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CS 2100: Discrete Structures

Homework 5: Probability

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1. **Exercises 3.b and 3.d on page 446**
   If two dice, one red and one green, are rolled, find the probability that:

   (b) The sum of the dice is 5

   (d) The value on the red dice is less than the value on the green dice.

   Solution:

b) $x_1 + x_2 = 5$

$x_1' + x_2' = 3$

$C(r+n-1, r) = C(4,3) = 4$

$6 \times 6 = 36$ possible

$\frac{4}{36} = \boxed{\frac{1}{9}}$

d)

green	solutions
1	0
2	1
3	2
4	3
5	4
6	+5
	15

$\frac{15}{36} = \boxed{\frac{5}{12}}$

2. **Exercise 19.b on page 448**
Suppose Jessica rolls an eight-sided die and John rolls a six-sided die. What is the probability that John's roll is higher than Jessica's?

Solution:

$6 \times 8 = 48$

John	Sol's
1	0
2	1
3	2
4	3
5	4
6	+ 5
	15

$$\frac{15}{48} = \boxed{\frac{5}{16}}$$

3. **Exercise 20 on page 448**
Suppose a non-negative integer solution to the equation $w + x + y + z = 10$ is chosen at random (each one being equally likely to be chosen). What is the probability that in this particular solution w is less than or equal to 2?

Solution:

num basic solutions is $C(13,10) = a$

num where $w \geq 2$:

$w' = w - 3$

$w' + x + y + z = 7$

$\therefore C(10,7) = b$

num where $w \leq 2 = a - b = C(13,10) - C(10,7)$

prob.: $\frac{C(13,10) - C(10,7)}{C(13,10)} = \frac{286 - 120}{286} = \frac{166}{286} = \boxed{\frac{83}{143}}$

4. **Exercises 5.b and 5.d on page 458**
Two cards are drawn from a deck, with replacement. Show how to use the product rule to find the probability that:

   (b) Both cards are spades

   (d) At least one card is an ace. (HINT: Consider the complementary problem)

Solution:

b) $\text{Prob}(\text{card is spade}) \times \text{Prob}(\text{card is spade})$

$= \frac{1}{4} \times \frac{1}{4} = \left(\frac{1}{4}\right)^2 = \boxed{\frac{1}{16}}$

d) $\text{Prob}(\text{at least 1 ace}) = 1 - \text{Prob}(\text{no aces})$

$\text{Prob}(\text{no aces}) = \text{Prob}(\text{not an ace})^2$

$= \left(\frac{12}{13}\right)^2$

$\text{Prob}(\text{at least one ace}) = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169}$

$= \boxed{\frac{25}{169}}$

5. **Exercise 10 on page 459**
John tosses a penny four times and Jessica tosses a nickel four times. What is the probability that one of them gets four results of heads?

Solution:

$$P = \text{prob}(\text{all heads}) \times \text{prob}(\text{not all heads}) + \text{prob}(\text{not all heads}) \times \text{prob}(\text{all heads})$$

$$= 2 \cdot \text{prob}(\text{all heads}) \times (1 - \text{prob}(\text{all heads}))$$

$$= 2\,\text{prob}(\text{all heads}) - 2\,\text{prob}(\text{all heads})^2$$

$$\text{prob}(\text{all heads}) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$P = 2\left(\frac{1}{16}\right) - 2\left(\frac{1}{16}\right)^2$$

$$= \frac{1}{8} - \frac{2}{16^2} = \frac{1}{8} - \frac{2}{256} = \frac{1}{8} - \frac{1}{128} = \frac{16}{128} - \frac{1}{128} = \boxed{\frac{15}{128}}$$

6. **Exercises 22.a and 22.b on page 459**
When choosing a committee of three people from a club of 8 mean and 12 women, let E_1 be the event that the committee has a woman and let E_2 be the even that the committee has a man.

$P(a|b) = \frac{P(a \cap b)}{P(b)}$

(a) Find $Prob(E_1|\bar{E}_2)$

(b) Find $Prob(E_2|\bar{E}_1)$

Solution:

a) $P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{\frac{12}{20} \cdot \frac{8}{20}}{}$

?

7. **Exercise 4 on page 464**
What is the probability that in 10 tosses of a fair coin, the result of head appears at least eight times?

Solution:

$$P(8 \text{ heads}) + P(9 \text{ heads}) + P(10 \text{ heads})$$

$$= \frac{C(10,8)}{2^{10}} + \frac{C(10,9)}{2^{10}} + \frac{C(10,10)}{2^{10}}$$

$$= \frac{45 + 10 + 1}{1024} = \frac{56}{1024} = \boxed{\frac{7}{128}}$$

8. **exercise 8 on page 465**
What is the probability of getting more heads than tails on four tosses of a fair coin?

Solution:

$$P(3 \text{ heads}) + P(4 \text{ heads})$$

$$= \frac{C(4,3)}{2^4} + \frac{C(4,4)}{2^4} = \boxed{\frac{5}{16}}$$

9. **Exercise 19.b on page 465**
Assume Kenny always gets a hit with probability 1/3 and a base-on-balls with probability 1/6. Determine the probability that in five plate appearances, Kenny gets a base-on-balls at least twice.

Solution:

$$P(BOB \geq 2) = 1 - P(BOB < 2) = 1 - \left(P(BOB=1) + P(BOB=0)\right)$$

$$P(BOB=1) = P(E_{5,1}) = C(5,1) \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^4 = 5 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^4 = \left(\frac{5}{6}\right)^5$$

$$P(BOB=0) = P(E_{5,0}) = C(5,0) \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^5 = 1 \cdot 1 \cdot \left(\frac{5}{6}\right)^5$$

$$P(BOB \geq 2) = 1 - \left(\left(\frac{5}{6}\right)^5 \cdot 2\right) = 1 - 2\left(\frac{5}{6}\right)^5 = \boxed{0.1962}$$

10. **Exercise 4 on page 474**
Suppose a three-person committee is formed for a club by drawing names out of a hat. If the expected number of women on the committee is 2.0 and there are 10 men in the club, how many women are in the club?

Solution:

11. **Exercise 8 on page 474**
Assume Kenny always gets a hit with probability 1/3 and a base-on-balls with probability 1/6. Suppose that he negotiates his contract for an end-of-the-season bonus of \$1,000 for each hit and \$100 for each base-on-balls he gets. If he can reasonably expect 600 plate appearances this year, how much does he expect his bonus will be?

Solution:

Hits: $n \cdot p \cdot \$ = 600 \cdot 1/3 \cdot \$1{,}000 = \$200{,}000$

BoB: $n \cdot p \cdot \$ = 600 \cdot 1/6 \cdot \$100 = \$10{,}000$

Bonus = \$210,000

12. **Exercise 16 on page 474**
If the Bisons have a $\frac{3}{5}$ probability of winning any given game against the Mustangs, what will be the average length of a best-of-five series between these two teams?

Solution:

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