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Math 2270
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Assignment 5

2.3

1) $ad - bc = 5(-6) - 7(-3) = -30 + 21 = -7 \neq 0$

invertible

3) $\begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

invertible

5) $\begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ -4 & -9 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ -4 & -9 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ 0 & -9 & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & 0 \end{bmatrix}$

not invertible

7) $\begin{bmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 14 & 8 & 0 \\ 0 & 0 & 3 & 4 \\ 0 & -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 36 & 14 \\ 0 & 0 & 3 & 4 \\ 0 & -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & -34 \\ 0 & 0 & 3 & 4 \\ 0 & -1 & 2 & 1 \end{bmatrix}$

invertible

9) $\begin{bmatrix} 4 & 0 & -7 & -7 \\ -6 & 1 & 11 & 9 \\ 7 & -5 & 10 & 19 \\ -1 & 2 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 0 & 8 & 5 & -11 \\ -6 & 1 & 11 & 9 \\ 0 & -2 & 24 & 27 \\ 1 & -2 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 101 & 97 \\ 0 & -11 & -7 & 15 \\ 0 & -2 & 24 & 27 \\ 1 & -2 & -3 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 0 & 0 & 101 & 97 \\ 0 & 0 & -139 & -133.5 \\ 0 & 1 & -12 & -\frac{27}{2} \\ 1 & -2 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -0.96.. \\ 0 & 1 & -12 & -\frac{27}{2} \\ 1 & -2 & -3 & 1 \end{bmatrix}$

not invertible

11) T, IMT

13) T, IMT

15) F, could be linearly dependent

17) T, IMT

22) (For lower triangular matrix C) When there is an upper triangular matrix D s.t. $CD = I$

25) A^{-1} is also an invertible matrix as $AA^{-1} = I$ & $A^{-1}A = I$, So by IMT A^{-1} has linearly independent columns

35) $(AB)^{-1}$ exists, $(AB)^{-1}W = I \therefore ABV = I \therefore A(BV) = I$
$\therefore AZ = I \therefore A^{-1}$ exists

41) $M = \begin{bmatrix} -5 & 9 \\ 4 & -7 \end{bmatrix}$

$ad - bc = (-5)(-7) - 9(4) = 35 - 36 = -1 \neq 0$
invertible

$T^{-1}(x_1, x_2) = \frac{1}{-1}\begin{bmatrix} -7 & -9 \\ -4 & -5 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 4 & 5 \end{bmatrix}$

47) $\vec{v} = T(\vec{x}) = A\vec{x}$, as $A\vec{x} = \vec{b}$ has at least one sol. if invertible

$S(T(\vec{x})) = \vec{x} = S(\vec{v})$
$U(T(\vec{x})) = \vec{x} = U(\vec{v})$
$S(\vec{v}) = U(\vec{v})$
$S(\vec{z}) = U(\vec{z})$

2.4

1) $$\begin{bmatrix} I & 0 \\ E & I \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ EA+C & EB+D \end{bmatrix}$$

3) $$\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} W & X \\ Y & Z \end{bmatrix} = \begin{bmatrix} Y & Z \\ W & X \end{bmatrix}$$

5) $$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} I & 0 \\ X & Y \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

$0 = A + BX$

$I = BY$

$Z = C$

$BX = -A$

$B^{-1}(BX) = B^{-1}(-A)$

$X = -B^{-1}A$

$BY = I$

$Y = B^{-1}$

$Z = C$

7) $$\begin{bmatrix} X & 0 & 0 \\ Y & 0 & I \end{bmatrix}\begin{bmatrix} A & Z \\ 0 & 0 \\ B & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$I = XA \therefore$ $\boxed{\begin{matrix} X = A^{-1} \\ Y = A^{-1} \end{matrix}}$ $XZ = 0$

$0 = XZ$ $A^{-1}Z = 0$

$0 = YA + B \therefore 0 = I + B \therefore B = -I$

$I = YZ + I \therefore YZ = 0 = XZ \therefore Y = X$

$$\begin{bmatrix} A & Z \\ 0 & 0 \\ B & I \end{bmatrix}\begin{bmatrix} X & 0 & 0 \\ Y & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$I = AX + ZY$

$0 = ZX$ $\qquad ZX = 0 = XZ$

$0 = AY + B$ $\qquad ZX = XZ$

$I = ZY + I$ $\qquad X = Z$

$\qquad Z = A^{-1}$

10) $$\begin{bmatrix} I & 0 & 0 \\ C & I & 0 \\ A & B & I \end{bmatrix}\begin{bmatrix} I & 0 & 0 \\ Z & I & 0 \\ X & Y & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

$0 = C + Z \therefore Z = -C$

$0 = A + BZ + X \therefore X = -A - BZ = BC - A$

$0 = B + Y \therefore Y = -B$

$$\boxed{\begin{array}{l} X = BC - A \\ Y = -B \\ Z = -C \end{array}}$$

11) T, Adding is element-wise

13) Yes, As long as the partitions are conformable

15) If B^{-1} exists & C^{-1} exists, $RREF(B^{-1}) = I$, $RREF(C^{-1}) = I$, $\therefore$

$RREF\left(\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}\right) = I$, $\therefore \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}^{-1}$ exists

2.5

1) $L(U\vec{x})=\vec{b}$

$U\vec{x}=\vec{y}$

$$\left[\begin{array}{ccc|c} 3 & -7 & -2 & -7 \\ 0 & -2 & -1 & 5 \\ 0 & 0 & -1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 3 & -7 & 0 & -11 \\ 0 & -2 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 3 & -7 & 0 & -11 \\ 0 & 1 & 0 & -3/2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\sim \; y_3 = -2$

$y_2 = -3/2$

$3y_1 - 7y_2 = -11$

$3y_1 - 7(-3/2) = -11$

$3y_1 + 21/2 = -11$

$3y_1 = -\frac{22}{2} - \frac{21}{2}$

$y_1 = -\frac{43}{6}$

$\vec{y} = \begin{bmatrix} -43/6 \\ -3/2 \\ -2 \end{bmatrix}$

$L\vec{y}=\vec{b}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -43/6 \\ -1 & 1 & 0 & -3/2 \\ 2 & -5 & 1 & -2 \end{array}\right] \sim$$

$x_1 = -43/6$

$-x_1 + x_2 = -3/2$

$43/6 + x_2 = -9/6$

$x_2 = -52/6 = -26/3$

$2x_1 - 5x_2 + x_3 = -2$

$2(-43/6) - 5(-26/3) + x_3 = -2$

$-43/3 + 130/3 + x_3 = -2$

$x_3 = -2 - 87/3$

$x_3 = -\frac{6}{3} - \frac{87}{3}$

$x_3 = -\frac{93}{3}$

$x_3 = -31$

$\vec{x} = \begin{bmatrix} -43/6 \\ -26/3 \\ -31 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 3 & -7 & -2 & -7 \\ -3 & 5 & 1 & 5 \\ 6 & -4 & 0 & 2 \end{array}\right] \sim \begin{bmatrix} 3 & -7 & -2 & -7 \\ 0 & -2 & -1 & -2 \\ 0 & 10 & 4 & 16 \end{bmatrix} \sim \begin{bmatrix} 3 & -7 & -2 & -7 \\ 0 & 1 & 1/2 & 1 \\ 0 & 5 & 2 & 8 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -7 & -2 & -7 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & -1/2 & 3 \end{bmatrix} \sim \left[\begin{array}{ccc|c} 1 & -7/3 & -2/3 & -7/3 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 1 & -6 \end{array}\right] \sim \Big[$$

$x_3 = -6$?

3) $$\begin{bmatrix} 1 & 0 & 0 & 1 \\ -3 & 1 & 0 & 0 \\ 4 & -1 & 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & -1 & 1 & 0 \end{bmatrix} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 2 & 1 \\ 0 & -3 & 4 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right] \sim \begin{bmatrix} 2 & -1 & 0 & -5 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 2 & 1 \\ -6 & 0 & -2 & 0 \\ 8 & -1 & 5 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 2 & 1 \\ 0 & -3 & 4 & 3 \\ 0 & 3 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 2 & 1 \\ 0 & -3 & 4 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 0 & -5 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & -1 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \vec{x} = \begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix} \checkmark$$

5) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 7 \\ -1 & 0 & 1 & 0 & 0 \\ -4 & 3 & -5 & 1 & 3 \end{array}\right]$

$y_1 = 1$

$2y_1 + y_2 = 7$

$y_2 = 5$

$-y_1 + y_3 = 0$

$y_3 = -y_1$

$y_3 = -1$

$\left[\begin{array}{cccc|c} 1 & -2 & -4 & -3 & 1 \\ 0 & -3 & 1 & 0 & 5 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 0 & 0 & 1 & -13 \end{array}\right]$

$-4y_1 + 3y_2 - 5y_3 + y_4 = 3$

$-4 + 15 + 5 + y_4 = 3$

$y_4 + 16 = 3$

$y_4 = -13$

$x_4 = -13$

$2x_3 + x_4 = -1$

$2x_3 - 13 = -1$

$2x_3 = 12$

$x_3 = 6$

$-3x_2 + x_3 = 5$

$-3x_2 + 6 = 5$

$-3x_2 = -1$

$x_2 = 1/3$

$\vec{x} = \begin{bmatrix} -40/3 \\ 1/3 \\ 6 \\ -13 \end{bmatrix}$

$x_1 - 2x_2 - 4x_3 - 3x_4 = 1$

$x_1 - 2/3 - 24 + 39 = 1$

$x_1 - 2/3 + 15 = 1$

$x_1 = 2/3 - 14$

$x_1 = 2/3 - \frac{42}{3}$

$x_1 = -\frac{40}{3}$

$\left[\begin{array}{cccc|c} 1 & -2 & -4 & -3 & 1 \\ 2 & -7 & -7 & -6 & 7 \\ -1 & 2 & 6 & 4 & 0 \\ -4 & -1 & 9 & 8 & 3 \end{array}\right] \sim \begin{bmatrix} 1 & -2 & -4 & -3 & 1 \\ 0 & -3 & 1 & 0 & 5 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & -9 & -7 & -4 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -4 & -3 & 1 \\ 0 & -3 & 1 & 0 & 5 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & -4 & -4 & 22 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -2 & -4 & -3 & 1 \\ 0 & -3 & 1 & 0 & 5 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & -2 & 24 \end{bmatrix}$

$x_4 = -12$?

7) $\begin{bmatrix} 2 & 5 \\ -3 & -4 \end{bmatrix}$ $R_2 = R_2 - -3/2\, R_1$ $\sim \begin{bmatrix} 2 & 5 \\ 0 & 7/2 \end{bmatrix} = U$

$k = -3/2$

$5(3/2) = 15/2$

$-4 + 15/2 = -8/2 + 15/2 = 7/2$

$L = \begin{bmatrix} 1 & 0 \\ -3/2 & 1 \end{bmatrix}$

11) $\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$ $\begin{matrix} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - -1/3\, R_1 \end{matrix}$ $\sim \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 5 & 1 \end{bmatrix}$ $R_3 = R_3 - 1R_2$ $\sim \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 1 & 1 \end{bmatrix}$

15) $\begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix}$ $\begin{matrix} R_2 - 3R_1 \\ R_3 - -1/2\, R_1 \end{matrix}$ $\rightarrow \begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & -6 & 10 & -1 \end{bmatrix}$ $R_3 - -2R_2$ $\sim$

$\begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix} = U$ $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -1/2 & -2 & 1 & 0 \end{bmatrix}$

19) IMT?