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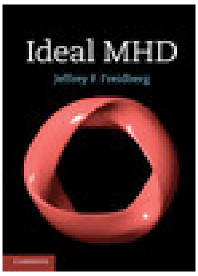

Ideal MHD

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Introduction

1.1 The role of MHD in fusion energy

Magnetohydrodynamics (MHD) is a fluid model that describes the macroscopic equilibrium and stability properties of a plasma. Actually, there are several versions of the MHD model. The most basic version is called "ideal MHD" and assumes that the plasma can be represented by a single fluid with infinite electrical conductivity and zero ion gyro radius. Other, more sophisticated versions are often referred to as "extended MHD" or "generalized MHD" and include finite resistivity, two-fluid effects, and kinetic effects (e.g. finite ion gyro radius, trapped particles, energetic particles, etc.). The present volume is focused on the ideal MHD model.

Most researchers agree that MHD equilibrium and stability are necessary requirements for a fusion reactor. If an equilibrium exists but is MHD unstable the result is almost always very undesirable. There can be a violent termination of the plasma known as a major disruption. If no disruption occurs, the result is likely to be a greatly enhanced thermal transport which is highly detrimental to fusion power balance. In order to avoid MHD instabilities it is necessary to limit the regimes of operation so that the plasma pressure and current are below critical values. However, these limiting values must still be high enough to meet the needs of producing fusion power. In fact it is fair to say that the main goal of ideal MHD is the discovery of stable, magnetically confined plasma configurations that have sufficiently high plasma pressure and current to satisfy the requirements of favorable power balance in a fusion reactor.

1.1.1 The plasma pressure in a fusion reactor

To put the role of MHD in context with respect to fusion it is useful to quantify the value of plasma pressure required in a reactor. This is easily done by considering

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simple power balance in a deuterium–tritium (D–T) fusion plasma where the heating power produced by fusion alpha particles balances the thermal conduction losses due to classical collisions and plasma turbulence.[1] This balance must be achieved at an optimum temperature that maximizes fusion energy production. The resulting "ignited" plasma is self-sustaining, requiring no external heating sources. The power balance condition is given by

$$\begin{gathered}\text{alpha heating} = \text{thermal loss}\\ \frac{E_\alpha}{4} n^2 \langle \sigma v \rangle = \frac{3}{2} \frac{p}{\tau_E}\end{gathered} \tag{1.1}$$

where $E_\alpha = 3.5$ MeV, n is the electron number density, $\langle \sigma v \rangle$ is the velocity averaged D–T fusion cross section, p is the plasma pressure, and τ_E is the thermal conduction energy confinement time. For a plasma with equal temperatures $T_D = T_T = T_e \equiv T$, the plasma pressure is equal to $p = 2nT$, where T is measured in units of energy. Some simple manipulations allow Eq. (1.1) to be rewritten in terms of one version of the Lawson (1957) parameter as follows:

$$p\tau_E = \frac{24}{E_\alpha} \frac{T^2}{\langle \sigma v \rangle} \tag{1.2}$$

For many years this fundamental requirement has divided fusion research into three main areas of study: heating, transport, and MHD. The reasoning for this division starts with the recognition that the function $T^2/\langle \sigma v \rangle$ has a minimum at approximately $T = 15\,\text{keV}$. It is important to operate at this temperature or else p and/or τ_E would have to raised, both of which lead to increased costs. It is the job of the heating community to provide ways to heat the plasma to about 15 keV.

At this temperature ignition requires

$$(p\tau_E)_{\min} \approx 8 \text{ atm-sec} \tag{1.3}$$

Learning how to produce a plasma with a sufficiently long τ_E is the job of the transport community. Learning how to produce plasmas with a sufficiently large p is the job of the MHD community. For many years these three areas of research were reasonably separated. As fusion research has progressed, longer duration, high-performance plasmas have been produced and these three areas have started to overlap. The reason is that plasma–wall interactions have become increasingly important and have a large, simultaneous impact on heating, transport, and MHD. For the moment it is, nonetheless, still useful to think of the three separate plasma requirements for an ignited plasma.

[1] Readers unfamiliar with fusion reactor power balance should refer to the Further reading at the end of the chapter.

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One might think on the basis of Eq. (1.3) that it might be possible to make tradeoffs between p and τ_E in order to reach ignition in as easy a way as possible. In practice there is not much room for tradeoffs. The reason is that if one wants to construct a standard base-load reactor with a power output of 1 GWe as economically possible, this actually requires a specific value of p. The reasoning behind this conclusion is based on (1) the intuition that "most economical" translates into smallest size and (2) the smallest size is set by the maximum neutron flux passing through the first wall. The maximum allowable neutron wall loading as set by material limitations is typically assumed to be $P_W \approx 4\,\mathrm{MW/m^2}$. The condition that the neutron flux not exceed the wall loading limit in a toroidal reactor is given by

$$\begin{gathered}\text{fusion neutron flux} = \text{wall loading} \\ \frac{E_n}{16}p^2\frac{\langle\sigma v\rangle}{T^2}\left(2\pi^2R_0a^2\right) = P_W\left(4\pi^2R_0a\right)\end{gathered} \tag{1.4}$$

Here, $E_n = 14.1$ MeV, R_0 is the major radius of the torus, and a is the minor radius. Solving for p yields

$$p = \left(32\frac{T^2}{\langle\sigma v\rangle}\frac{P_W}{E_na}\right)^{1/2} \tag{1.5}$$

The minor radius of the plasma appearing in Eq. (1.5) can be accurately approximated by assuming that most of the electric power is produced by the fusion neutrons with a conversion efficiency $\eta \approx 0.4$. Thus, Eq. (1.4) can be rewritten as

$$\begin{gathered}\text{electric power} = \eta(\text{neutron power}) \\ P_E = \eta P_n = \eta P_W\left(4\pi^2R_0a\right)\end{gathered} \tag{1.6}$$

Now, the minor radius a can be rewritten in terms of the dimensionless inverse aspect ratio a/R_0

$$a = \left(\frac{1}{4\pi^2}\frac{a}{R_0}\frac{P_E}{\eta P_W}\right)^{1/2} \tag{1.7}$$

Typically $R_0/a \sim 3$. The exact value is not too critical since it enters the value of the pressure as a fourth root. For the parameters under consideration one finds $a \approx 2.3$ m, which when substituted into Eq. (1.5) leads to

$$p \approx 7\ \text{atm} \tag{1.8}$$

The conclusion is that a fusion plasma must have a pressure of about 7 atm and a corresponding energy confinement time equal to 1.1 sec. In general there is some, but not a lot, of flexibility in these values.

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1.1.2 The dimensionless pressure, β

The analysis of MHD is almost always carried out in terms of a dimensionless pressure denoted by β. There are various detailed definitions in the literature, the most important of which are discussed in the text. All definitions involve the ratio of plasma pressure to applied magnetic pressure:

$$\beta \equiv \frac{p}{B^2/2\mu_0} \tag{1.9}$$

In configurations with a large toroidal magnetic field and an aspect ratio $R_0/a \sim 3$, the corresponding reactors typically require $\beta \sim 5-10\%$, values that have been already achieved experimentally. In tighter aspect ratio devices, higher stable β values are attainable, but often the pressure is not higher because, for engineering and geometric reasons, the magnetic field is smaller. Other concepts do not rely on a large toroidal magnetic field, which is an important engineering advantage. As a result their required and achieved MHD β values are higher. However, such configurations typically have poorer MHD stability behavior leading to enhanced thermal transport. Almost all discussions of MHD in the literature involve β, but readers should stay alert to the fact that it is pressure that is the critical parameter for a fusion reactor.

1.1.3 A variety of fusion concepts

What is the best magnetic geometry for a fusion reactor from the point of view of MHD? Over the years many ideas have been tried. A list is given below:

Belt pinch
Cusp
Elmo bumpy torus
Field reversed configuration
Force-free pinch
Heliac
High β stellarator
Levitated dipole
Mirror
Octopole
Perhapsatron
Plasma focus
Reversed field pinch
Screw pinch
Spherical tokamak
Spheromak
Stellarator
Stuffed caulked cusp
Tandem mirror
Theta pinch
Tokamak
Tormac
Z-pinch
Z-pinch – hard-core

Clearly there has not been a shortage of imagination in inventing new concepts. Of this long list two concepts have risen to the top, largely because of superior overall plasma physics performance. These are the tokamak and the stellarator. It should be noted that while these configurations have the best plasma physics performance,

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they may not be the optimized choice from an engineering point of view. Both of these concepts have a large toroidal magnetic field which adds to the cost and complexity of a fusion reactor. Still, unless other concepts can overcome the plasma physics challenges their more desirable engineering features cannot be utilized. So far, while progress has been made, they have not yet been able to overcome these challenges, thereby explaining why tokamaks and stellarators remain at the top of the list.

1.1.4 Structure of the textbook

The basic structure of the textbook is straightforward. The discussion begins with a description of the ideal MHD model and some of its general properties. This is followed by a discussion of MHD equilibrium in simple and general geometries. The last main topic discussed involves MHD stability.

There are many examples presented, although the bulk of the actual applications involve tokamaks and stellarators. There is also a substantial discussion of the reversed field pinch, a concept that is not as yet quite as advanced as tokamaks and stellarators in terms of performance. Still, it does hold some promise and its relatively simple geometric properties make it an ideal example to help understand MHD equilibrium and stability.

The overall purposes of the textbook are to provide both a qualitative and quantitative understanding of ideal MHD theory as applied to magnetic fusion. The specific goals are to discover concepts capable of achieving MHD stable, high-pressure, fusion-grade plasmas.

1.2 Units

The basic units used throughout the textbook are the usual SI units. The one exception is temperature, which always appears in conjunction with Boltzmann's constant, k. This constant is always absorbed into the temperature which then has the units of energy: $kT \to T$.

In the course of the text a number of relations are derived in terms of practical units as defined below:

Number density	n	$10^{20}\ \mathrm{m}^{-3}$
Temperature	T	keV
Magnetic field	B	T (tesla)
Current	I	MA (megamperes)
Minor radius	a	m
Major radius	R_0	m

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