{vmatrix} 1 & 5 & -4 \\ -1 & -4 & 5 \\ -2 & -8 & 7 \end{vmatrix} \underset{+=-2R_2}{} \rightarrow \begin{vmatrix} 1 & 5 & -4 \\ -1 & -4 & 5 \\ 0 & 0 & -3 \end{vmatrix} \underset{+=R_1}{} \rightarrow \begin{vmatrix} 1 & 5 & -4 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{vmatrix} = \boxed{-3}$

7) $\begin{vmatrix}1&3&0&2\\-2&-5&7&4\\3&5&2&1\\1&-1&2&-3\end{vmatrix} \begin{matrix}\\+2R_1\\-3R_1\\-R_1\end{matrix} \rightarrow \begin{vmatrix}1&3&0&2\\0&1&7&8\\0&-4&2&-5\\ \cancel{0}&\cancel{-4}&\cancel{2}&\cancel{-5}\end{vmatrix}$ $\quad \boxed{0}$

11) $\begin{vmatrix}3&4&-3&-1\\3&0&1&-3\\-6&0&-4&3\\6&8&-4&-1\end{vmatrix} \begin{matrix}-R_2\\ \\+R_4\\ \\ \end{matrix} \rightarrow \begin{vmatrix}0&4&-4&2\\3&0&1&-3\\0&8&-8&2\\6&8&-4&-1\end{vmatrix} \rightarrow \begin{vmatrix}6&8&-4&-1\\3&0&1&-3\\0&8&-8&2\\0&4&-4&2\end{vmatrix} \rightarrow$

$-\cancel{\det}$ $\quad -\cancel{\det}$

$\begin{vmatrix}3&0&1&-3\\6&8&-4&-1\\0&8&-8&2\\0&4&-4&2\end{vmatrix} \begin{matrix}\\-2R_1\\ \\ \\ \end{matrix} \rightarrow \begin{vmatrix}3&0&1&-3\\0&8&-6&5\\0&8&-8&2\\0&4&-4&2\end{vmatrix} = 3\begin{vmatrix}8&-6&5\\8&-8&2\\4&-4&2\end{vmatrix} \begin{matrix}\\-R_1\\ \\ \end{matrix} \rightarrow 3\begin{vmatrix}8&-6&5\\0&-2&-3\\4&-4&2\end{vmatrix} \begin{matrix}\\ \\-\frac{R_1}{2}\end{matrix} \rightarrow$

$3\begin{vmatrix}8&-6&5\\0&-2&-3\\0&-1&-1/2\end{vmatrix} = 3\cdot 8\left(-2\cdot -1/2 - (-3\cdot -1)\right) = 24(1-3) = 24(-2) = \boxed{-48}$

15) $7\cdot 3 = \boxed{21}$

17) $\boxed{7}$

25) $\begin{vmatrix}7&-8&7\\-4&5&0\\-6&7&-5\end{vmatrix} = 7(-4\cdot 7 - 5\cdot -6) - 5(7\cdot 5 - -8\cdot -4)$
$= 7(-28+30) - 5(35-32) = 7(2) - 5(3)$
$= 14 - 15 = -1$ $\quad \boxed{\text{yes}}$

35) $|B^4| = |B|^4$
$|B| = \begin{vmatrix}1&0&1\\1&1&2\\1&2&1\end{vmatrix} = 1(1\cdot 1 - 2\cdot 2) + 1(1\cdot 2 - 1\cdot 1) = (1-4) + (2-1) = -3+1 = -2$
$|B|^4 = (-2)^4 = \boxed{16}$

3.3

Cramer's: $A\vec{x}=\vec{b}$ ∴

$$x_i = \frac{|A_i(b)|}{|A|}$$

$$A^{-1} = \frac{\text{adj } A}{\det A} = \frac{C(A)^T}{\det A}$$

1) $A=\begin{bmatrix}5 & 7\\ 2 & 4\end{bmatrix}, \vec{b}=\begin{bmatrix}3\\1\end{bmatrix}$

$|A| = 5\cdot4-7\cdot2 = 20-14 = 6$

$$\vec{x} = \begin{bmatrix}\frac{1}{6}\begin{vmatrix}3 & 7\\ 1 & 4\end{vmatrix}\\ \frac{1}{6}\begin{vmatrix}5 & 3\\ 2 & 1\end{vmatrix}\end{bmatrix} = \begin{bmatrix}(3\cdot4-7\cdot1)/6\\ (5\cdot1-3\cdot2)/6\end{bmatrix} = \begin{bmatrix}5/6\\ -1/6\end{bmatrix}$$

3) $A=\begin{bmatrix}3 & -2\\ -4 & 6\end{bmatrix}, \vec{b}=\begin{bmatrix}3\\-5\end{bmatrix}$

$|A| = 3\cdot6 - -2\cdot -4 = 18-8 = 10$

$$\vec{x} = \begin{bmatrix}\frac{1}{10}\begin{vmatrix}3 & -2\\ -5 & 6\end{vmatrix}\\ \frac{1}{10}\begin{vmatrix}3 & 3\\ -4 & -5\end{vmatrix}\end{bmatrix} = \begin{bmatrix}(3\cdot6--2\cdot-5)/10\\ (3\cdot-5-3\cdot-4)/10\end{bmatrix} = \begin{bmatrix}4/5\\ -3/10\end{bmatrix}$$

5) $A=\begin{bmatrix}1^+ & 1^- & 0\\ -3 & 0 & 2\\ 0 & 1 & -2\end{bmatrix}, \vec{b}=\begin{bmatrix}3\\0\\2\end{bmatrix}$

$|A| = 1(-2\cdot1) - 1(-3\cdot-2) = -2-6 = -8$

$$\vec{x} = \begin{bmatrix}\frac{1}{-8}\begin{vmatrix}3 & 1 & 0\\ 0 & 0 & 2^-\\ 2 & 1 & -2\end{vmatrix}\\ -\frac{1}{8}\begin{vmatrix}1^+ & 3^- & 0\\ -3 & 0 & 2\\ 0 & 2 & -2\end{vmatrix}\\ -\frac{1}{8}\begin{vmatrix}1 & 1 & 3\\ -3^- & 0 & 0\\ 0 & 1 & 2\end{vmatrix}\end{bmatrix} = \begin{bmatrix}-\frac{1}{8}(-2(3\cdot1-1\cdot2))\\ -\frac{1}{8}(1(-2\cdot2)-3(-3\cdot-2))\\ -\frac{1}{8}(-3(1\cdot2-3\cdot1))\end{bmatrix} = \begin{bmatrix}1/4\\ 11/4\\ -3/8\end{bmatrix}$$

7) $\begin{vmatrix} 6s & 4 \\ 9 & 2s \end{vmatrix} = 12s^2 - 4\cdot 9 \neq 0 \therefore 12s^2 \neq 36 \therefore s^2 \neq 3$

$\boxed{s \neq \pm\sqrt{3}}$

11) $A = \begin{bmatrix} 0 & -2 & -1 \\ 5 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$ $C(A) = \begin{bmatrix} 0 & -5 & 5 \\ 3 & -1 & 2 \\ 0 & -5 & 10 \end{bmatrix}$ $|A| = 5(3) = 15 = \det A$

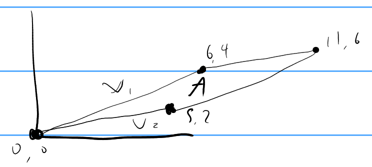
$\text{adj}(A) = C(A)^T = \begin{bmatrix} 0 & 3 & 0 \\ -5 & -1 & -5 \\ 5 & 2 & 10 \end{bmatrix}$

$A^{-1} = \frac{\text{adj}(A)}{\det(A)} = \longrightarrow \boxed{\begin{bmatrix} 0 & 1/5 & 0 \\ -1/3 & -1/15 & -1/3 \\ 1/3 & 2/15 & 2/3 \end{bmatrix}}$

15) $A = \begin{bmatrix} 5 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 3 & -1 \end{bmatrix}$ $|A| = -5$ $C(A) = \begin{bmatrix} -1 & -1 & -1 \\ 0 & -5 & -15 \\ 0 & 0 & 5 \end{bmatrix}$ $C(A)^T = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -5 & 0 \\ -1 & -15 & 5 \end{bmatrix} = \text{adj} A$

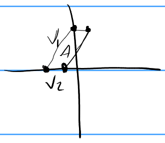
$A^{-1} = \frac{C(A)^T}{-5} = \boxed{\begin{bmatrix} 1/5 & 0 & 0 \\ 1/5 & 1 & 0 \\ 1/5 & 3 & -1 \end{bmatrix}}$

19)



$v_1 = (6,4)$
$v_2 = (5,2)$

$A = \left\| \begin{matrix} 6 & 5 \\ 4 & 2 \end{matrix} \right\| = |6\cdot 2 - 5\cdot 4| = |12 - 20| = 8$

21)  $V_1=\begin{bmatrix}0\\3\end{bmatrix}-\begin{bmatrix}-2\\0\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}$ $V_2=\begin{bmatrix}-1\\0\end{bmatrix}-\begin{bmatrix}-2\\0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

$A=\left|\begin{vmatrix}2&1\\3&0\end{vmatrix}\right|=|-1\cdot 3|=\boxed{3}$

27) $S=\begin{bmatrix}-2&-2\\3&5\end{bmatrix}$ $A=\begin{bmatrix}6&-3\\-3&2\end{bmatrix}$

$|\det S\cdot\det A|=|(-2\cdot 5--2\cdot 3)(6\cdot 2--3\cdot -3)|$
$=|(-10+6)(12-9)|=|(-4)(3)|=\boxed{12}$

4.1

1) a) yes, any positive or 0 numbers added will always be positive or 0.

b) $\vec{u}=\begin{bmatrix}1\\1\end{bmatrix}$ $c=-1$ $c\vec{u}=\begin{bmatrix}-1\\-1\end{bmatrix}$

3) $c=1{,}000$ $\vec{v}=\begin{bmatrix}0\\1\end{bmatrix}$ $c\vec{v}=\begin{bmatrix}0\\1{,}000\end{bmatrix}$ $c\vec{v}\notin H$

5) $(\mathbb{P}_2)$ $p(\cancel{t})$ ~~$=at^2$ is a subspace of $\mathbb{P}_2$, s $p(t)=a_1+a_2t+a_3t^2$~~
~~where $a_1=0$ & $a_2=0$ & $a_3\in\mathbb{R}$ & $a_2\in\mathbb{R}$~~

a) $p(0)=0$ ✓

b) $p(x+y)=a(x+y)^2=az^2\in\mathbb{P}_2$ ✓

c) $-1\cdot p(x)=-ax^2$ $-a=b$ $-p(x)=bx^2\in\mathbb{P}_2$ ✓

yes

7) a) $p(t) = z_1 + z_2 t + z_3 t^2 + z_4 t^3,\ z_x \in \mathbb{Z}$

$p(0) = z_1$, not always 0 ✗

not a subspace of $\mathbb{P}_3$

9) $H = \left\{ \begin{bmatrix} s \\ 3s \\ 2s \end{bmatrix}, s \in \mathbb{R} \right\}$

$V = \begin{bmatrix} 3 \\ 9 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, shows closed under addition and multiplication

11) $W = \left\{ \begin{bmatrix} 5b+2c \\ b \\ c \end{bmatrix}, b,c \in \mathbb{R} \right\}$ $u = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, v = \begin{bmatrix} 9 \\ 1 \\ 2 \end{bmatrix}$

u&v are linearly independent,

span $\left\{ \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 9 \\ 1 \\ 2 \end{bmatrix} \right\}$ spans $\mathbb{R}_2$,

$u, v \in \mathbb{R}_3$ & $u, v \in W$

13) a) no, 3

b) ∞

c) yes, $w = v_1 + v_2$

15) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \notin W$, not vector space

17) $v_1: a = 1, b = 2, c = 3$ $v_2: a = 4, b = 2, c = 1$

$v_1 = \begin{bmatrix} -1 \\ -1 \\ 2 \\ 2 \end{bmatrix}$ $v_2 = \begin{bmatrix} 2 \\ 1 \\ -3 \\ 2 \end{bmatrix}$

$S = \left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ 2 \end{bmatrix} \right\}$