q1-sol

March 11, 2019

1 Q1

$$\tilde{\ell} = \mathbb{P}(\tilde{M} > 3) = 1 - \mathbb{P}(\tilde{M} \leq 3) = 1 - \mathbb{P}(\tilde{X}_1 \leq 3, \tilde{X}_2 \leq 3) = 1 - \mathbb{P}(\tilde{X}_1 \leq 3)\,\mathbb{P}(\tilde{X}_2 \leq 3) = 1 - \Phi(3)^2$$

```
In [1]: 1 - pnorm(3)^2
```

0.0026979738385644

2 Q2

```
In [2]: library(cubature)
        library(mvtnorm)

        mu <- c(0, 0)
        sigma <- matrix(c(1, 0.8, 0.8, 1), 2)

        methods <- c("cuhre", "hcubature")

        cat("Integrate over the region of interest.\n\n")
        for (method in methods) {
            res <- cubintegrate(f = function(x) { (max(x) > 3) * dmvnorm(x, mu, sigma) },
                                lower=rep(-5,2), upper=rep(5,2), method = method)
            cat("Method: ", method, "\n")
            cat('Estimate:', res$integral, 'with error', res$error, 'using',
                res$neval, 'function calls\n\n')
        }

        cat("\nIntegrate over the complement of the region of interest.\n\n")
        for (method in methods) {
            res <- cubintegrate(f = function(x) { dmvnorm(x, mu, sigma) },
                                lower=rep(-100,2), upper=rep(3,2), method = method)
            cat("Method: ", method, "\n")
            cat('Estimate:', 1 - res$integral, 'with error', res$error, 'using',
                res$neval, 'function calls\n\n')
        }
```

```
Integrate over the region of interest.

Method:  cuhre
Estimate: 0.002327154 with error 2.323471e-08 using 42575 function calls

Method:  hcubature
Estimate: 0.002327123 with error 2.323542e-08 using 10285 function calls


Integrate over the complement of the region of interest.

Method:  cuhre
Estimate: 0.00232782 with error 8.524119e-06 using 6435 function calls

Method:  hcubature
Estimate: 0.002327477 with error 9.958947e-06 using 7769 function calls
```

3 Q3

```
In [3]: library(mvtnorm)

        R <- 10^6
        mu <- c(0, 0)
        sigma <- matrix(c(1, 0.8, 0.8, 1), 2)

        xs <- rmvnorm(R, mu, sigma)
        ms <- apply(xs, 1, max)
        estMean <- mean(ms>3)
        estVar <- var(ms>3)
        estMean
```

0.002282

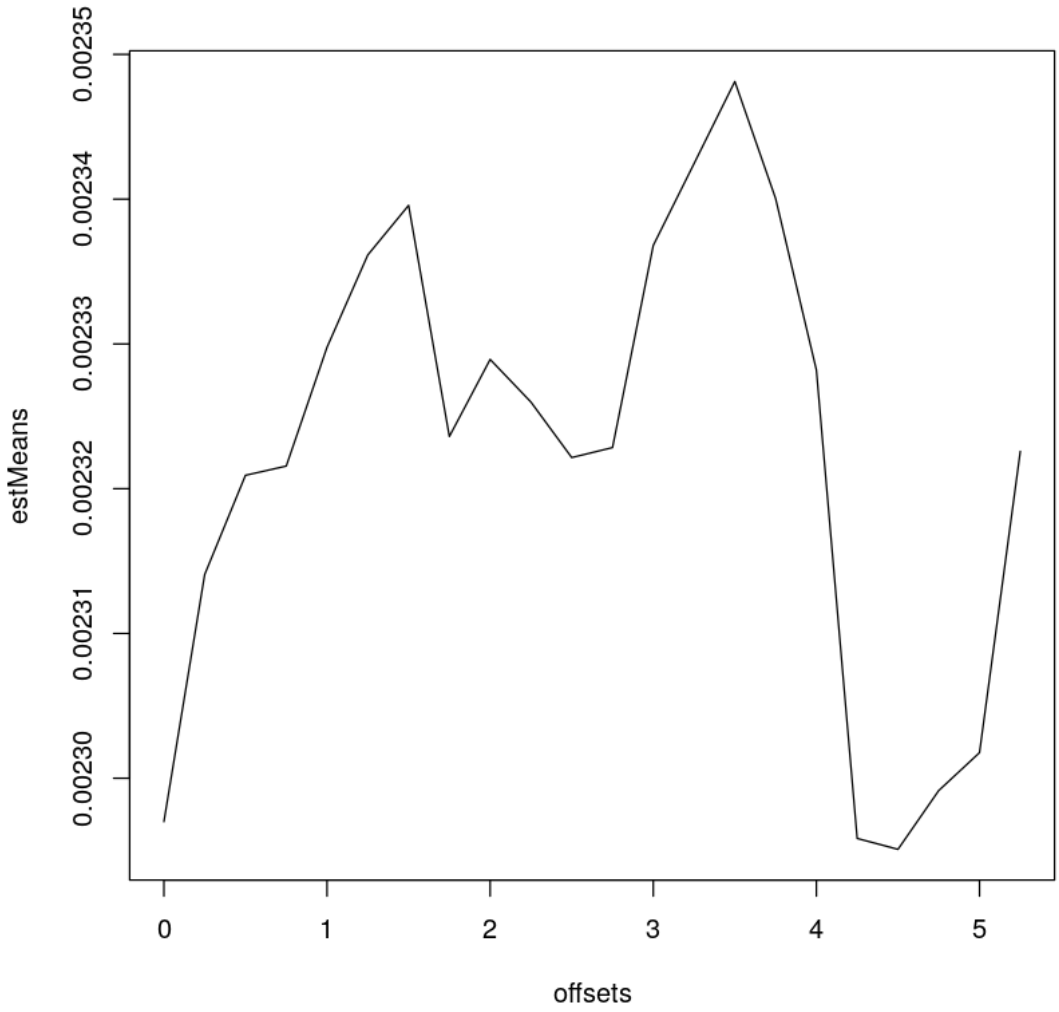
```
In [4]: q <- abs(qnorm(0.01/2))

        lowerCI <- estMean - q * sqrt(estVar / R)
        upperCI <- estMean + q * sqrt(estVar / R)

        c(lowerCI, upperCI)
```

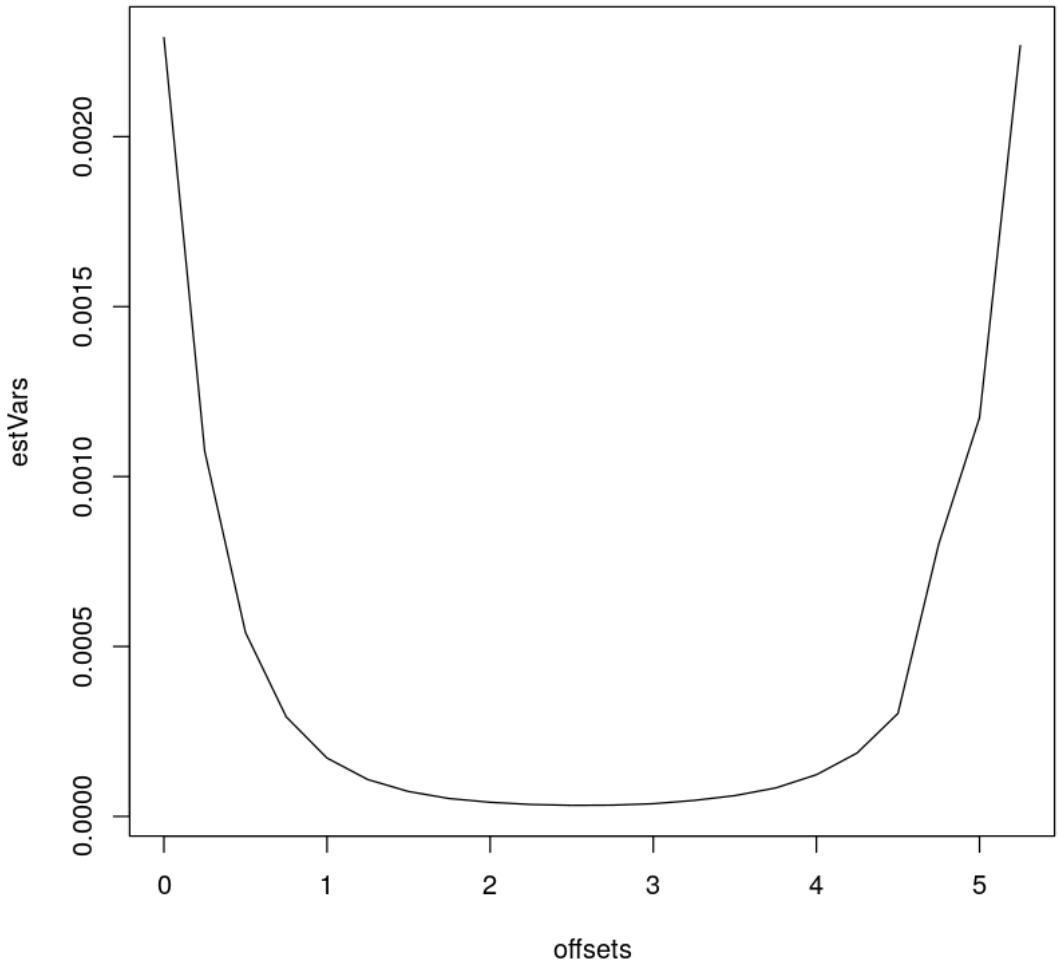
1. 0.00215909232014774 2. 0.00240490767985226

4 Q4

```
In [5]: library(mvtnorm)
```

```r
R <- 10^6

offsets <- seq(0, 5.25, 0.25)

mu <- c(0, 0)
sigma <- matrix(c(1, 0.8, 0.8, 1), 2)

xsOrig <- rmvnorm(R, mu, sigma)

estMeans <- rep(NA, length(offsets))
estVars <- rep(NA, length(offsets))

for (i in 1:length(offsets)) {
    offset <- offsets[i]

    xs <- xsOrig + offset

    origPDF <- dmvnorm(xs, mu, sigma)
    newPDF <- dmvnorm(xs, mu + offset, sigma)
    LRs <- origPDF / newPDF

    ms <- apply(xs, 1, max)
    ests <- (ms > 3) * LRs

    estMeans[i] <- mean(ests)
    estVars[i] <- var(ests)
}

plot(offsets, estMeans, type="l")
plot(offsets, estVars, type="l")
```


estMeans
0.00230
0.00231
0.00232
0.00233
0.00234
0.00235
0
1
2
3
4
5
offsets


estVars
0.0000
0.0005
0.0010
0.0015
0.0020
0
1
2
3
4
5
offsets