CS 2100: Discrete Structures

Homework 1: Logic Spring 2023

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Section 1.4

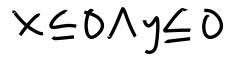
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1. Exercise 10.d on page 38

Suppose x and y indicate particular real numbers. Write conditions that express the following statement: "Neither x nor y is positive.", by using comparisons (such as x > 0) and the basic operations of logic.



2. Exercise 17 on page 39

Use truth tables to verify the following statement:

$$p \vee (p \wedge q) \equiv p$$

establishing the second absorption property in Theorem 2.

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3. Exercise 21.e on page 39

Use truth tables to check if the following statement is true:

$$(p \lor q) \land (q \lor r) \equiv (p \land r) \lor q$$

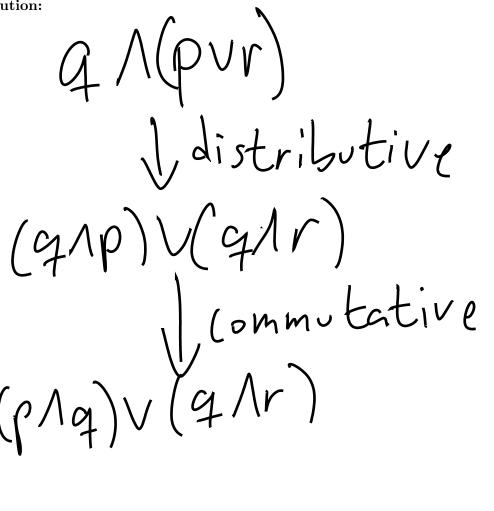
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4. Exercise 24.c on page 40

By quoting parts of Theorem 2, verify the following logical equivalence:

$$q \mathrel{\wedge} (p \mathrel{\vee} r) \equiv (p \mathrel{\wedge} q) \mathrel{\vee} (q \mathrel{\wedge} r)$$

Start with the left side and use parts of Theorem 2 to change the problem, ending with the right side.



5. Exercises 14.d on page 52

For the following statement:

$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, \frac{x}{y} = 2$$

Write the negation of the statement using Proposition 1 and the rules for negating to simplify each to the point that no \neg symbol occurs to the left of a quantifier. (Recall that $\mathbb Z$ denotes the set of all integers, and $\mathbb R$ denotes the set of all real numbers.)



6. Exercise 15.d on page 52

For the following statement:

$$\forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, \frac{x}{y} = 2$$

Decide whether the original statement or its negation is true.

negation:

$$\exists x \in \mathbb{Z}, \forall y \in \mathbb{R}, \overset{\sim}{\to} = \mathbb{Z}$$

the original is true.

7. Exercise 17.c on page 52

Write the negation of the following statement as an English sentence:

"For every positive integer x, there is a positive integer y such that y is smaller than x and y is a factor of x."

Original. $\forall x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+, y \in \mathbb{Z}^+, y \in \mathbb{Z}^+, y \in \mathbb{Z}^+, \forall y \in \mathbb{Z}^$

there is a positive integer x, for all positive integers y, y is greater or equal to x or y is not a factor of x.

8. Exercise 20.b on page 53

For the statement:

"For every two real numbers x and y, there is an integer n such that x < n < y."

Decide whether the original statement or its negation is true.

original: $\forall (x,y) \in \mathbb{R}, \exists n \in \mathbb{Z}, \times \langle n < y \rangle$ negation: $\exists (x,y) \in \mathbb{R}, \forall n \in \mathbb{Z}, \times \geq n \geq y$ the negation is true.

9. Exercise 11.e on page 66

Let D be the set $\{1, 3, 5, 7, 8, 10, 11, 12\}$. Decide whether the following statement is true for all the elements of D. If it is not, give a counterexample.

"If x > 5 and x < 7, then x is negative."

10. Exercise 14.d on page 67

Express the following statement using predicates and the quantifier \forall

"For every positive real number x, if $x < \sqrt{2}$, then $\frac{2}{x} > \sqrt{2}$."

11. Exercise 17.d on page 67

Express the following statement using predicates and the quantifiers \forall and \exists

"For every real number y, if $y \ge 0$, then there exists $x \in \mathbb{R}$ such that $x^2 = y$."

$$\forall y \in \mathbb{R}, y \geq 0 \rightarrow \exists x \in \mathbb{R}, x^2 = y$$

12. Exercise 25.e on page 67

Form the contrapositive of the following statement:

"If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle."

Solution:

if a triangle is not an isosedes then it does not have two equal sides nor angles.

13. Exercise 26.e on page 67

Form the converse of the following statement:

"If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle."

it has either two equal sides or angles.

14. Exercise 27.e on page 67

Form the inverse of the following statement:

"If a triangle has either two equal sides or two equal angles, then it is an isosceles triangle."

Solution:

if a triangle bes not have two equal sides nor angles, then it is not isosceles.