Appendix to *HyperLogLog in Practice: Algorithmic Engineering of a State of the Art Cardinality Estimation Algorithm*

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This is an online appendix to the research paper *HyperLogLog in Practice: Algorithmic Engineering of a State of the Art Cardinality Estimation Algorithm* by Stefan Heule, Marc Nunkesser and Alexander Hall. Here, we list some more related work, show more information on accuracy and we give the values of all empirically determined parameters of our implementation to allow easier reproduction of our results.

Contents

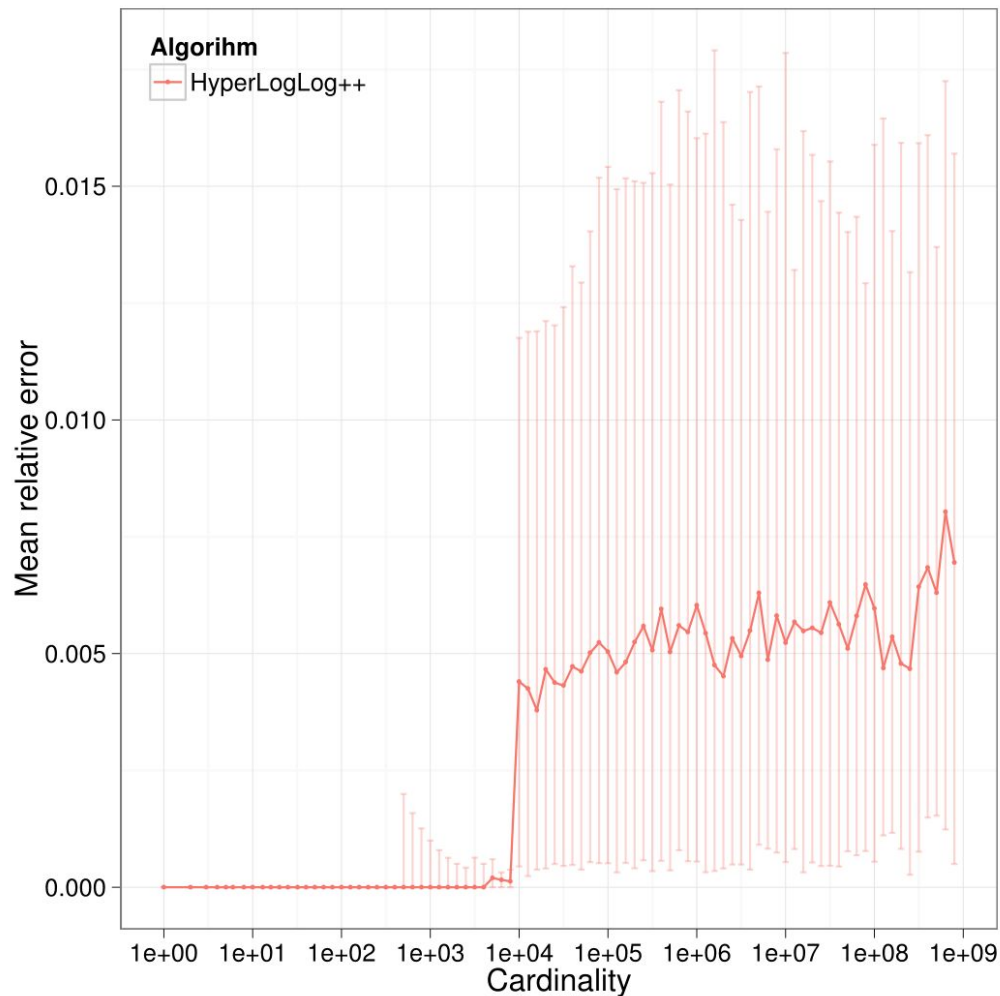## More Related Work

Marianne Durand, in her thesis (in French), had a subsection 5.7 p136 about compression. She remarked that due to the concentrated distribution of the registers' value, it was possible to encode them using very few bits, and she suggested using a variable bit prefix encoding, and showed how to implement it; this was used in Pranav Kashyap's implementation of HyperLogLog, used for the simulation in Flajolet et al.'s paper, although out of convenience Pranav only implemented it as an offline algorithm (i.e., compression after processing the stream). Thanks to Jeremie Lumbroso for pointing this out.

Sebastiano Vigna points out that the HyperANF algorithm has been using HyperLogLog counters based on 64-bit hash functions. The algorithm is available in the DSI utilities as part of the IntHyperLogLogCounterArray class and has been published at WWW 2011 by Paolo Boldi, Marco Rosa, and Sebastiano Vigna

More Accuracy Information on HyperLogLog++

To give a more complete picture of the accuracy we show the results of an experiment over a larger range of cardinalities (note the logarithmic scale on the horizontal axis). We used the HyperLogLog++ algorithm with precision 14, and show the mean relative error as well as top and bottom 5%.

Bias-Correction Data

In Section 5.2 we presented the bias-correction to reduce the error of the raw estimate. In the following we provide the necessary data; `rawEstimateData[p-4]` contains an array of the 200 interpolation points, storing the average raw estimate we have measured at this point over 5000 different data sets. Similarly, `biasData[p-4]` contains the corresponding 200 measured biases. Both values are for precision `p`.

Note that there are fewer than 200 interpolation points if $5m$ (which is the maximum value for which

we still do bias correction) is smaller than 200.

To give an example how to read this:

For a precision of 5, if the rawEstimate is 27.5, we are between 27.3742 and 27.9336 in the rawEstimateData array at indices 9 and 10. Looking at biasData[9] = 17.3742 and biasData[10] = 16.9336. Thus on average the correct estimates at indices 9 and 10 are between 27.3742 - 17.3742 and 27.9336 - 16.9336. In our implementation we use k-nearest neighbor interpolation between 6 neighbors to the compute the estimate. A linear interpolation between the two points in this example would give very similar values.

To determine whether or not to use this estimate, we use the linear counting estimate . Let that be 10.21. As 10.21 is far below the threshold of 20 for p=5, we would actually use the linear counting estimate here.

As pointed out by Tom Dial, there are very few cases, where the numbers in rawEstimateData are not monotonically increasing. This can be expected from running this many experiments over the 5000 data points and should have negligible impact on the accuracy of estimates.

```
double[][] rawEstimateData = {
  // precision 4
  { 11, 11.717, 12.207, 12.7896, 13.2882, 13.8204, 14.3772, 14.9342, 15.5202, 16.161, 16.7722,
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      505374.3855, 508550.9915, 511806.7305, 515026.776, 518217.0005, 521523.9855, 524705.9855,
      527950.997, 531210.0265, 534472.497, 537750.7315, 540926.922, 544207.094, 547429.4345,
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      573447.425, 576689.6245, 579874.5745, 583202.337, 586503.0255, 589715.635, 592910.161,
      596214.3885, 599488.035, 602740.92, 605983.0685, 609248.67, 612491.3605, 615787.912,
      619107.5245, 622307.9555, 625577.333, 628840.4385, 632085.2155, 635317.6135, 638691.7195,
      641887.467, 645139.9405, 648441.546, 651666.252, 654941.845, },
    // precision 18
    { 189084, 192250.913, 195456.774, 198696.946, 201977.762, 205294.444, 208651.754,
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    240543.233, 244279.475, 248044.27, 251854.588, 255693.2, 259583.619, 263494.621, 267445.385,
    271454.061, 275468.769, 279549.456, 283646.446, 287788.198, 291966.099, 296181.164,
    300431.469, 304718.618, 309024.004, 313393.508, 317760.803, 322209.731, 326675.061,
    331160.627, 335654.47, 340241.442, 344841.833, 349467.132, 354130.629, 358819.432, 363574.626,
    368296.587, 373118.482, 377914.93, 382782.301, 387680.669, 392601.981, 397544.323, 402529.115,
    407546.018, 412593.658, 417638.657, 422762.865, 427886.169, 433017.167, 438213.273,
    443441.254, 448692.421, 453937.533, 459239.049, 464529.569, 469910.083, 475274.03, 480684.473,
    486070.26, 491515.237, 496995.651, 502476.617, 507973.609, 513497.19, 519083.233, 524726.509,
    530305.505, 535945.728, 541584.404, 547274.055, 552967.236, 558667.862, 564360.216,
    570128.148, 575965.08, 581701.952, 587532.523, 593361.144, 599246.128, 605033.418, 610958.779,
    616837.117, 622772.818, 628672.04, 634675.369, 640574.831, 646585.739, 652574.547, 658611.217,
    664642.684, 670713.914, 676737.681, 682797.313, 688837.897, 694917.874, 701009.882,
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    756504.185, 762712.579, 768876.985, 775167.859, 781359, 787615.959, 793863.597, 800245.477,
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    850642.711, 856959.756, 863322.774, 869699.931, 876102.478, 882355.787, 888694.463,
    895159.952, 901536.143, 907872.631, 914293.672, 920615.14, 927130.974, 933409.404, 939922.178,
    946331.47, 952745.93, 959209.264, 965590.224, 972077.284, 978501.961, 984953.19, 991413.271,
    997817.479, 1004222.658, 1010725.676, 1017177.138, 1023612.529, 1030098.236, 1036493.719,
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    1088426.025, 1094776.005, 1101327.448, 1107901.673, 1114423.639, 1120884.602, 1127324.923,
    1133794.24, 1140328.886, 1146849.376, 1153346.682, 1159836.502, 1166478.703, 1172953.304,
    1179391.502, 1185950.982, 1192544.052, 1198913.41, 1205430.994, 1212015.525, 1218674.042,
    1225121.683, 1231551.101, 1238126.379, 1244673.795, 1251260.649, 1257697.86, 1264320.983,
    1270736.319, 1277274.694, 1283804.95, 1290211.514, 1296858.568, 1303455.691, }
  };

  double[][] biasData = {
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    { 10, 9.717, 9.207, 8.7896, 8.2882, 7.8204, 7.3772, 6.9342, 6.5202, 6.161, 5.7722, 5.4636,
    5.0396, 4.6766, 4.3566, 4.0454, 3.7936, 3.4856, 3.2666, 2.9946, 2.766, 2.4692, 2.3638, 2.0764,
    1.7864, 1.7602, 1.4814, 1.433, 1.2926, 1.0664, 0.999600000000001, 0.7956, 0.5366,
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-0.648600000000002, -0.919800000000002, -0.851799999999997, -0.962400000000002, -0.6402,
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-1.2072, -1.1752, -1.16759999999999, -1.14019999999999, -1.3754, -1.29859999999999, -1.607,
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8.687, 8.5398, 8.2462, 7.857, 7.6606, 7.4168, 7.1248, 6.9222, 6.6804, 6.447, 6.3454, 5.9594,
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3.755, 3.472, 3.2076, 3.1024, 2.8908, 2.7338, 2.7728, 2.629, 2.413, 2.3266, 2.1524, 2.2642,
2.1806, 2.0566, 1.9192, 1.7598, 1.3516, 1.5802, 1.43859999999999, 1.49160000000001, 1.1524,
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0.765600000000006, 0.331000000000003, 0.591399999999993, 0.601200000000006, 0.701599999999999,
0.558199999999999, 0.339399999999998, 0.354399999999998, 0.491200000000006, 0.308000000000007,
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0.132199999999997, 0.394400000000005, -0.241200000000006, 0.242000000000004,
0.191400000000002, 0.253799999999998, -0.122399999999999, -0.370800000000003,
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  // precision 6
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  // precision 7
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49.9092, 48.6306, 47.5294, 46.5756, 45.6508, 44.662, 43.552, 42.3724, 41.617, 40.5754,
39.7872, 38.8444, 37.7988, 36.8606, 36.2118, 35.3566, 34.4476, 33.5882, 32.6816, 32.0824,
31.0258, 30.6048, 29.4436, 28.7274, 27.957, 27.147, 26.4364, 25.7592, 25.3386, 24.781,
23.8028, 23.656, 22.6544, 21.996, 21.4718, 21.1544, 20.6098, 19.5956, 19.0616, 18.5758,
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85.7668, 83.298, 81.6688, 79.9466, 77.9746, 76.1672, 74.3474, 72.3028, 70.8912, 69.114,
67.4646, 65.9744, 64.4092, 62.6022, 60.843, 59.5684, 58.1652, 56.5426, 55.4152, 53.5388,
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176.2574, 172.302, 168.1644, 164.0056, 160.3802, 156.7192, 152.5234, 149.2084, 145.831,
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};
```

Sub-Algorithm Threshold

Our proposed algorithm HLLꜰɪɴᴀʟ uses both LɪɴᴇᴀʀCᴏᴜɴᴛɪɴɢ as well as the bias-corrected raw estimate as sub-algorithms, and uses the one that provides the better estimate for a given cardinality. To this end, a threshold is used that determines at which cardinality the switch from LɪɴᴇᴀʀCᴏᴜɴᴛɪɴɢ to the bias-corrected raw estimate is performed. The following table lists the empirically determined thresholds used in our implementation.

p	Threshold	p	Threshold	p	Threshold
4	10	9	400	14	11500
5	20	10	900	15	20000
6	40	11	1800	16	50000
7	80	12	3100	17	120000
8	220	13	6500	18	350000

Typos in the paper

The equation for the raw estimate in Section 4 (page 3, upper left) is missing a $.-1$ It appears

correctly in Figure 1, line 10.
As pointed out by Dave Revell, there are off by one errors in the indexing in the implementation of GetIndex in Figure 7, it should read as follows:
```
Define GetIndex(k,p)
    if <k_0> = 1 then
       return <k_{p+6},...,.k_7>
```

```
    else
       return <k_p,...,k_1>
    end if
```
This implementation will now correctly return p bits, whereas the one we give in the paper returns p+1 bits (in both cases), which is wrong.