Results for the relaxation techniques

This brief report presents the results for a single running of Matlab® implementation of algorithms presented on "Structured robust control against mixed uncertainty"[1] for inner, hybrid and outer relaxations methods for the problem of synthesize a structured robust controller for an uncertain system subjected to both parametric and dynamic uncertainties. Evaluation was based on the best worst-case H_∞-performance, or gain, achieved by each method.

Table 1 recalls the structure of each test case [2]. The column labeled 'Δ-structure' shows the structure of the uncertainty Δ. Positive numbers give the size of the square complex blocks Δ_d of dynamic uncertainties, negative numbers represent a real parametric uncertainty and its repetition. For instance, case 9 has $N_d = 0$, $N_p = 2$ with $r_1 = 18$ and $r_2 = 2$. Column n_P is the order of the generalized plant P, column z-w shows the number of exogenous outputs and inputs and column y-u shows the number of control outputs and inputs.

In all plants the performance channel $w \to z$ was scaled so that the worst case performance of the closed-loop system computed by the inner approach was close to the value one, in order to render comparison between the different techniques more straightforward. All performance values were certified by the `WCGAIN` Matlab® routine, with the exception of case 17, where `WCGAIN` failed, indicated by '–' in column 4 of table 2.

Table 2 shows the results for the inner relaxation technique, while tables 3 and 4 present the results for the hybrid and outer relaxation techniques, respectively. For each table, column n_K gives the order of the synthesized controller, which is the same for the 3 approaches, column named 'gain' corresponds to the worst-case H_∞-performance found by the routine implemented and T_{syn} the time spent in doing so. Global certification uses `WCGAIN`, and the result is given in column labeled 'certified' while the time needed is given in coulumn T_{cft}.

For the methods *inner* and *hybrid*, in tables 2 and 3, respectively, also is indicated the number $|\mathbf{\Delta}_a|$ of scenarios needed for the synthesis, which also corresponds to the number of times the local loop was executed. For instance, in case study no. 1, the inner method - table 2, found the gain 1.003

[1] Aguiar, R.S.S., Apkarian, P. and Noll, D. - "Structured robust control against mixed uncertainty". Submitted to IEEE TCST.

[2] Test case set available in http:// .

Table 1: Test cases

No.	Δ-structure		n_P	z-w	y-u
1	complex	1	9	3 3	2 1
2	complex	3	7	1 2	1 1
3	complex	3	8	4 4	3 1
4	complex	8	12	6 2	2 2
5	complex	1,1	22	2 2	2 2
6	complex	1	3	1 1	1 1
7	complex	2	26	6 5	5 2
8	real	-1	3	2 3	1 1
9	real	-18,-2	23	3 2	3 1
10	real	-20	10	2 1	1 1
11	real	-21	5	2 2	1 1
12	mixed	1,-1,-1	9	1 1	1 1
13	mixed	1,-2,-2	7	4 2	1 1
14	mixed	1,-1,-3	8	3 4	2 1
15	mixed	1,-1	8	2 4	2 2
16	mixed	2,-1,-1,-1,-1,-1,-1	14	6 2	2 2
17	mixed	1,-1,-1,-1,-1,-3	9	2 1	1 1
18	mixed	1,-1,-2,-2	6	2 1	1 1
19	mixed	1,-1,-1,-3,-3,-3	6	2 2	1 1
20	mixed	1,-1,-1,-1,-1,-1,-3,-3,-3,-3,-3	11	2 1	1 1
21	mixed	4,-1,-1,-1,-1	8	6 2	2 2
22	mixed	1,-1,-1,-1,-1,-1,-1,-1,-2,-2,-2,-2	19	2 3	1 1
23	mixed	3,-1,-1,-6	8	4 4	3 1
24	mixed	3,-1	7	1 2	1 1
25	mixed	1,-1,-1,-1,-6,-6,-6	24	3 2	3 1
26	mixed	1,1,1,1,-1,-1,-1,-1	8	6 2	2 2
27	mixed	1,-1	7	2 2	1 1
28	mixed	1,-1,-5	7	2 3	2 1
29	mixed	1,-1	4	2 3	1 1
30	mixed	1,-1	8	2 2	1 1

with a controller of order $n_K = 2$ in 133.6s, and with a total of $|\mathbf{\Delta}_a| = 10$ scenarios. Certification with `WCGAIN` confirmed this value as correct in 5.7s.

Table 2: Results for the inner relaxation techniques

No.	n_K	gain	certified	$\|\Delta_a\|$	T_{syn}	T_{cft}
1	2	1.003	1.003	10	133.6	5.7
2	1	1.000	1.000	6	27.7	2.8
3	4	0.977	0.978	28	988.4	3.5
4	3	0.999	1.000	2	18.2	6.7
5	5	0.989	0.991	23	1706.3	5.0
6	1	1.000	1.000	3	4.2	1.9
7	4	1.027	1.026	26	5667.0	3.1
8	2	1.000	1.000	3	4.9	1.7
9	2	0.999	0.998	5	72.9	1947.8
10	2	1.000	1.000	2	10.4	1450.6
11	1	1.000	1.000	1	1.0	3260.4
12	3	1.000	0.998	10	133.2	6.0
13	1	1.000	0.993	2	4.2	14.8
14	2	1.000	1.000	3	9.2	23.5
15	3	1.000	1.000	8	68.9	26.4
16	4	1.178	1.230	16	2408.2	20.3
17	4	0.906	–	13	396.2	75.0
18	3	0.992	0.992	6	43.0	21.1
19	1	1.000	1.000	1	1.8	127.9
20	2	1.210	1.210	5	69.3	104.6
21	1	1.000	1.005	2	27.8	29.5
22	3	0.976	0.976	10	305.7	233.0
23	4	1.070	1.079	37	556.3	73.9
24	4	0.997	0.997	8	48.7	11.8
25	3	1.000	0.999	8	517.4	269.6
26	1	1.000	1.000	2	6.7	21.0
27	3	0.998	0.997	7	88.2	12.2
28	4	1.001	1.001	5	27.9	48.9
29	2	1.020	1.020	5	14.7	6.1
30	5	1.088	1.085	14	346.9	6.7

Table 3: Results for the hybrid relaxation techniques

No.	n_K	gain	certified	$\|\Delta_a\|$	T_{syn}	T_{cft}
1	2	1.008	0.999	2	68.5	3.0
2	1	1.001	0.999	2	22.2	2.1
3	4	1.444	1.521	2	98.8	3.7
4	3	1.006	0.999	2	34.1	4.2
5	5	1.182	1.209	2	138.3	4.4
6	1	1.008	1.003	2	20.4	1.7
7	4	1.045	1.043	2	137.2	3.5
8	2	1.000	1.000	3	22.7	1.2
9	2	1.001	1.001	5	157.2	1511.3
10	2	1.000	1.000	2	28.5	1469.9
11	1	1.000	1.000	2	20.0	3211.3
12	3	1.052	1.052	3	50.8	5.3
13	1	1.002	0.994	2	21.0	13.9
14	2	1.000	1.000	3	43.8	22.9
15	3	1.001	1.000	3	64.8	24.9
16	4	1.238	1.225	14	5001.9	20.5
17	4	1.155	1.143	3	124.3	34.8
18	3	1.093	1.090	5	109.3	17.8
19	1	1.000	1.000	2	15.3	106.9
20	2	1.223	1.222	3	84.7	68.7
21	1	1.005	1.000	2	18.9	42.3
22	3	1.042	1.041	4	137.4	199.7
23	4	4.238	4.168	2	73.0	84.7
24	4	0.997	0.994	3	60.5	11.8
25	3	1.103	1.103	4	157.1	455.3
26	1	1.007	0.999	2	22.6	41.3
27	3	1.000	0.999	5	123.7	11.6
28	4	1.000	0.999	6	60.8	50.9
29	2	1.011	1.019	3	15745.8	6.2
30	5	1.071	1.057	3	144.7	4.9

Table 4: Results for the outer relaxation techniques

No.	n_K	gain	certified	T_{syn}	T_{cft}
1	2	0.989	0.996	21.6	6.0
2	1	0.991	0.999	21.4	2.8
3	4	1.488	1.493	78.5	3.9
4	3	0.991	0.999	30.1	5.5
5	5	1.601	1.555	81.2	4.8
6	1	0.991	1.000	20.5	2.1
7	4	1.035	1.043	117.6	3.4
8	2	1.116	1.126	39.9	1.9
9	2	–	86.26	71.9	0.02
10	2	–	4.687	779.2	0.02
11	1	–	1.000	835.9	769.1
12	3	1.217	1.227	118.9	8.2
13	1	18.83	1.135	84.1	14.5
14	2	1.005	1.000	111.3	21.5
15	3	9.999	6.373	30.3	18.2
16	4	2.227	1.732	687.3	20.5
17	4	6.449	2.475	310.7	34.3
18	3	1817	1785	138.2	16.6
19	1	10.190	1.692	389.0	155.6
20	2	20.00	18.52	1211.2	81.0
21	1	0.990	1.000	144.3	27.4
22	3	–	36339	2339.1	258.9
23	4	10.180	7.881	130.0	0.02
24	4	1.813	1.762	59.0	12.0
25	3	–	60.338	725.1	0.02
26	1	0.990	0.999	122.7	29.0
27	3	1.598	1.589	56.2	10.4
28	4	10.02	3.027	49.0	46.1
29	2	1.228	1.188	113.9	6.3
30	5	6.935	6.721	83.9	6.1