Assignment 4

1.9)

1) $\begin{bmatrix} 2 & -5 \\ 1 & 2 \\ 2 & 0 \\ 1 & 0 \end{bmatrix}$

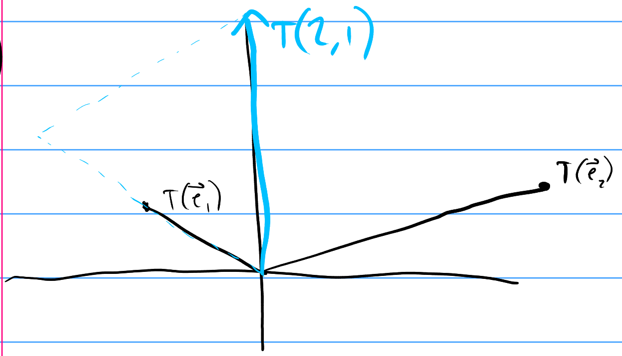
2) $\begin{bmatrix} 1 & 4 & -5 \\ 3 & 2 & 4 \end{bmatrix}$

3) $\begin{bmatrix} \cos\frac{3\pi}{2} & -\sin\frac{3\pi}{2} \\ \sin\frac{3\pi}{2} & \cos\frac{3\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

4) $\begin{bmatrix} \cos-\frac{\pi}{4} & -\sin-\frac{\pi}{4} \\ \sin-\frac{\pi}{4} & \cos-\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

8) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

13)



15) $\begin{bmatrix} 2 & 0 & -3 \\ 4 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - 3x_3 \\ 4x_1 \\ x_1 - x_2 + x_3 \end{bmatrix}$

17) $T(\vec{x}) = A\vec{x}$

$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

19) $A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

23) T, the identity matrix is just a composite of the unit vectors the matrix already acts on.

33) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ - free variable

not inconsistent

1-1: F
onto: T

2.1)

1) $-2A = -2\begin{bmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 2 \\ -8 & 6 & -4 \end{bmatrix}$

$B - 2A = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 2 \\ -8 & 6 & -4 \end{bmatrix} = \begin{bmatrix} 3 & -5 & 3 \\ -7 & 2 & -7 \end{bmatrix}$

AC = not defined
A: 2x3
C: 2x2

$CD = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$

4) $A-5I_3=\begin{bmatrix}9&-1&3\\-8&7&-3\\-4&1&8\end{bmatrix}-\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}=\begin{bmatrix}4&-1&3\\-8&2&-3\\-4&1&3\end{bmatrix}$

$(5I_3)A=\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}\begin{bmatrix}9&-1&3\\-8&7&-3\\-4&1&8\end{bmatrix}=\begin{bmatrix}45&-5&15\\-40&35&-15\\-20&5&40\end{bmatrix}$

5) a) $\begin{bmatrix}-1&2\\5&4\\2&-3\end{bmatrix}\begin{bmatrix}3\\-2\end{bmatrix}=\begin{bmatrix}-7\\7\\12\end{bmatrix}$ $\quad\begin{bmatrix}-1&2\\5&4\\2&-3\end{bmatrix}\begin{bmatrix}-4\\1\end{bmatrix}=\begin{bmatrix}10\\-16\\-11\end{bmatrix}\rightarrow\begin{bmatrix}-7&10\\7&-16\\12&-11\end{bmatrix}$

b) $\begin{bmatrix}-1&2\\5&4\\2&-3\end{bmatrix}\begin{bmatrix}3&-4\\-2&1\end{bmatrix}=\begin{bmatrix}-7&10\\7&-16\\12&-11\end{bmatrix}$

7) 3×7

9) $AB=\begin{bmatrix}2&5\\-3&1\end{bmatrix}\begin{bmatrix}4&-5\\3&k\end{bmatrix}=\begin{bmatrix}23&-10+5k\\-9&15+k\end{bmatrix}$

$BA=\begin{bmatrix}4&-5\\3&k\end{bmatrix}\begin{bmatrix}2&5\\-3&1\end{bmatrix}=\begin{bmatrix}23&15\\6-3k&15+k\end{bmatrix}$

$15=-10+5k$

$25=5k$

$\boxed{k=5}$

$6-3k=6-3(5)=6-15=-9$

$15+k=15+k$

15) $A=[a_1\ \ a_2]$

$B=[b_1\ \ b_2]$

F Matrix multiplication is not column-wise

21) $(AB)C = (AC)B$

F, Matrix multiplication is not commutative

35) $\vec{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$\vec{u}^T = [-2 \ \ 3 \ \ -4]$

$\vec{v}^T = [a \ \ b \ \ c]$

$$u^T v = [-2 \ \ 3 \ \ -4] \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \boxed{-2a + 3b - 4c}$$

$$v^T u = [a \ \ b \ \ c] \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix} = \boxed{-2a + 3b - 4c}$$

$$uv^T = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix} [a \ \ b \ \ c] = \begin{bmatrix} -2a & -2b & -2c \\ 3a & 3b & 3c \\ -4a & -4b & -4c \end{bmatrix}$$

$$vu^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix} [-2 \ \ 3 \ \ -4] = \begin{bmatrix} -2a & 3a & -4a \\ -2b & 3b & -4b \\ -2c & 3c & -4c \end{bmatrix}$$

36)

$$uv^T = (vu^T)^T$$

$$u^T v = v^T u$$

2.2) 1) $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}^{-1} \rightarrow \frac{1}{1} \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}$

$ad - bc = 16 - 15 = 1 \neq 0$

5) $\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}^{-1} \rightarrow \frac{1}{1} \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$

$ad - bc = 16 - 15 = 1$

7) $\vec{x} = A^{-1} b = \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \boxed{\begin{bmatrix} 7 \\ -18 \end{bmatrix}}$

11) T, definition of inverse

13) F, $(AB)^{-1} = B^{-1}A^{-1}$

15) T, definition of inverse

17) T, $Ax=b$ has unique $\vec{x}$ for all $\vec{b}$

23) $AB=AC$

A: $n\times n$, invertible

$B \& C$: $n\times p$

$AB = [Ab_1 \; Ab_2 \; Ab_3 \ldots Ab_p]$

where $b_x \in \mathbb{R}^n$

Ab_x has unique solution (theorem 5)

so if $AC = AB = [Ac_1 \ldots Ac_p]$, $b_x = c_x$, $B=C$

not always true if A^{-1} d.n.e.