$$A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \qquad \det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 3-\lambda \end{bmatrix}, \quad |A-\lambda I| = (3-\lambda)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix} + \begin{vmatrix} -1 & -1 \\ -1 & 3-\lambda \end{vmatrix} - \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left((3-\lambda)^2 - 1\right) + (\lambda - 3) - 1 - (1 + 3 - \lambda)$$

$$= (3-\lambda)^3 - 1 + \lambda - 3 - 1 - 1 - 3 + \lambda$$

$$= (3-\lambda)^3 + 2\lambda - 9 = (-\lambda+3)^3 + 2\lambda - 9$$

$$= (-\lambda+3)(\lambda^2 - 6\lambda + 9) + 2\lambda - 9$$

$$= -\lambda^3 + 6\lambda^2 - 9\lambda + 3\lambda^2 - 18\lambda + 27 + 2\lambda - 9$$

$$= -\lambda^3 + 9\lambda^2 - 25\lambda + 18$$

Only 1 real eigenvalue
$\therefore$ not diagonalizable