Power Logistic Regression

Patrick Wilson 4/17/2019

Power a logistic regression model for Observational research

When designing an observational study many times a sample size will need to be planned for. You may have the ability to delay for greater accrual of study subjects or you may have a cost function like chart abstraction that you need to optimize.

Observational models tend to be more complex than trial models. Usually this is due to the need to adjust for confounding.

Here is an toy example on how one would power a design for an exposure/outcome relationship where there is a suspected confounder called "Age".

$$P(\text{Disease} = 1) = \frac{\exp(\beta_0 + \beta_1 \text{Exposure} + \beta_2 \text{Age})}{1 + \exp(\beta_0 + \beta_1 \text{Exposure} + \beta_2 \text{Age})}$$

Design Your Model, Simulate Your Model, and Record Your Result

```
sims <- 10000
alpha <-0.05
pvalues <- replicate(sims,{</pre>
  #simulate parameter values
  N <-300
  age <- round(runif(N, min = 60, max = 80), digits = 1)
  #first simulate your exposure model
  base_exposure <- -2.5
  beta0 <- -3.0
  b.ex.age <- 0.03
  b.age <- 0.02
  b.x < -1
  linpred_exposure <- base_exposure + b.ex.age * age</pre>
  prob_exposure <- exp(linpred_exposure)/(1 + exp(linpred_exposure))</pre>
  exposure <- rbinom(N, 1, prob_exposure)</pre>
  #simulate your outcome model
  linpred <- beta0 + b.x * exposure + b.age * age</pre>
  prob <- exp(linpred)/(1 + exp(linpred))</pre>
  y <- rbinom(N, 1, prob)
```

```
data <- data.frame(y, exposure, age)

model <-glm(y ~ exposure + age, data = data, family = binomial)
p.value <- summary(model)$coefficients[,4]
})</pre>
```

The proportion of statistically significant effects is your power

```
signficant.experiment <- matrix(data=NA, nrow=3, ncol=sims)
for (i in 1:3){
    signficant.experiment[i,] <- (pvalues[i,] <= alpha)
}
power <-rowMeans(signficant.experiment)
power[2]</pre>
```

[1] 0.9534

Caveats

There is a lot to be desired with this quick example. In this simple example there is no demonstration of how to gauge the magnitude of a confounder in an intuitive way and the subsequent impact on power. Furthermore in designing a study you would need to be able to have realistic scenarios informed by the science.