

Rare event estimation: Quiz 2

Email answers & code to Patrick Laub before class on March 22

Version 2

1. Imagine performing the cross-entropy method where the family of distributions to search inside are the $\text{Pareto}(\alpha)$ distributions, with p.d.f. $f(x; v) = vx^{-(v+1)}$ for $x \geq 1$. Solve algebraically

$$\sum_{r=1}^R 1\{X_r > \gamma_i\} \frac{f(X_r)}{f(X_r; v_{i-1})} \frac{d}{dv} \left\{ \log[f(X_r; v)] \right\} = 0.$$

2. Estimate $\ell = \mathbb{P}(S > 10^5)$ where $S = X_1 + X_2 + X_3$ where $X_i \sim \text{Pareto}(i + 10)$ and the X_i 's are independent of each other. Use the cross-entropy method to find the best IS proposal distribution. Look in the family of distributions

$$f(\mathbf{x}; \mathbf{v}) = f_{\text{Pareto}(v_1)}(x_1) f_{\text{Pareto}(v_2)}(x_2) f_{\text{Pareto}(v_3)}(x_3)$$

where \mathbf{x} denotes (x_1, x_2, x_3) and $f_{\text{Pareto}(v)}(x) = vx^{-(v+1)}$ is the p.d.f. of a $\text{Pareto}(v)$ variable.

Hint: In R, the `actuar` package has some simple methods for Pareto variables (and more complex ones for generalised Pareto). However, note that their definition of Pareto variables is shifted so that they have support $[0, \infty)$ whereas the ones above are defined on $[1, \infty)$.

3. Use cross-entropy to find the minimum of Rastrigin's function

$$S(\mathbf{x}) = 30 + (x_1^2 - 10 \cos(2\pi x_1)) + (x_2^2 - 10 \cos(2\pi x_2)).$$

Use the family of bivariate normal distributions, i.e.

$$f(x; v_i) = f(x; (\mu_i, \Sigma_i)) = (2\pi)^{-1} \Sigma_i^{-1/2} \exp \left\{ -\frac{1}{2} (x - \mu_i)^\top \Sigma_i^{-1} (x - \mu_i) \right\}.$$

Start with $\mu_0 = (150, 150)$, and choose your own Σ_0 starting matrix. Recreate the some of the plots from the lecture slides:

- scatterplots of the samples for a few different iterations, marking the elite samples differently to the others
- a plot of the best objective value found across each of the iterations (on log scale for y axis)

4. *Unmarked question:* Think about anything related to computers which you don't understand. Email me one or more computer-related questions that you'd like answered before class on Monday 18 March.