# Empirical Dynamic Modelling

Automatic Causal Inference and Forecasting

Dr Patrick Laub

Time-Series and Forecasting Symposium

December 2, 2022



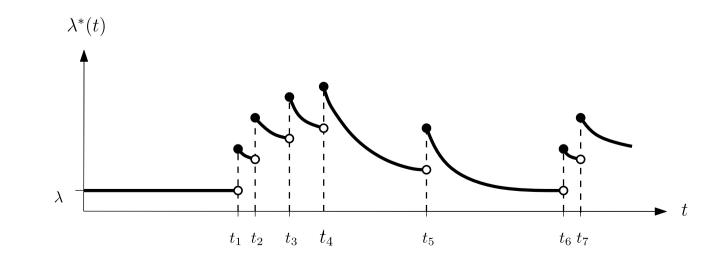
# Introduction

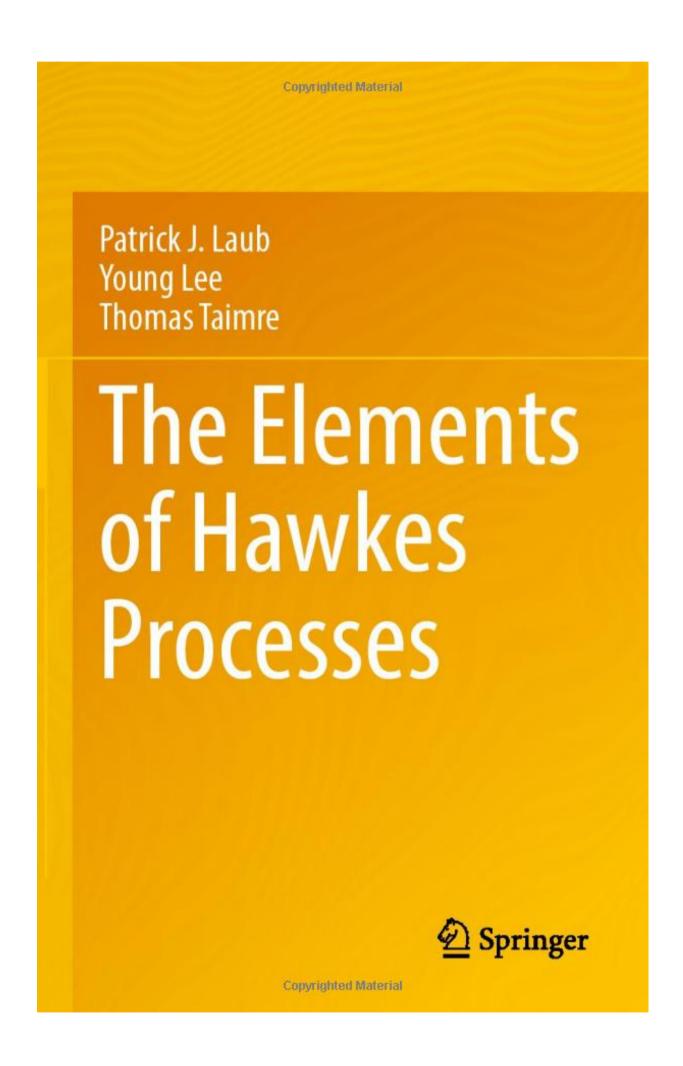


## About me

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- Software engineer & maths (UQ)
- PhD in computational applied probability (Aarhus)
- Actuarial science post-doc (Lyon)
- Research software engineer (Uni Melbourne)
- Actuarial science lecturer (UNSW)





## Other unrelated past work

approxbayescomp Python Package

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Approximate Bayesian Computation Python Package

### Package Description

Approximate Bayesian Computation (ABC) is a statistical method to fit a

Bayesian model to data when the likelihood function is hard to compute.

The approxbayescomp package implements an efficient form of ABC—

the sequential Monte Carlo (SMC) algorithm. While it can handle any general statistical problem, we built in some models so that fitting insurance loss distributions is particularly easy.

#### Installation

To install simply run

pip install -U approxbayescomp

Soon, it will be possible to install using conda; at that point the preferred

## **Deep Learning for Actuaries**

Lecture slides from UNSW's ACTL3143 & ACTL5111 courses

AUTHOR

Dr Patrick Laub

#### Overview

These are the lecture slides from my recent "Deep Learning for Actuaries" courses (coded ACTL3143 & ACTL5111) at UNSW. They can be used to see what topics I covered in these courses. The slides are not intended to be used to learn deep learning from scratch. For that, you need to attend the lectures & complete the assessment.

### Lecture slides

Course overview slides, source



## Goal: automatic causal inference

```
1 df <- read.csv("chicago.csv")</pre>
  head(df)
  #> Time Temperature Crime
                24.08
                      1605
5 #> 2 2 19.04 1119
  #> 3 28.04 1127
7 #> 4 4 30.02 1154
  9 #> 6 6 33.08 1276
10
  library(fastEDM)
  crimeCCMCausesTemp <- easy_edm("Crime", "Temperature", data=df)</pre>
14 #> * No evidence of CCM causation from Crime to Temperature found.
  tempCCMCausesCrime <- easy_edm("Temperature", "Crime", data=df)</pre>
  #> ✓ Some evidence of CCM causation from Temperature to Crime found.
```

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# Beyond linearity, stability, and equilibrium: The edm package for empirical dynamic modeling and convergent cross-mapping in Stata

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#### Acknowledgments

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## A different view of causality

Imagine  $x_t$ ,  $y_t$ ,  $z_t$  are interesting time series...

If the data is generated according to the nonlinear system:

$$egin{aligned} x_{t+1} &= \sigma(y_t - x_t) \ y_{t+1} &= x_t(
ho - z_t) - y_t \ z_{t+1} &= x_t y_t - eta z_t \end{aligned}$$

then  $y \Rightarrow x$ , both  $x, z \Rightarrow y$ , and both  $x, y \Rightarrow z$ .

# Empirical Dynamic Modelling (EDM)





## Create lagged embeddings

Given two time series, create E-length trajectories

$$\overleftarrow{\mathbf{x}}_t = (\mathrm{Temp}_t, \mathrm{Temp}_{t-1}, \dots, \mathrm{Temp}_{t-(E-1)}) \in \mathbb{R}^E$$

and targets

$$y_t = \text{Crime}_t$$
.

## (i) Note

The  $\overleftarrow{\mathbf{x}}_t$ 's are called *points* (on the shadow manifold).

## Split the data

• 
$$\mathcal{L} = \{(\overleftarrow{\mathbf{x}}_1, y_1), \ldots, (\overleftarrow{\mathbf{x}}_n, y_n)\}$$
 is library set,

• 
$$\mathcal{P} = \{(\overleftarrow{\mathbf{x}}_{n+1}, y_{n+1}), \dots, (\overleftarrow{\mathbf{x}}_T, y_T)\}$$
 is prediction set.

For point  $\mathbf{x}_s \in \mathcal{P}$ , pretend we don't know  $y_s$  and try to predict it.

$$orall \mathbf{x} \in \mathcal{L} \quad ext{find} \quad d(\overleftarrow{\mathbf{x}}_s, \overleftarrow{\mathbf{x}})$$

This is computationally demanding.

## Non-parametric prediction: simplex

For point  $\overleftarrow{\mathbf{x}}_s \in \mathcal{P}$ , find k nearest neighbours in  $\mathcal{L}$ .

Say, e.g., k=2 and the neighbours are

$$\mathcal{NN}_k = ((\overleftarrow{\mathbf{x}}_3, y_3), (\overleftarrow{\mathbf{x}}_5, y_5))$$

The simplex method predicts

$$\hat{y}_s = w_1 y_3 + w_2 y_5.$$

## Non-parametric prediction: S-map

Sequential Locally Weighted Global Linear Maps (S-map)

Weight the points by distance

$$w_i = \exp\{-\theta d(\overleftarrow{\mathbf{x}}_s, \overleftarrow{\mathbf{x}}_i)\}.$$

Build a local linear system

$$\widehat{y}_s = \mathbf{A}_s \overleftarrow{\mathbf{x}}_s.$$

For all  $s \in \mathcal{P}$ , compare  $\widehat{y}_s$  to true  $y_s$ , and calculate  $\rho$ .

## Convergent cross mapping

- If  $Temp_t$  causes  $Crime_t$ , then information about  $Temp_t$  is somehow embedded in  $Crime_t$ .
- By observing  $Crime_t$ , we should be able to forecast  $Temp_t$ .
- By observing more of  $Crime_t$  (more "training data"), our forecasts of  $Temp_t$  should be more accurate.

Example: Chicago crime and temperature.



# Software



## Stata package

**EDM Stata Package** 

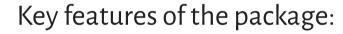


**Q** Search

## Empirical Dynamic Modeling Stata Package

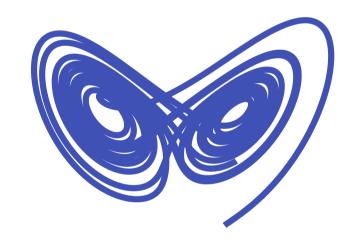
### Package Description

Empirical Dynamic Modeling (EDM) is a way to perform causal analysis on time series data. The edm Stata package implements a series of EDM tools, including the convergent cross-mapping algorithm.



- powered by a fast multi-threaded C++ backend,
- able to process panel data, a.k.a. multispatial EDM,
- able to handle missing data using new dt algorithms or by dropping points,
- factor variables can be added to the analysis,
- multiple distance functions available (Euclidean, Mean Absolute Error, Wasserstein),
- GPU acceleration available.

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Patrick Laub, Time-Series and Forecasting Symposium, University of Sydney

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## Rpackage

Thanks to Rishi Dhushiyandan for his hard work on easy\_edm.

fastEDM 0.1 Reference Articles Changelog

fastEDM

The fastEDM R package implements a series of *Empirical Dynamic Modeling* tools that can be used for *causal analysis of time series* data.

Key features of the package:

- powered by a fast multi-threaded C++ backend,
- able to process panel data, a.k.a. multispatial EDM,
- able to handle *missing data* using new dt algorithms or by dropping points.

### Installation

You can install the development version of fastEDM from GitHub with:

Search for

#### License

MIT + file LICENSE

#### Citation

fastEDM

**Citing fastEDM** 

#### Developers

Patrick Laub

Author, maintainer

Jinjing Li

Author

Michael Zyphur

Author

More about authors...

# Python package

fastEDM Python Package



**Q** Search

## fastEDM Python Package

### Package Description

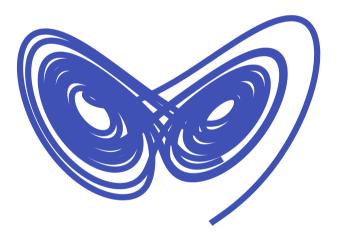
Empirical Dynamic Modeling (EDM) is a way to perform causal analysis on time series data. The fastEDM Python package implements a series of EDM tools, including the convergent cross-mapping algorithm.

Key features of the package:

- powered by a fast multi-threaded C++ backend,
- able to process panel data, a.k.a. multispatial EDM,
- able to handle missing data using new dt algorithms or by dropping points.

#### Installation

To install the latest version from Github using pip run:



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Example: Chicago crime levels and temperature

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## Modern engineering

- Open code (9,745 LOC) on MIT License,
- unit & integration tests (5,342 LOC),
- documentation (3,157 LOC),
- Git (1,198 commits),
- Github Actions (11 tasks),
- vectorised, microbenchmarking, ASAN, linting,
- all C++ compilers, WASM, all OSs.

## Get involved!

Give it a try, feedback would be very welcome.

Use If you're talented in causal inference or programming (Stata/Mata, R, Javascript, C++, Python), we'd love contributions!

"We actually also have a PhD scholarship if we have a domestic applicant who can commit like in next week" (need  $\geq$  85 in a 4 year degree)







# Linear/nonlinear dynamical systems

Say  $\mathbf{x}_t = (x_t, y_t, z_t)$ , then if:

$$\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t$$

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t)$$

we have a linear system. we have a nonlinear system.

Using a term like nonlinear science is like referring to the bulk of zoology as the study of non-elephant animals. (Stanisław Ulam)

## Noise / unobserved variables?

Takens' theorem to the rescue, though...

Takens' theorem is a deep mathematical result with farreaching implications. Unfortunately, to really understand it, it requires a background in topology. (Munch et al. 2020)

#### Takens's theorem

From Wikipedia, the free encyclopedia

In the study of dynamical systems, a **delay embedding theorem** gives the conditions under which a chaotic dynamical system can be reconstructed from a sequence of observations of the state of a dynamical system. The reconstruction preserves the properties of the dynamical system that do not change under smooth coordinate changes (i.e., diffeomorphisms), but it does not preserve the geometric shape of structures in phase space.

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#### Simplified, slightly inaccurate version [edit]

Suppose the d-dimensional state vector  $x_t$  evolves according to an unknown but continuous and (crucially) deterministic dynamic. Suppose, too, that the one-dimensional observable y is a smooth function of x, and "coupled" to all the components of x. Now at any time we can look not just at the present measurement y(t), but also at observations made at times removed from us by multiples of some lag  $\tau:y_{t+\tau},y_{t+2\tau}$ , etc. If we use k lags, we have a k-dimensional vector. One might expect that, as the number of lags is increased, the motion in the lagged space will become more and more predictable, and perhaps in the limit  $k \to \infty$  would become deterministic. In fact, the dynamics of the lagged vectors become deterministic at a finite dimension; not only that, but the deterministic dynamics are completely equivalent to those of the original state space (More exactly, they are related by a smooth, invertible change of coordinates, or diffeomorphism.) The magic embedding dimension k is at most 2d+1, and often less.<sup>[1]</sup>

