Assignment 3

1.6) 7) $Na HCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$

$$\begin{bmatrix} Na \\ H \\ C \\ O \end{bmatrix} \quad x_1\begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2\begin{bmatrix}0\\8\\6\\7\end{bmatrix} = x_3\begin{bmatrix}3\\5\\6\\7\end{bmatrix} + x_4\begin{bmatrix}0\\1\\0\\2\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\2\end{bmatrix}$$

$$x_1\begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2\begin{bmatrix}0\\8\\6\\7\end{bmatrix} + x_3\begin{bmatrix}-3\\-5\\-6\\-7\end{bmatrix} + x_4\begin{bmatrix}0\\-1\\0\\-2\end{bmatrix} + x_5\begin{bmatrix}0\\0\\-1\\-2\end{bmatrix} = \vec{0}$$

$$\begin{bmatrix}1&0&-3&0&0&0\\1&8&-5&-1&0&0\\1&6&-6&0&-1&0\\3&7&-7&-2&-2&0\end{bmatrix} \sim \begin{bmatrix}1&0&-3&0&0&0\\0&8&-2&-1&0&0\\0&6&-3&0&-1&0\\0&7&2&-2&-2&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&-3&0&0&0\\0&1&-1/4&-1/8&0&0\\0&1&-1/2&0&-1/6&0\\0&1&2/7&-2/7&-2/7&0\end{bmatrix} \sim \begin{bmatrix}1&0&-3&0&0&0\\0&1&-1/4&-1/8&0&0\\0&0&-1/4&1/8&-1/6&0\\0&0&11/28&-9/56&-2/7&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&-3&0&0&0\\0&1&-1/4&-1/8&0&0\\0&0&1&-1/2&2/3&0\\0&0&11/28&-9/56&-2/7&0\end{bmatrix} \sim \begin{bmatrix}1&0&-3&0&0&0\\0&1&-1/4&-1/8&0&0\\0&0&1&-1/2&2/3&0\\0&0&0&1/28&-23/42&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&-3&0&0&0\\0&1&-1/4&-1/8&0&0\\0&0&1&-1/2&2/3&0\\0&0&0&1&-46/3&0\end{bmatrix} \sim \begin{bmatrix}1&0&0&-3/2&2&0\\0&1&0&-1/4&1/6&0\\0&0&1&0&-7&0\\0&0&0&1&-46/3&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&0&0&-21&0\\0&1&0&0&-11/3&0\\0&0&1&0&-7&0\\0&0&0&1&-46/3&0\end{bmatrix} \quad \begin{aligned} x_1 - 21x_5 &= 0 \\ x_2 - \tfrac{11x_5}{3} &= 0 \\ x_3 - 7x_5 &= 0 \\ x_4 - \tfrac{46x_5}{3} &= 0 \end{aligned} \quad \begin{aligned} x_1 &= 21x_5 \\ x_2 &= \tfrac{11}{3}x_5 \\ x_3 &= 7x_5 \\ x_4 &= \tfrac{46}{3}x_5 \end{aligned}$$

$$21t\,NaHCO_3 + \frac{11}{3}t\,H_3C_6H_5O_7 \rightarrow 7t\,Na_3C_6H_5O_7 + \frac{46}{3}t\,H_2O + 6t\,CO_2$$

t is a multiple of 3

11) A: $x_1 + x_3 = 20$ | $x_1 + x_3 = 20$

B: $x_2 = x_3 + x_4$ | $x_2 - x_3 - x_4 = 0$

C: $80 = x_1 + x_2$ | $x_1 + x_2 = 80$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 60 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccccc|l|l} 1 & 0 & 1 & 0 & 20 & x_1 + x_3 = 20 & x_1 = 20 - x_3 \\ 0 & 1 & -1 & 0 & 60 & x_2 - x_3 = 60 & x_2 = x_3 + 60 \\ 0 & 0 & 0 & 1 & 60 & x_4 = 60 & x_3 = x_3 \\ & & & & & & x_4 = 60 \end{array}\right]$$

largest x_3 is 20

13) A: $30 + x_2 = 80 + x_1$ | $-x_1 + x_2 = 50$

B: $x_3 + x_5 = x_2 + x_4$ | $-x_2 + x_3 - x_4 + x_5 = 0$

C: $100 + x_6 = x_5 + 40$ | $-x_5 + x_6 = -60$

D: $40 + x_4 = x_6 + 90$ | $x_4 - x_6 = 50$

E: $x_1 + 60 = x_3 + 20$ | $x_1 - x_3 = -40$

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & -1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ -1 & 1 & 0 & 0 & 0 & 0 & 50 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & -1 & 1 & 0 & 10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & -1 & 1 & -50 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \end{bmatrix}$$

$x_1 - x_3 = -40$ | $x_1 = x_3 - 40$

$x_2 - x_3 = 10$ | $x_2 = x_3 + 10$

$x_4 - x_6 = 50$ | $x_3 = x_3$

$x_5 - x_6 = 60$ | $x_4 = x_6 + 50$

| | $x_5 = x_6 + 60$

| | $x_6 = x_6$

mins:	
x_1	0
x_2	10
x_3	0
x_4	50
x_5	60
x_6	0

1.7)

1) $\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} \neq c\begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix} \neq c\begin{bmatrix} -7 \\ -1 \\ 6 \end{bmatrix}$ ✓

3) $\begin{bmatrix} 1 \\ -3 \end{bmatrix} \neq c\begin{bmatrix} -3 \\ 6 \end{bmatrix}$ ✓

5) $\begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix} \neq c\begin{bmatrix} -8 \\ -7 \\ 5 \\ -3 \end{bmatrix} \neq c\begin{bmatrix} 5 \\ 4 \\ -4 \\ 2 \end{bmatrix}$ ✓

7) $\begin{bmatrix}1\\-2\\-4\end{bmatrix} \neq c\begin{bmatrix}4\\-7\\-5\end{bmatrix} \pm c\begin{bmatrix}-3\\5\\7\end{bmatrix} \neq c\begin{bmatrix}0\\1\\5\end{bmatrix}$ ✓

9) $c_1\begin{bmatrix}1\\-3\\2\end{bmatrix} + c_2\begin{bmatrix}-3\\10\\-6\end{bmatrix} = \begin{bmatrix}2\\-7\\h\end{bmatrix}$

$\begin{bmatrix}1&-3&2\\-3&10&-7\\2&-6&h\end{bmatrix} \sim \begin{bmatrix}1&-3&2\\-3&10&-7\\0&0&h-4\end{bmatrix} \sim \begin{bmatrix}1&-3&2\\0&1&-1\\0&0&h-4\end{bmatrix}$

a) $\boxed{h=4}$ b) $h=4$ as well?

11) $\begin{bmatrix}1&3&-1\\-1&-5&5\\4&7&h\end{bmatrix} \sim \begin{bmatrix}1&3&-1\\-1&5&5\\0&-5&h+4\end{bmatrix} \sim \begin{bmatrix}1&3&-1\\0&8&4\\0&-5&h+4\end{bmatrix}$

$\sim \begin{bmatrix}1&3&-1\\0&1&1/2\\0&-5&h+4\end{bmatrix} \sim \begin{bmatrix}1&3&-1\\0&1&1/2\\0&0&h+4+5/2\end{bmatrix}$

$h+4+5/2=0$

$h+\frac{8}{2}+5/2=0$

$h+\frac{13}{2}=0$

$\boxed{h=-\frac{13}{2}}$

13) $\begin{bmatrix}1&-2&3\\5&-9&h\\-3&6&-9\end{bmatrix} \sim \begin{bmatrix}1&-2&3\\5&-9&h\\1&-2&3\end{bmatrix}$

$\boxed{\text{any } h}$

15) $\begin{bmatrix}5\\1\end{bmatrix} \neq c\begin{bmatrix}7\\8\end{bmatrix} \neq c\begin{bmatrix}1\\3\end{bmatrix} \neq c\begin{bmatrix}-1\\7\end{bmatrix}$ ✓

17) $\begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$ ∴ ✗

19) $\begin{bmatrix} -8 \\ 12 \\ -4 \end{bmatrix} / -4 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \neq c \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$ ✓

21) T by definition of linear independence

23) F only 1 has to be

25) F not necessarily

1.8)

1) $T(\vec{x}) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \vec{x}$

$T(\vec{u}) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

$T(\vec{v}) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$

3) $\begin{bmatrix} 1 & 0 & -2 & -1 \\ -2 & 1 & 6 & 3 \\ 3 & -2 & -5 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & -2 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 5 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \therefore \boxed{\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \quad \vec{x} \text{ is unique}}$$

5) $$\begin{bmatrix} 1 & 5 & -7 & -2 \\ -3 & -7 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & -7 & -2 \\ 0 & 8 & -16 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & -7 & -2 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \end{bmatrix} \quad \begin{array}{l|l} x_1 + 3x_3 = 3 & x_1 = 3 - 3x_3 \\ x_2 - 2x_3 = -1 & x_2 = 2x_3 - 1 \end{array}$$

$$\vec{x} = x_3 \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} \qquad \boxed{\vec{x} \text{ is not unique}}$$

7) $a = 5, b = 6$

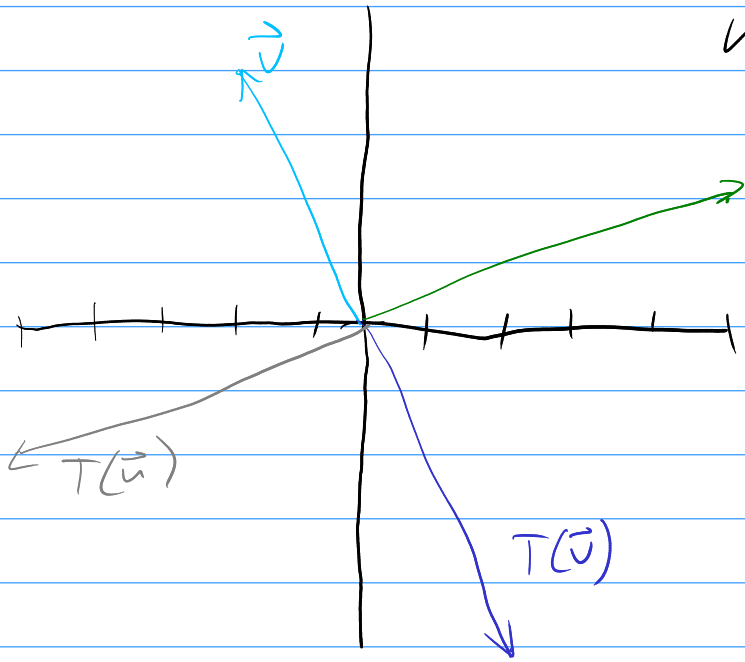
9) $$\begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 2 & -8 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -9 & 7 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \quad \begin{array}{l|l} x_1 - 9x_3 + 7x_4 = 0 & x_1 = 9x_3 - 7x_4 \\ x_2 - 4x_3 + 3x_4 = 0 & x_2 = 4x_3 - 3x_4 \end{array}$$

$$\boxed{\vec{x} = x_3 \begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ -3 \\ 0 \\ 1 \end{bmatrix}}$$

11) no, $\vec{x}$ is in $\mathbb{R}^4$, b is in $\mathbb{R}^3$

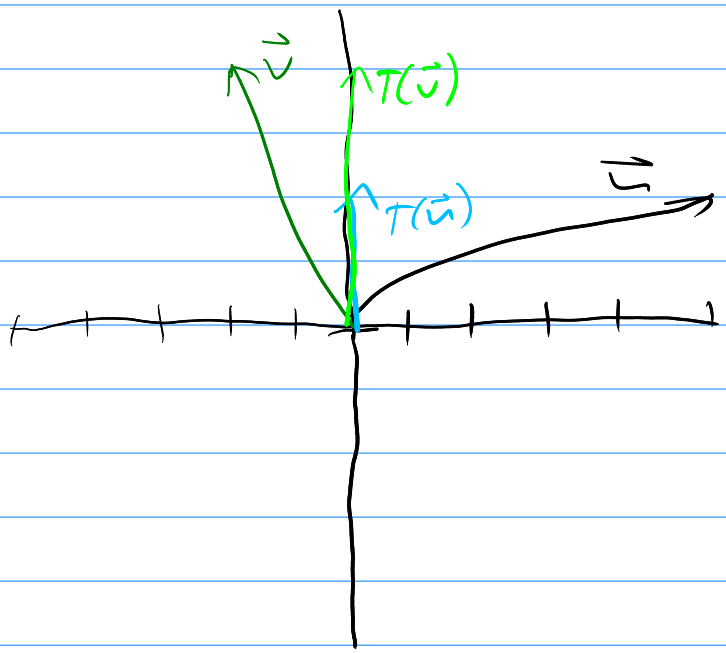
13) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$



negates the vector

15) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

gets the height of the vector



19) $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\begin{bmatrix}2\\5\end{bmatrix}$, $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\begin{bmatrix}-1\\6\end{bmatrix}$

A is 2×2

$$A=\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}a_1\\a_2\end{bmatrix}=\begin{bmatrix}2\\5\end{bmatrix}$$

$$\begin{bmatrix}a_1 & b_1\\ a_2 & b_2\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}b_1\\b_2\end{bmatrix}=\begin{bmatrix}-1\\6\end{bmatrix}$$

$$A=\begin{bmatrix}2 & -1\\ 5 & 6\end{bmatrix} \qquad T(\vec{x})=\begin{bmatrix}2 & -1\\ 5 & 6\end{bmatrix}\vec{x}$$

$$T\left(\begin{bmatrix}5\\-3\end{bmatrix}\right)=\begin{bmatrix}2 & -1\\ 5 & 6\end{bmatrix}\begin{bmatrix}5\\-3\end{bmatrix}=\begin{bmatrix}13\\-11\end{bmatrix}$$

$$T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{bmatrix}2 & -1\\ 5 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}2x_1-x_2\\5x_1+6x_2\end{bmatrix}$$

21) T, yup

23) F, it's $\mathbb{R}^5$