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CS 2100: Discrete Structures

Homework 3: Graph theory

Spring 2023

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Section 7.1

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Section 7.3

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Section 7.4

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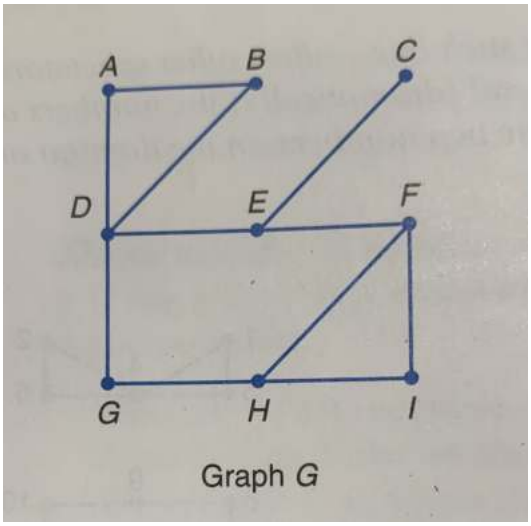
1. **Exercise 1.a and 1.b on page 515**
   For the graph below:

   a. Find the number of nodes and the number of edges
   b. Find the degree of each node.



Graph *G*

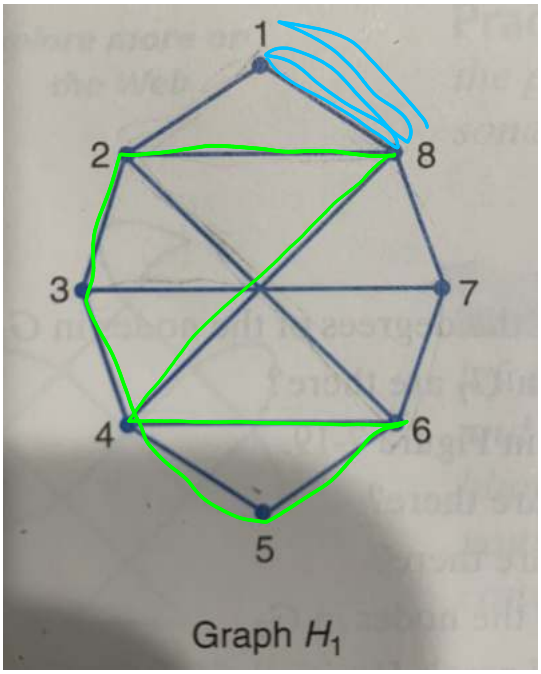
Solution:

a) nodes: 9
edges: 11

b) A: 2, B: 2, C: 1, D: 4, E: 3, F: 3,
G: 2, H: 3, I: 2

2. **Exercise 4.b and 4.d on page 516**
For the graph below:

b. Find a walk from 1 to 8 that uses five edges and is not a trail.
d. Find a circuit starting and ending at 8 that is not a cycle.



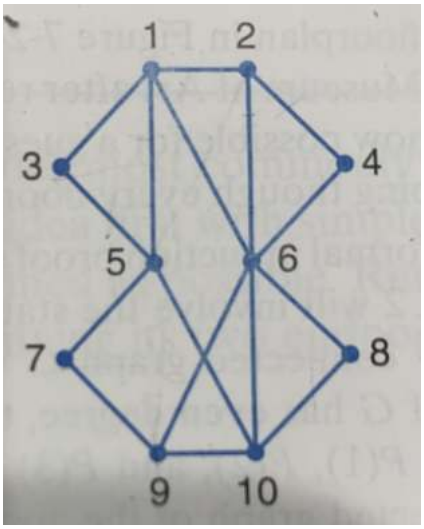
Graph H_1

Solution:

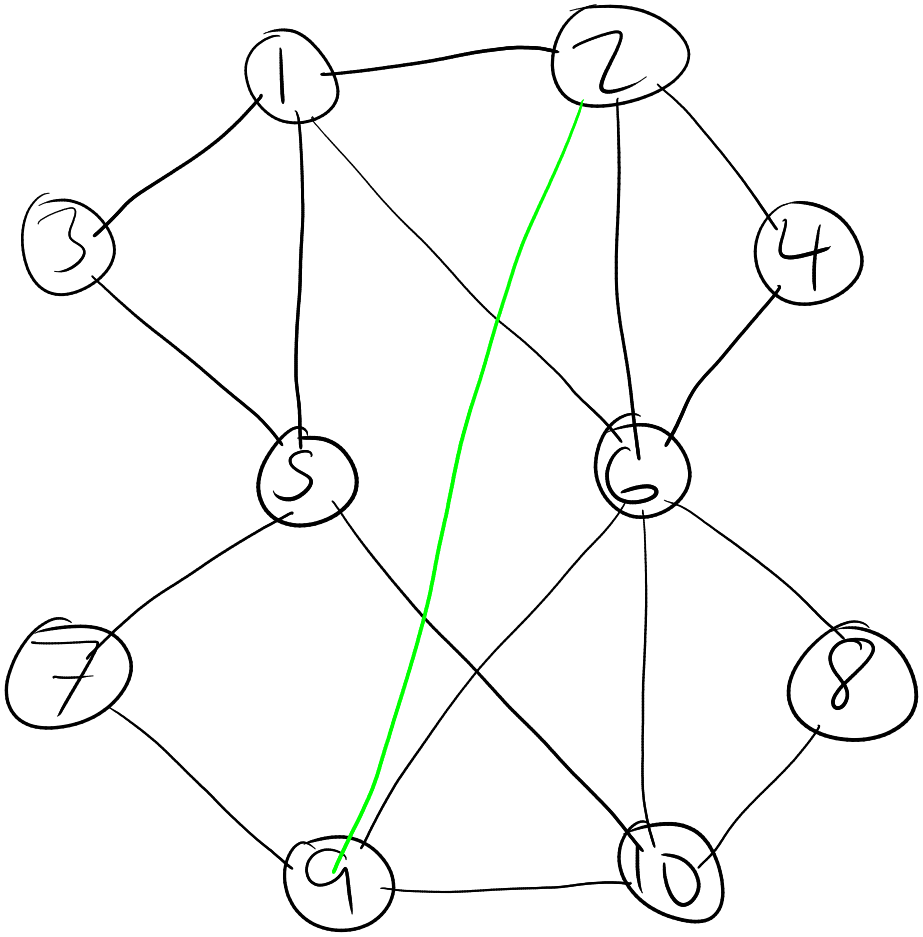
b) 1 8 1 8 1 8

d) 8 2 3 4 5 6 4 8

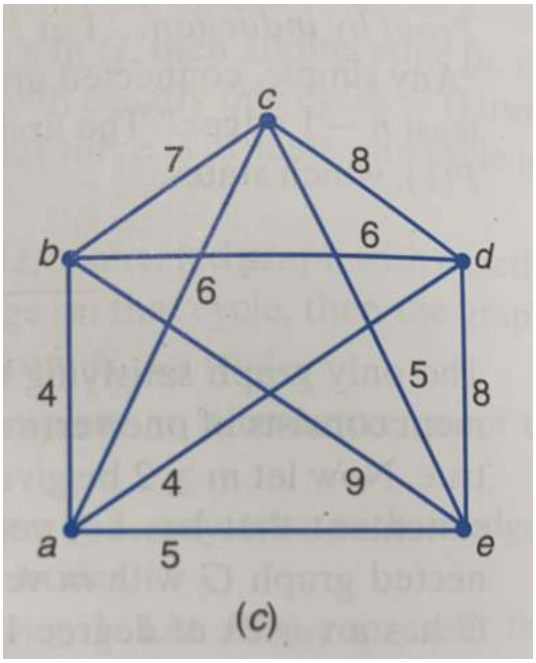
3. **Exercise 8.d on page 516 and 517**
The graph below is not Eulerian. Add the fewest edges possible to create a graph that is Eulerian.



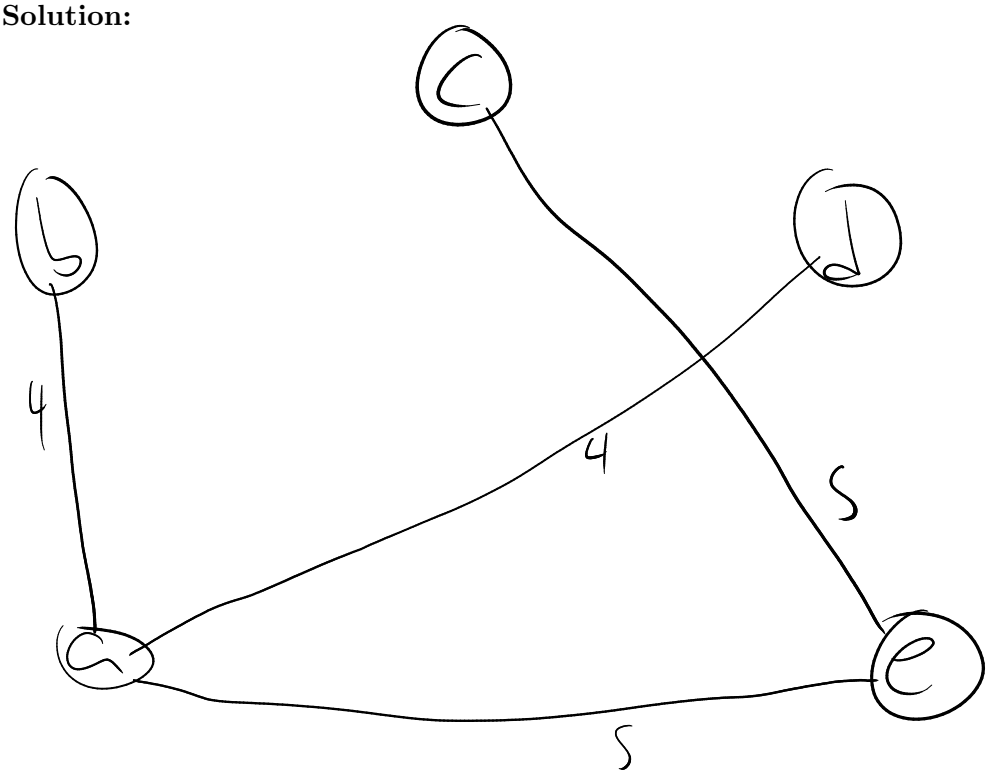
Solution:



4. **Exercise 23.c on page 532**
Find the minimal spanning tree for the graph below.



Solution:



Sum cost: 18

5. **Exercise 29 on page 532**
The graph below shows the mains roads in the Borough of Boatsville, along with the cost of plowing each street during a heavy snowstorm. Find the cheapest set of roads that must be plowed in order for everyone to be able to travel from any point to any other point in town.

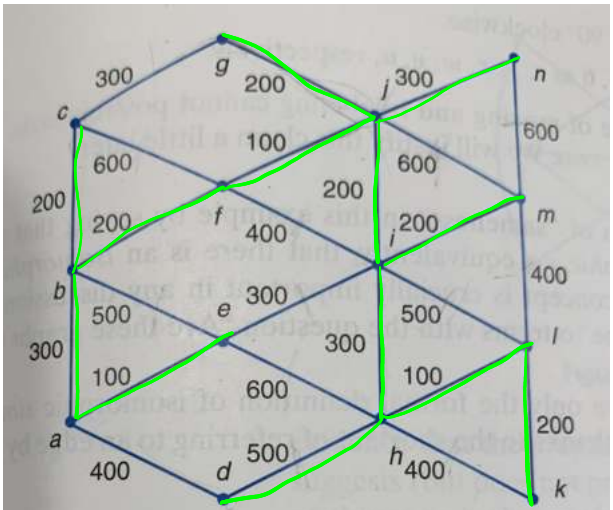
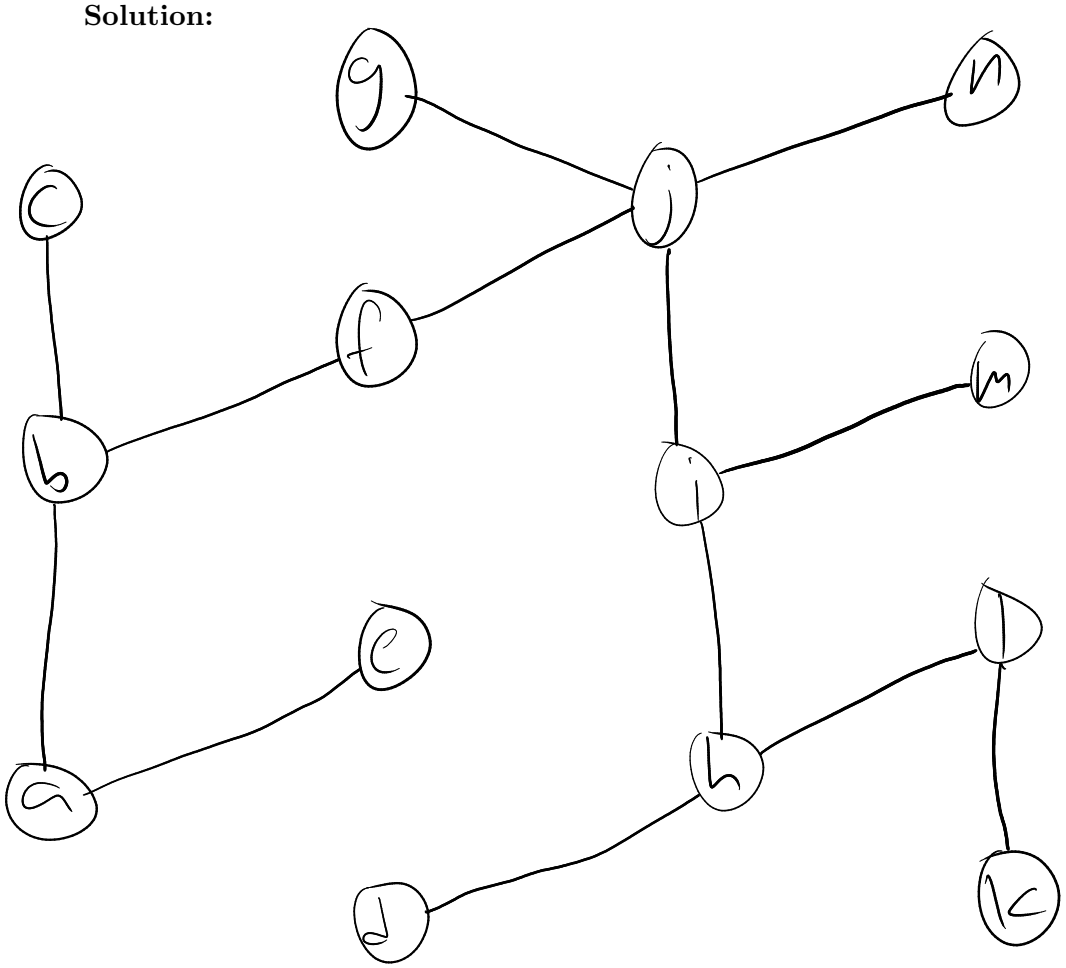


Figure 7-42 Graph for Exercise 29.

Solution:



6. **Exercise 30 on pages 532 and 533**
Consider the following algorithm on a simple connected graph:

- Let $G_0 = G$
- Repeat the following as long as possible: If G_i has a cycle, let e be the most expensive edge on that cycle, and let G_{i+1} be the graph obtained by removing e from G_i.
- The resulting graph has no cycles. Return this as the result T

Find a minimal spanning tree of the graph below using this algorithm.

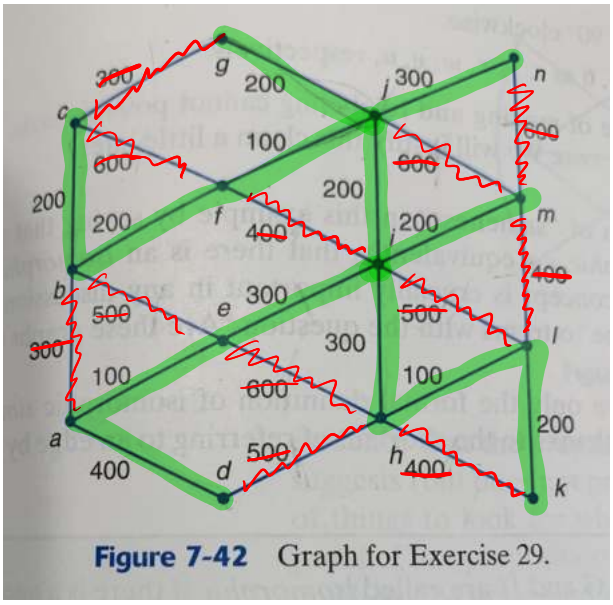
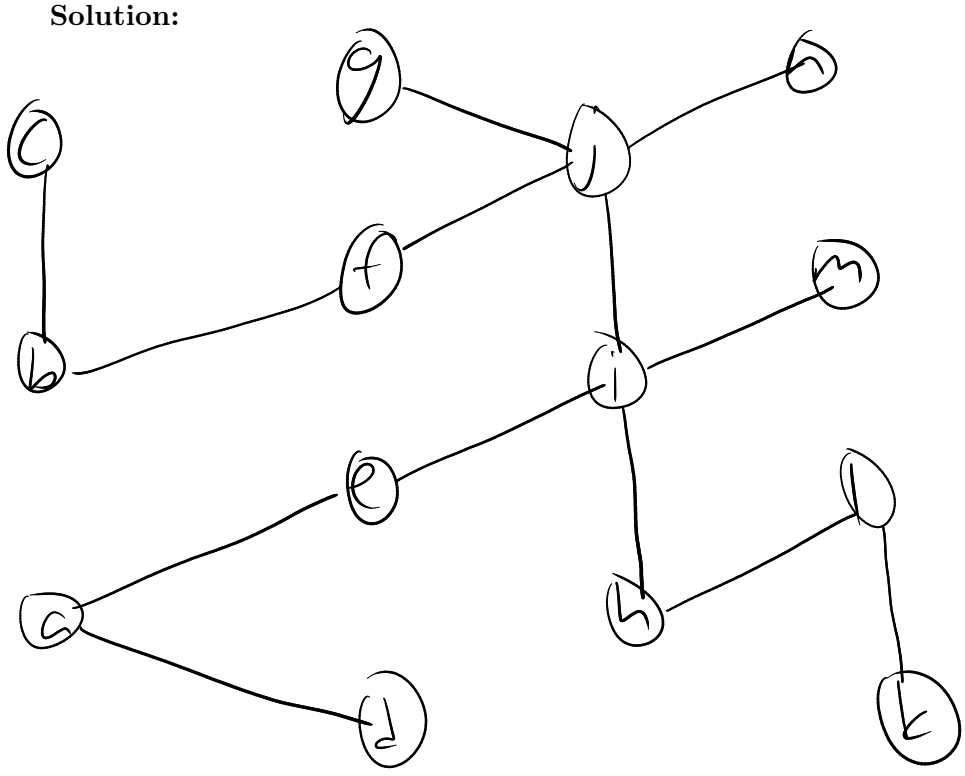
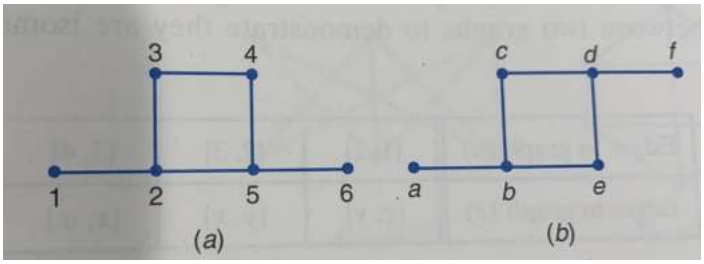


Figure 7-42 Graph for Exercise 29.

Solution:



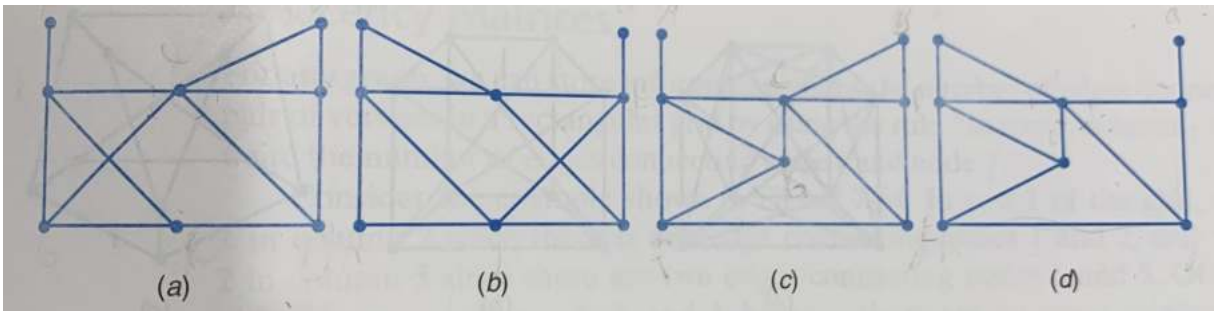
7. **Exercise 2 on page 544**
Explain why the two graphs below are not isomorphic.



Solution:

There is no function to translate from one set of vertices to the other w/o retaining adjacency.

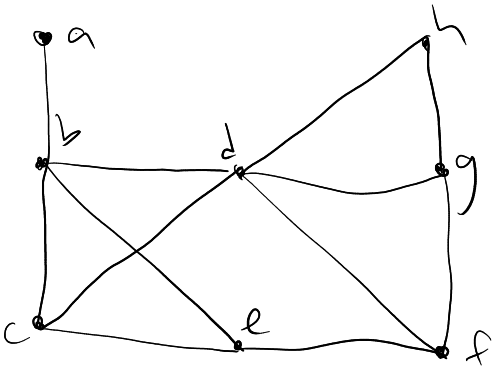
8. **Exercise 3 on page 544 - Modified**
Two of the graphs below are isomorphic to each other. Which ones? After deciding, label each of the nodes of the isomorphic grabs and provide an isomorphism for them.



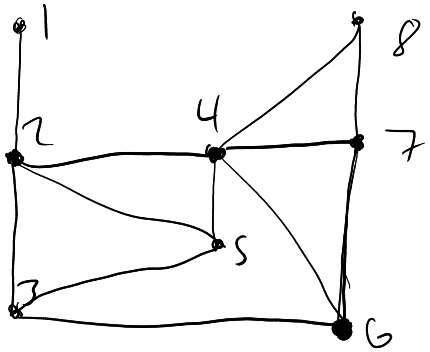
Solution:

a & c

a)



c)



a → 1

b → 2

c → 5

d → 4

e → 3

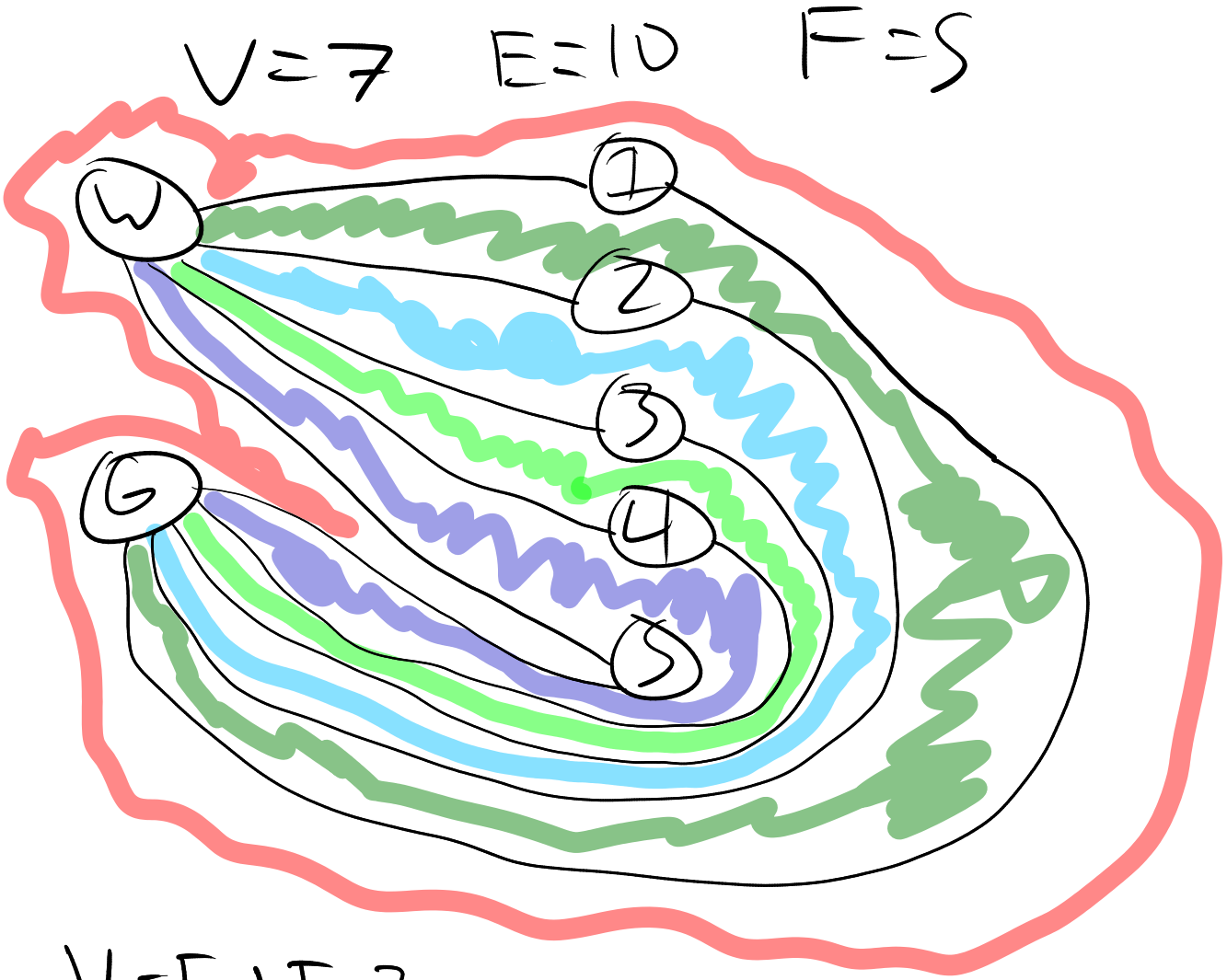
f → 6

g → 7

h → 8

9. **Exercise 8 on page 545 - Modified**
Show that each of 5 houses can be connected to 2 utilities (for example: water and gas) without lines crossing. Draw the planar graph for this situation and use Euler's Formula to validate your answer

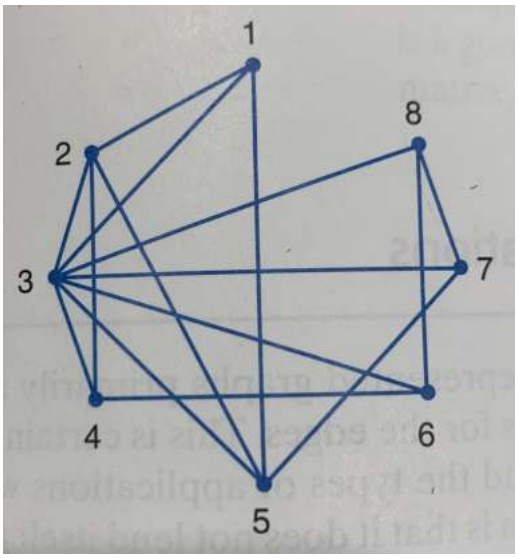
Solution:



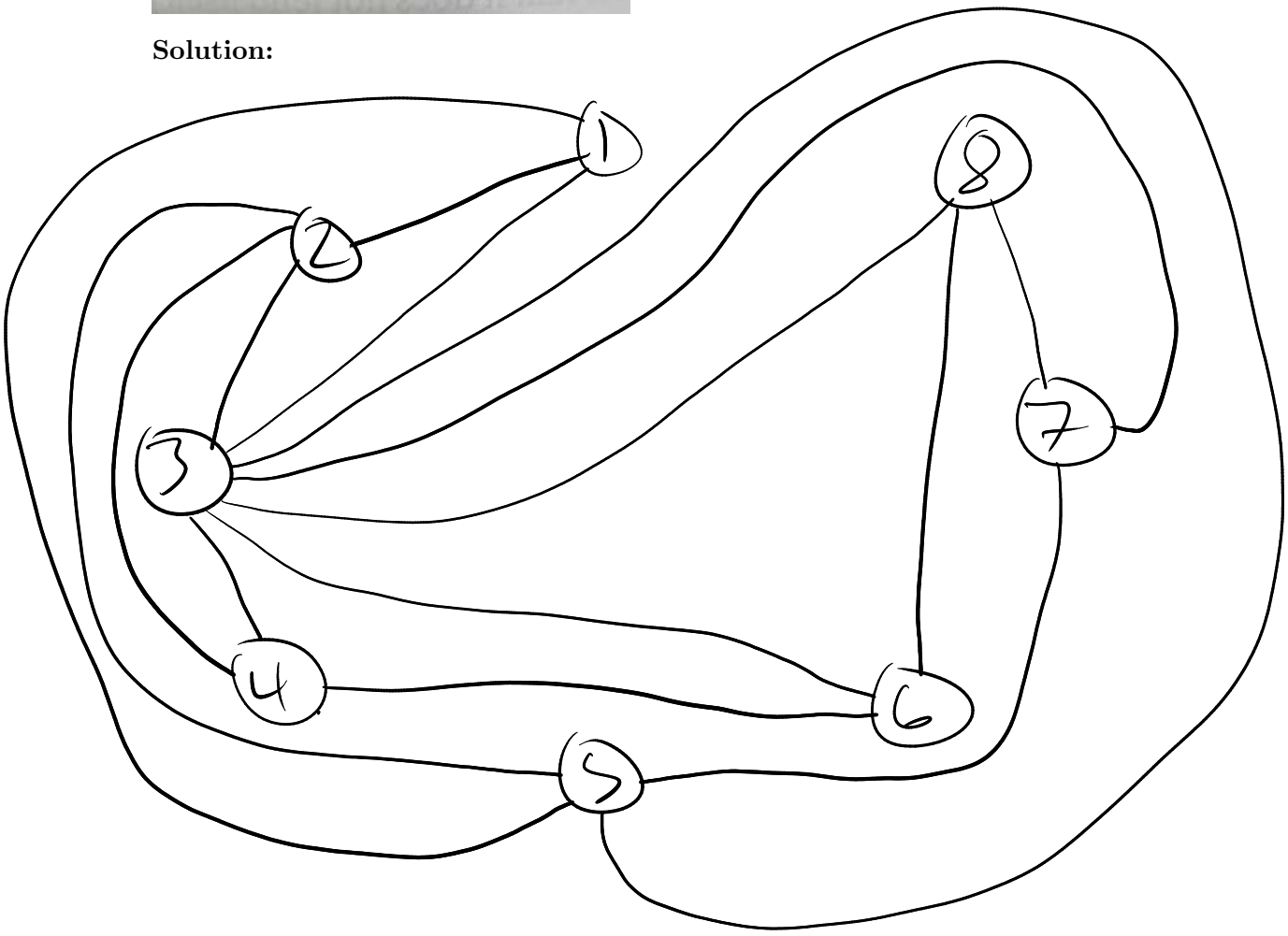
$V - E + F = 2$

$7 - 10 + 5 = 2$

10. **Exercise 10.a on page 545**
Show that the graph below is planar by redrawing it in the plane with no edge crossings.



Solution:



11. **Exercise 3 on page 564**
Explain how to use the adjacency matrix to calculate the degree of a node for these situations:

- A simple graph (no parallel edges, no loops).
- Loops are allowed but not parallel edges.
- Parallel edges are allowed but not loops.
- Both loops and parallel edges are allowed.

Solution:

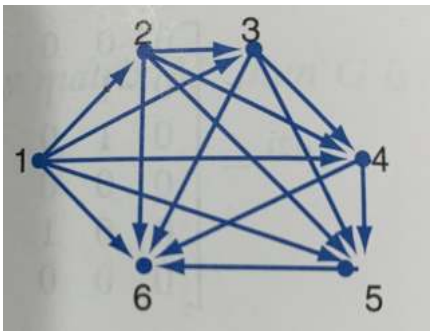
a) Sum the row of the node, ignoring the diagonal.

b) Sum the row of the node, except double any number on the diagonals first.

c) sum the row & column of the node, ignoring the diagonal

d) sum the row & column of the node, including the diagonal on both sums

12. **Exercise 6.b on page 564**
For the graph below, give the corresponding relation on the set $\{1, 2, 3, 4, 5, 6\}$ as a set of ordered pairs.



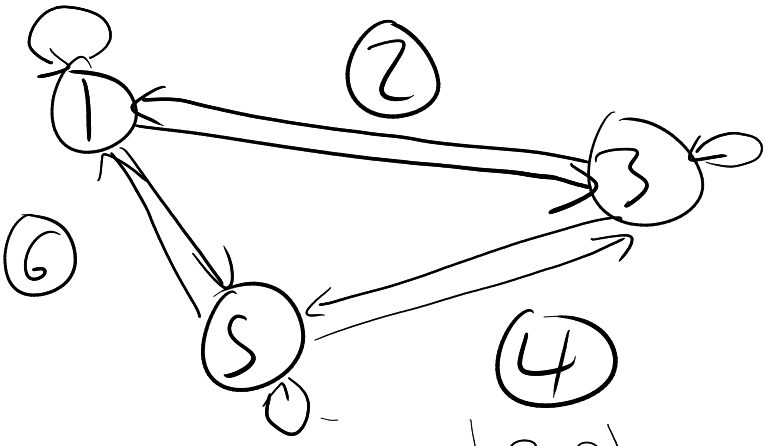
Solution:

$\{(1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6)\}$

13. **Exercise 8.b on page 564**
For the following relation on the set $A = \{1, 2, 3, 4, 5, 6\}$, draw the graph and give the adjacency matrices for relations R and $R \circ R$.

$$R = \{(1,3), (3,1), (1,5), (5,1), (3,5), (5,3), (1,1), (3,3), (5,5)\}$$

Solution:



R)

	1	2	3	4	5	6
1	1	0	1	0	1	0
2	0	0	0	0	0	0
3	1	0	1	0	1	0
4	0	0	0	0	0	0
5	1	0	1	0	1	0
6	0	0	0	0	0	0

R∘R)

	1	2	3	4	5	6
1	3	0	3	0	3	0
2	0	0	0	0	0	0
3	3	0	3	0	3	0
4	0	0	0	0	0	0
5	3	0	3	0	3	0
6	0	0	0	0	0	0