Causal Data Science:
A general framework for data fusion and causal inference

Elias Bareinboim
Columbia University
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Machine Learning Research School
Bangkok, August, 2019

Logistics

Logistics

- Website
  - https://causalai.net/mlss19/ (soon)
  - slides – l: mlss19, p: mlss19

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1. prob. & statistics
2. graphical models
3. d-separation
4. experimental design
5. causal inference

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- The Book of Why: The New Science of Cause and Effect
  - J. Pearl and D. Mackenzie
  - Basic Books, 2018

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- The Book of Why: The New Science of Cause and Effect
  - J. Pearl and D. Mackenzie
  - Basic Books, 2018
- Causal Inference in Statistics: A primer
  - J. Pearl, M. Glymour, N. Jewel
  - Cambridge Press, 2016

References (cont'd)

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- Causality: Models, Reasoning, and Inference
  - J. Pearl
  - Cambridge Press, 2000

References (cont'd)

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- General Identifiability with Arbitrary Surrogate Experiments
  - S. Lee, J. Correa, and E. Bareinboim
  - Proc. of 35th UAI conference

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- General Identifiability with Arbitrary Surrogate Experiments
  - S. Lee, J. Correa, and E. Bareinboim
  - Proc. of 35th UAI conference
- Probabilistic Graphical Models
  - D. Koller and N. Friedman
  - MIT Press, 2009

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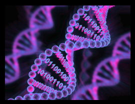

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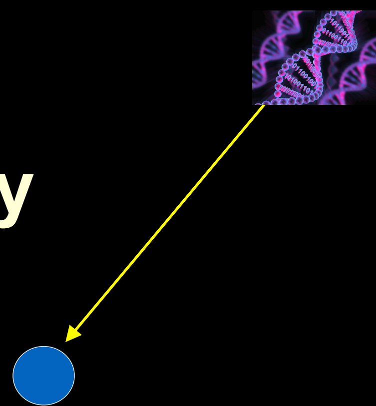

1. Explainability

(Effect identification and decomposition, Bias Analysis and Fairness, Robustness and Generalizability)

Research in CausalAI - Big Picture

Structural Causal Models

1. Explainability

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2. Decision-Making

(Reinforcement Learning,
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Personalized Decision-Making)

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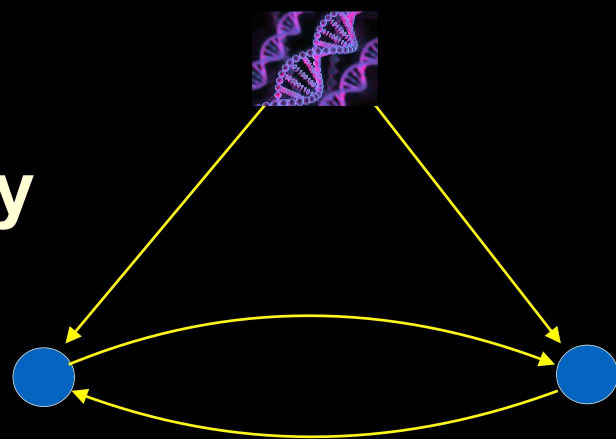

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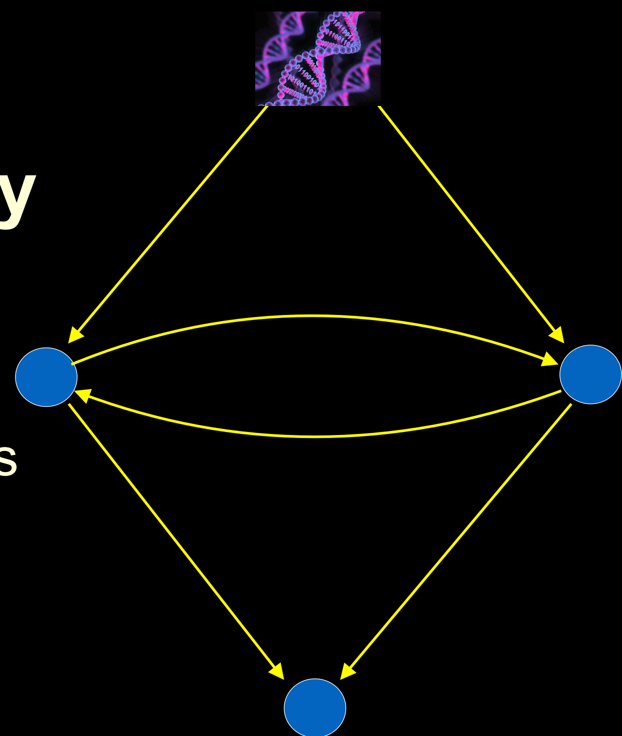

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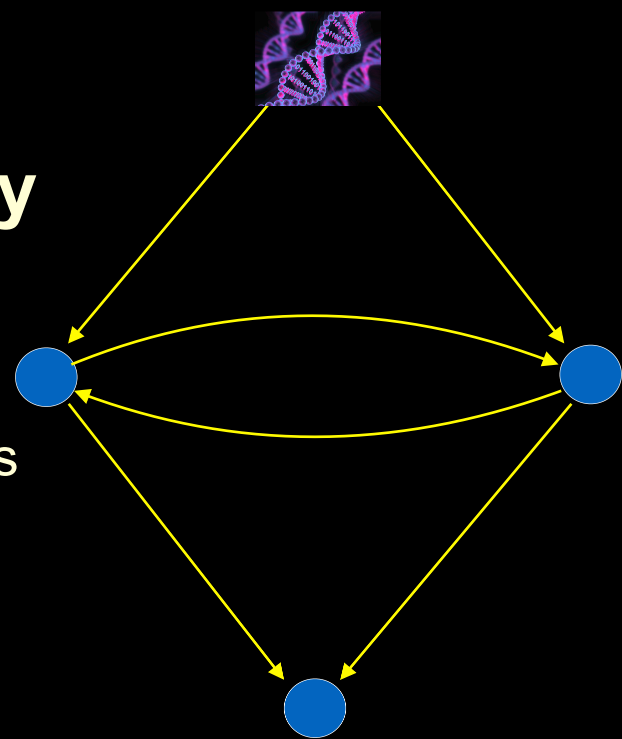

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Principles and tools for designing robust and adaptable learning systems.

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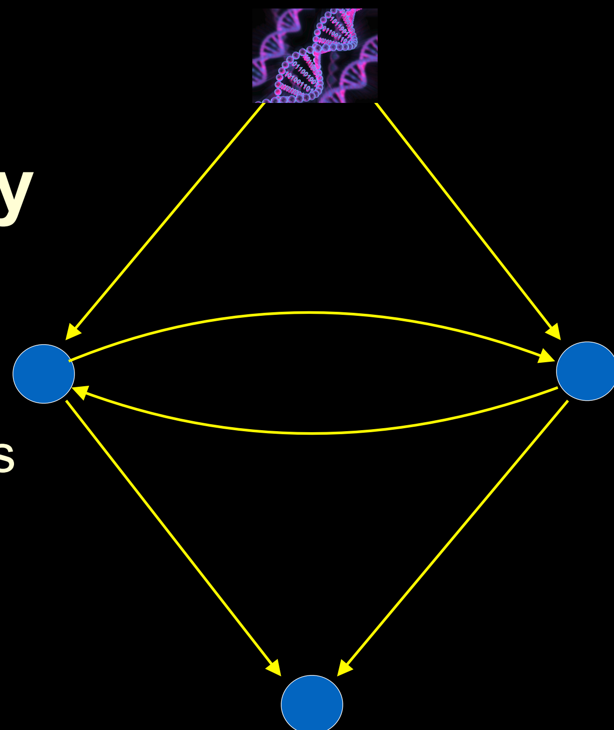

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Principles and tools for designing robust and adaptable learning systems.

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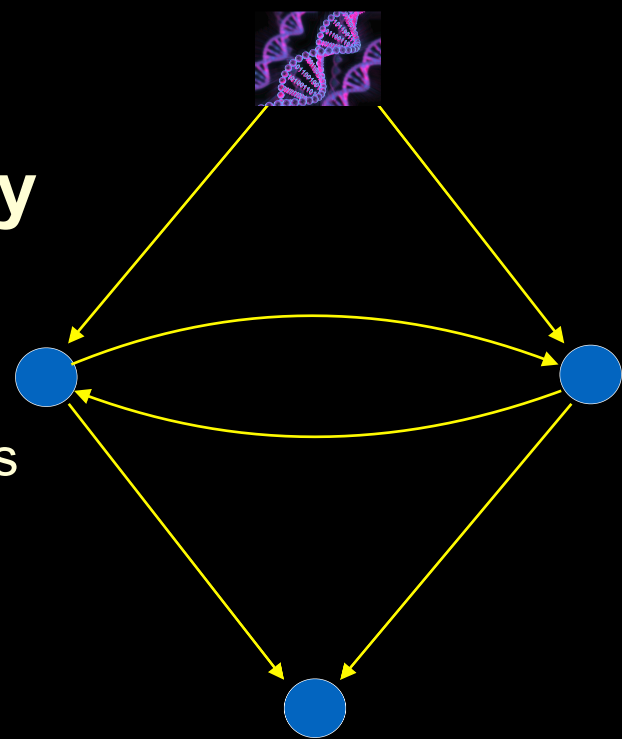

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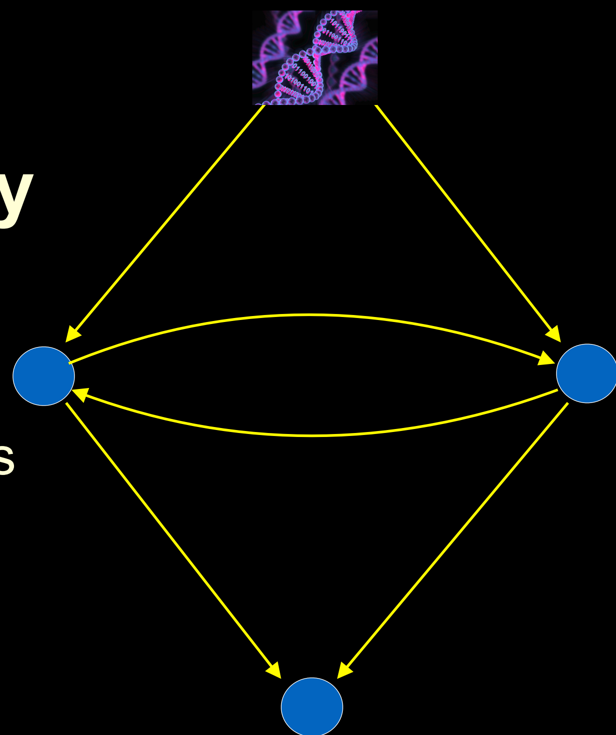

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MOTIVATION

The aspirations & challenges of modern Data Science (circa 2019)

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- *"The ability to take data — to be able to understand it, to extract value from it, to visualize it, to communicate it - that's going to be hugely important in the next decades"*

  Hal Varian, chief economist at Google and UC Berkeley
  Professor of Information Sciences, Business, and Economics.

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- *“Big data is not about the data!”*

  Gary King, Political Scientist, University Professor, Harvard University.

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- *“Big data is not about the data!”*

  Gary King, Political Scientist, University Professor, Harvard University.

- *“Data Science is only as much of a science as it facilitates the interpretation of data - a two body problem, connecting data to reality”.*

  Judea Pearl, Professor of Computer Science & Statistics, UCLA.

CURRENT STATE OF AFFAIRS
(REPORT FROM THE TRENCHES)


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The Relationship of Coffee Consumption with Mortality
-Garcia, PhD; Rob M. van Dam, PhD; Tricia Y. Li, MD; Fernando Rodriguez-Artalejo, MD, PhD; and
B. Hu, MD, PhD
[+] Article and Author Information
Ann Intern Med. 2008;148(12):904-914. doi:10.7326/0003-4819-148-12-200806170-00003
Article Figures Tables References Audio/Video Summary for Patients Comments (2)
Text Size: A A A
Abstract
Abstract | Context | Contribution | Caution | Methods | Results | Discussion | References
Background: Coffee consumption has been linked to various beneficial and detrimental health effects,
but data on its relation with mortality are sparse.


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Video
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RELAX AND
ENJOY THE LUXURY.
A full-size sedan designed with you in mind.
By MICHELLE CASTILLO / CBS NEWS / February 15, 2013, 3: 36 PM
Alcohol causes 20,000 cancer deaths in the U.S. annually
In Texas it is illegal to take more than three sips of beer at a time while standing. AP / FILE
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One Drink Of Red Wine Or Alcohol Is Relaxing To Circulation, But Two Drinks Are Stressful

Feb. 13, 2008 — One drink of either red wine or alcohol slightly benefits the heart and blood vessels, but the positive effects on specific biological markers disappear with two drinks, say researchers at the Peter Munk Cardiac Centre of the Toronto General Hospital.

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Why Do Heavy Drinkers Outlive Nondrinkers?
One of the most contentious issues in the vast literature about alcohol consumption has been the consistent finding that those who don't drink tend to die sooner than those who do
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Correction Appended: Aug. 31, 2010
One of the most contentious issues in the vast literature about alcohol consumption has been the consistent finding that those who don't drink tend to die sooner than those who do. The standard Alcoholics Anonymous explanation for this finding is that many of those who show up as abstainers in such research are actually former hard-core drunks who had already incurred health problems associated with drinking.
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Grapefruit Is a Culprit in More Drug Reactions
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The patient didn't overdose on medication. She overdosed on grapefruit juice.
The 42-year-old was barely responding when her husband brought her to the emergency room. Her heart rate was slowing, and her blood pressure was falling. Doctors had to insert a breathing tube, and then a pacemaker, to revive her.
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ORIGINAL ARTICLE
Association of Nut Consumption with Total and Cause-Specific Mortality
Ying Bao, M.D., Sc.D., Jiali Han, Ph.D., Frank B. Hu, M.D., Ph.D., Edward L. Giovannucci, M.D., Sc.D., Meir J. Stampfer, M.D., Dr.P.H., Walter C. Willett, M.D., Dr.P.H., and Charles S. Fuchs, M.D., M.P.H.
N Engl J Med 2013; 369:2001-2011 | November 21, 2013 | DOI: 10.1056/NEJMoa1307352
Abstract
Article
References
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BACKGROUND
Increased nut consumption has been associated with a reduced risk of major chronic diseases, including cardiovascular disease and type 2 diabetes mellitus. However, the association between nut consumption and mortality remains unclear.
Full Text of Background...
METHODS
We examined the association between nut consumption and
MEDIA IN THIS ARTICLE
Video
Nuts
Nuts and Death.

Other Applications

- Evaluating the impact of a bilingual education program

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- Evaluating the impact of a bilingual education program
- Evaluating the impact of a new taxation program

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- Assessing the strength of the causal relationship between a set of genes and a certain type of cancer

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Tangled web of cause-effect relationships

Why study causality?

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- Cognitive observation

Why study causality?

- Cognitive observation
  - human: sensors + agency + retrospection
  - ability to observe the world and infer the ‘next’ stage
  - ability to intervene in the world and change it
  - ability to reflect about what we did and infer new behavior

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  - to be able to understand nature and the human being
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- Scientific perspective
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  - relies on empirical data, logic/math, and scientific method

- Pragmatic perspective (e.g., AI, Econ, Medicine)
  - to be able to create "truly" intelligent systems
  - to be able to create and implement policies optimizing certain {social, economical, political} goals

OUTLINE (PART I)

- Introduction
  - Data, data, data…
  - Basic definitions
- Technical Results
  - The truncated product formula
  - The back-door adjustment formula
  - The front-door adjustment formula
  - The do-calculus
- Tasks
  - Policy evaluation
  - Generalizability & Robustness
  - Decision-making & RL

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  * The do-calculus
- Tasks
  * Policy evaluation
  * Generalizability & Robustness
  * Decision-making & RL

Data, data, data…

Ex 1. Data analysis in a 2x2 table...

(Simpson, 1951; Ch. 6, Pearl, 2000)

	surv. (Y)	not-surv. (¬Y)		Recovery Rate
drug (X)	20	20	40	50%
no-drug (¬X)	16	24	40	40%
	36	44		

Stratifying by sex...

	surv. (Y)	not-surv. (¬Y)		Recovery Rate
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	36	44		

male (¬F)	surv. (Y)	not-surv.		Recovery Rate
drug (X)	18	12	30	60%
no-drug (¬X)	7	3	10	70%
	25	15	40	

female (F)	surv. (Y)	not-surv. (¬Y)		Recovery Rate
drug (X)	2	8	10	20%
no-drug (¬X)	9	21	30	30%
	11	29	40	

Which table should the physician consult?

	surv. (Y)	not-surv. (¬Y)		Recovery Rate
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	36	44		

$P(Y \mid F, X) < P(Y \mid F, \neg X)$

$P(Y \mid \neg F, X) < P(Y \mid \neg F, \neg X)$

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$P(Y \mid \neg F, X) < P(Y \mid \neg F, \neg X)$

but

$P(Y \mid X) > P(Y \mid \neg X)$!!

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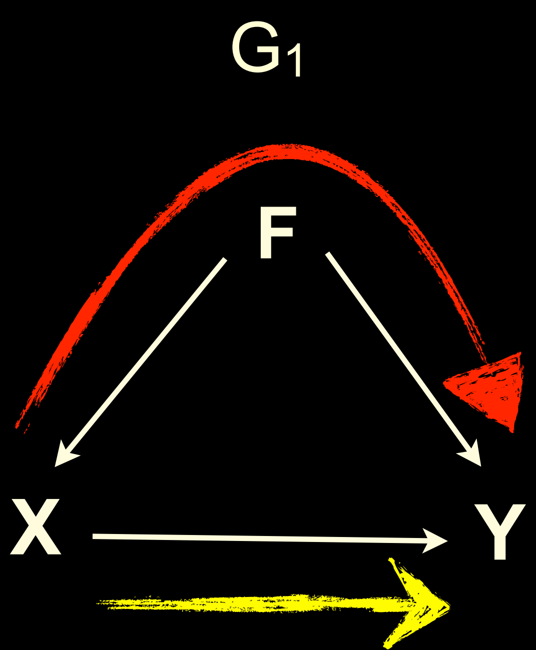
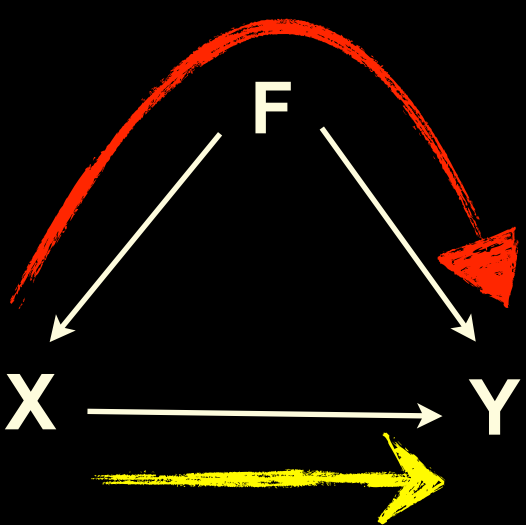


WOULD THE RESULT BE DIFFERENT WITH A DIFFERENT STORY?

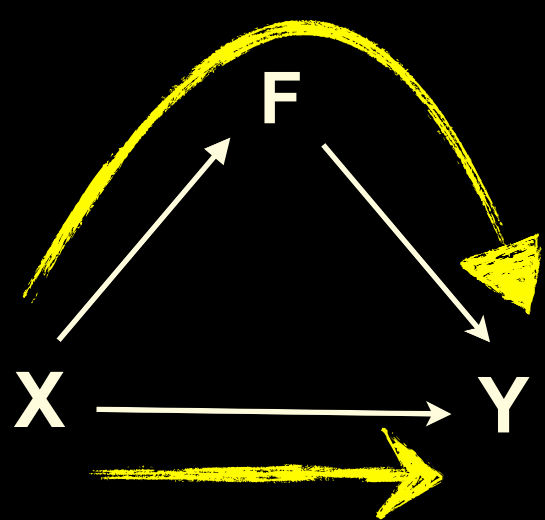
male (¬F)	surv. (Y)	not-surv.		Recovery Rate
drug (X)	18	12	30	60%
no-drug	7	3	10	70%
	25	15	40	

female (F)	surv. (Y)	not-surv.		Recovery Rate
drug (X)	2	8	10	20%
no-drug	9	21	30	30%
	11	29	40	

G_1



G_2



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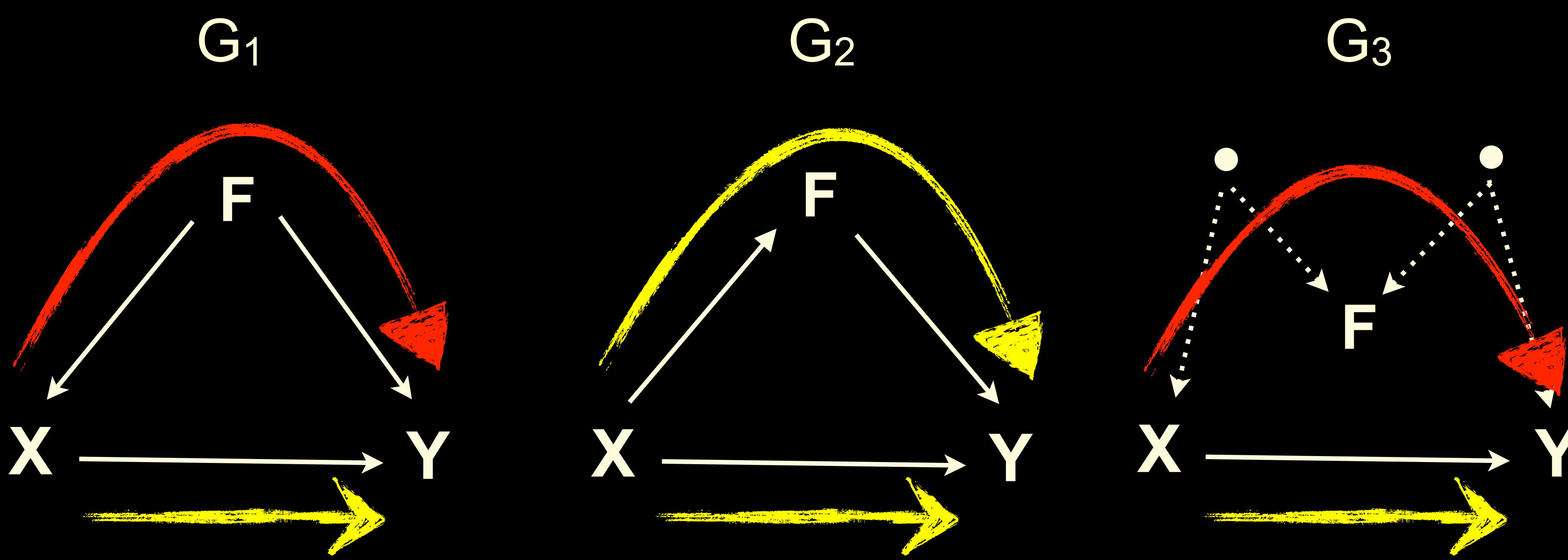


Ex 2. Trivial data analysis (Thrun)?

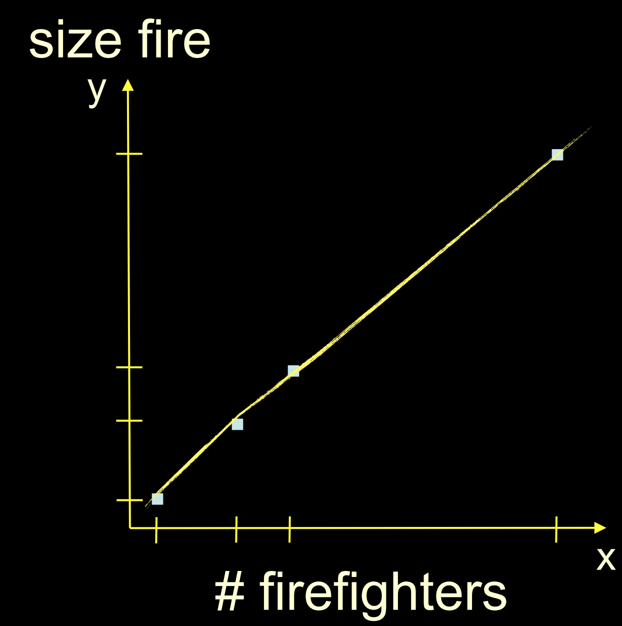
#Firefighters (X)	#Size Fire (Y)
10	100
40	400
60	600
200	2000

Is

$Y = \beta X + \epsilon$ a good fit?

Simple linear regression...

#Firefighters (X)	#Size Fire (Y)
10	100
40	400
60	600
200	2000



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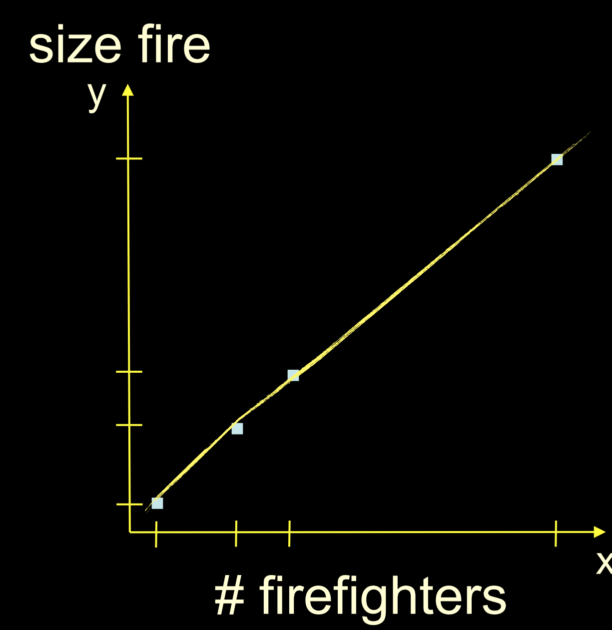

Is

$Y = \beta X + \epsilon$ a good fit?

X and Y are perfectly correlated...

but does X cause Y?

Simple linear regression...

#Firefighters (X)	#Size Fire (Y)
10	100
40	400
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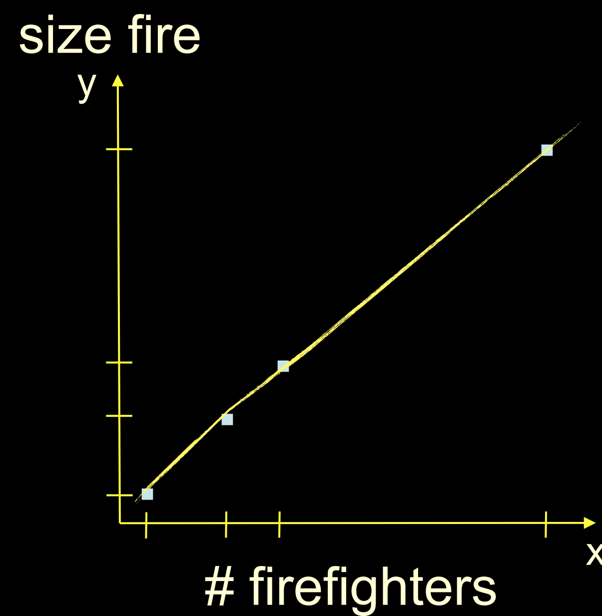

Is

$Y = \beta X + \epsilon$ a good fit?

after all,

what's the meaning of β?

Simple linear regression...

#Firefighters (X)	#Size Fire (Y)
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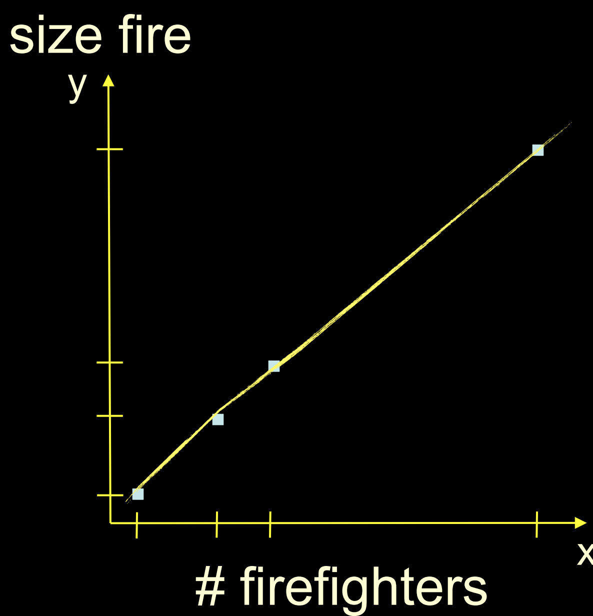

Is

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Lasso?

after all,

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Simple linear regression...

#Firefighters (X)	#Size Fire (Y)
10	100
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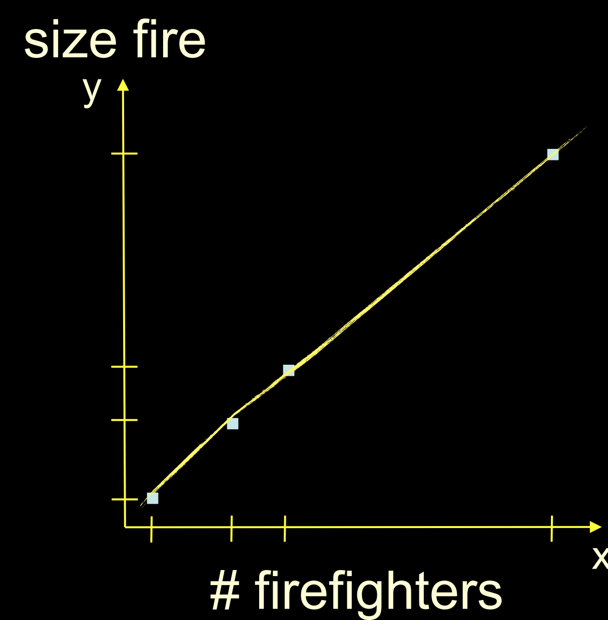

Is

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Lasso?

DNN?

after all,

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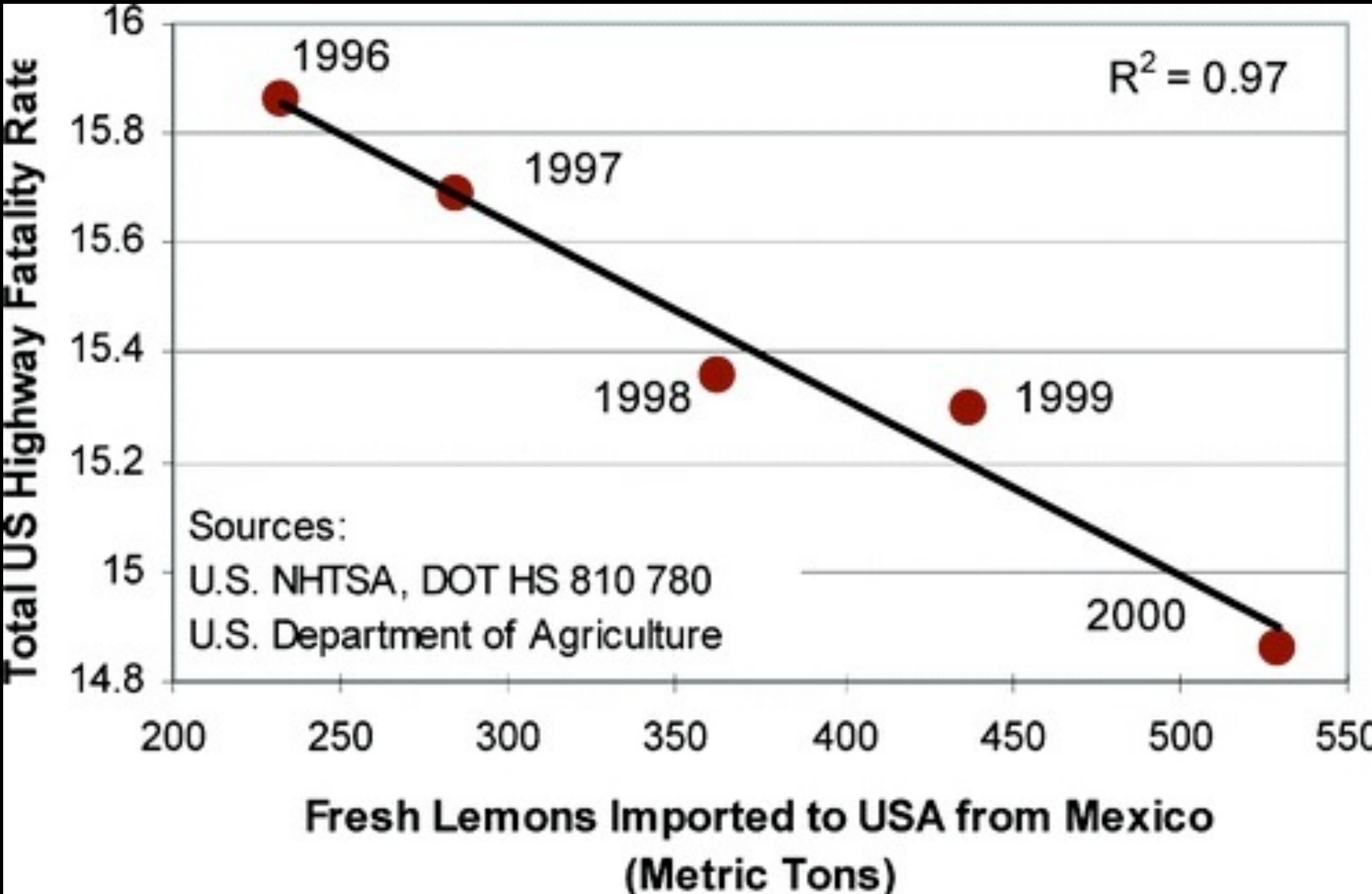
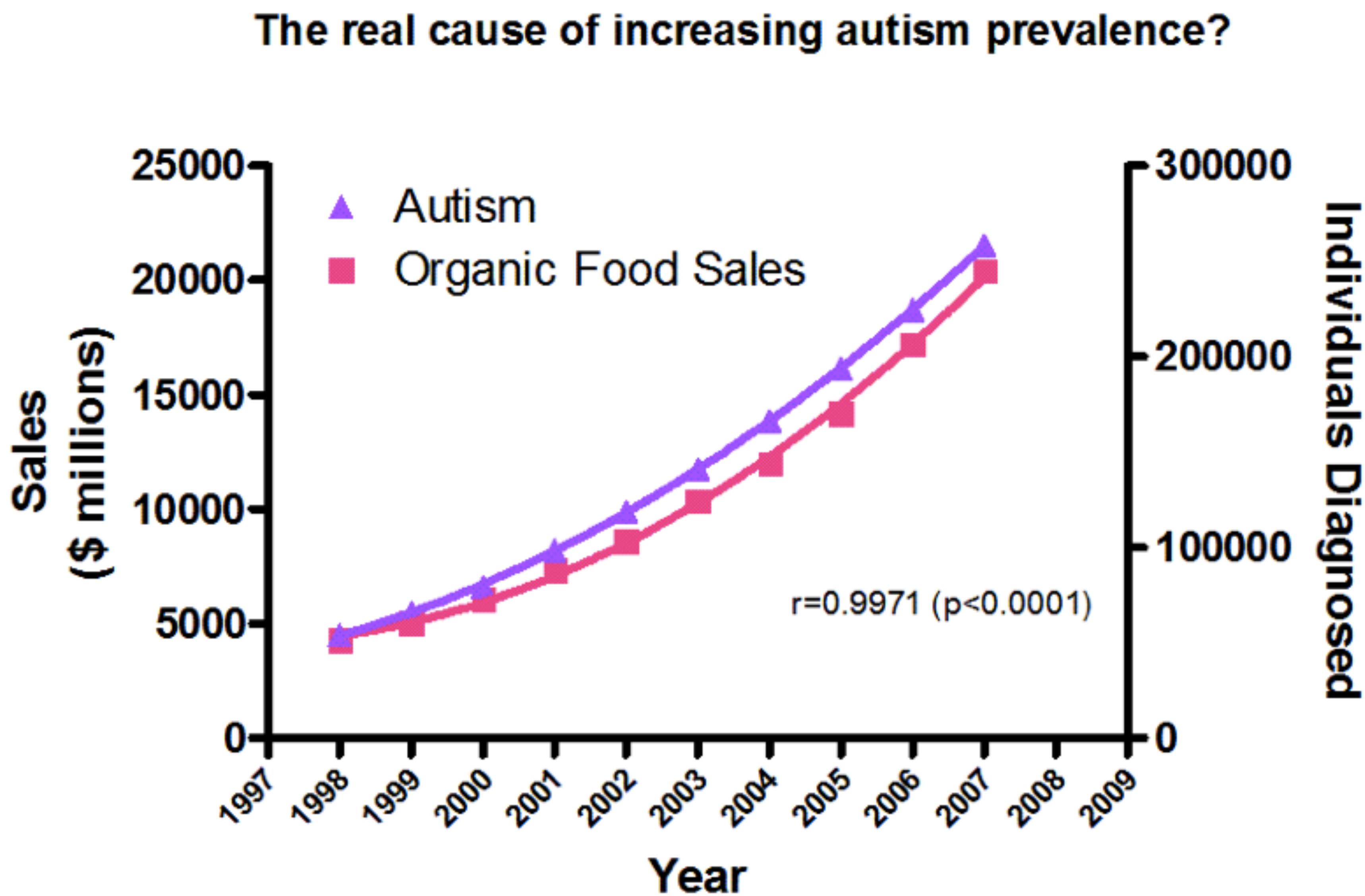
Ex 2b. Trivial data analysis (?)



Ex 2b. Trivial data analysis (?)



Sources: Organic Trade Association, 2011 Organic Industry Survey; U.S. Department of Education, Office of Special Education Programs, Data Analysis System (DANS), OMB# 1820-0043: "Children with Disabilities Receiving Special Education Under Part B of the Individuals with Disabilities Education Act

What’s going on here...?

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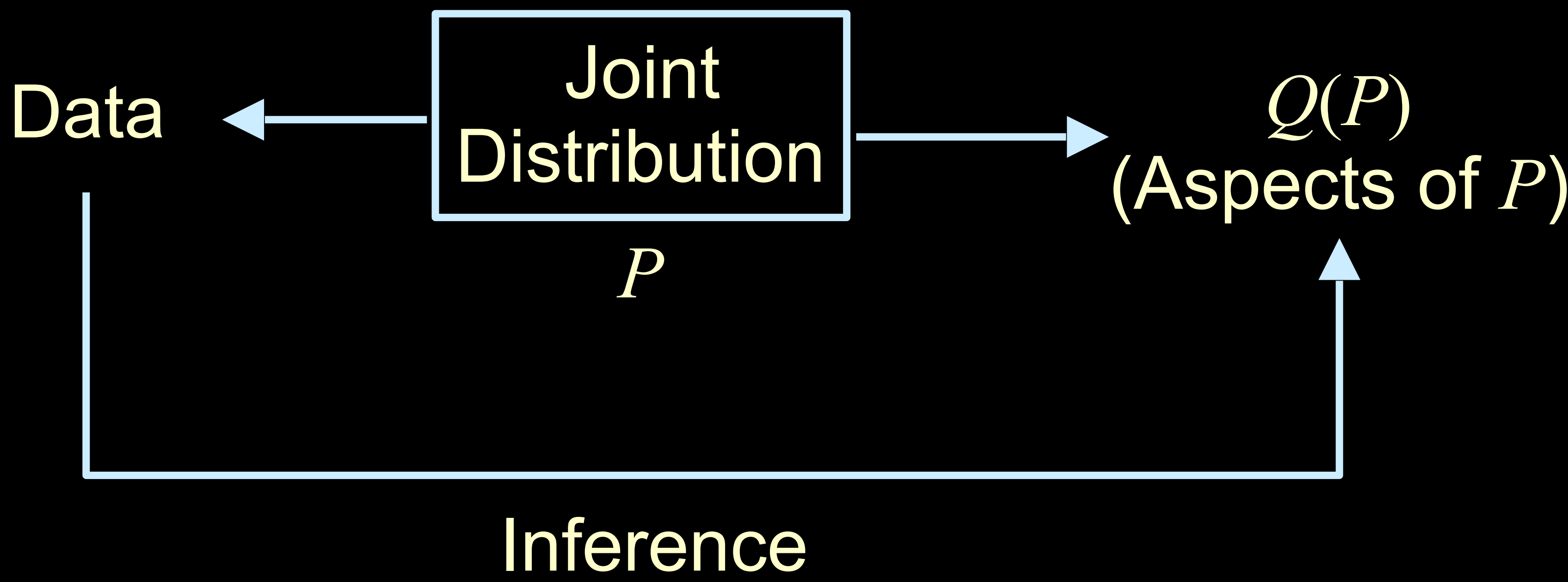
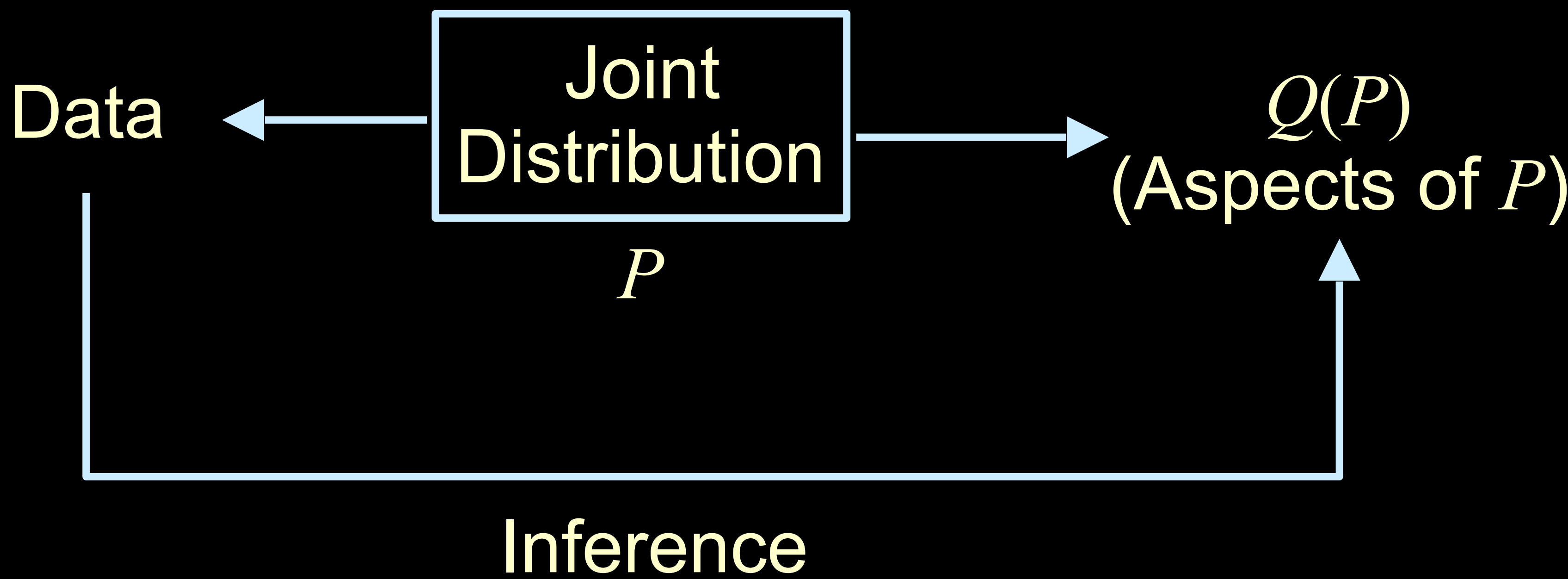
🤷🏻‍♂️

TRADITIONAL STATISTICAL/ MACHINE LEARNING PARADIGM

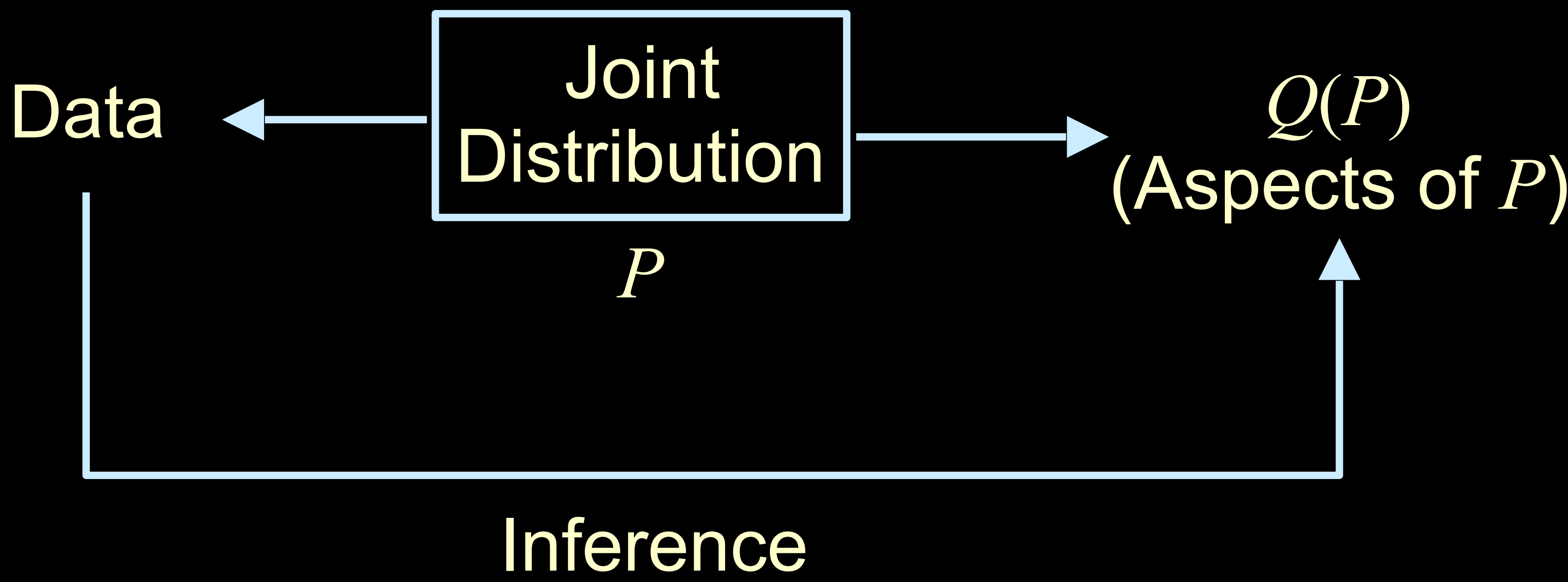


TRADITIONAL STATISTICAL/ MACHINE LEARNING PARADIGM



e.g., Infer whether customers who bought product A would also buy product B — or, compute $Q = P(B \mid A)$.

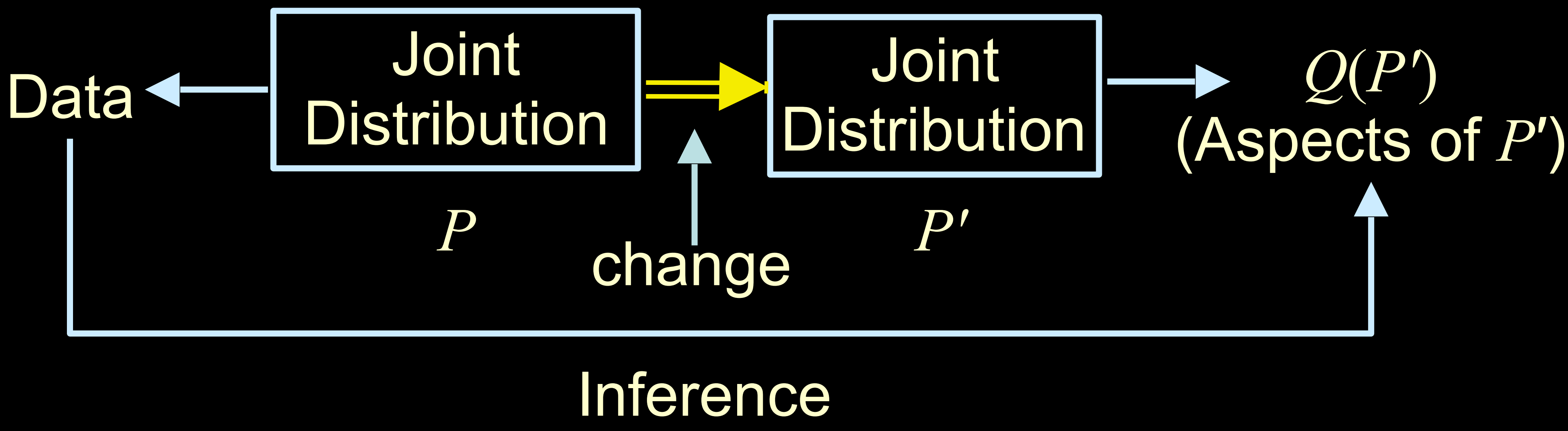
TRADITIONAL STATISTICAL/ MACHINE LEARNING PARADIGM



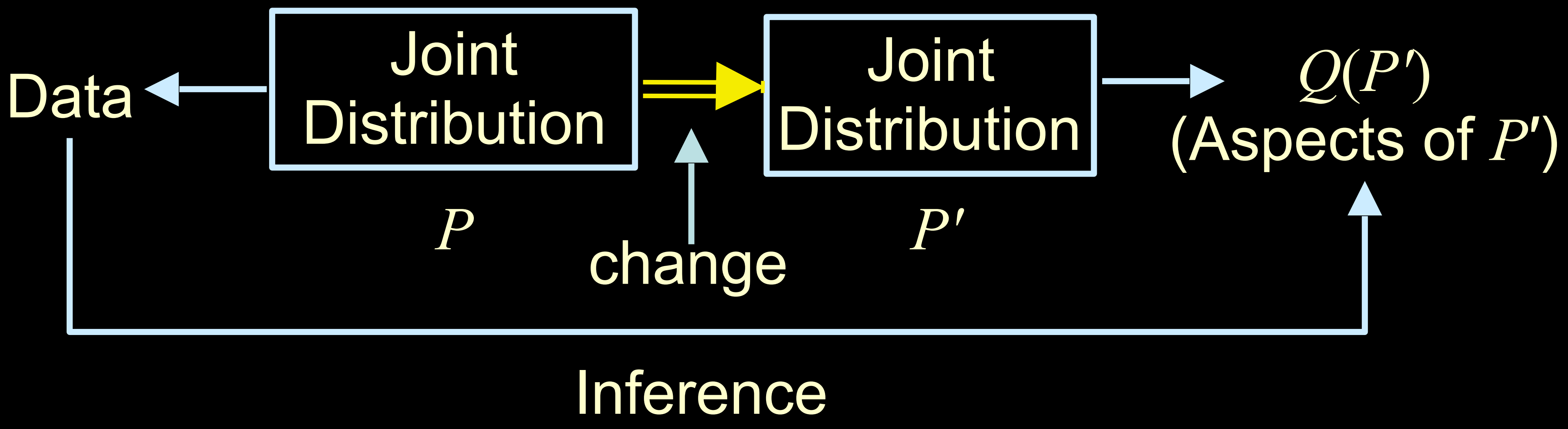
e.g., Infer whether customers who bought product A would also buy product B — or, compute $Q = P(B \mid A)$.

- capturing regularities of the original P (LLN, CLT)

FROM STATISTICAL TO CAUSAL ANALYSIS



FROM STATISTICAL TO CAUSAL ANALYSIS

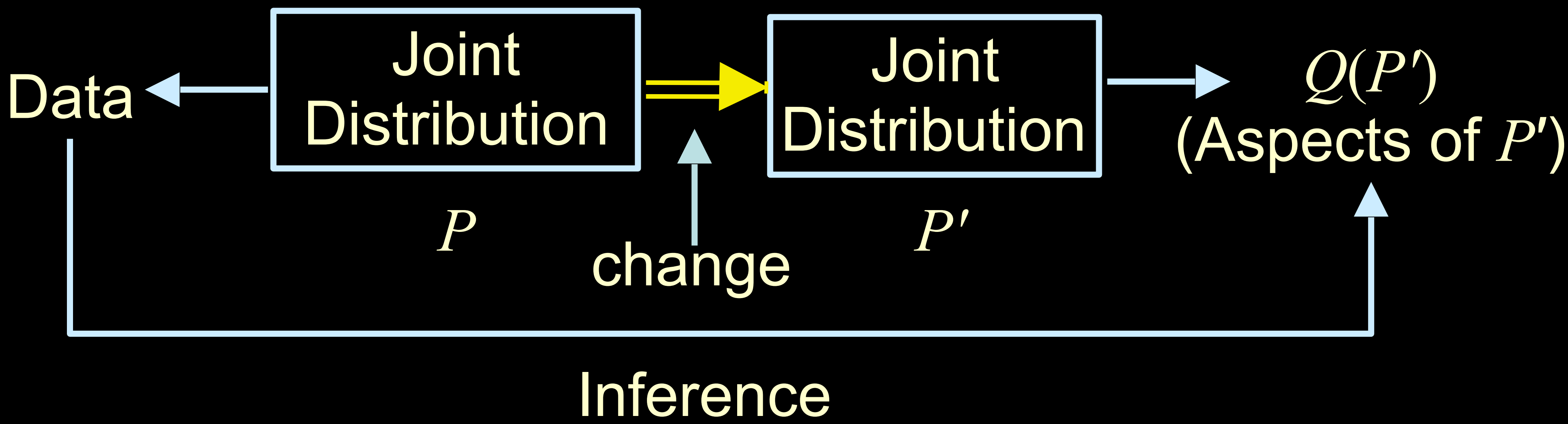


e.g., Estimate $P'(sales)$ if we double the price.

Estimate $P'(cancer)$ if we ban smoking.

How does P change to P'? New oracle

FROM STATISTICAL TO CAUSAL ANALYSIS



e.g., Estimate $P'(sales)$ if we double the price.

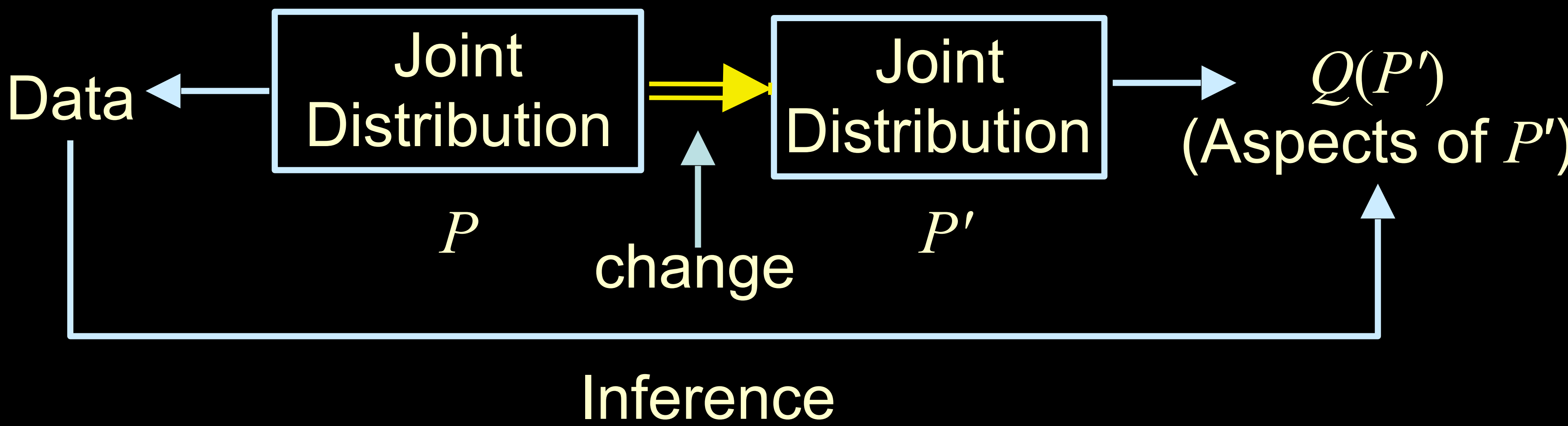
Estimate $P'(cancer)$ if we ban smoking.

What about the CLT?

How does P change to P'? New oracle

FROM STATISTICS TO COUNTERFACTUALS

Probability and statistics deal with static relations



FROM STATISTICS TO COUNTERFACTUALS

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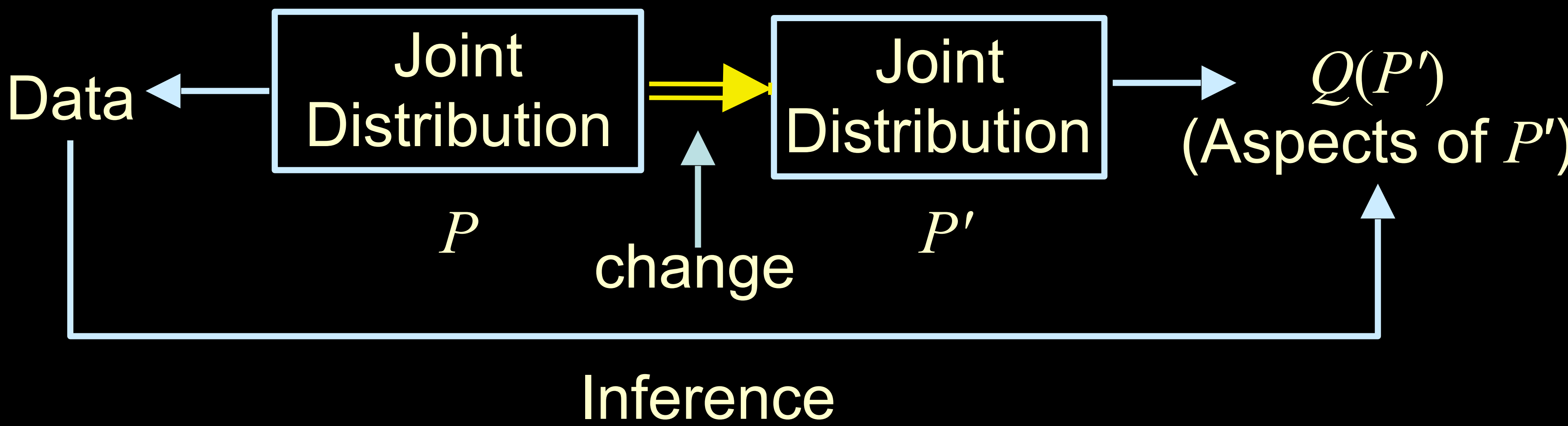
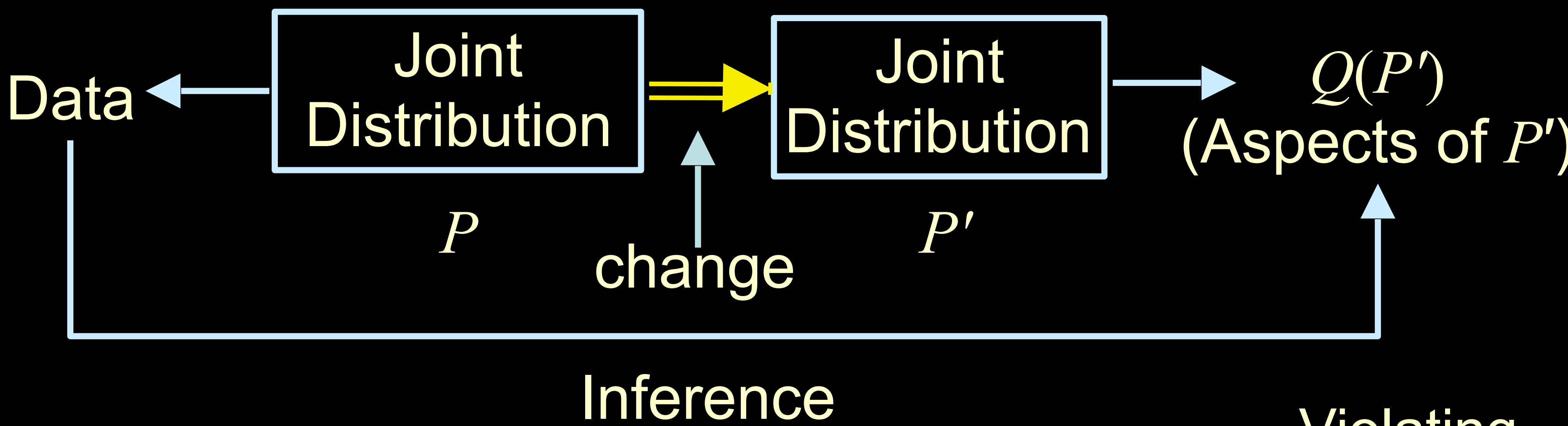


What happens when P changes?

e.g., Estimate the probability that a patient who took a drug and died would be alive had he not taken the drug.

FROM STATISTICS TO COUNTERFACTUALS

Probability and statistics deal with static relations



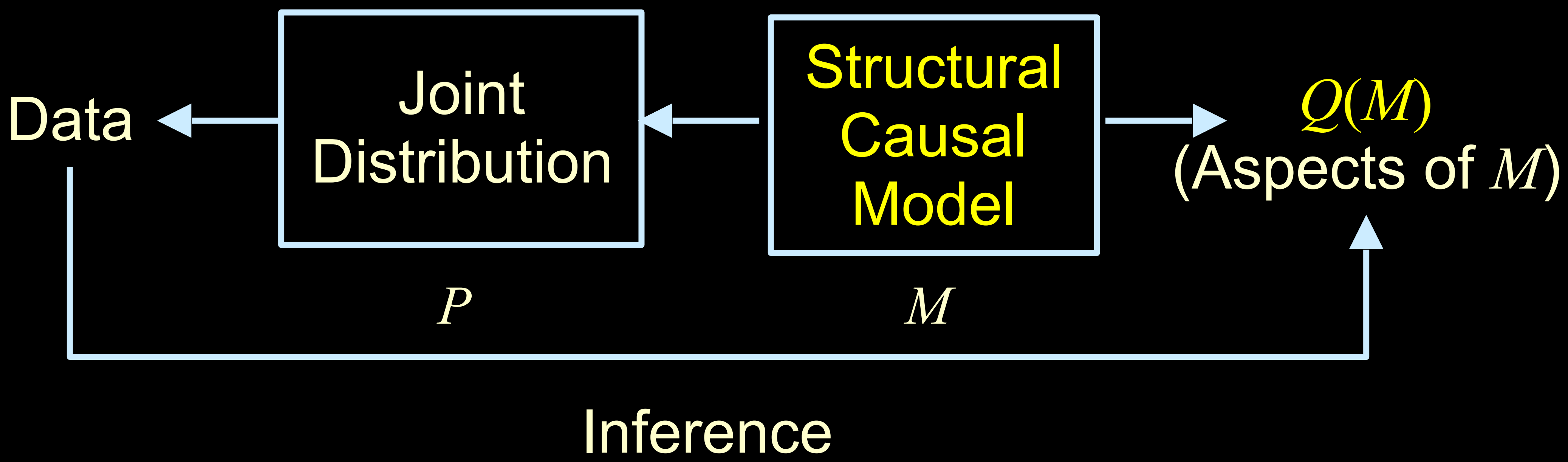
Violating
the laws
of physics?

What happens when P changes?

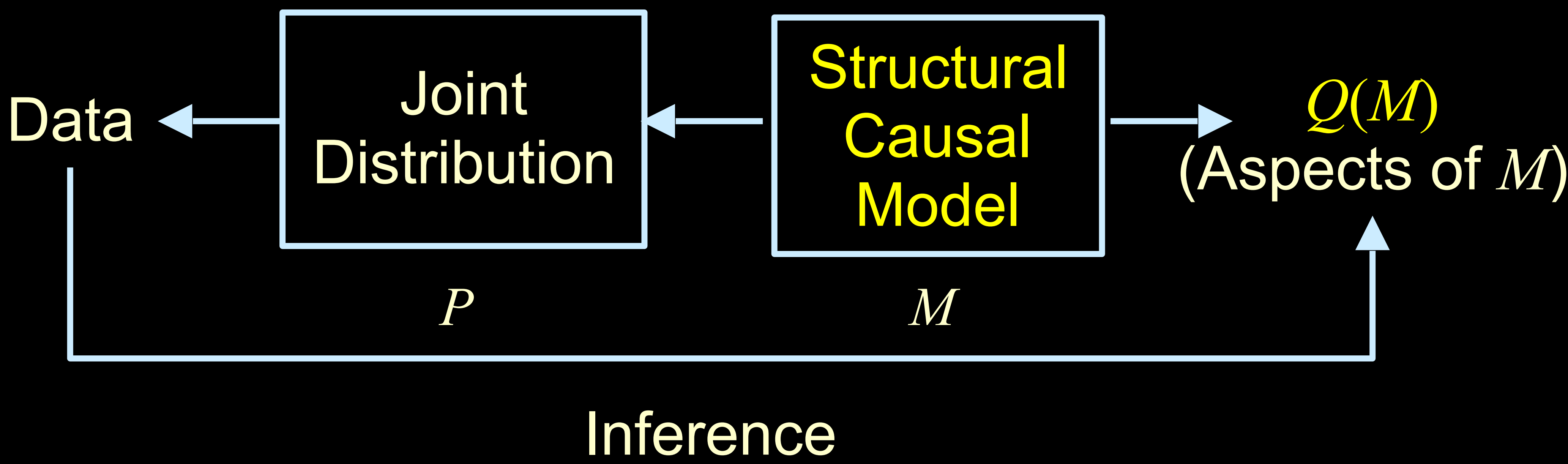
e.g., Estimate the probability that a patient who took a drug and died would be alive had he not taken the drug.

Instead of thinking about multiple individual distributions, let's dig deeper…

THE STRUCTURAL MODEL PARADIGM

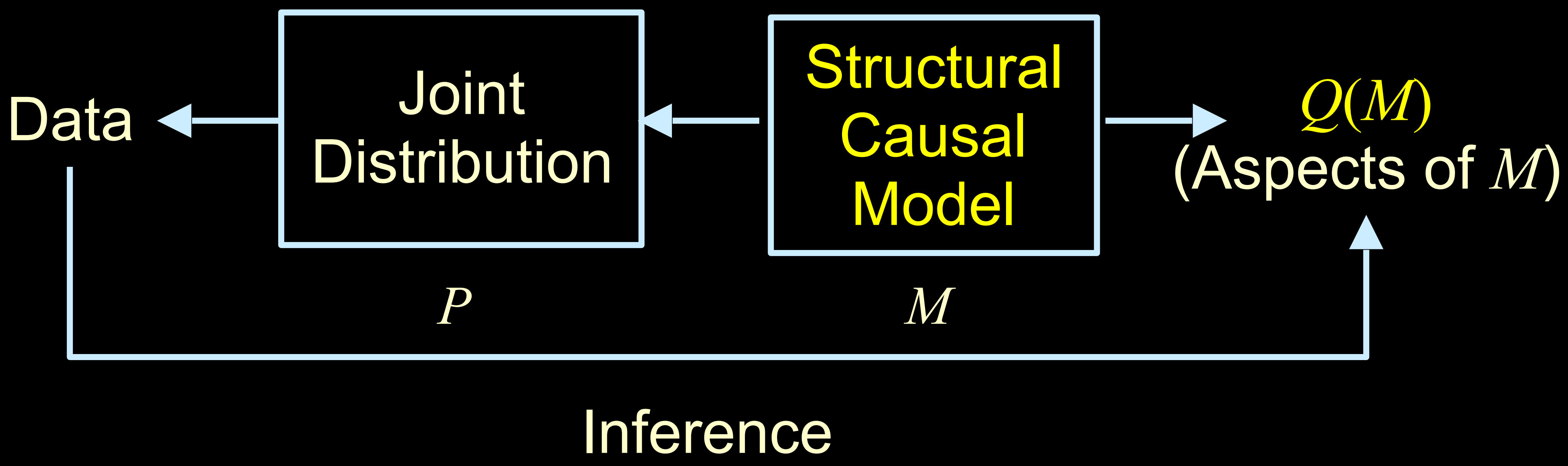


THE STRUCTURAL MODEL PARADIGM



SCM M – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

THE STRUCTURAL MODEL PARADIGM



SCM M – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

P — model of data
M — model of reality

REPRESENTING THE DATA GENERATING MODEL

- Processes

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- Processes

  Edu $\leftarrow f_E\,(SES,\, U_e)$

REPRESENTING THE DATA GENERATING MODEL

- Processes

$$\text{Edu} \leftarrow f_E\,(SES, U_e)$$
$$\text{Income} \leftarrow f_I\,(Edu, SES, U_i)$$

REPRESENTING THE DATA GENERATING MODEL

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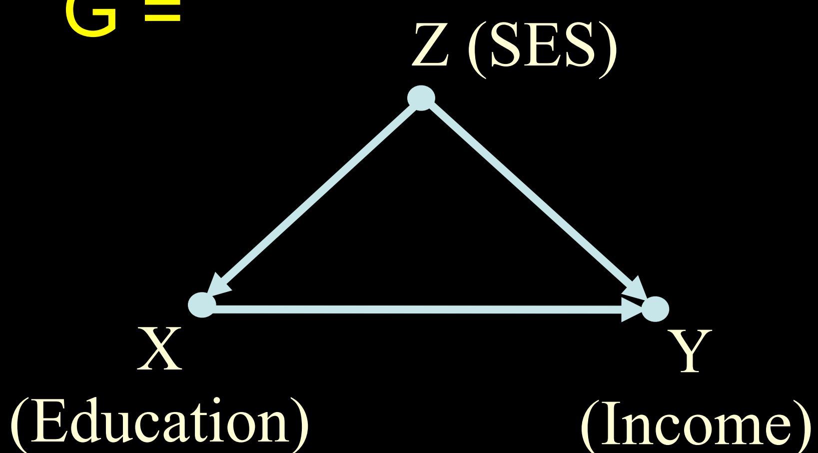
G =



P(Z, X, Y)
(observational)

REPRESENTING THE DATA GENERATING MODEL

- Processes

$$\text{Edu} \leftarrow f_E(SES, U_e)$$
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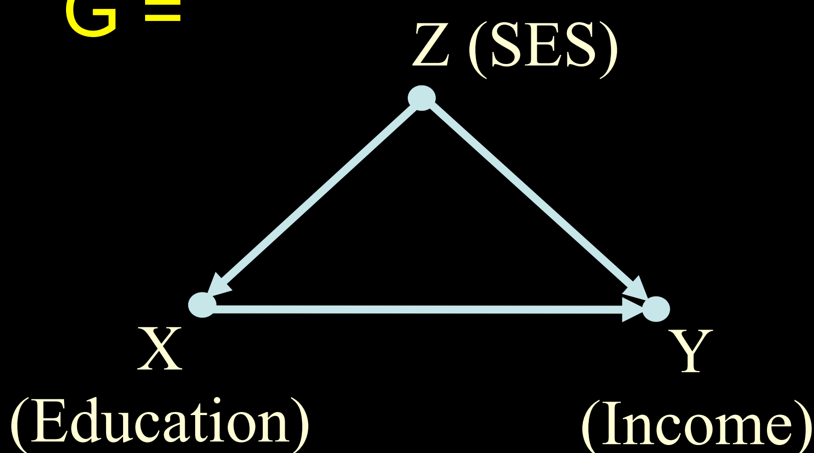


P(Z, X, Y)
(observational)

- Intervention

REPRESENTING THE DATA GENERATING MODEL

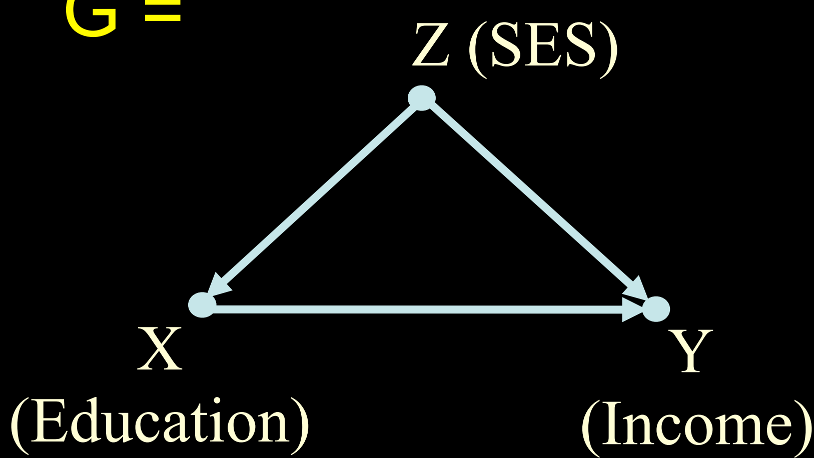
- Processes

$$\text{Edu} \leftarrow f_E\,(\text{SES}, U_e)$$
$$\text{Income} \leftarrow f_I\,(\text{Edu}, \text{SES}, U_i)$$

- Intervention

$$\text{Edu} \leftarrow \text{Yes}$$

G =



P(Z, X, Y)
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REPRESENTING THE DATA GENERATING MODEL

- Processes

  Edu $\leftarrow f_E$ (SES, U_e)

  Income $\leftarrow f_I$ (Edu, SES, U_i)

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  Edu $\leftarrow$ Yes

  Income $\leftarrow f_I$ (Edu, SES, U_i)

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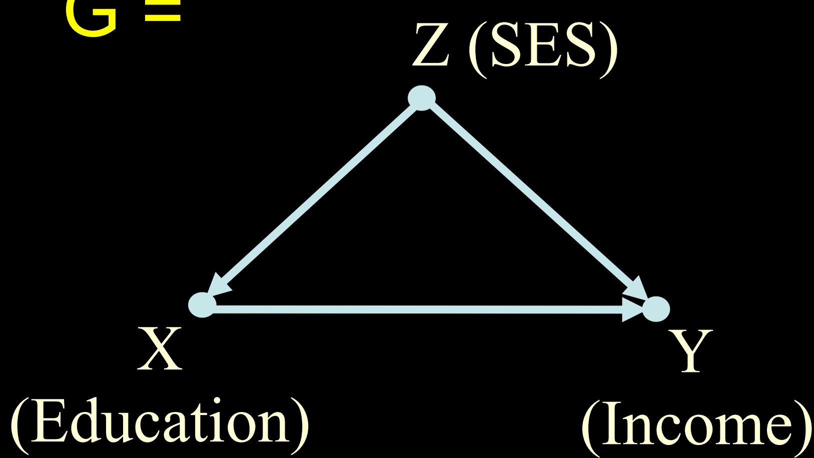

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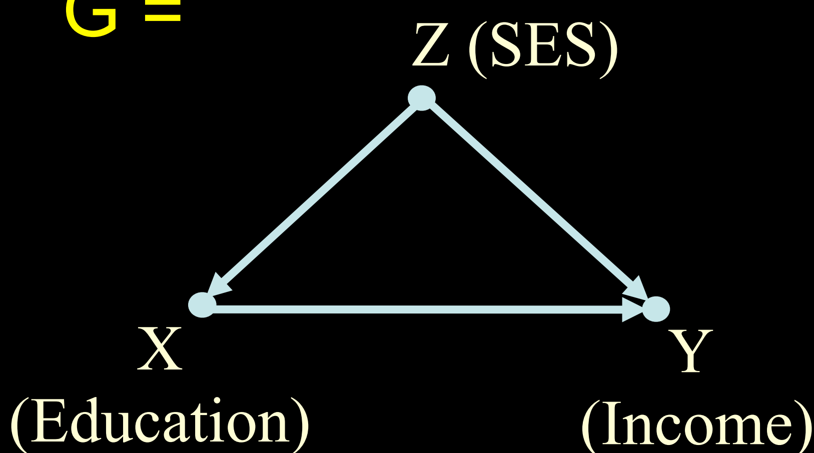
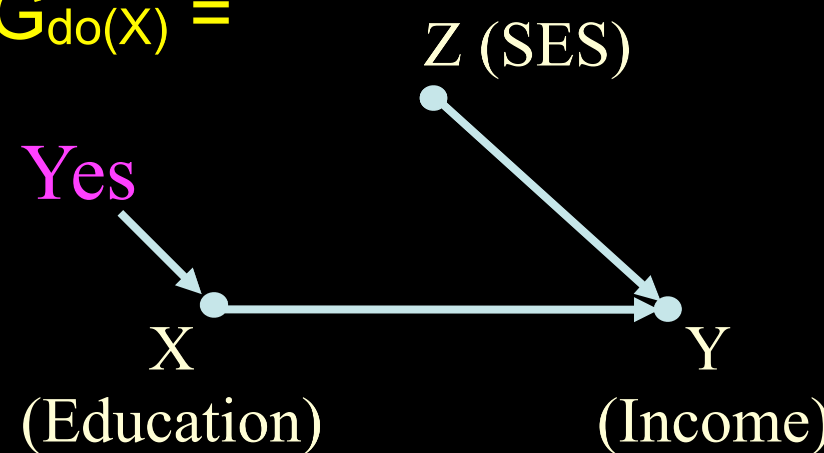


$P(Z, X, Y)$
(observational)

- Intervention

$$\text{Edu} \leftarrow \text{Yes}$$
$$\text{Income} \leftarrow f_I(\text{Edu}, \text{SES}, U_i)$$

$G_{do(X)} =$



$P(Z, Y \mid do(X = \text{Yes}))$
(interventional)

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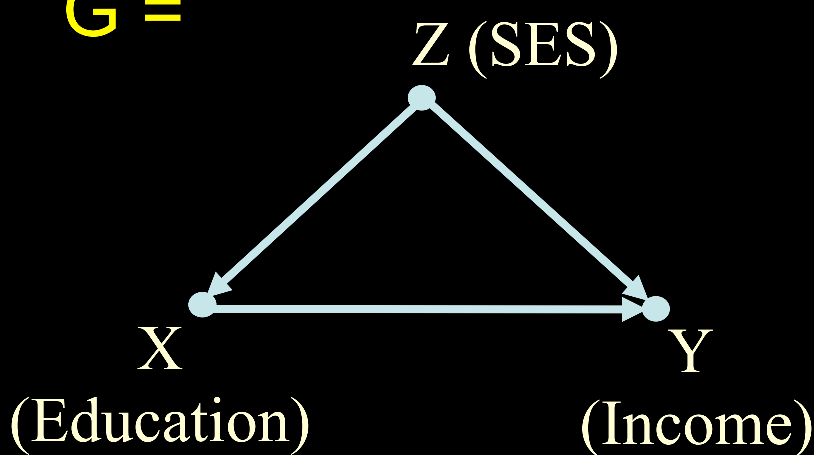
$$\text{Income} \leftarrow f_I(Edu, SES, U_i)$$

$G =$

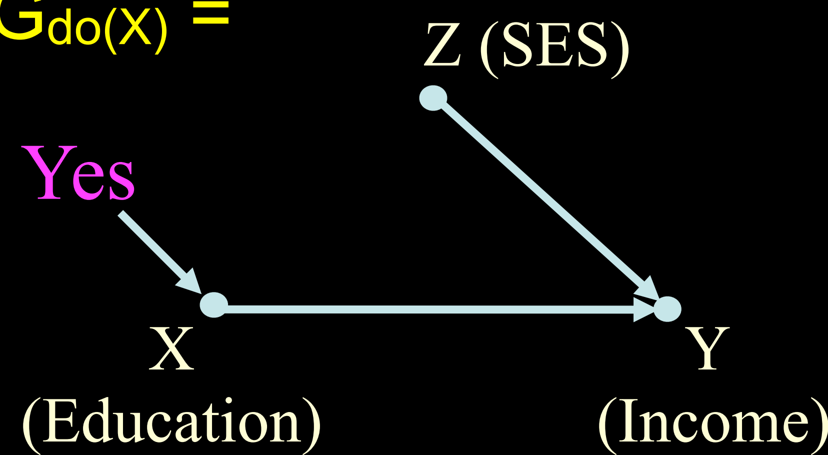


$P(Z, X, Y)$
(observational)

- Intervention

$$\text{Edu} \leftarrow \text{Yes}$$
$$\text{Income} \leftarrow f_I(Edu, SES, U_i)$$

$G_{do(X)} =$



$P(Z_{X=Yes}, Y_{X=Yes}) = P(Z, Y \mid do(X = Yes))$
(interventional)

REPRESENTING
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- $F = \{f_1,\ldots, f_n\}$ are functions determining V,

  for each V_i, $V_i \leftarrow f_i(Pa_i, U_i)$, where $Pa_i \subset V$, $U_i \subset U$.

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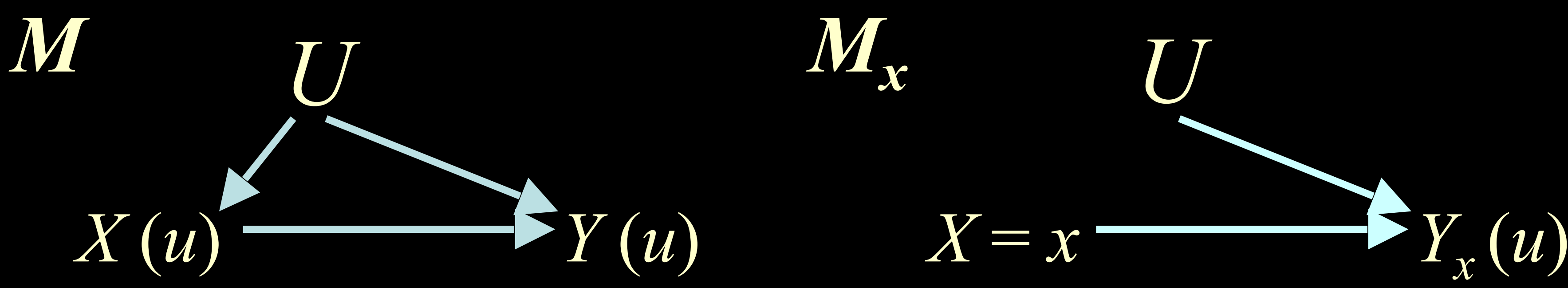
(Axiomatic characterization [Halpern, Galles, Pearl, 1998].)

COUNTERFACTUALS IN A NUTSHELL

Definition:
Given a SCM model M, the potential outcome $\boldsymbol{Y_x(u)}$ for unit u is equal to the solution for Y in a mutilated model M_x, in which the equation for X is replaced by $X = x$.



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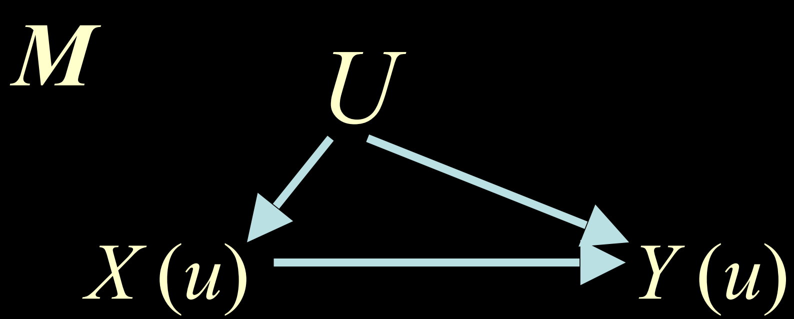

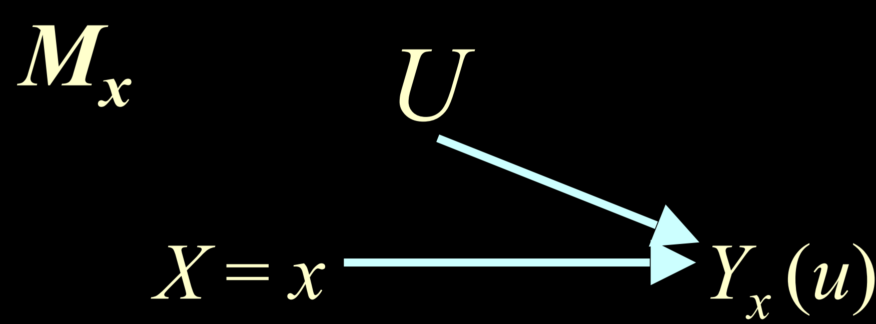

The Law of Structural Counterfactuals:

$$\boxed{Y_x(u) \triangleq Y_{M_x}(u)}$$

COUNTERFACTUALS IN A NU

If $Y = V_i$,
and $X = V \setminus Y$,
$Y_x(u) = f_i(pa_i, u)$

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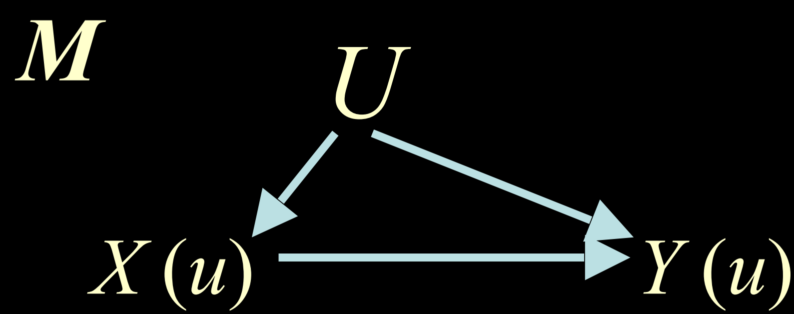

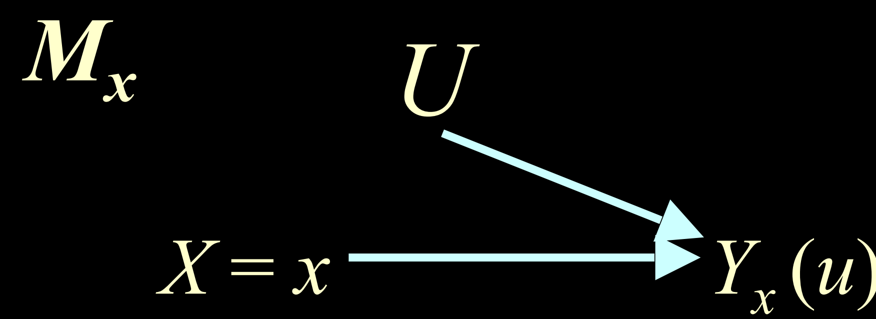

The Law of Structural Counterfactuals:

$$\boxed{Y_x(u) \triangleq Y_{M_x}(u)}$$

SCM → PEARL (CAUSAL) HIERARCHY

SCM → CAUSAL HIERARCHY

	Level (Symbol)	Typical Activity	Typical Question	Examples
1	Associational $P(y \mid x)$	Seeing	What is? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?

SCM → CAUSAL HIERARCHY

	Level (Symbol)	Typical Activity	Typical Question	Examples
1 👀	Associational $P(y \mid x)$	Seeing ML - (Un)Supervised DNN, Bayes Net Regression	What is? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?

SCM → CAUSAL HIERARCHY

	Level (Symbol)	Typical Activity	Typical Question	Examples
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2 💪	Interventional $P(y \mid do(x), c)$	Doing	What if? What if I do X?	What if I take aspirin, will my headache be cured?

SCM → CAUSAL HIERARCHY

	Level (Symbol)	Typical Activity	Typical Question	Examples
1 👀	Associational $P(y \mid x)$	Seeing ML - (Un)Supervised DNN, Bayes Net Regression	What is? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?
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SCM → CAUSAL HIERARCHY

Level (Symbol)	Typical Activity	Typical Question	Examples
1 👀 Associational $P(y \mid x)$	Seeing ML - (Un)Supervised DNN, Bayes Net Regression	What is? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?
2 💪 Interventional $P(y \mid do(x), c)$	Doing ML - Reinforcement Causal Bayes Net	What if? What if I do X?	What if I take aspirin, will my headache be cured?
3 🧠 Counterfactual $P(y_x \mid x', y')$	Imagining, Retrospection	Why? What if I had acted differently?	Was it the aspirin that stopped my headache?

SCM → CAUSAL HIERARCHY

Level (Symbol)	Typical Activity	Typical Question	Examples
1 Associational 👀 $P(y \mid x)$	Seeing ML - (Un)Supervised DNN, Bayes Net Regression	What is? How would seeing X change my belief in Y?	What does a symptom tell us about the disease?
2 Interventional 💪 $P(y \mid do(x), c)$	Doing ML - Reinforcement Causal Bayes Net	What if? What if I do X?	What if I take aspirin, will my headache be cured?
3 Counterfactual 🧠 $P(y_x \mid x', y')$	Imagining, Retrospection Structural Causal Model	Why? What if I had acted differently?	Was it the aspirin that stopped my headache?

SCM → CAUSAL HIERARCHY

	Level (Symbol)	Typical Activity	Typical Question	Examples
1 👀	Associational $P(y \mid x)$	Seeing ML - (Un)Supervised DNN, Bayes Net Regression	part 1	
2 💪	Interventional $P(y \mid do(x), c)$	Doing ML - Reinforcement Causal Bayes Net	What if? What if I do X?	What if I take aspirin, will my headache be cured?
3 🧠	Counterfactual $P(y_x \mid x', y')$	Imagining, Retrospection Structural Causal Model	Why? What if I had acted differently?	Was it the aspirin that stopped my headache?

SCM → CAUSAL HIERARCHY

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SCM → CAUSAL HIERARCHY

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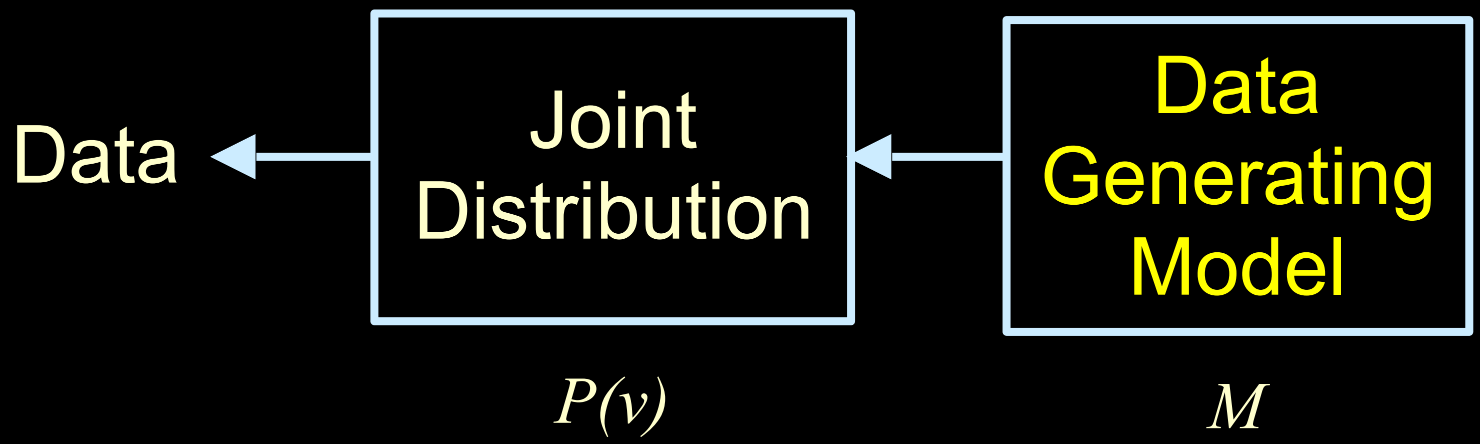
FIRST LAYER OF THE CAUSAL HIERARCHY

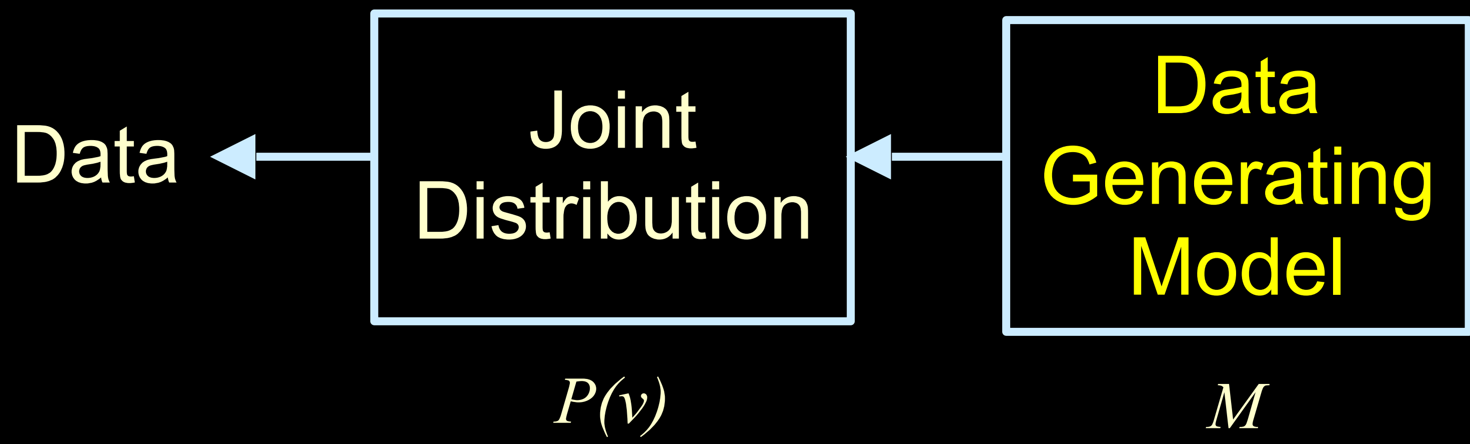
ASSOCIATIONAL

(What if I see $X=x$?)

THE EMERGENCE OF THE FIRST LAYER



THE EMERGENCE OF THE FIRST LAYER



Theorem (PV, 1991). Every Markovian structural causal model *M* (recursive, with independent disturbances) induces an observational distribution $P(v_1,\ldots, v_n)$ that can be factorized as

$$P(v_1, v_2, \ldots, v_n) = \prod_i P(v_i \mid pa_i)$$

where pa_i are the (values of) the parents of V_i in the causal diagram associated with *M*.

ENCODING PROBABILISTIC INVARIANCE (GRAPHOIDS)

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Statements of the form:

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"X is irrelevant to Y given that we know Z",

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The notion of irrelevance can be considered under different interpretations, including probabilistic, relational, and correlational.

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(See also J. Pearl (1988) and P. David (1979).)

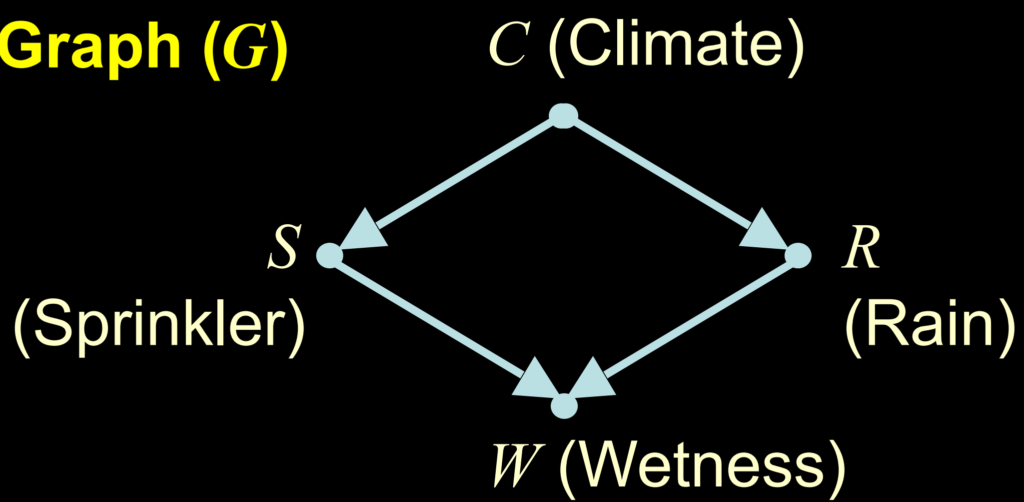
THE LAW OF CONDITIONAL INDEPENDENCE

SCM (M)

$C \leftarrow f_c(U_c)$
$S \leftarrow f_S(C, U_S)$
$R \leftarrow f_R(C, U_R)$
$W \leftarrow f_W(S, R, U_W)$

THE LAW OF CONDITIONAL INDEPENDENCE

Graph (G)



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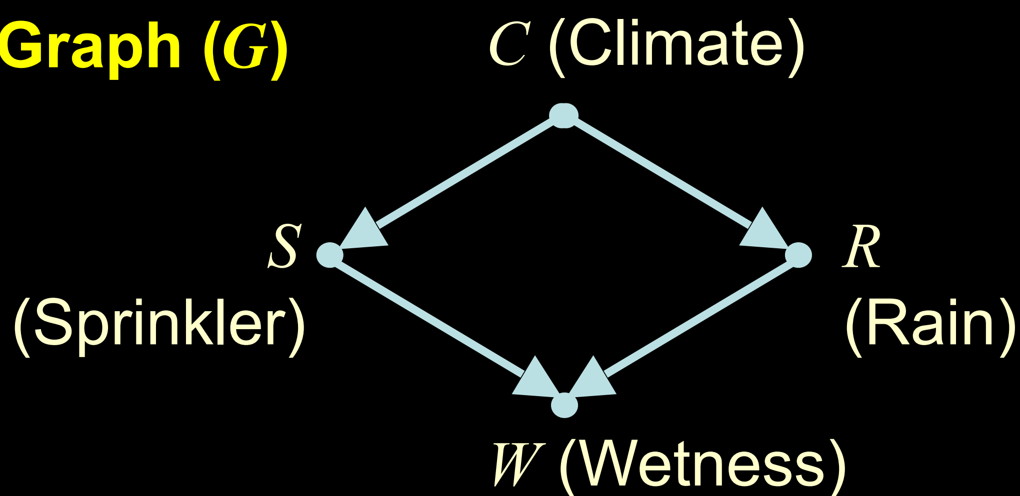

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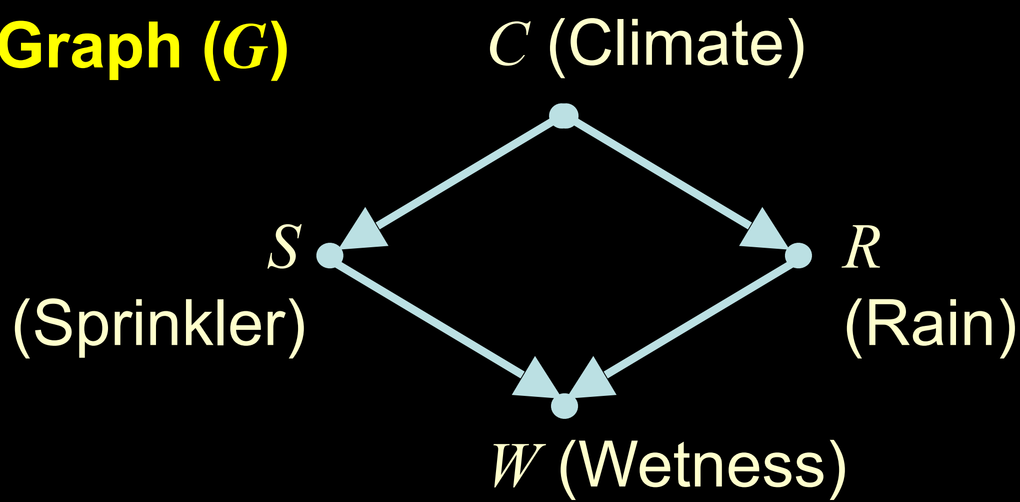
*** Surprising result:**

If the U's are independent, the observed distribution $P(c, r, s, w)$ satisfies constraints that are:

(1) independent of the f's and of $P(U)$,
(2) readable from the graph.

D-SEPARATION: READING CONSTRAINTS IMPLIED IN THE DATA

Graph (*G*)



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Every missing arrow advertises an independency, conditional on a separating set.

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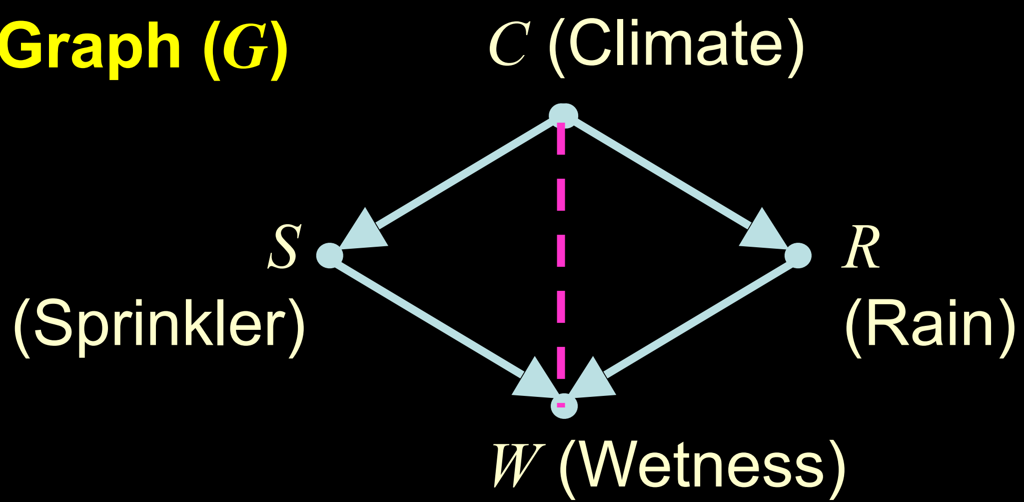

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$C \perp\!\!\!\perp W \mid (S, R)$

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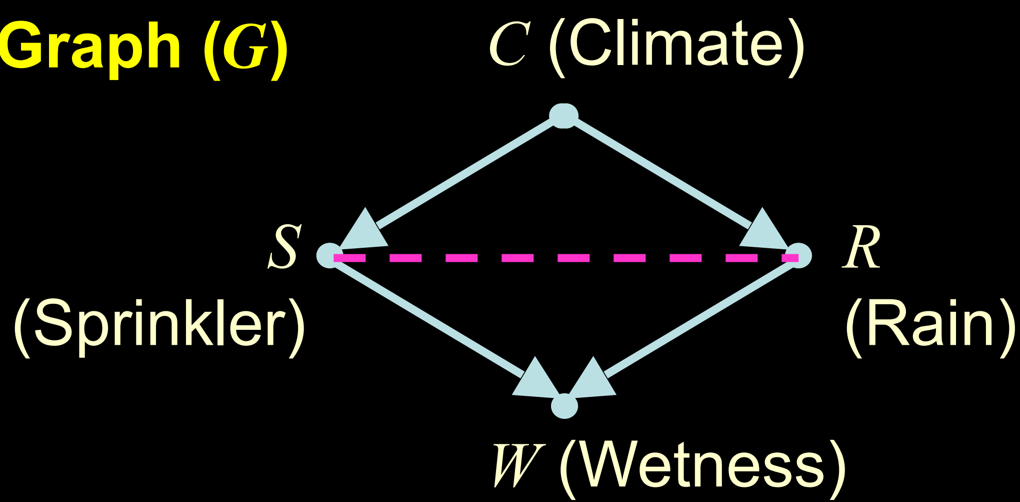

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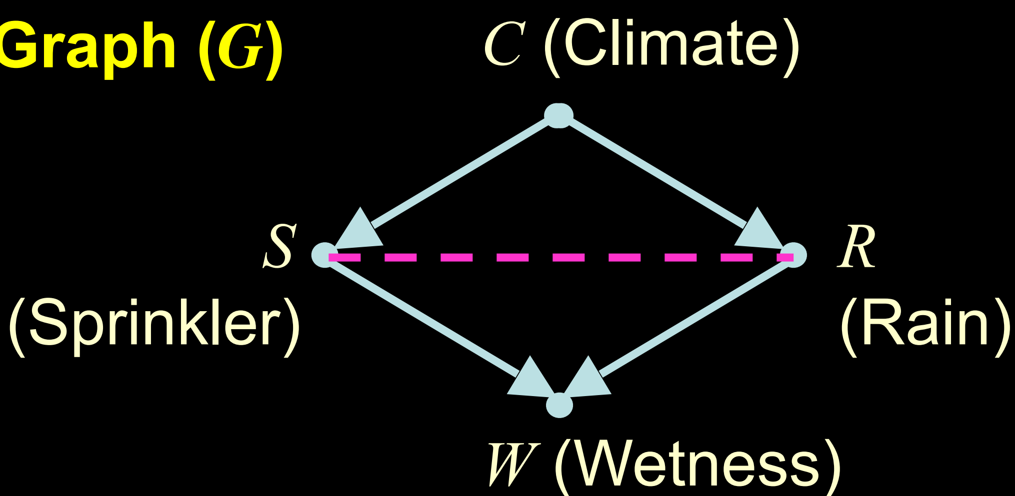

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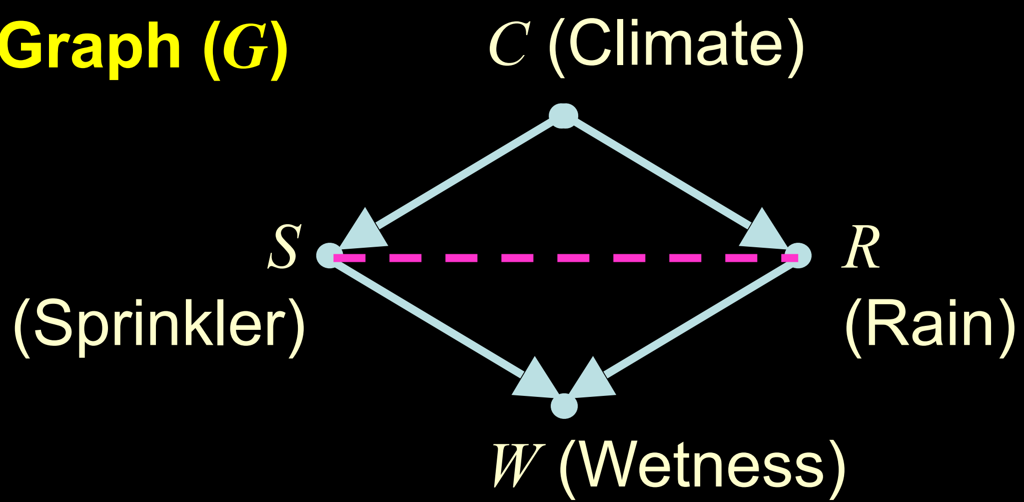
Applications:

D-SEPARATION: READING CONSTRAINTS IMPLIED IN THE DATA

Graph (G)



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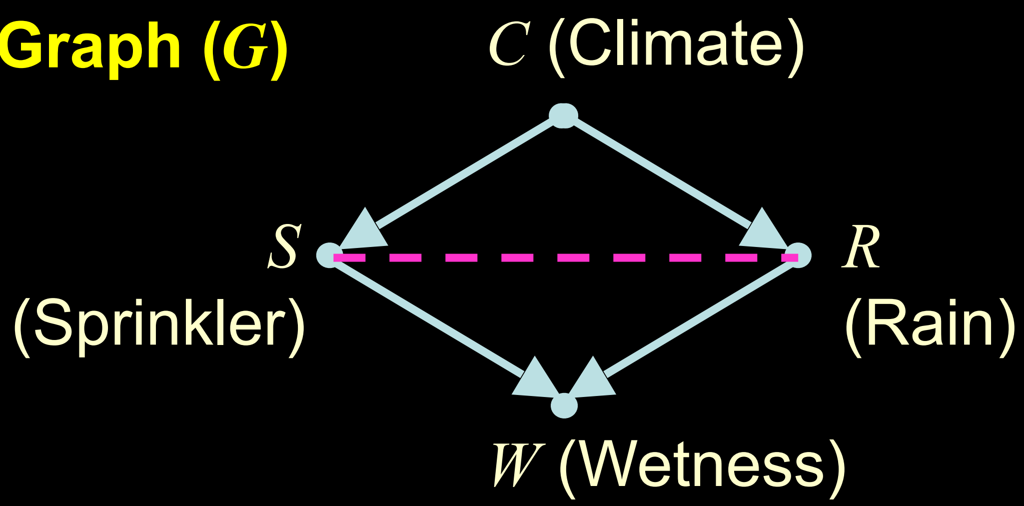

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Applications:

1. Structural learning

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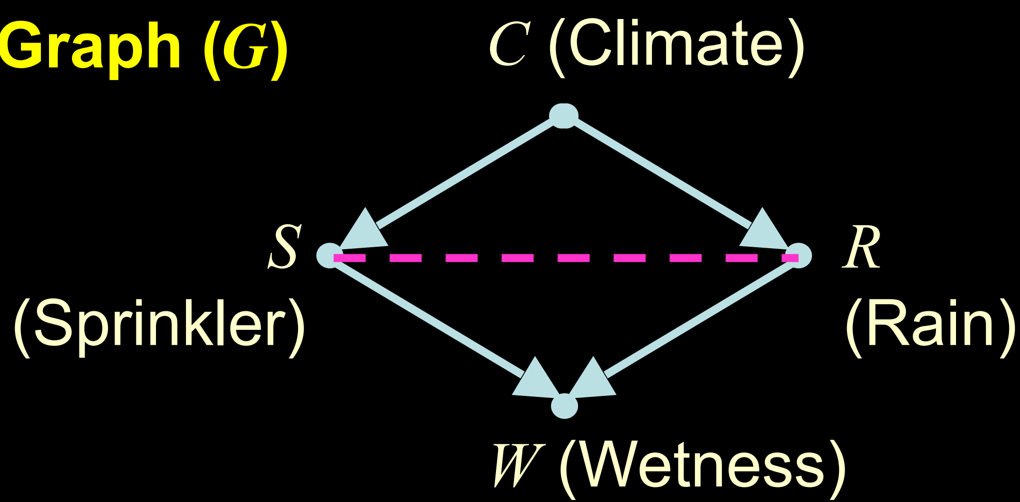

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2. Model testing

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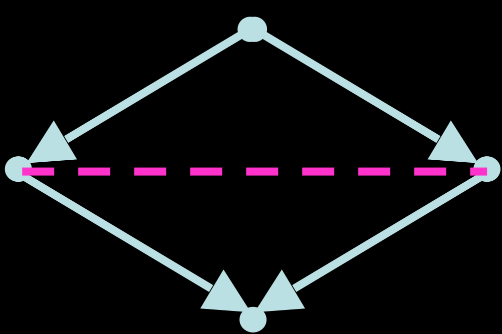

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$S \perp\!\!\!\perp R \mid C$

Applications:

1. Structural learning
2. Model testing
3. (Efficient) Probabilistic inference

GRAPH SEPARATION (D-SEPARATION)

GRAPH SEPARATION (D-SEPARATION)

$(X \perp\!\!\!\perp Y \mid Z)$

normal valve

$x \longleftarrow z \longrightarrow y$

$x \longrightarrow z \longrightarrow y$

$x \longleftarrow z \longleftarrow y$

abnormal valve

$x \longrightarrow z \longleftarrow y$

$$\begin{array}{ccccc} x & \longrightarrow & z & \longleftarrow & y \\ & & \downarrow & & \\ & & w & & \end{array}$$

$(X \perp\!\!\!\perp Y)$

$(X \not\perp\!\!\!\perp Y \mid Z)$

GRAPH SEPARATION (D-SEPARATION)

	normal valve	abnormal valve	
$(X \perp\!\!\!\perp Y \mid Z)$	$x \leftarrow z \rightarrow y$	$x \rightarrow z \leftarrow y$	$(X \perp\!\!\!\perp Y)$
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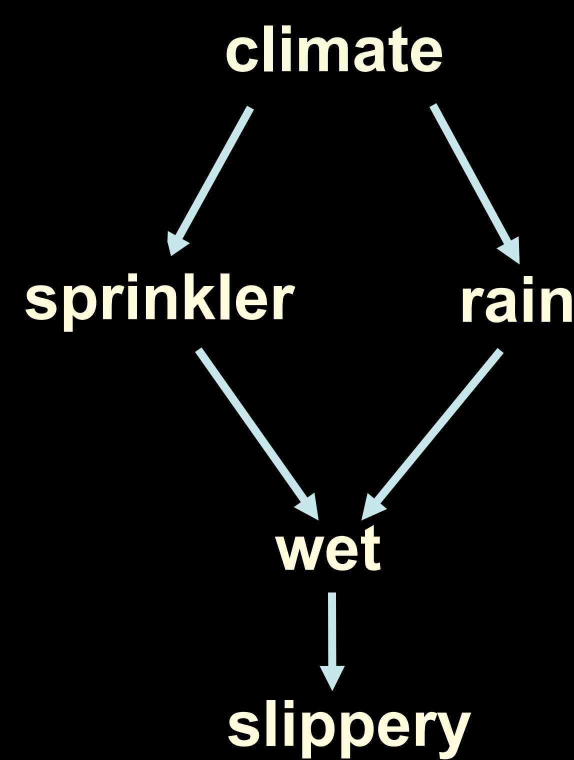


GRAPH SEPARATION (D-SEPARATION)

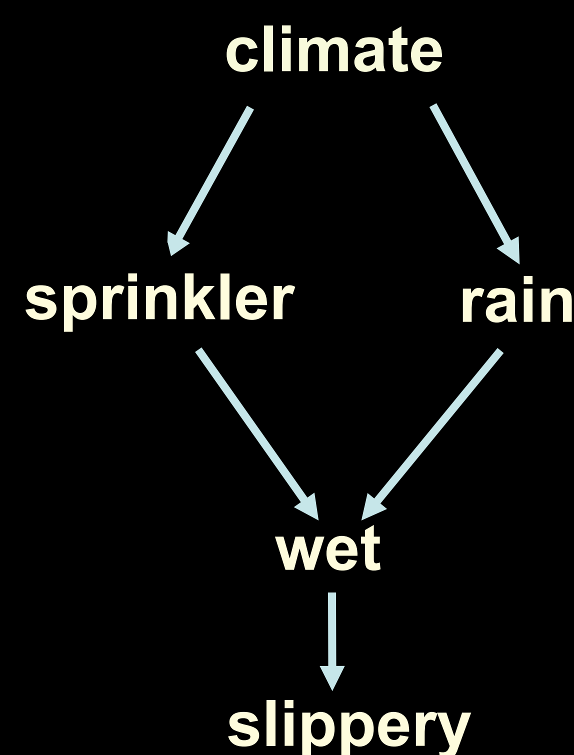
normal valve	abnormal valve
$x \longleftarrow z \longrightarrow y$	$x \longrightarrow z \longleftarrow y$
$x \longrightarrow z \longrightarrow y$	$x \longrightarrow z \longleftarrow y$, $z \longrightarrow w$
$x \longleftarrow z \longleftarrow y$	

$(X \perp\!\!\!\perp Y \mid Z)$

$(X \perp\!\!\!\perp Y)$

$(X \not\!\perp\!\!\!\perp Y \mid Z)$

$CI_1 : (\text{Wet} \perp\!\!\!\perp \text{Sprinkler})$



GRAPH SEPARATION (D-SEPARATION)

	normal valve	abnormal valve	
$(X \perp\!\!\!\perp Y \mid Z)$	$x \longleftarrow z \longrightarrow y$	$x \longrightarrow z \longleftarrow y$	$(X \perp\!\!\!\perp Y)$
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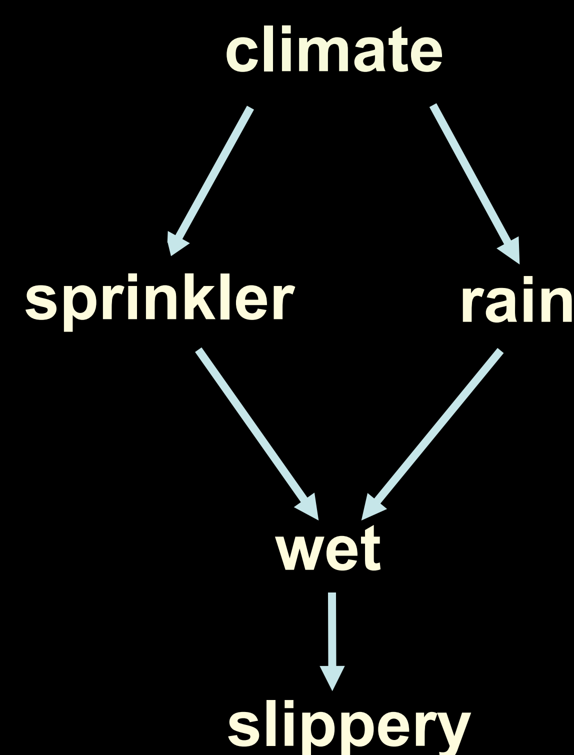
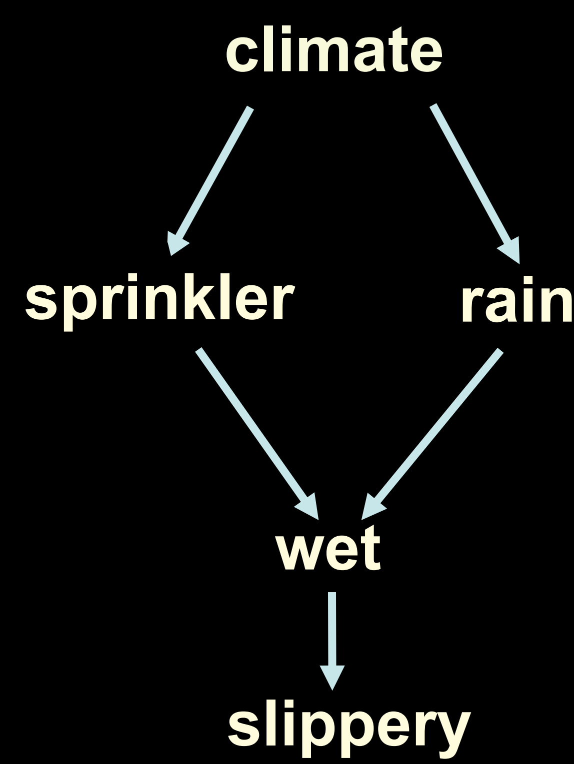


GRAPH SEPARATION (D-SEPARATION)

	normal valve	abnormal valve	
	$x \leftarrow z \rightarrow y$	$x \rightarrow z \leftarrow y$	$(X \perp\!\!\!\perp Y)$
$(X \perp\!\!\!\perp Y \mid Z)$	$x \rightarrow z \rightarrow y$	$x \rightarrow z \leftarrow y$, $z \rightarrow w$	$(X \not\!\perp\!\!\!\perp Y \mid Z)$
	$x \leftarrow z \leftarrow y$		

$CI_1 : (\text{Wet} \perp\!\!\!\perp \text{Sprinkler})$

$CI_2 : (\text{Wet} \perp\!\!\!\perp \text{Climate} \mid \text{Sprinkler})$



GRAPH SEPARATION (D-SEPARATION)

	normal valve	abnormal valve	
$(X \perp\!\!\!\perp Y \mid Z)$	$x \longleftarrow z \longrightarrow y$	$x \longrightarrow z \longleftarrow y$	$(X \perp\!\!\!\perp Y)$
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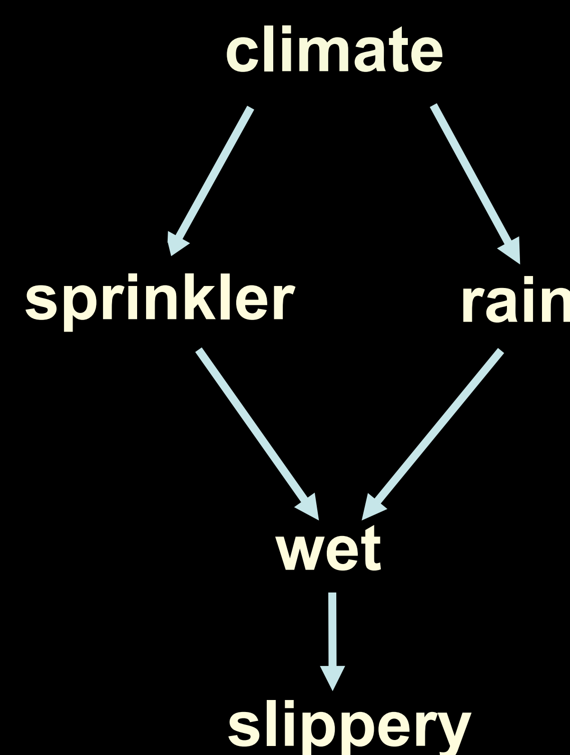
✗ CI_1 : (Wet $\perp\!\!\!\perp$ Sprinkler)

✗ CI_2 : (Wet $\perp\!\!\!\perp$ Climate | Sprinkler)



GRAPH SEPARATION (D-SEPARATION)

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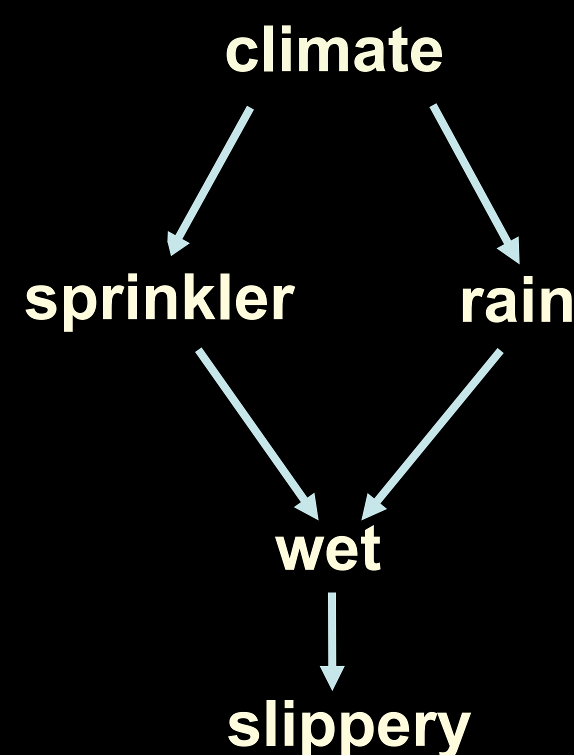
✘ CI_1 : (Wet $\perp\!\!\!\perp$ Sprinkler)

✘ CI_2 : (Wet $\perp\!\!\!\perp$ Climate | Sprinkler)

CI_3 : (Rain $\perp\!\!\!\perp$ Slippery | Wet)



GRAPH SEPARATION
(D-SEPARATION)

	normal valve	abnormal valve	
$(X \perp\!\!\!\perp Y \mid Z)$	$x \longleftarrow z \longrightarrow y$	$x \longrightarrow z \longleftarrow y$	$(X \perp\!\!\!\perp Y)$
	$x \longrightarrow z \longrightarrow y$	$\begin{tikzcd} x \arrow[r] & z \arrow[d] & y \arrow[l] \\ & w & \end{tikzcd}$	$(X \not\perp\!\!\!\perp Y \mid Z)$
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✘ CI_1 : (Wet $\perp\!\!\!\perp$ Sprinkler)

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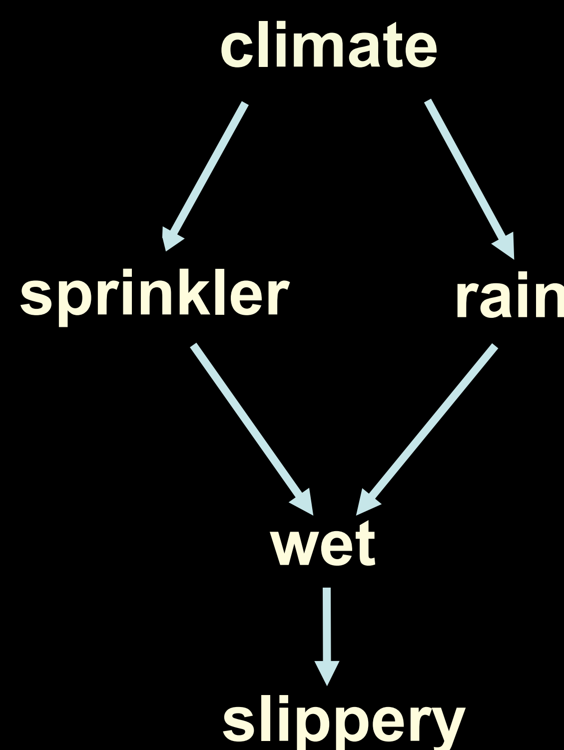
✔ CI_3 : (Rain $\perp\!\!\!\perp$ Slippery | Wet)



GRAPH SEPARATION (D-SEPARATION)

	normal valve	abnormal valve	
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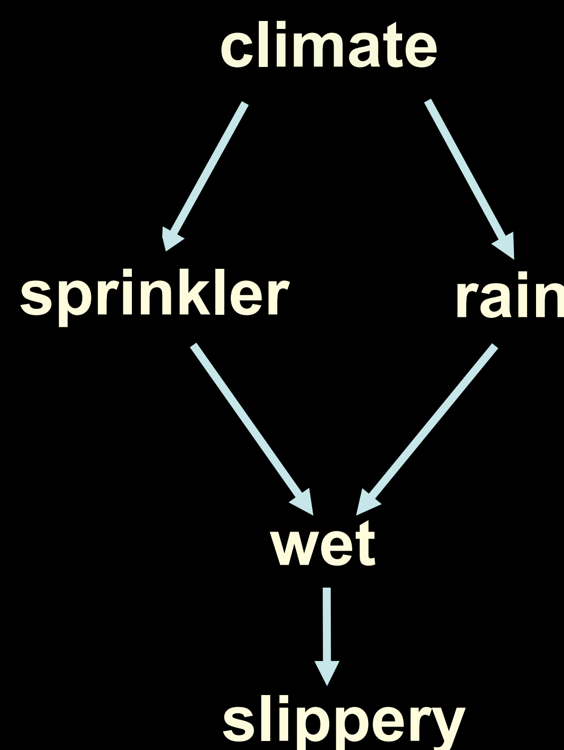
✗ CI_1 : (Wet $\perp\!\!\!\perp$ Sprinkler)

✗ CI_2 : (Wet $\perp\!\!\!\perp$ Climate | Sprinkler)

✔ CI_3 : (Rain $\perp\!\!\!\perp$ Slippery | Wet)

CI_4 : (Season $\perp\!\!\!\perp$ Wet | Sprinkler, Rain)



GRAPH SEPARATION (D-SEPARATION)

normal valve		abnormal valve	
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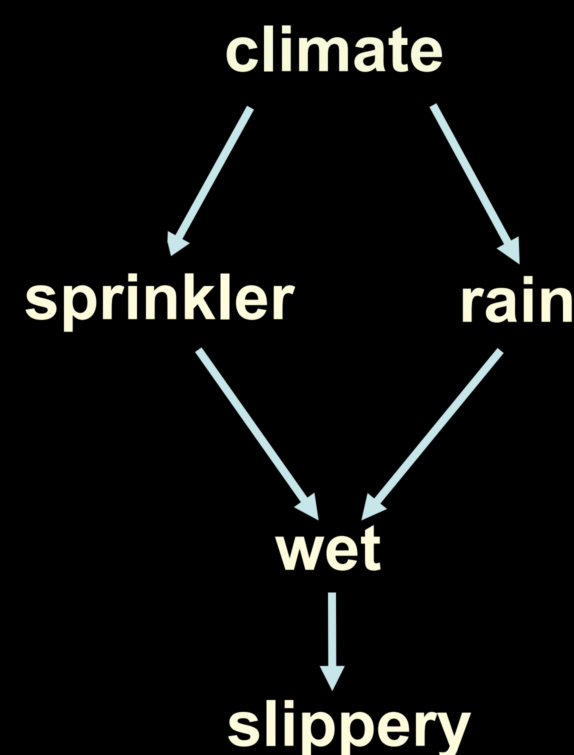
	$x \leftarrow z \leftarrow y$		

- ✘ CI_1 : (Wet $\perp\!\!\!\perp$ Sprinkler)
- ✘ CI_2 : (Wet $\perp\!\!\!\perp$ Climate | Sprinkler)
- ✔ CI_3 : (Rain $\perp\!\!\!\perp$ Slippery | Wet)
- ✔ CI_4 : (Season $\perp\!\!\!\perp$ Wet | Sprinkler, Rain)



GRAPH SEPARATION (D-SEPARATION)

	normal valve	abnormal valve	
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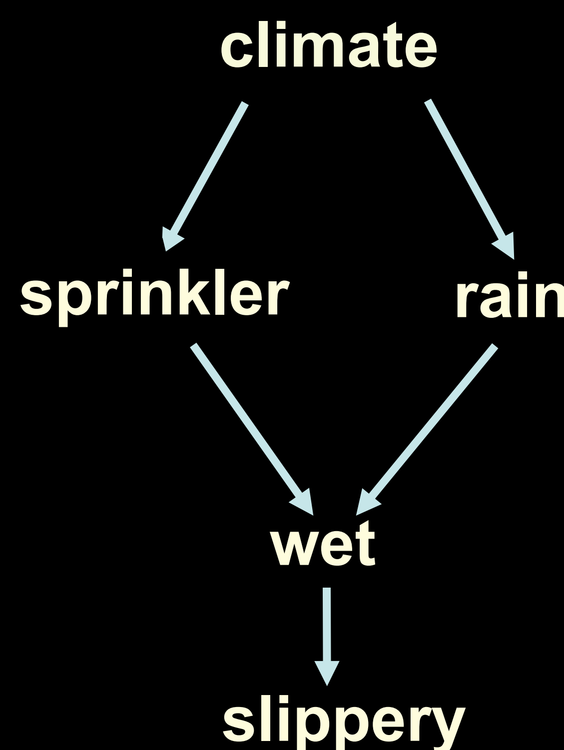
✘ CI_1 : (Wet $\perp\!\!\!\perp$ Sprinkler)

✘ CI_2 : (Wet $\perp\!\!\!\perp$ Climate | Sprinkler)

✔ CI_3 : (Rain $\perp\!\!\!\perp$ Slippery | Wet)

✔ CI_4 : (Season $\perp\!\!\!\perp$ Wet | Sprinkler, Rain)

CI_5 : (Sprinkler $\perp\!\!\!\perp$ Rain | Climate, Wet)



GRAPH SEPARATION (D-SEPARATION)

	normal valve	abnormal valve	
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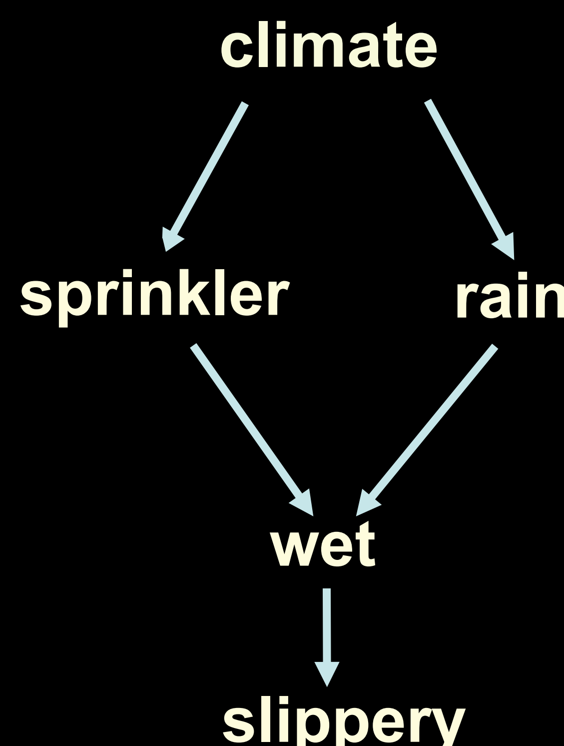
✗ CI_1 : (Wet $\perp\!\!\!\perp$ Sprinkler)

✗ CI_2 : (Wet $\perp\!\!\!\perp$ Climate | Sprinkler)

✓ CI_3 : (Rain $\perp\!\!\!\perp$ Slippery | Wet)

✓ CI_4 : (Season $\perp\!\!\!\perp$ Wet | Sprinkler, Rain)

✗ CI_5 : (Sprinkler $\perp\!\!\!\perp$ Rain | Climate, Wet)



THE SECOND LAYER ON CAUSAL HIERARCHY:

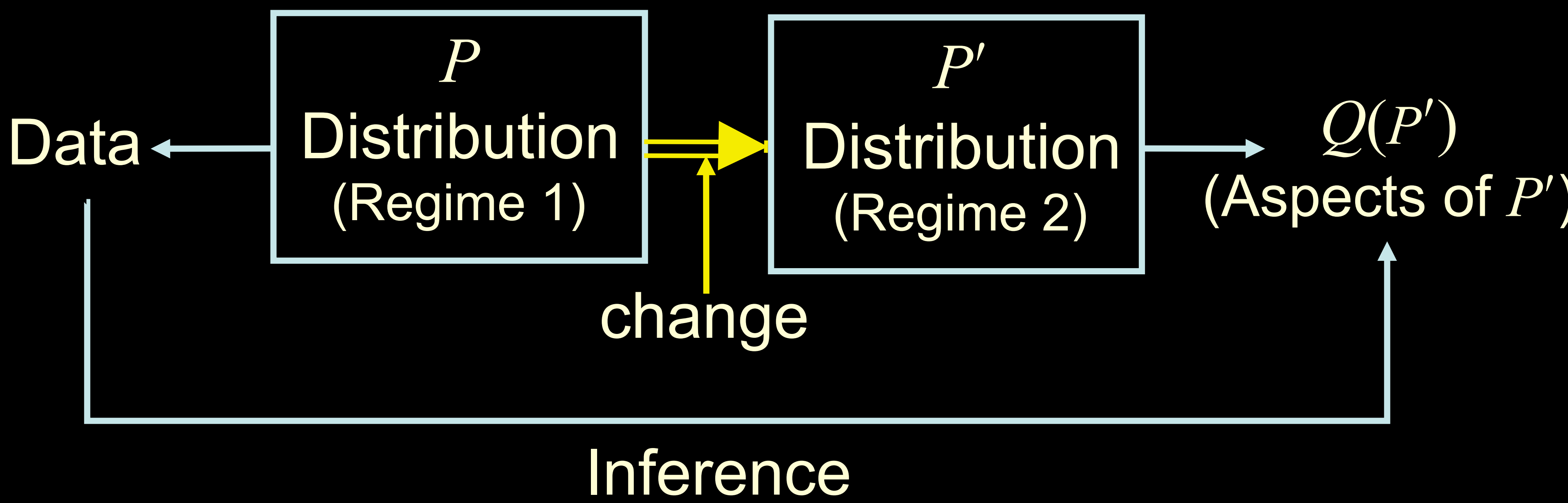
INTERVENTIONAL

(What if I do $X=x$?)

OUTLINE

- Introduction
  * Data, data, data…
  * Basic definitions
- Technical Results
  * The truncated product formula
  * The back-door adjustment formula
  * The front-door adjustment formula
  * The do-calculus
- Tasks
  * Policy evaluation
  * Generalizability & Robustness
  * Decision-making & RL

CAUSAL INFERENCE: MOVING ACROSS REGIMES



- What happens when ***P*** changes?

e.g., Infer whether less people would get cancer if we ban smoking.

$Q = P(Cancer = true \mid do(Smoking = no))$ Not an aspect of P.

Observation 1:

The distribution alone tells us nothing about change; it just describes static conditions of a population (under a specific regime).

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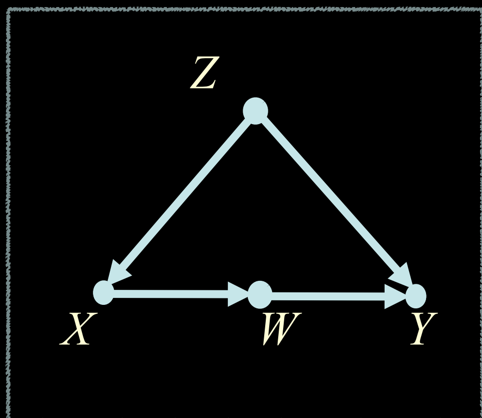
The distribution alone tells us nothing about change; it just describes static conditions of a population (under a specific regime).

Observation 2:

We need to be able to represent “change”, or how the population reacts when it undergoes change in regimes.

THE BIG PICTURE: THE CHALLENGE OF CAUSAL INFERENCE

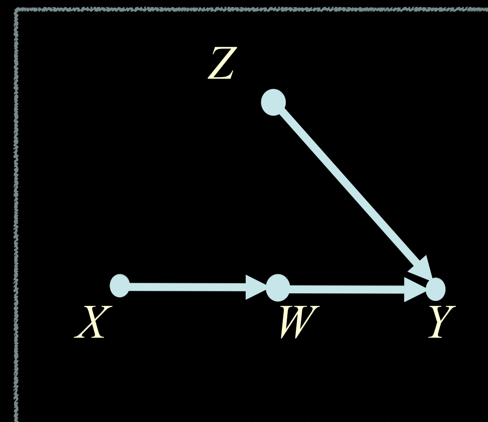
Real world



$P(z, x, w, y)$

change

Alternative world

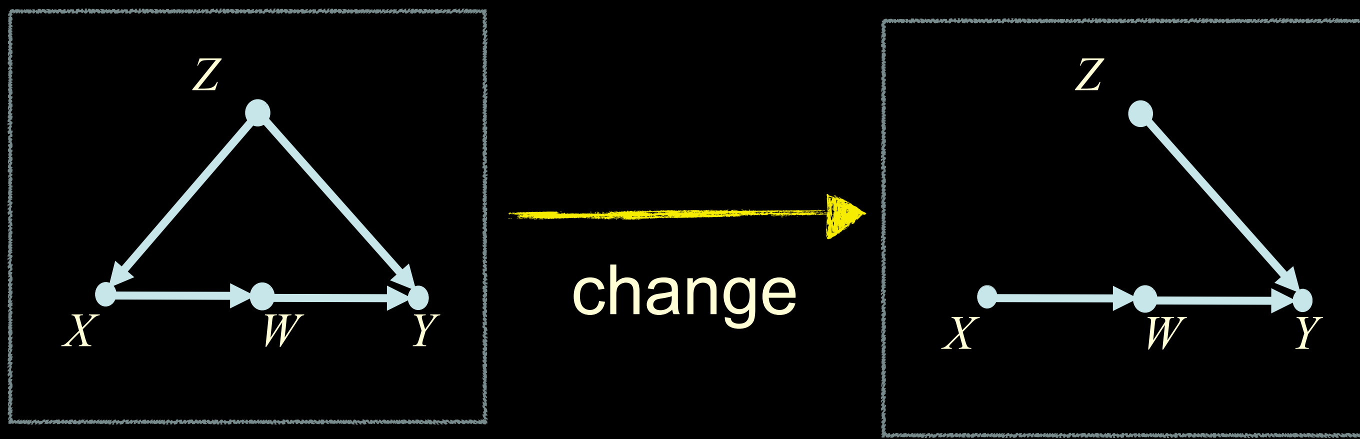


$P(y \mid do(x))$

Z: age, sex
X: action
W: mediator
Y: outcome

THE BIG PICTURE: THE CHALLENGE OF CAUSAL INFERENCE

Real world | Alternative world

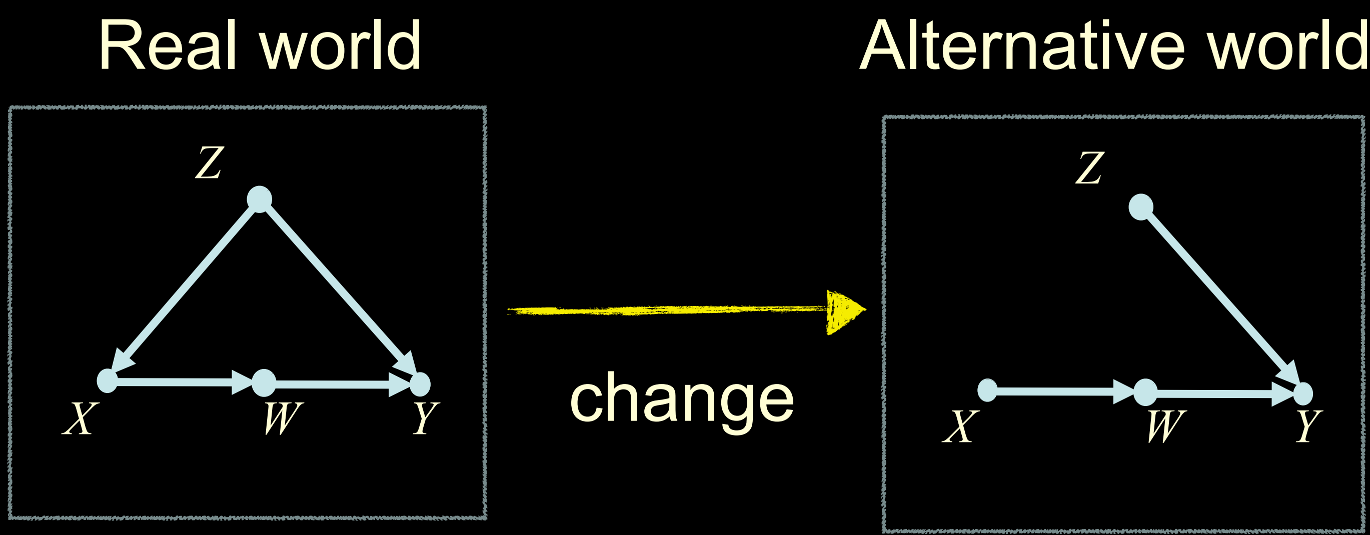


$P(z, x, w, y)$ | $P(y \mid do(x))$

Z: age, sex
X: action
W: mediator
Y: outcome

- Goal: how much Y changes with X if we vary X between two different constants free from the influence of Z.
- This is the definition of causal effect.

METHOD FOR COMPUTING CAUSAL EFFECTS: RANDOMIZED EXPERIMENTS



Z: age, sex
X: action
W: mediator
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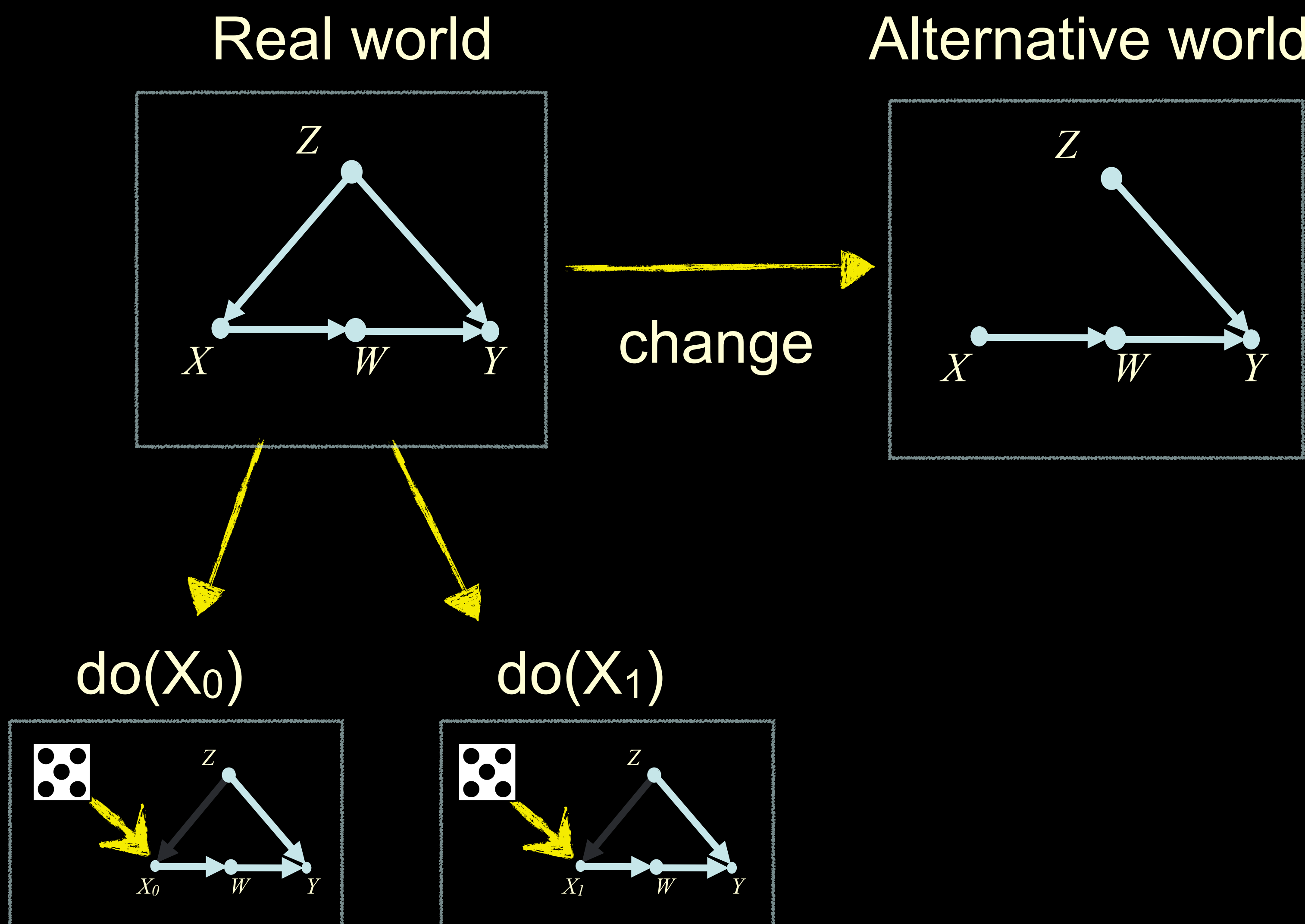

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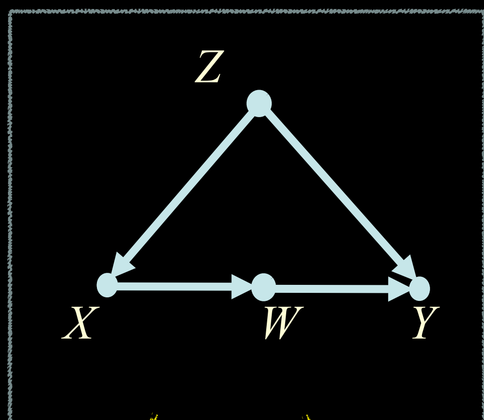
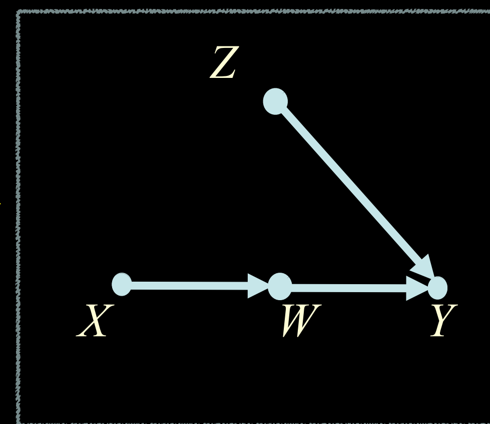
METHOD FOR COMPUTING CAUSAL EFFECTS: RANDOMIZED EXPERIMENTS

Real world



change

Alternative world



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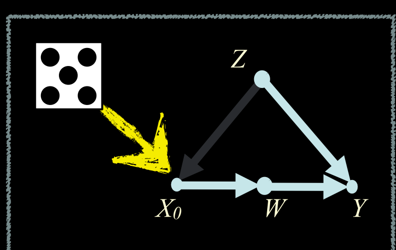
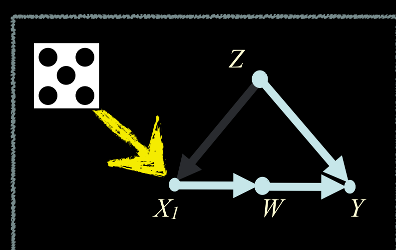
do(X_0)



do(X_1)

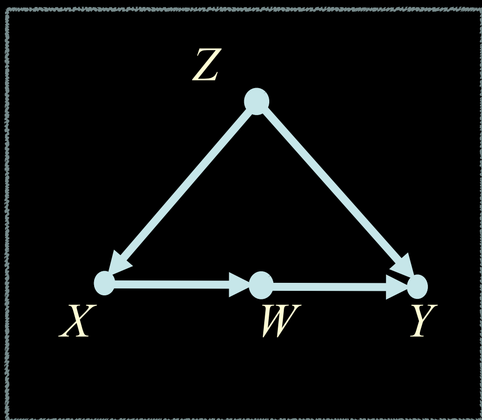


Randomization:

$P(y \mid do(X_0))$ $P(y \mid do(X_1))$

COMPUTING CAUSAL EFFECTS FROM OBSERVATIONAL DATA

Real world



change

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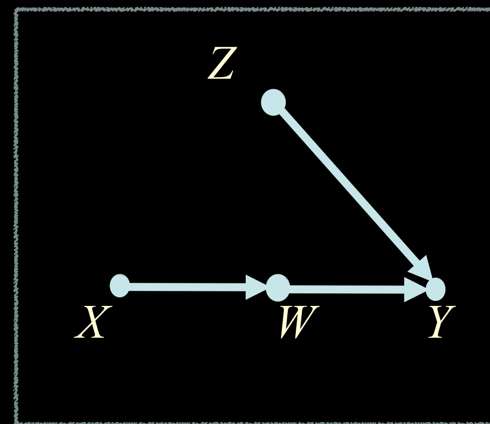

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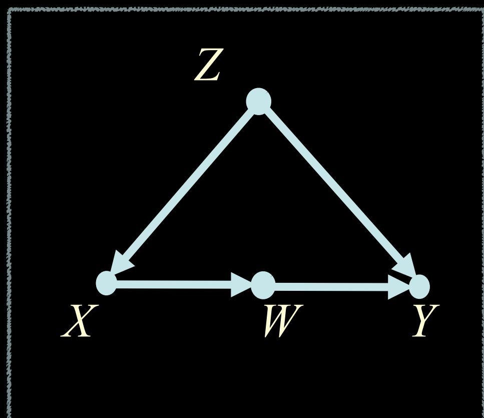
P(z, x, w, y) ? P(y | do(x))

COMPUTING CAUSAL EFFECTS FROM OBSERVATIONAL DATA

Real world



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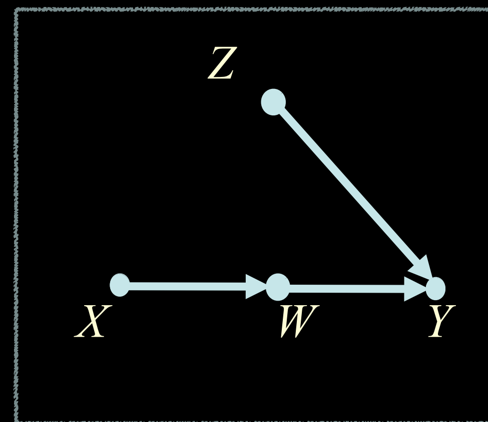

Z: age, sex
X: action
W: mediator
Y: outcome

P(z, x, w, y) ? P(y | do(x))

Questions:

* What is the relationship between P(z, x, w, y) and P(y | do(x))?
* Is P(y | do(x)) = P(y | x)?

EFFECT OF INTERVENTIONS

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- Def. 3.2.1 (Causal Effect)

Given two disjoint sets of variables, X and Y, the causal effect of X on Y, denoted as $P(y \mid do(x))$, is a function from X to the space of probability distributions of Y.

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- Def. 3.2.1 (Causal Effect)

Given two disjoint sets of variables, X and Y, the causal effect of X on Y, denoted as P(y | do(x)), is a function from X to the space of probability distributions of Y.

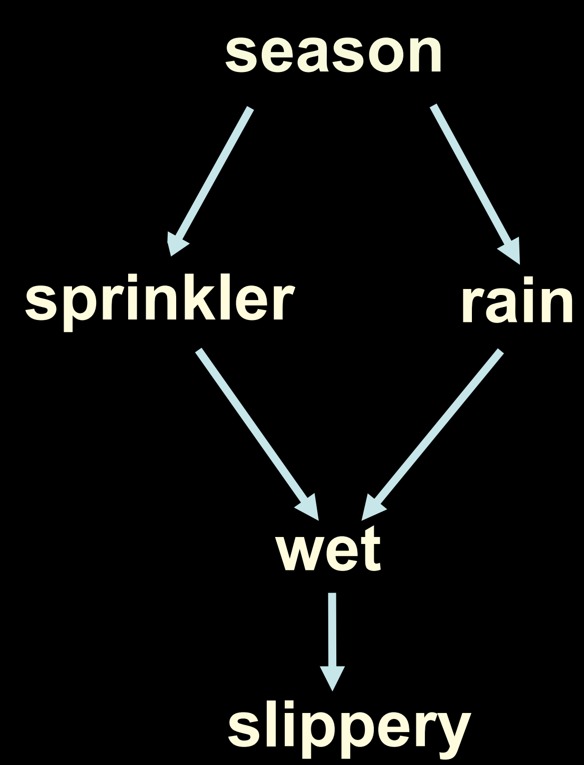
For each realization x of X, P(y | do(x)) gives the probability Y = y induced by deleting from the model all equations corresponding to variables in X and substituting X = x in the remaining equations.

COMPUTING CAUSAL EFFECTS FROM OBSERVATIONAL DATA

Queries:

Q_1 = Pr(wet | Sprinkler = on)

Q_2 = Pr(wet | do(Sprinkler = on))



COMPUTING CAUSAL EFFECTS FROM OBSERVATIONAL DATA

Queries:

Q_1 = Pr(wet | Sprinkler = on)
= P(p_1) + P(p_2)

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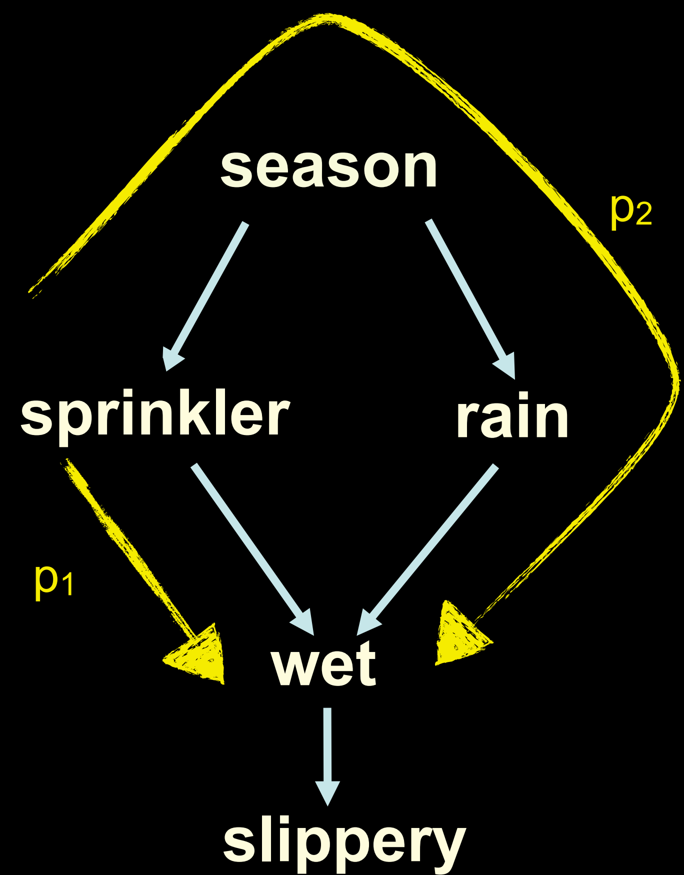

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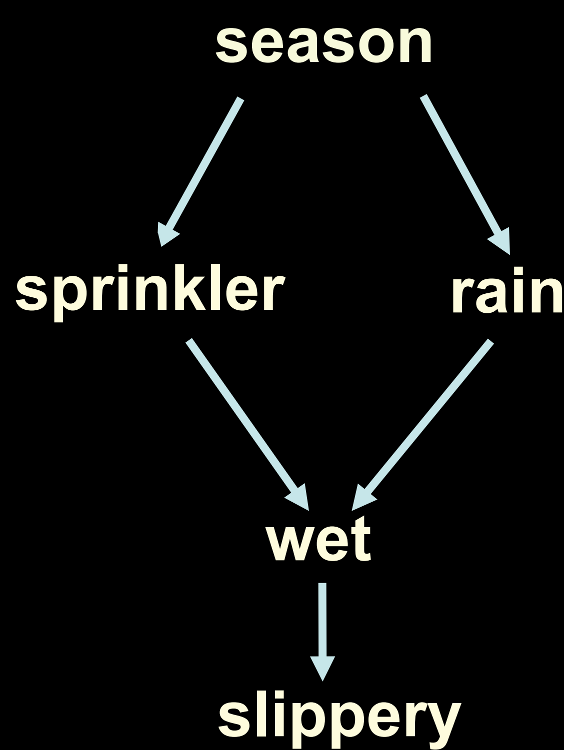

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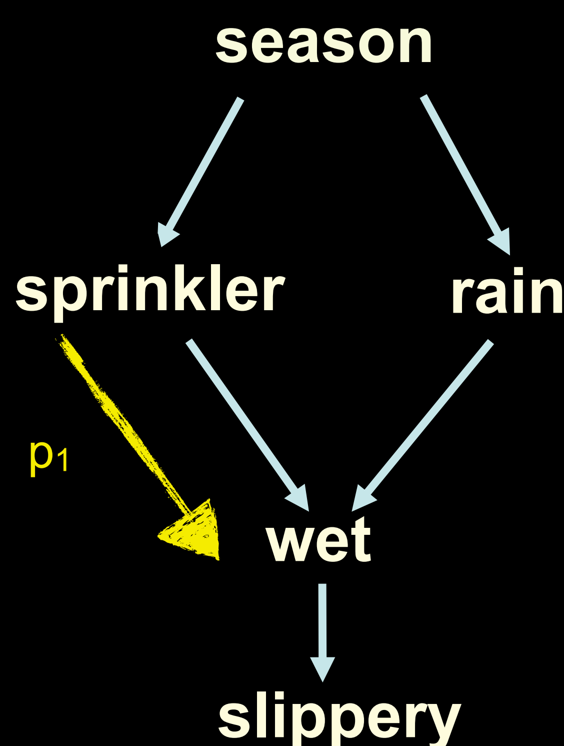

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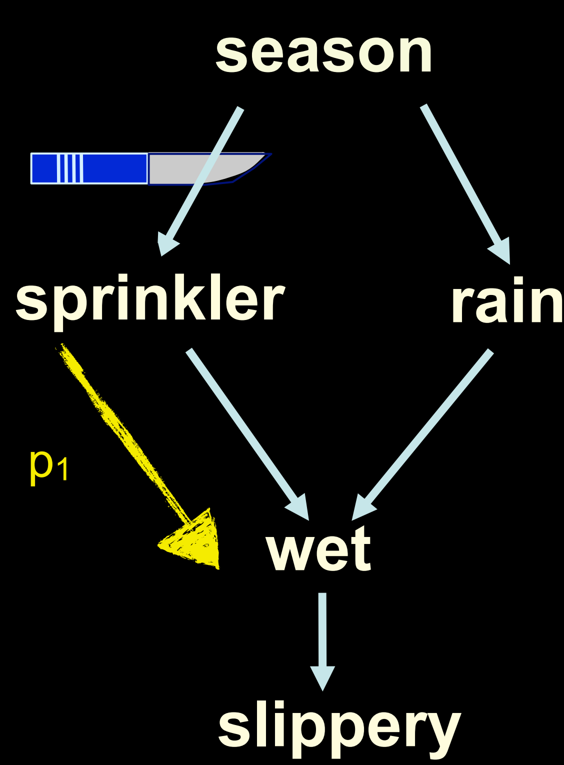

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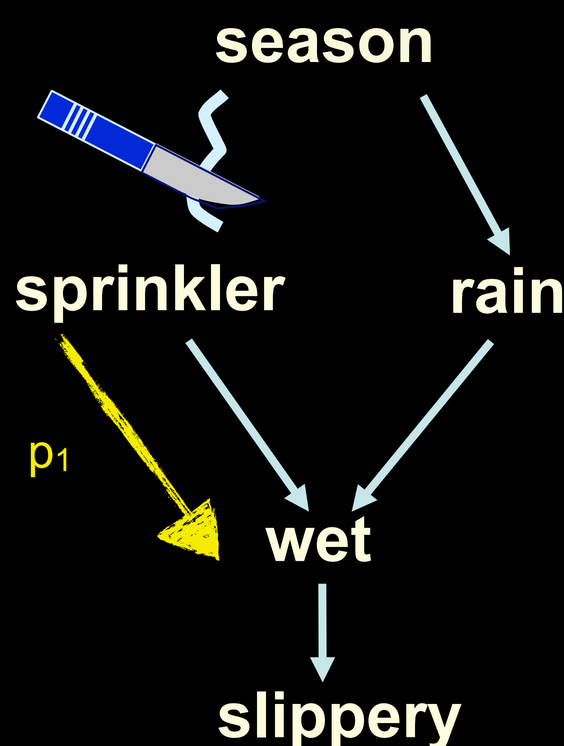

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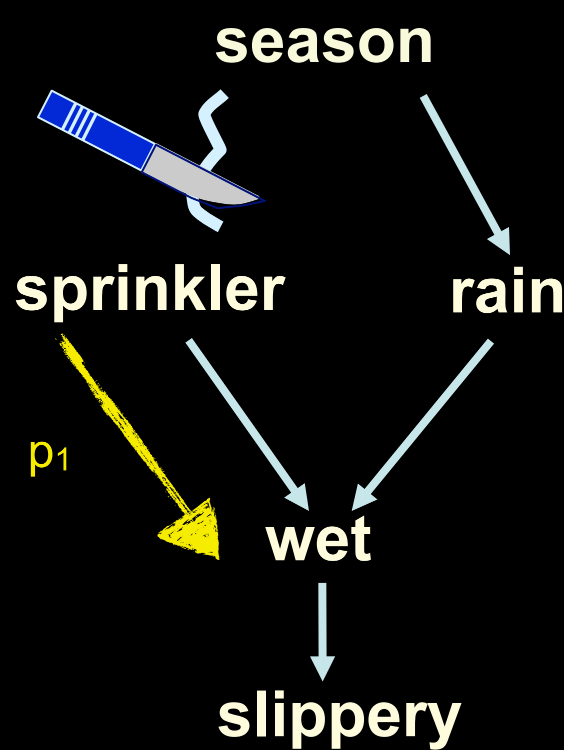

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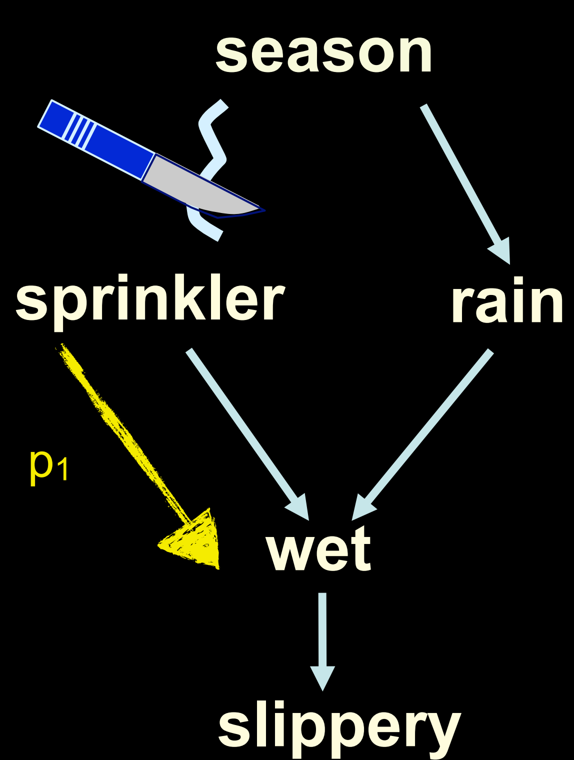

$$\sum_{Se,Ra,Sl} P(Se)\, P(Sp \mid Se)\, P(Ra \mid Se)\, P(We \mid Sp, Ra)\, P(Sl \mid We)$$

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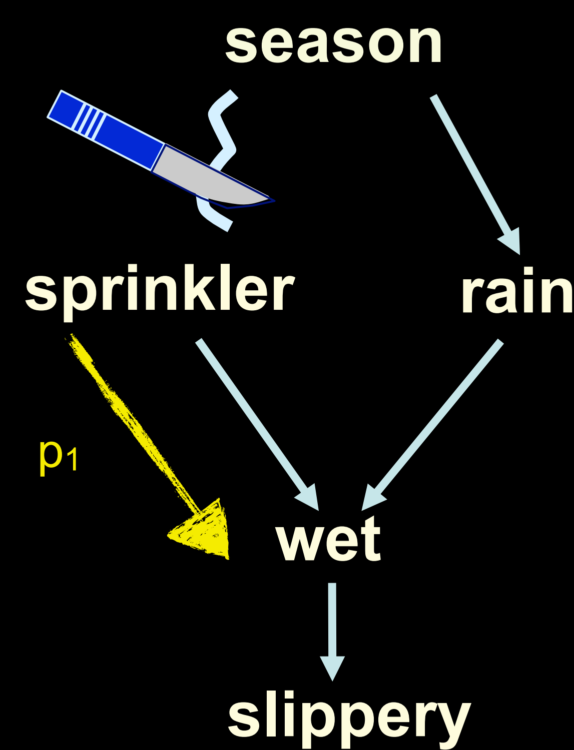

$$\sum_{Se,Ra,Sl} P(Se)\, \cancel{P(Sp \mid Se)}\, P(Ra \mid Se)\, P(We \mid Sp, Ra)\, P(Sl \mid We)$$

TRUNCATED FACTORIZATION PRODUCT (OPERATIONALIZING INTERVENTIONS)

Corollary (Truncated Factorization, Manipulation Thm., G-comp.):
The distribution generated by an intervention *do*(X=x)
(in a Markovian model M) is given by the truncated factorization:

$$P(v_1, v_2, \ldots, v_n \mid do(x)) = \prod_{i \mid V_i \notin X} P(v_i \mid pa_i) \Big|_{X = x}$$

THE IDENTIFIABILITY PROBLEM

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- Def. 3.2.2 (Causal Effect Identifiability)

The causal effect of X on Y is said to be identifiable from a graph G if the quantity P(y | do(x)) can be computed uniquely from a positive probability of the observed variables.

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That is, if $P_{M1}(y \mid do(x)) = P_{M2}(y \mid do(x))$ for every pair of models M_1 and M_2 with $P_{M1}(v) = P_{M2}(v) > 0$ and $G(M_1) = G(M_2) = G$.

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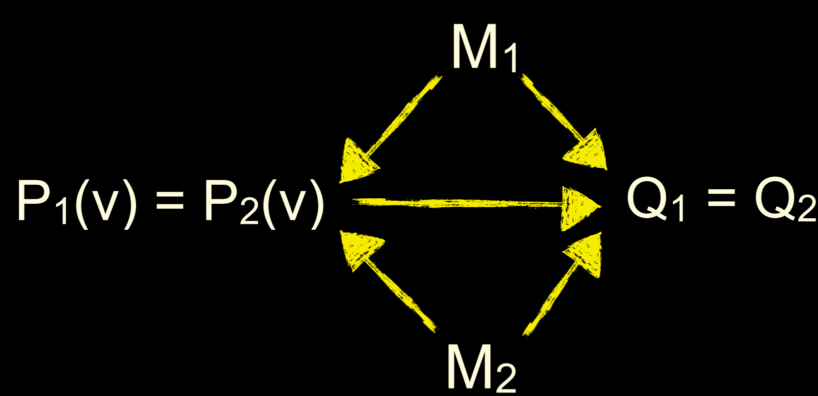

MEANING OF IDENTIFIABILITY

$P(y|do(x))$ is identifiable in G

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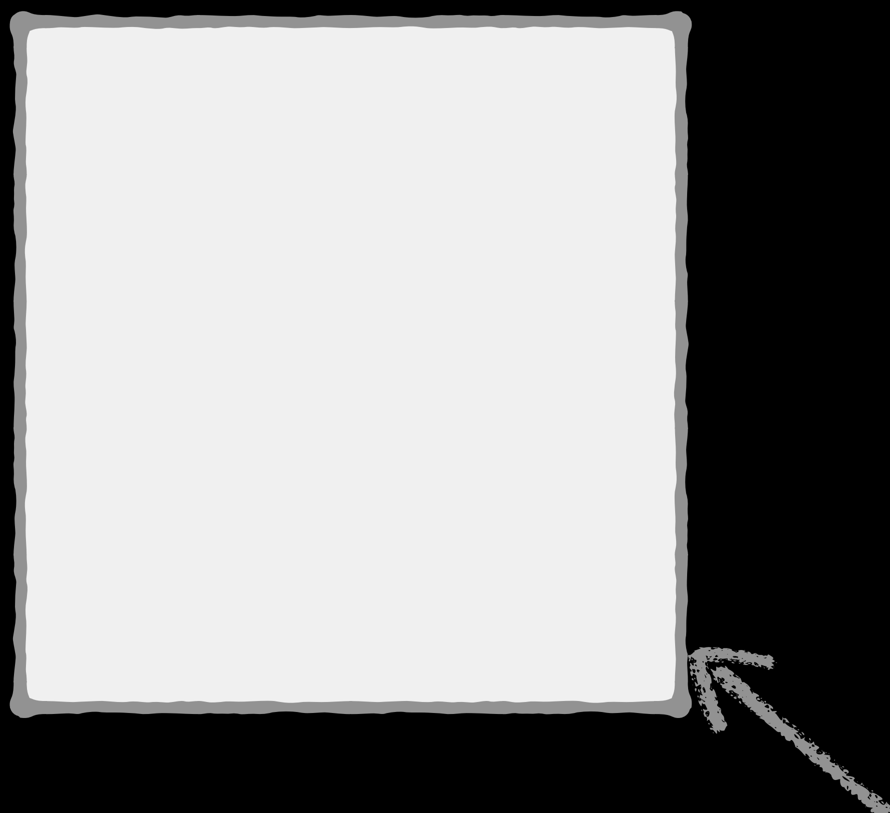
$P(y|do(x))$ is identifiable in *G*



All models

MEANING OF IDENTIFIABILITY

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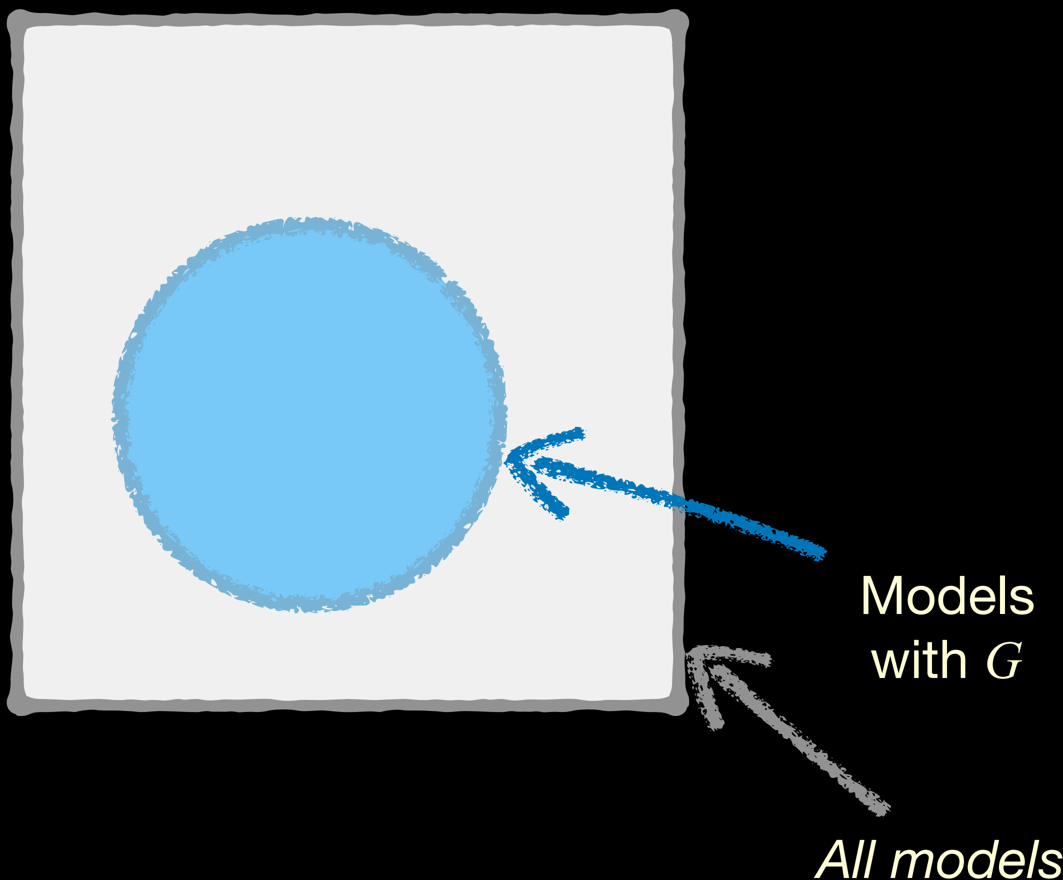

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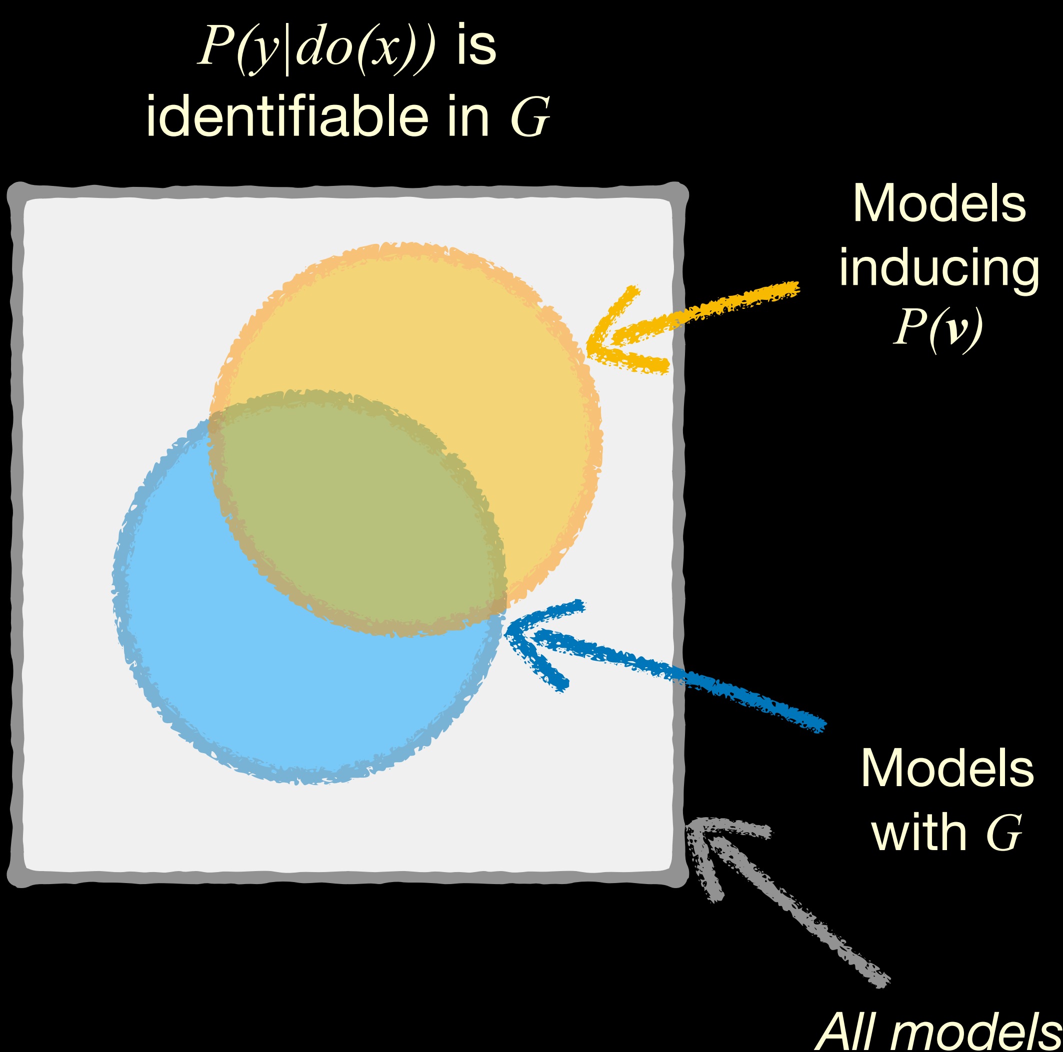

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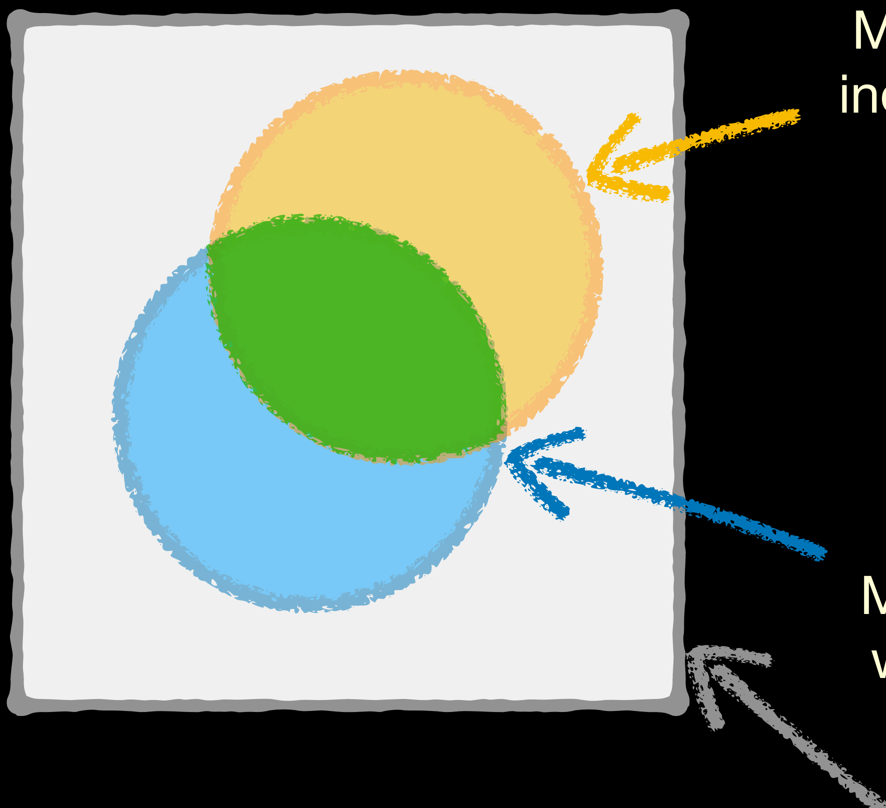

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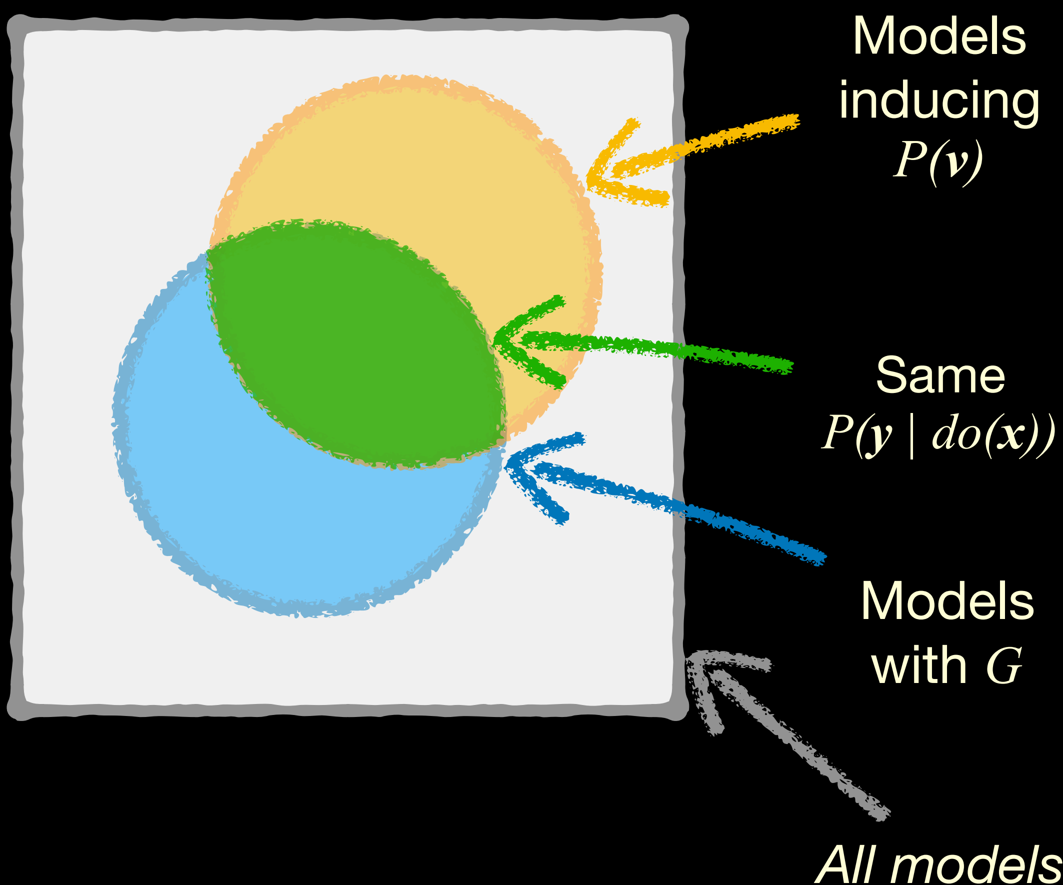

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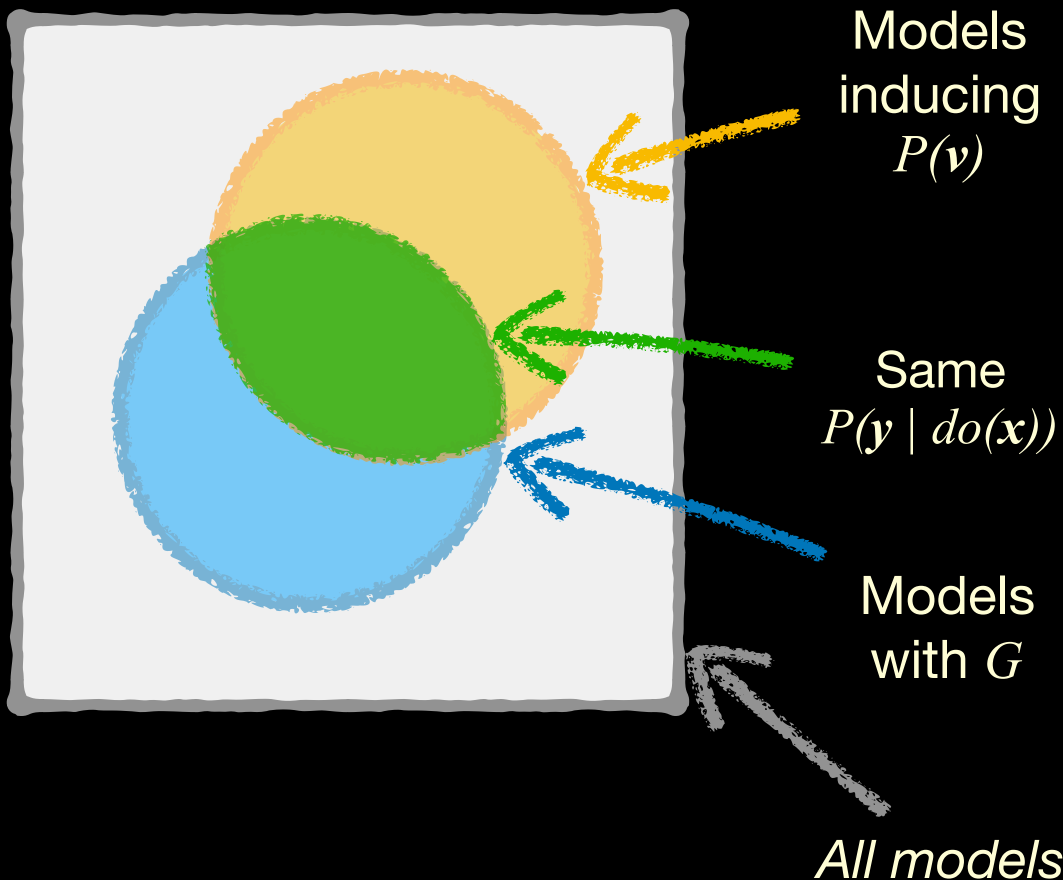

Formally,
for any two causal models (encoding the unobserved nature), N_1, N_2, s.t.

MEANING OF IDENTIFIABILITY

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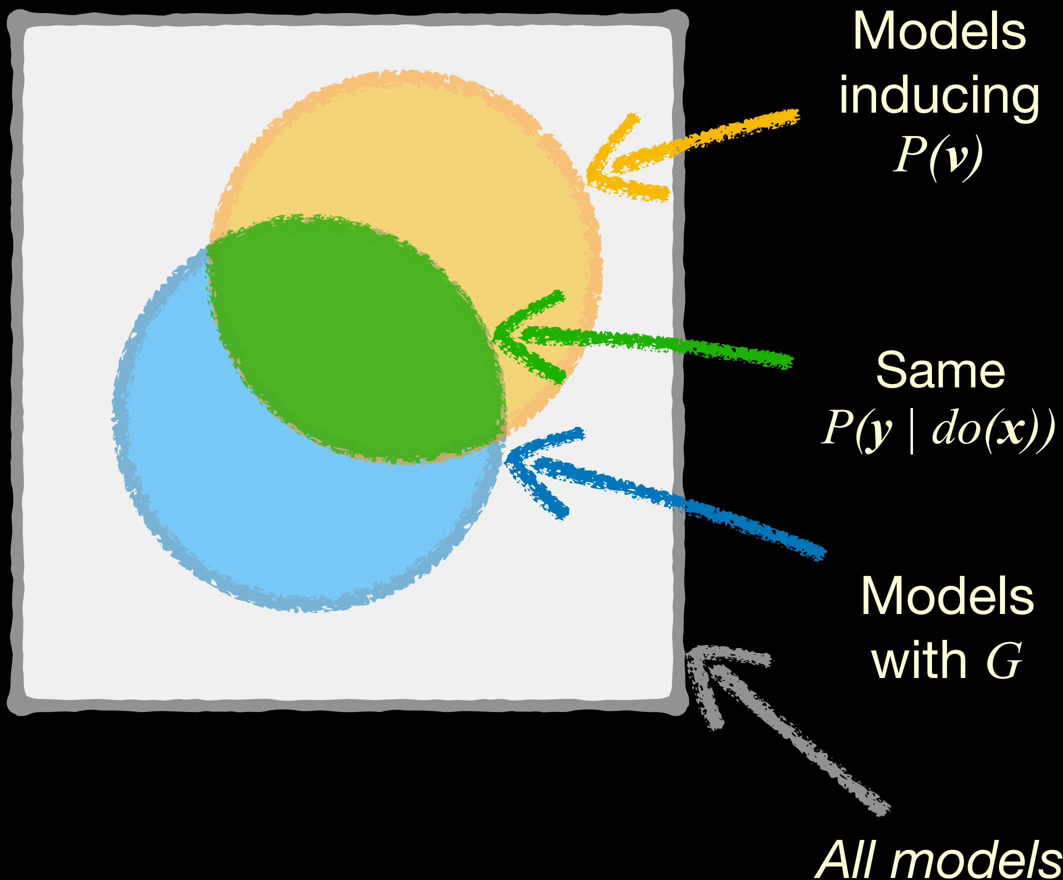

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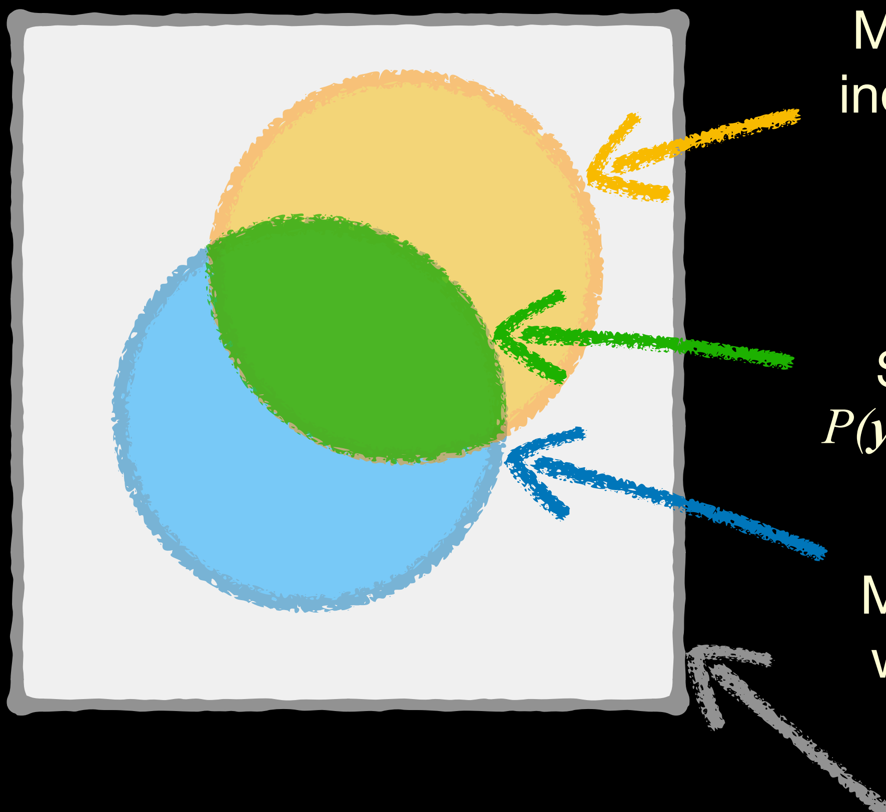

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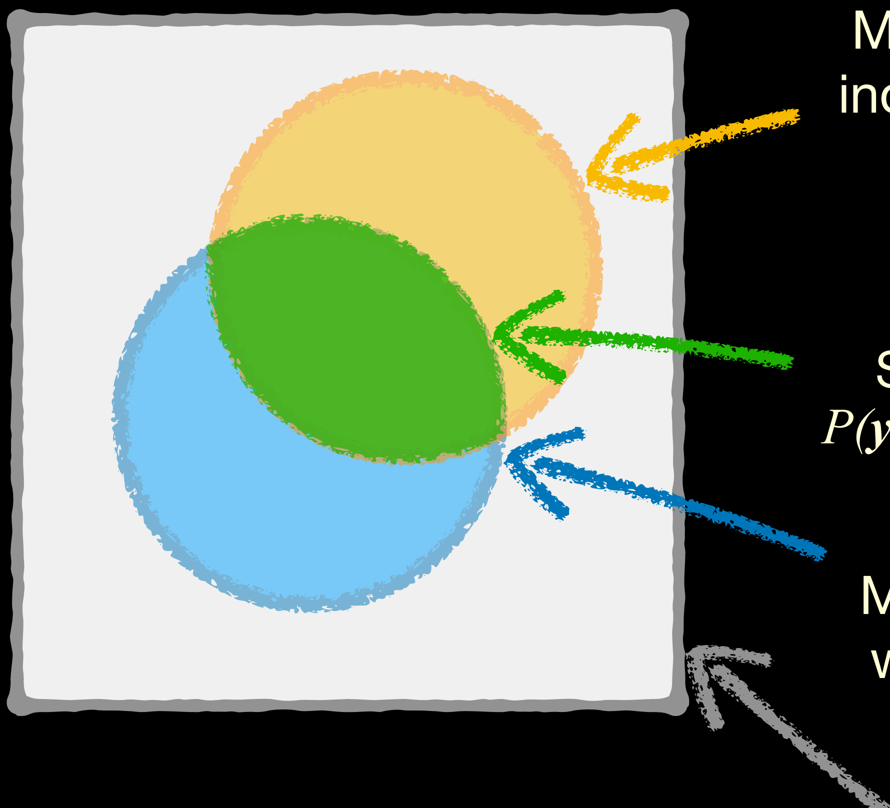

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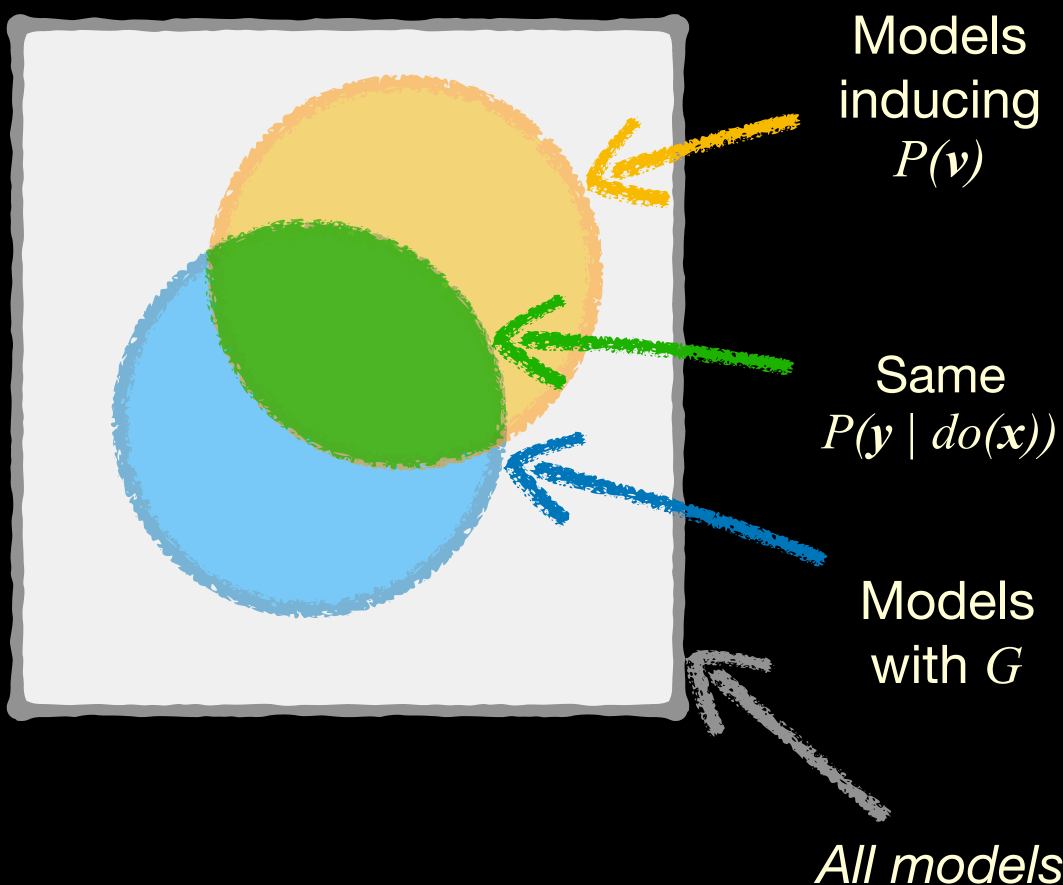

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MEANING OF IDENTIFIABILITY (II)

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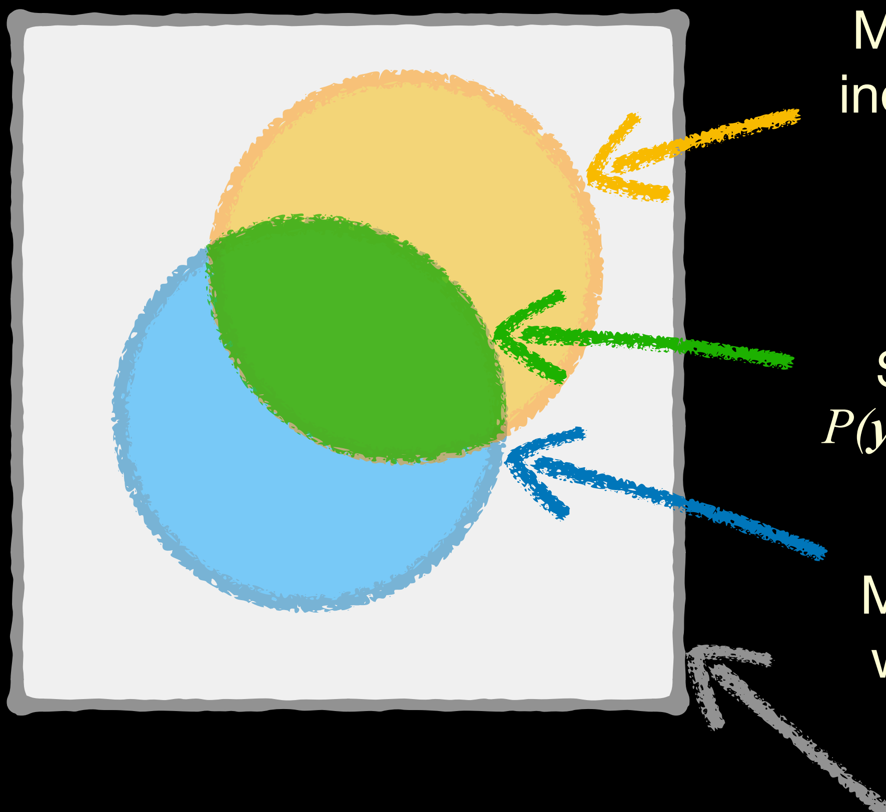

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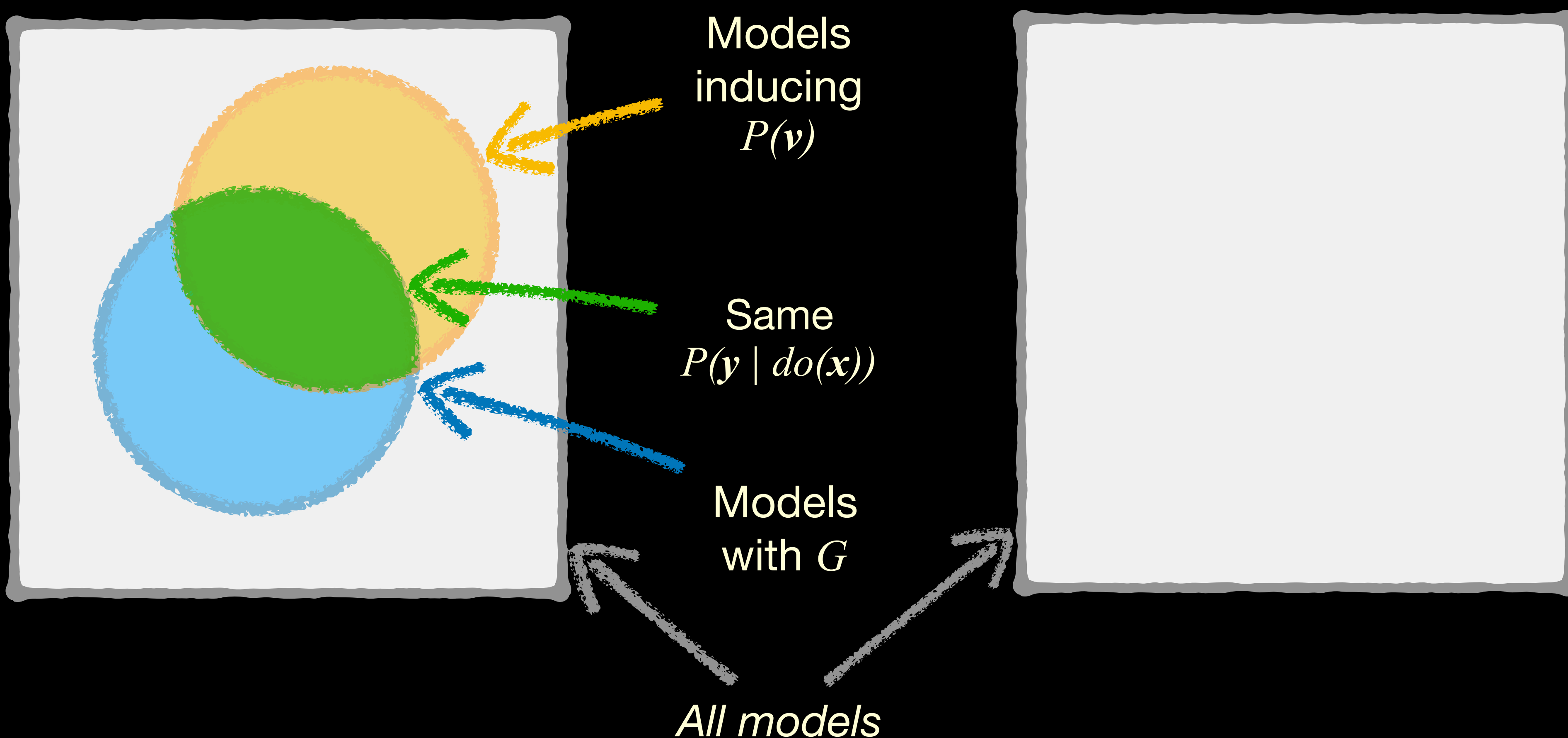

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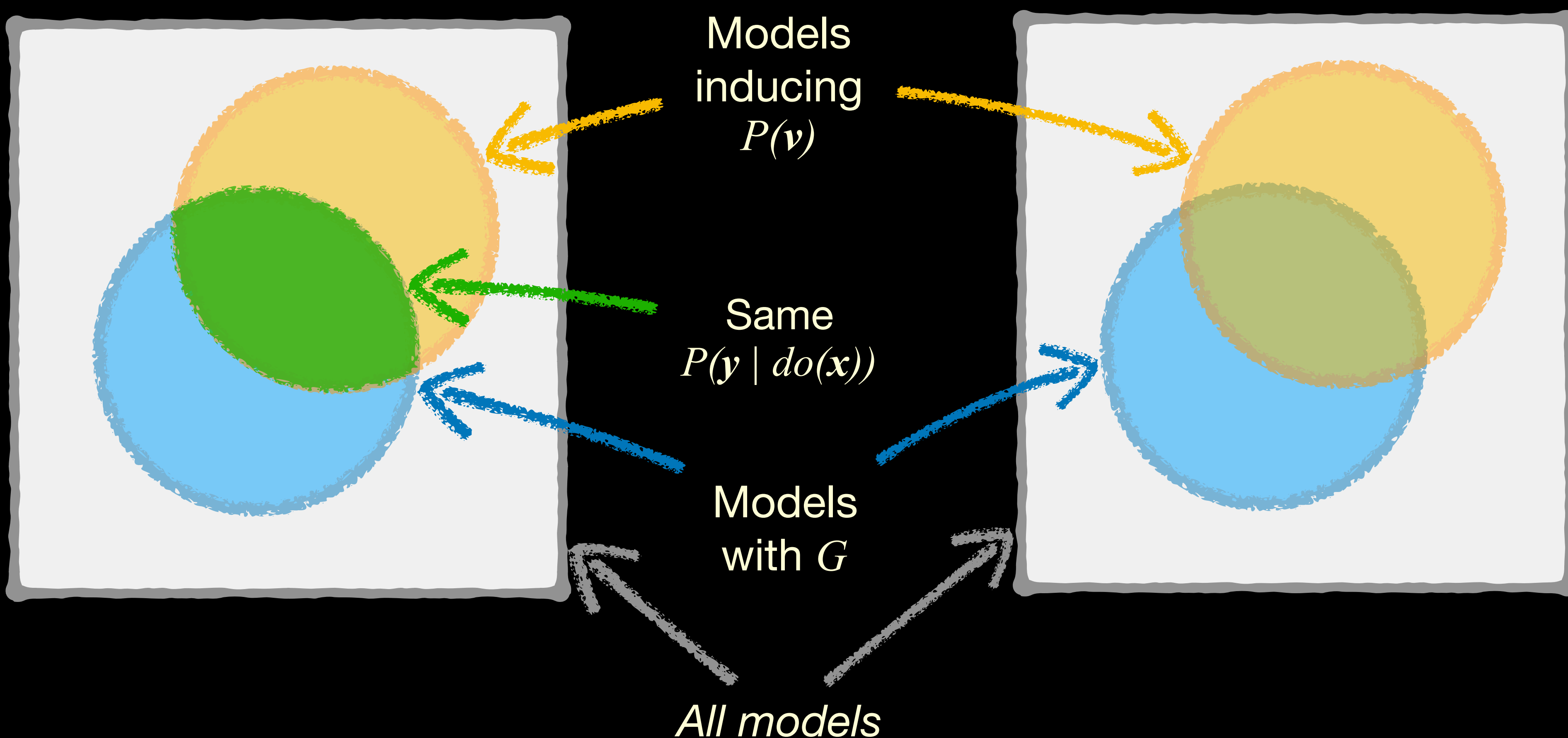

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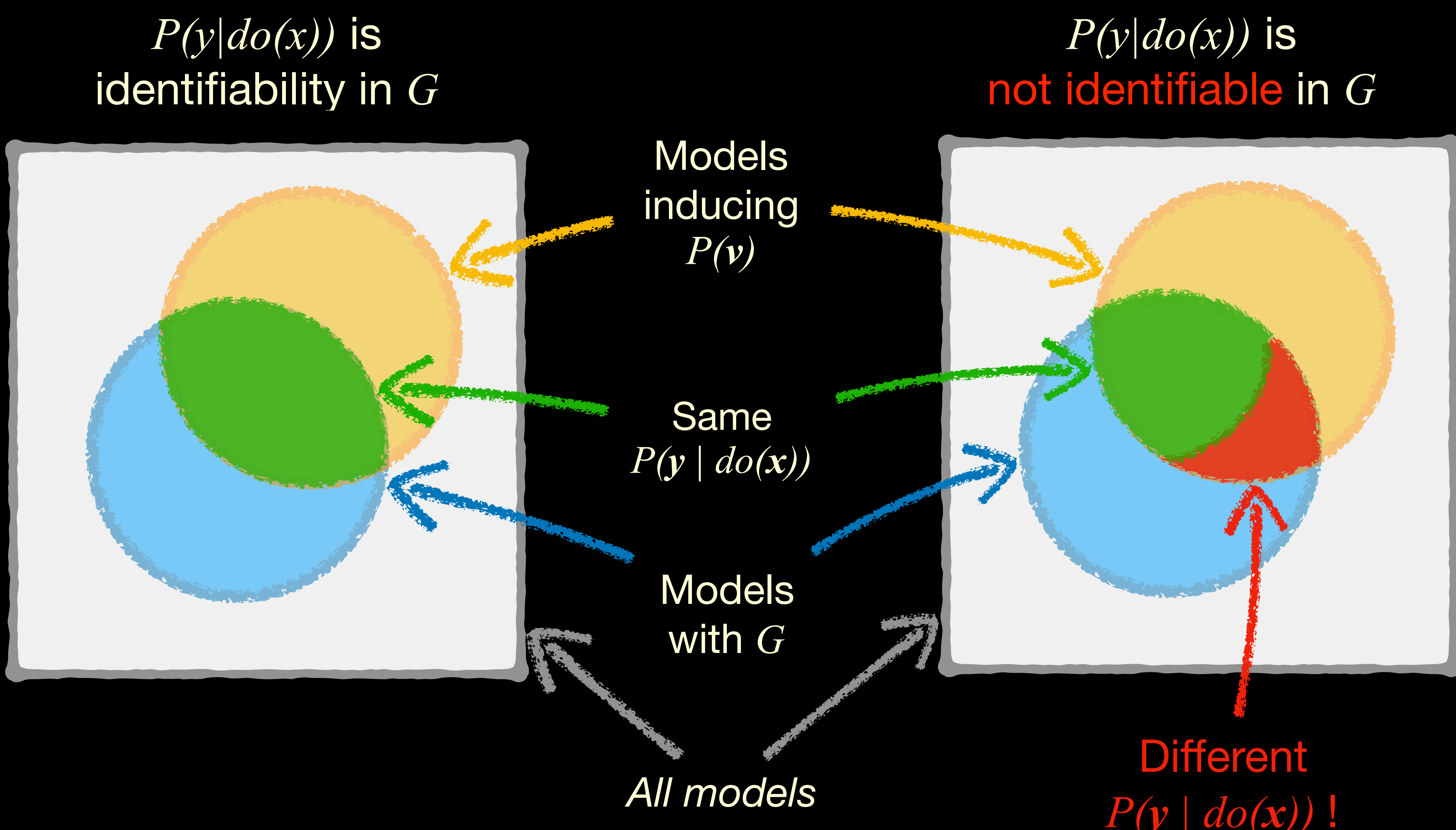

ADJUSTMENT BY DIRECT PARENTS

- Theorem 3.2.5

Given a causal diagram G of any Markovian model in which a subset V of variables are measured, the causal effect $Q = P(y \mid do(x))$ is identifiable whenever $\{X, Y, Pa_X\} \subseteq V$, that is, whenever X, Y, and all parents of variables X are measured. The expression of Q is then obtained by adjustment for PA_X, or

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$$Q = \sum_{PAx} P(y \mid x, PA_x = pa_x)\, P(PA_x = pa_x)$$

IDENTIFICATION IN MARKOVIAN MODELS

- Corollary 3.2.6

Given the causal diagram G of any Markovian model in which all variables are measured, the causal effect $Q = P(y \mid do(x))$ is identifiable for every subsets of variables X and Y and is obtained from the truncated factorization, i.e.,

IDENTIFICATION IN MARKOVIAN MODELS

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$$P(y \mid do(x)) = \sum_{V \setminus Y \setminus X} \prod_{\{i \mid X_i \notin X\}} P(x_i \mid pa_i)$$

IF SEASON IS LATENT, IS THE EFFECT STILL COMPUTABLE?

Queries:

Q_1 = Pr(wet | Sprinkler = on)
= $P(p_1) + P(p_2)$

Q_2 = Pr(wet | do(Sprinkler = on))
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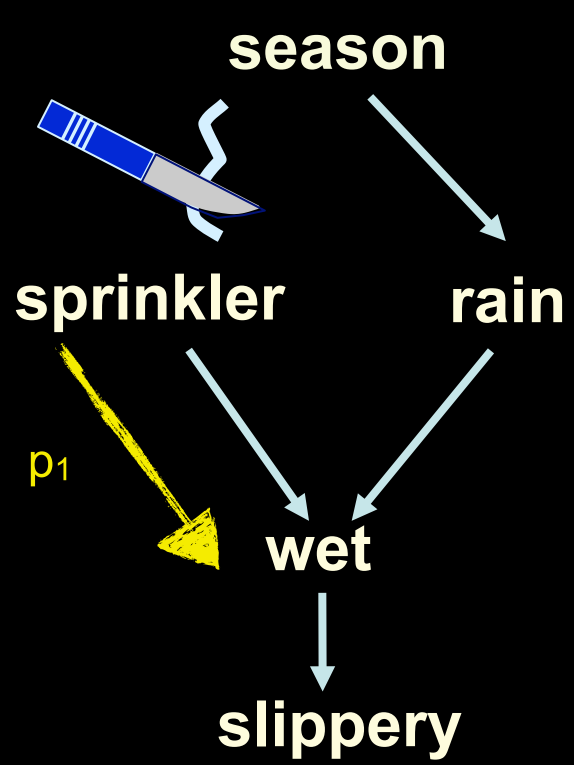

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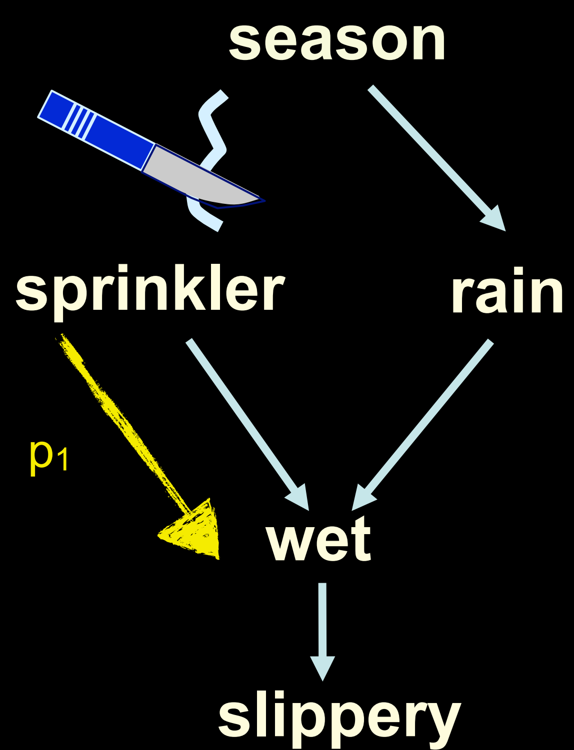

Q_2 $= \Pr(\text{wet} \mid do(\text{Sprinkler} = \text{on}))$
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$$\sum_{Se,Ra,Sl} P(Se)\, \cancel{P(Sp \mid Se)}\, P(Ra \mid Se)\, P(We \mid Sp, Ra)\, P(Sl \mid We)$$

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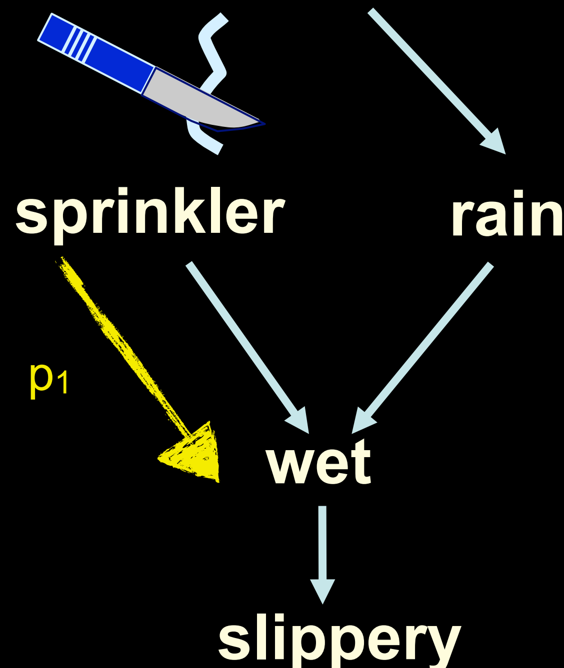

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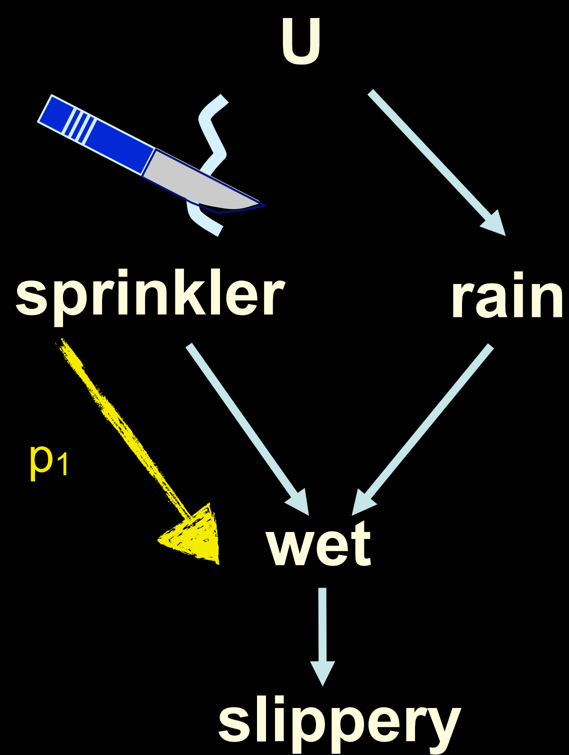

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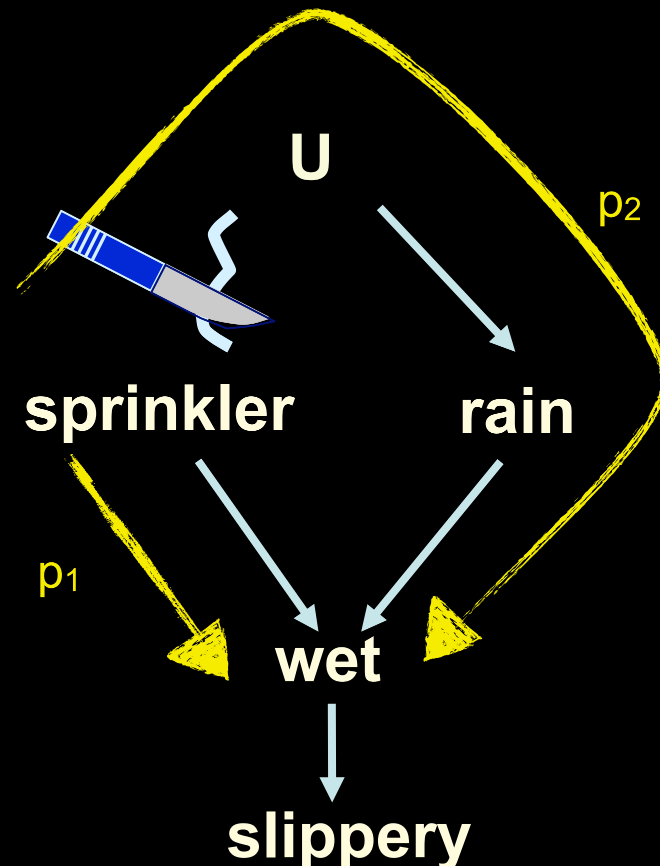

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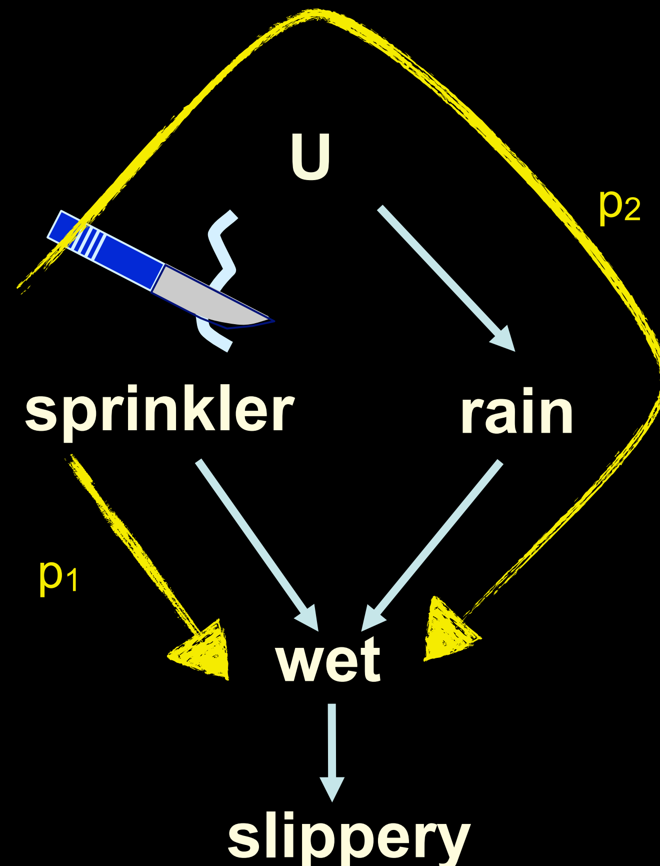

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$$= \sum_{Se} P(We \mid Sp, Se)\, P(Se)$$

IF SEASON IS LATENT, IS THE EFFECT STILL COMPUTABLE?

Queries:

$Q_1 = \Pr(\text{wet} \mid \text{Sprinkler} = \text{on})$
$\quad = P(p_1) + P(p_2)$

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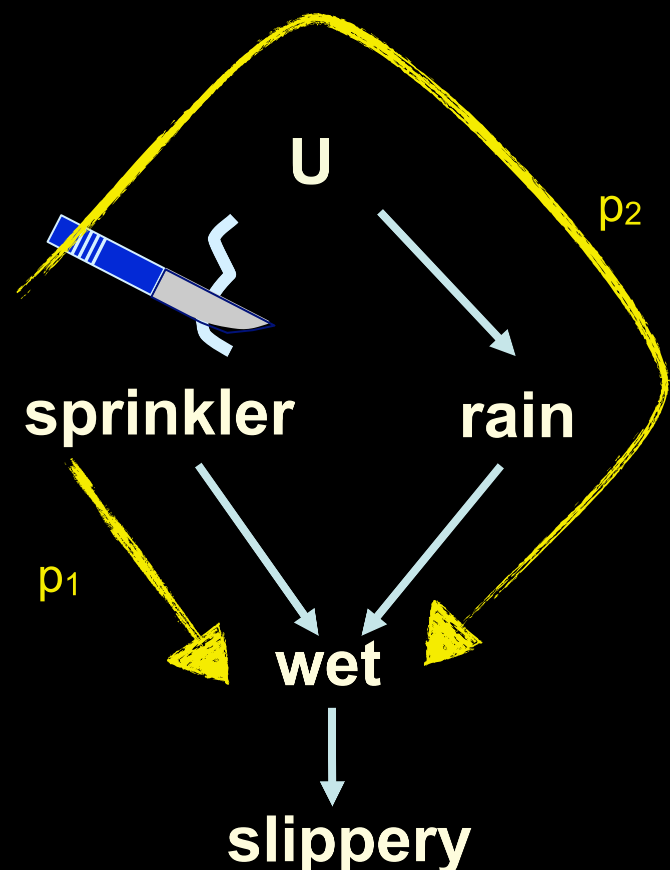

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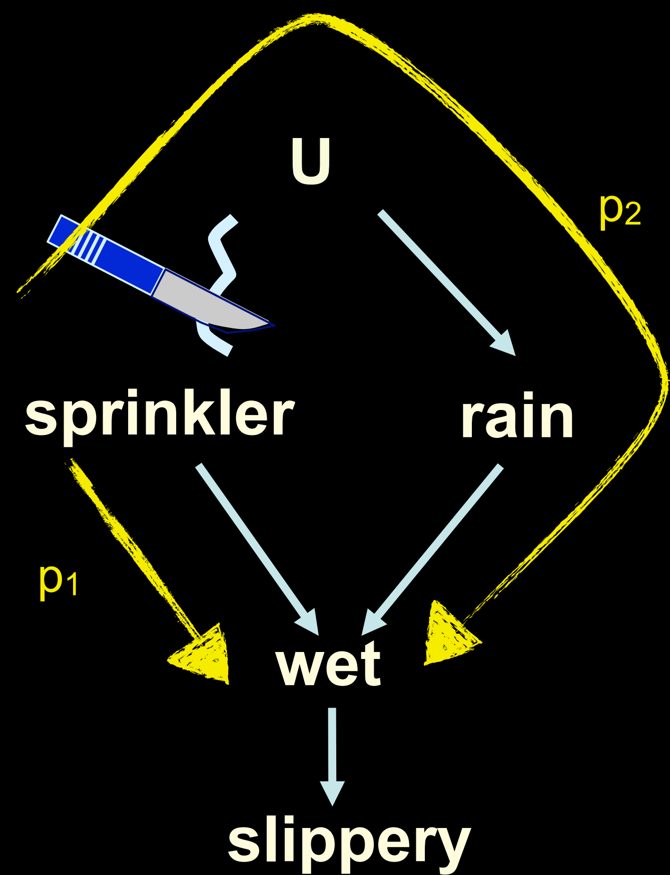

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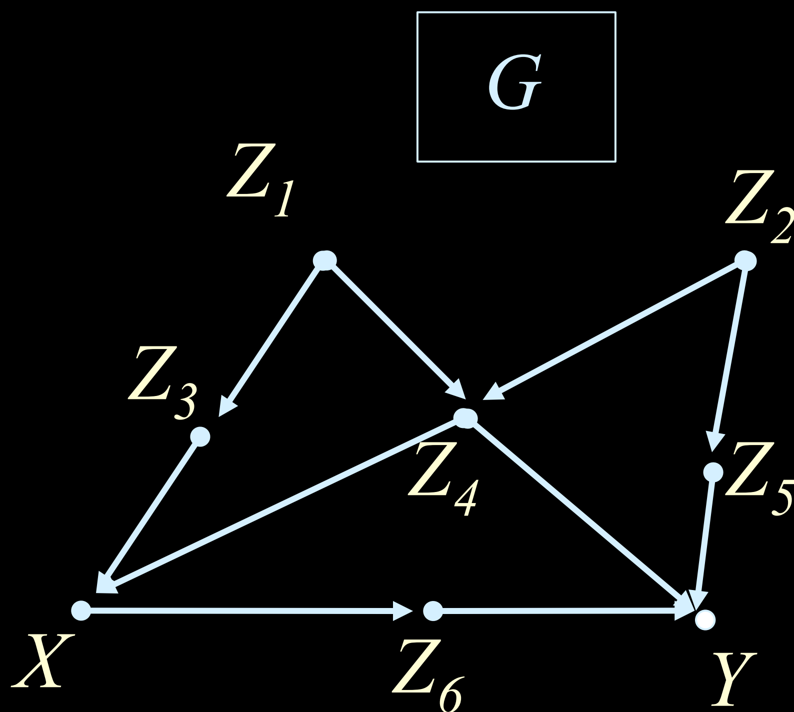
Adjustment formula

THE PROBLEM OF CONFOUNDING

Goal: Find the effect of X on Y, Q = $P(y|do(x))$, given measurements on auxiliary variables $Z_1,\ldots, Z_k$



Can Q be estimated if only a subset Z can be measured?

THE BACK-DOOR CRITERION

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- Definition 3.3.1 (Back-door)

A set Z satisfies the back-door criterion relative to an ordered pair of variables (X, Y) in a DAG G if:

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If a set Z satisfies the bd-c relative to (X, Y) then the causal effect of X on Y is identifiable and given by:

$$P(y \mid do(x)) = \sum_z P(y \mid x, z)\, P(z)$$

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(graphical condition)

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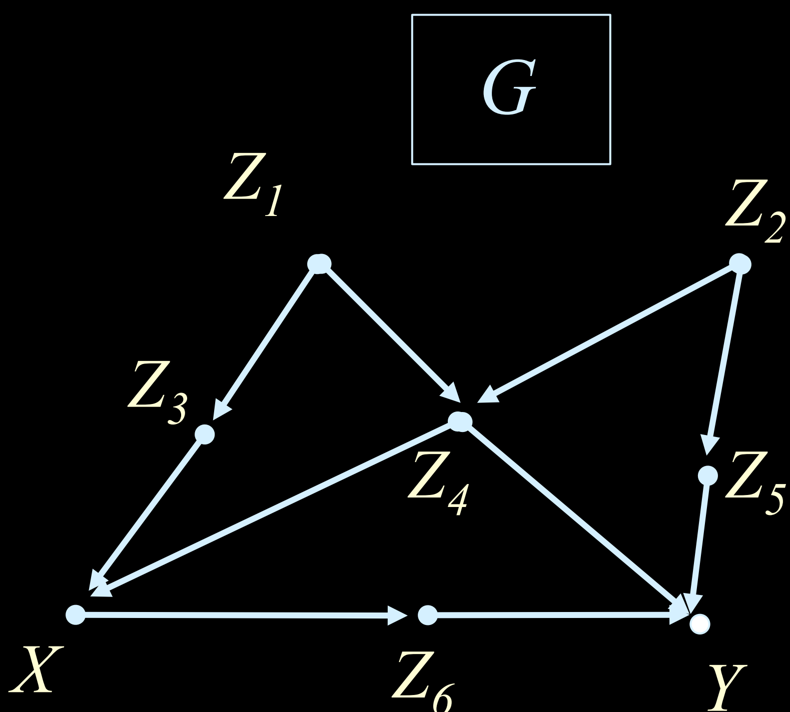

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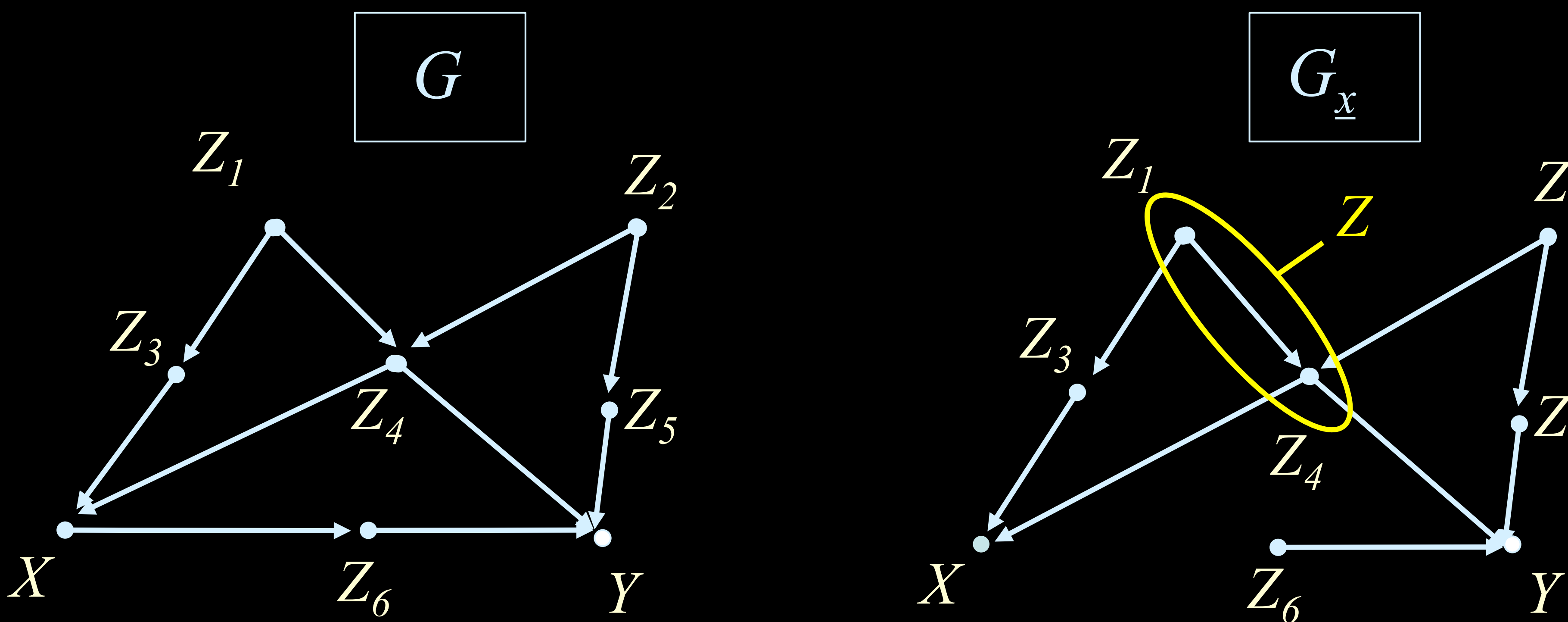
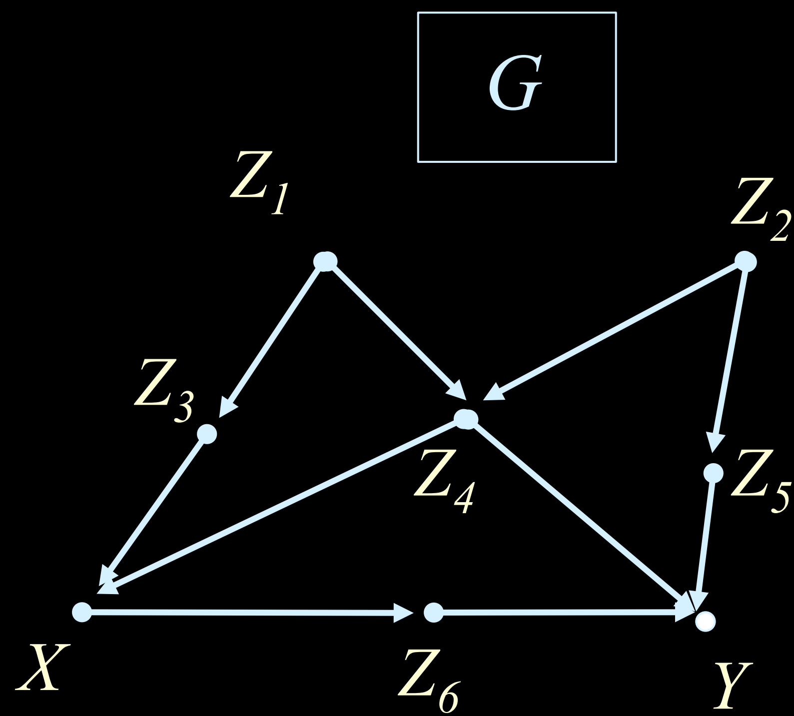


GOING BEYOND ADJUSTMENT: THE FRONT-DOOR CRITERION



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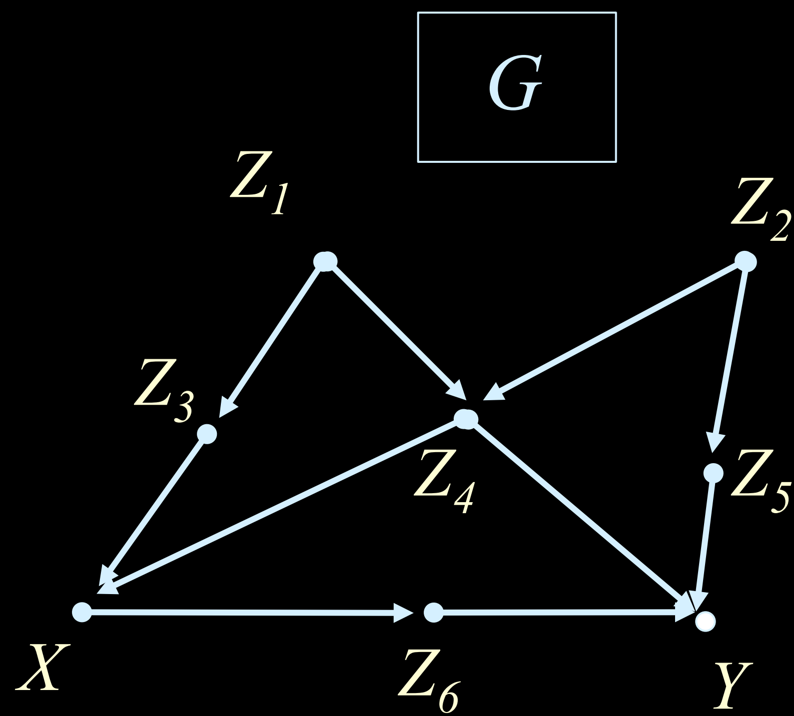

Suppose Z_1, Z_2, Z_3, Z_4, and Z_5 are not measured.

Can we compute $Q = P(y \mid do(x))$?

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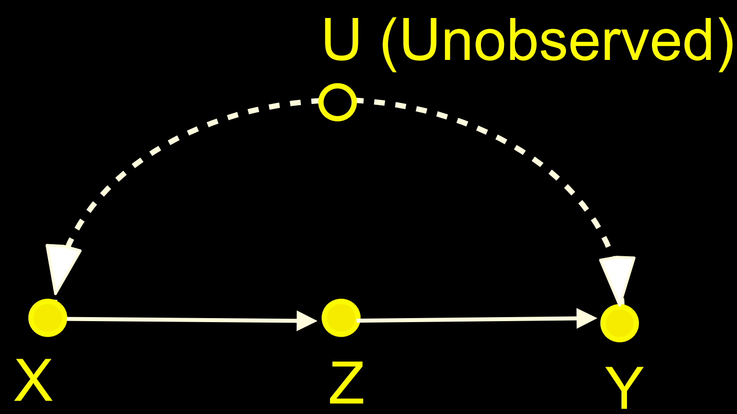

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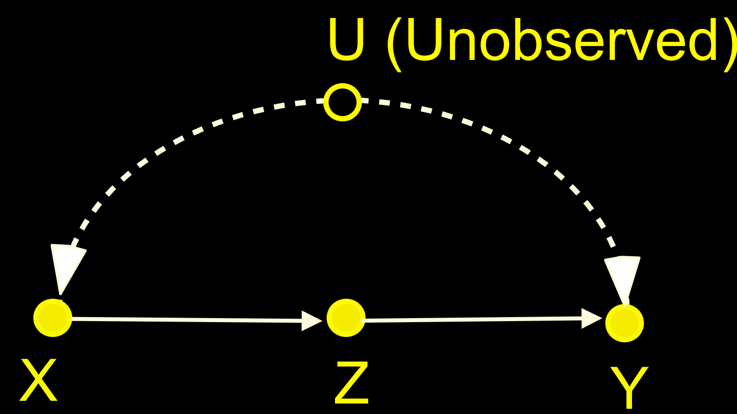

$$P(x, y, z, u) = P(u)\, P(x \mid u)\, P(z \mid x)\, P(y \mid z, u)$$

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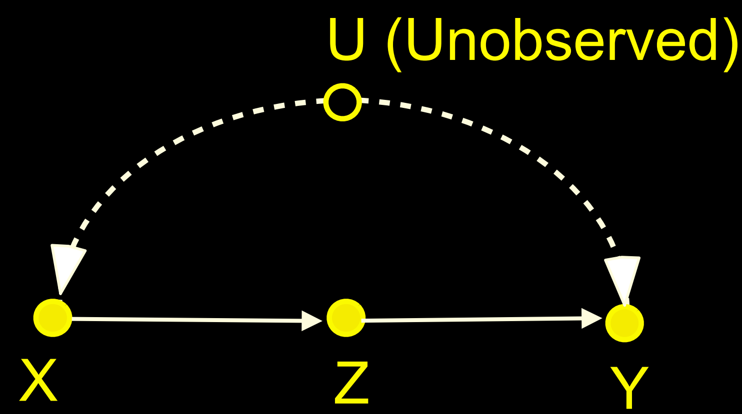

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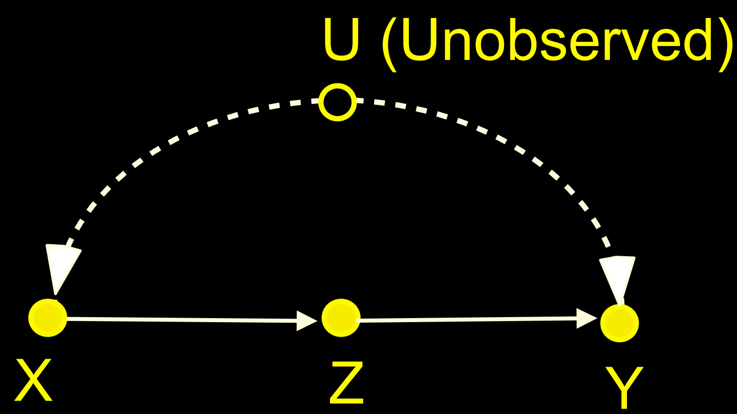

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(or, do-calculus)

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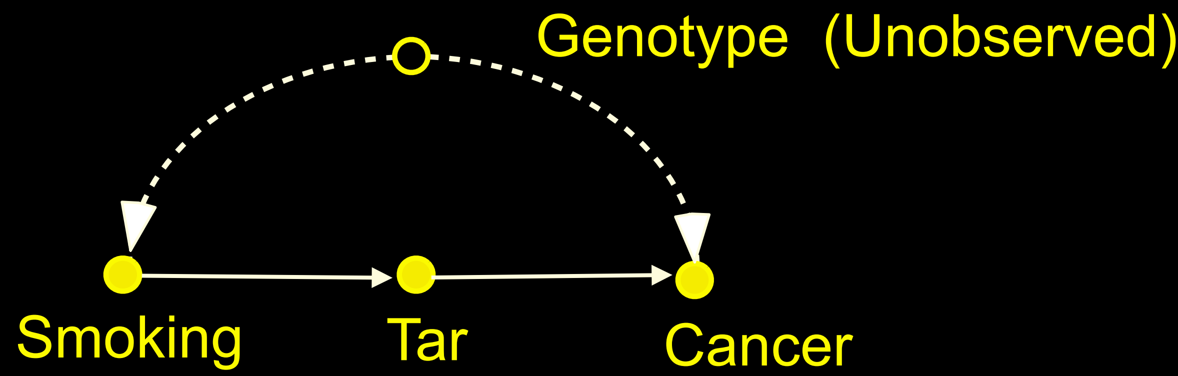
P(*y* | do(x), *do*(*z*), *w*) = *P*(*y* | *do*(*x*), *w*), if $(Y \perp\!\!\!\perp Z | X, W)_{G_{\overline{X}\,\overline{Z(W)}}}$

where Z(W) is the set of Z-nodes that are not ancestors of any W-node in G_X.

DERIVATION IN CAUSAL CALCULUS



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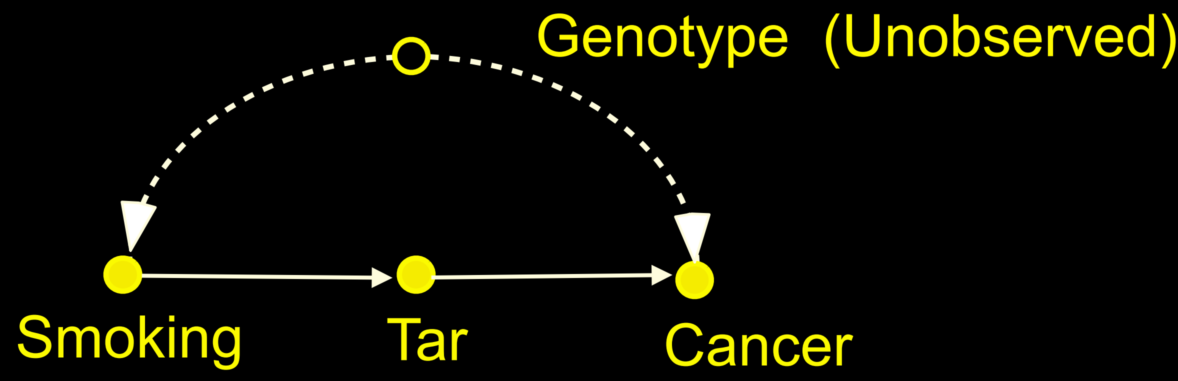
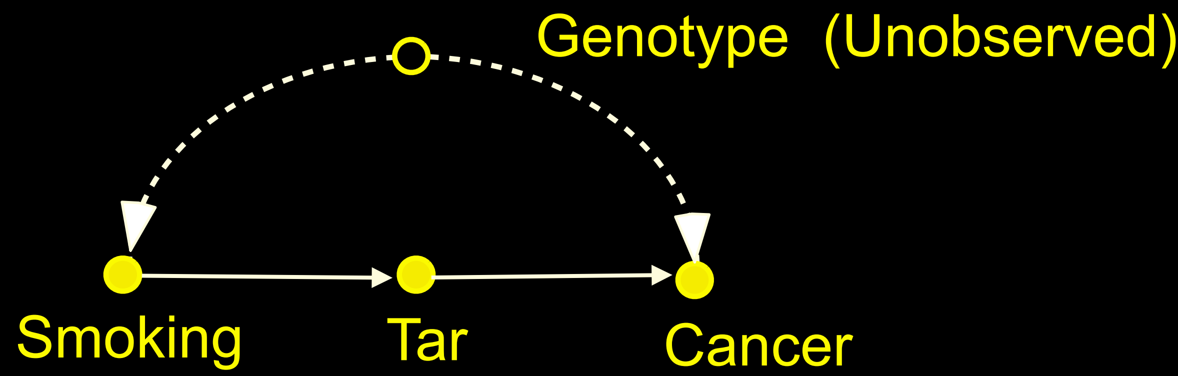
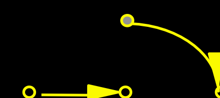


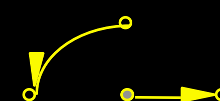
$P(c \mid do(s))$

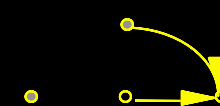
DERIVATION IN CAUSAL CALCULUS



$P(c \mid do(s)) = \Sigma_t\, P(c \mid do(s), t)\, P(t \mid do(s))$ Probability Axioms

$= \Sigma_t\, P(c \mid do(s), do(t))\, P(t \mid do(s))$ Rule 2 

$= \Sigma_t\, P(c \mid do(s), do(t))\, P(t \mid s)$ Rule 2 

$= \Sigma_t\, P(c \mid do(t))\, P(t \mid s)$ Rule 3 

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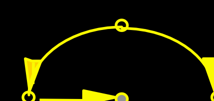
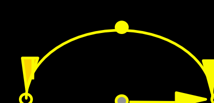
$= \Sigma_{s'} \Sigma_t\, P(c \mid t, s')\; P(s' \mid do(t))\, P(t \mid s)$ Rule 2 

$= \Sigma_{s'} \Sigma_t\, P(c \mid t, s')\; P(s')\, P(t \mid s)$ Rule 3 

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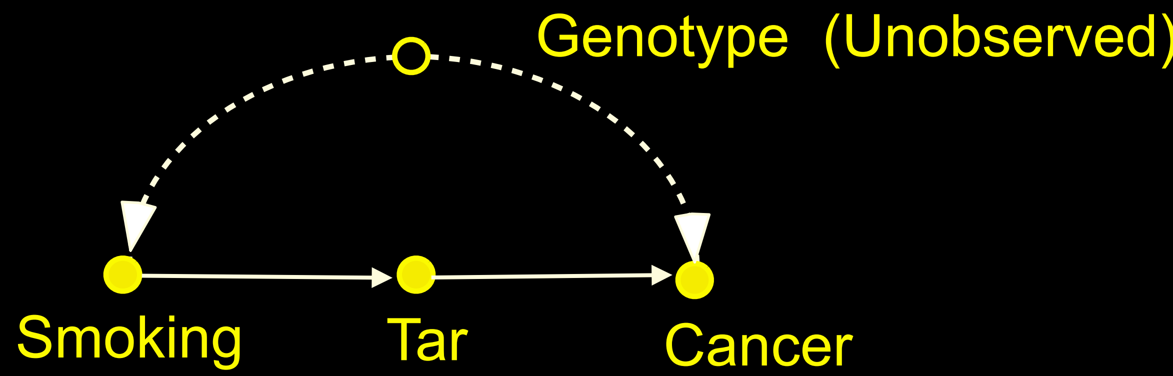

query

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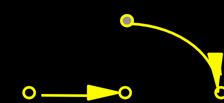
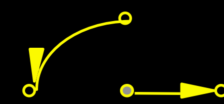
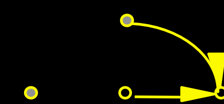
$= \Sigma_t P(c \mid do(s), do(t))\, P(t \mid do(s))$ Rule 2 

$= \Sigma_t P(c \mid do(s), do(t))\, P(t \mid s)$ Rule 2 

$= \Sigma_t P(c \mid do(t))\, P(t \mid s)$ Rule 3 

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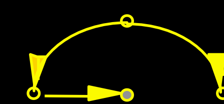
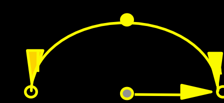
$= \Sigma_{s'} \Sigma_t P(c \mid t, s')\, P(s' \mid do(t))\, P(t \mid s)$ Rule 2 

$= \Sigma_{s'} \Sigma_t P(c \mid t, s')\, P(s')\, P(t \mid s)$ Rule 3 

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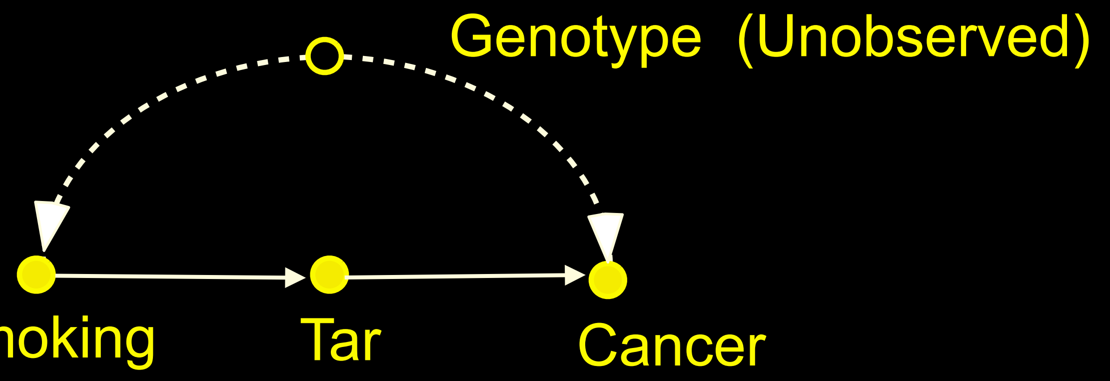

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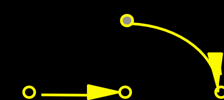
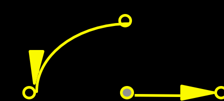
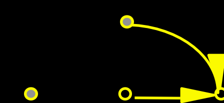
$$= \Sigma_t P(c \mid do(s), do(t))\, P(t \mid do(s))$$ Rule 2 

$$= \Sigma_t P(c \mid do(s), do(t))\, P(t \mid s)$$ Rule 2 

$$= \Sigma_t P(c \mid do(t))\, P(t \mid s)$$ Rule 3 

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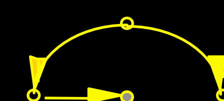
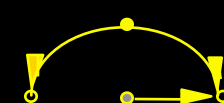
$$= \Sigma_{s'} \Sigma_t P(c \mid t, s')\; P(s' \mid do(t))\, P(t \mid s)$$ Rule 2 

$$= \Sigma_{s'} \Sigma_t P(c \mid t, s')\; P(s')\, P(t \mid s)$$ Rule 3 

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CBNs WITH LATENT VARIABLES

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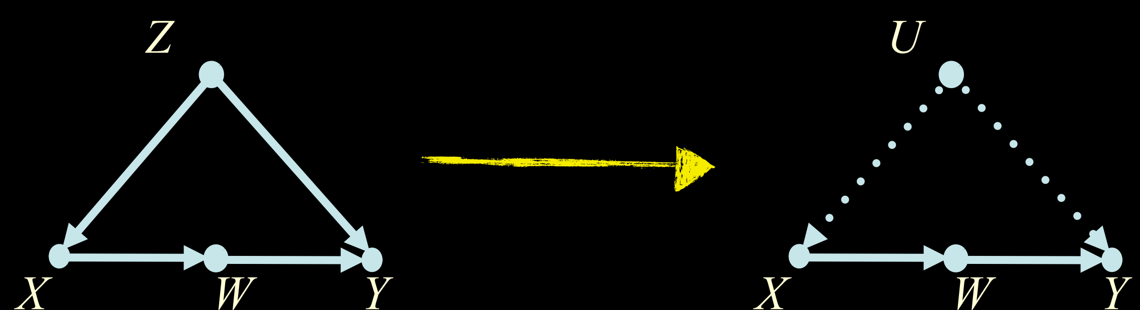

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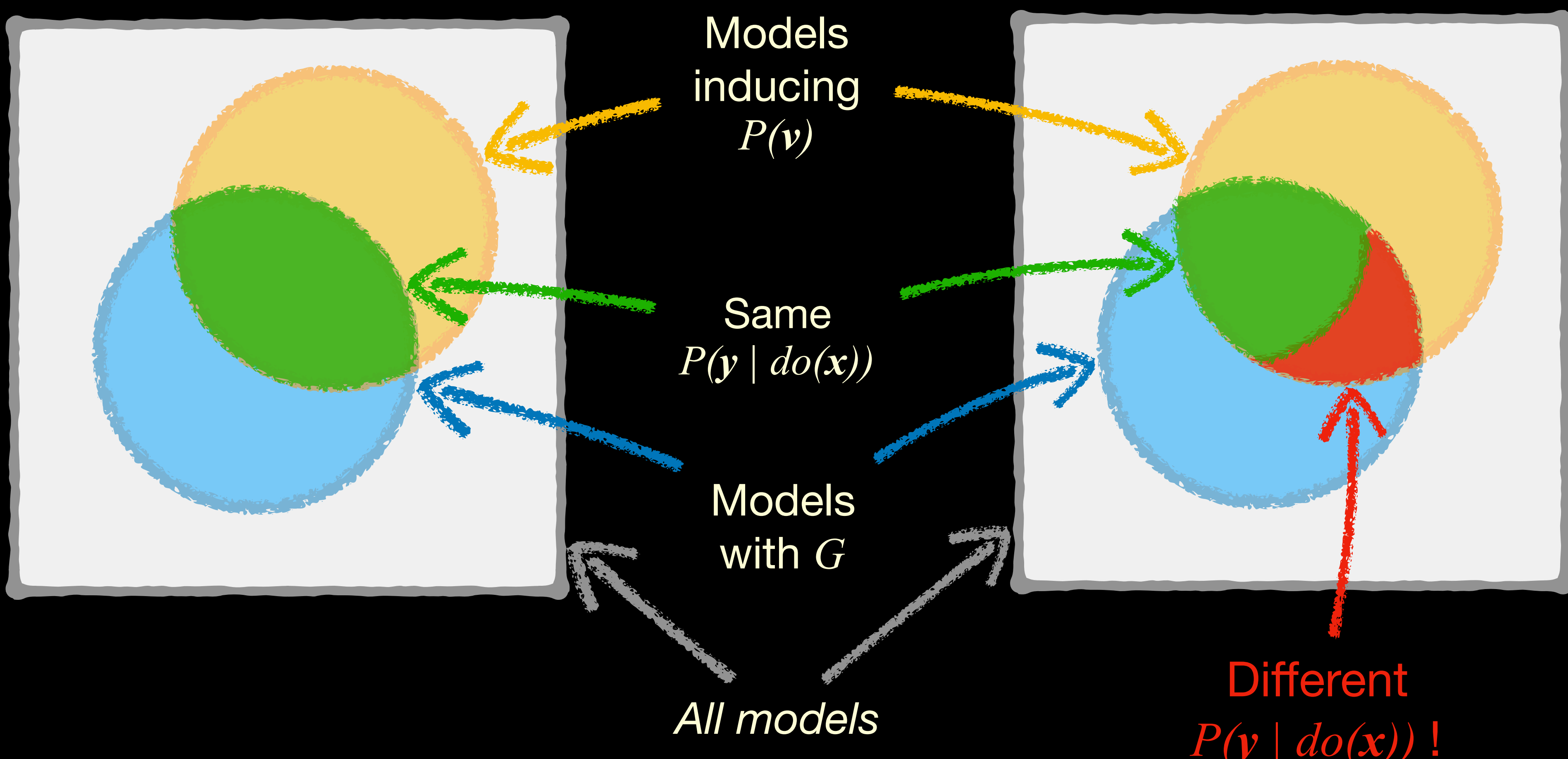

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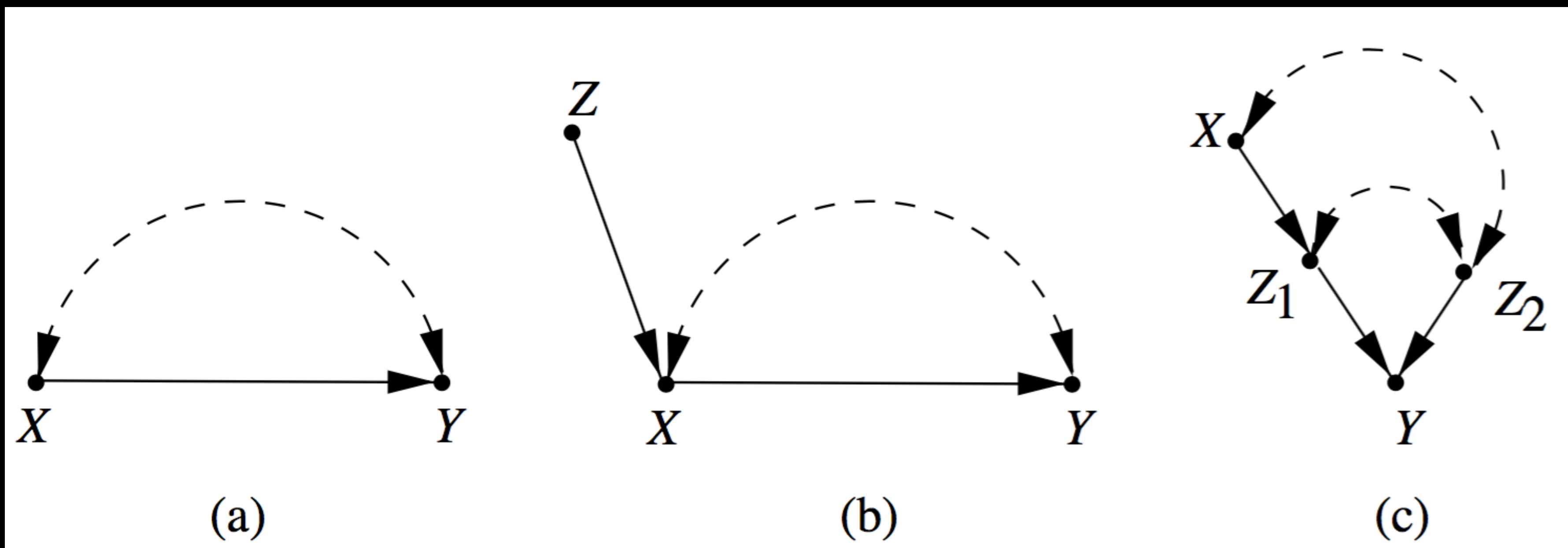

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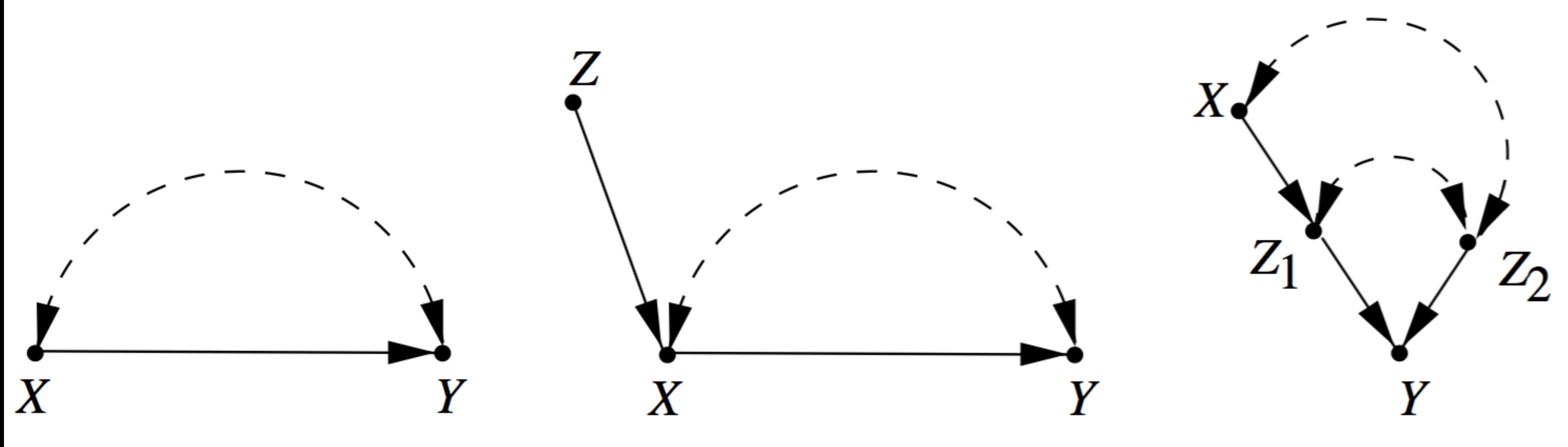

(a) (b) (c)

- Bow-arc observation

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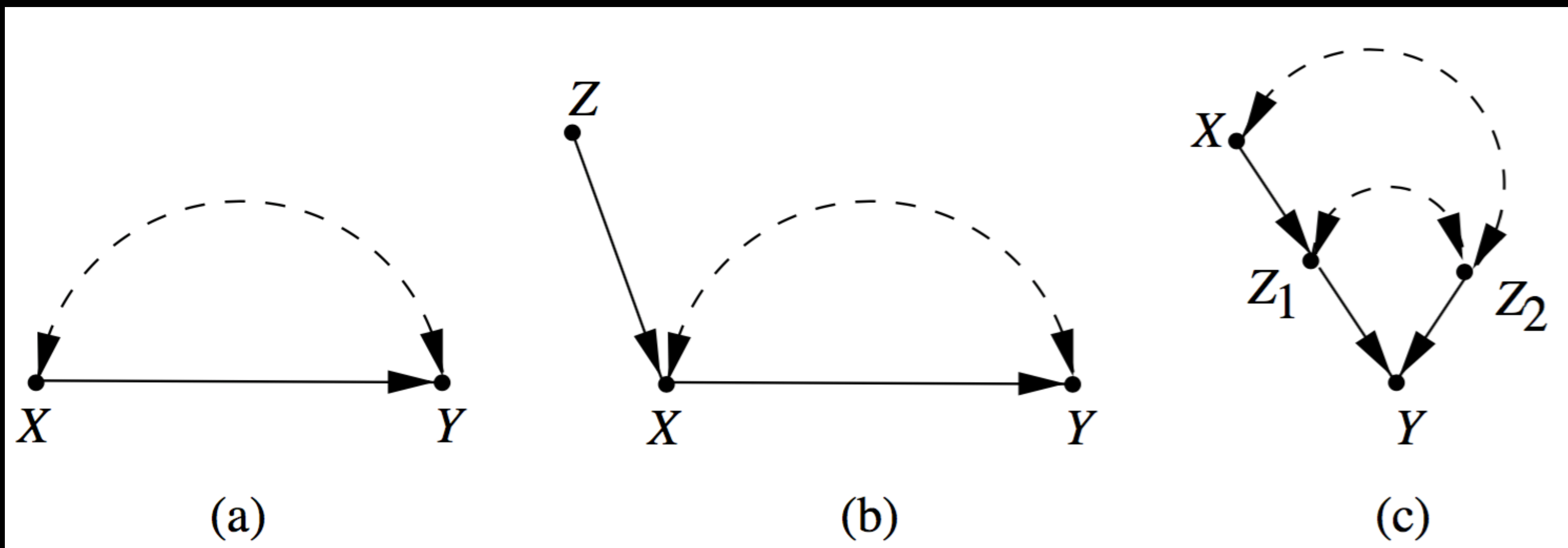
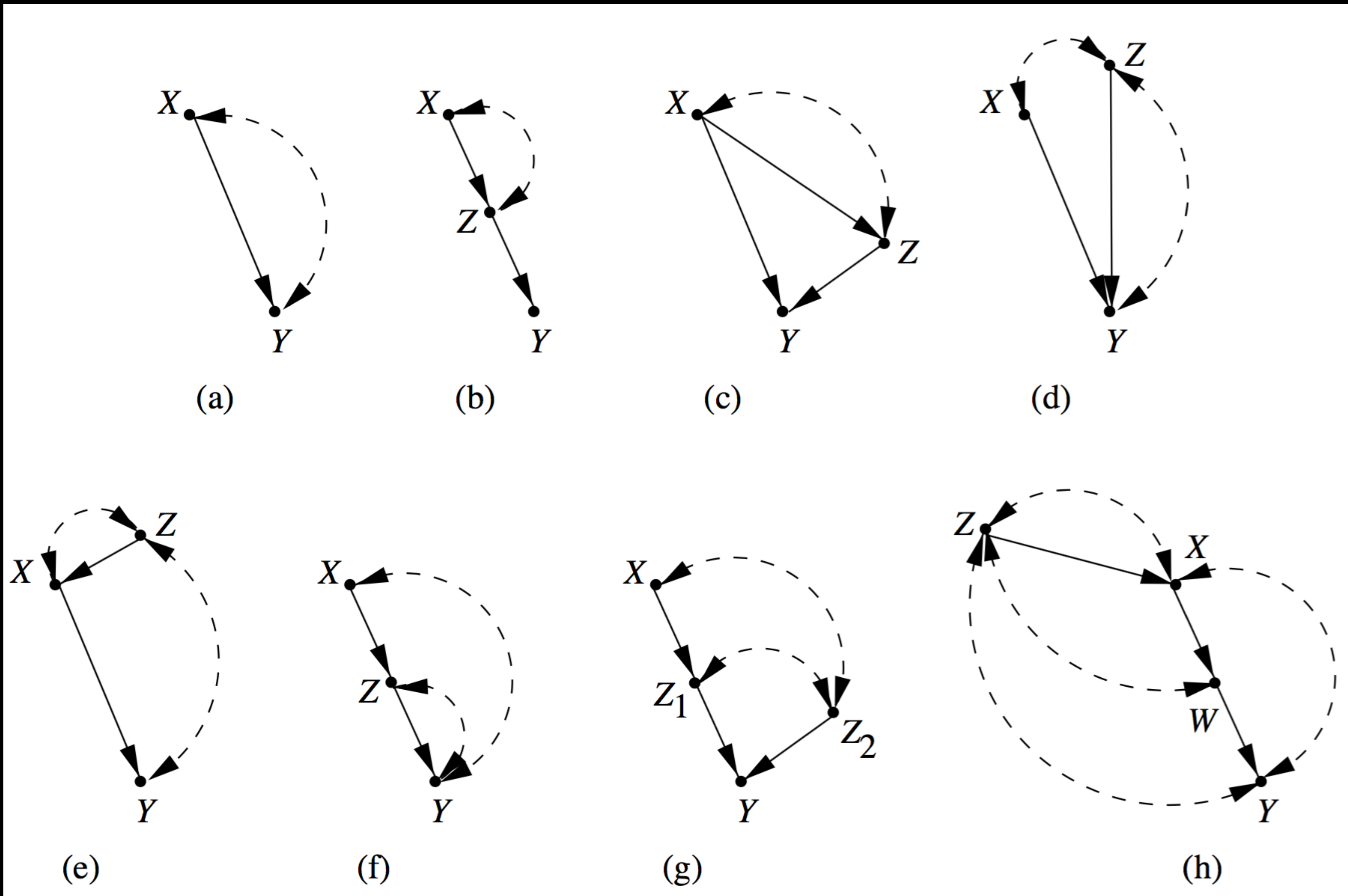


- Bow-arc observation
- $P(y \mid do(x, z_2))$ versus $P(y \mid do(x), z_2)$

NON-IDENTIFABLE MODELS (EXAMPLES)



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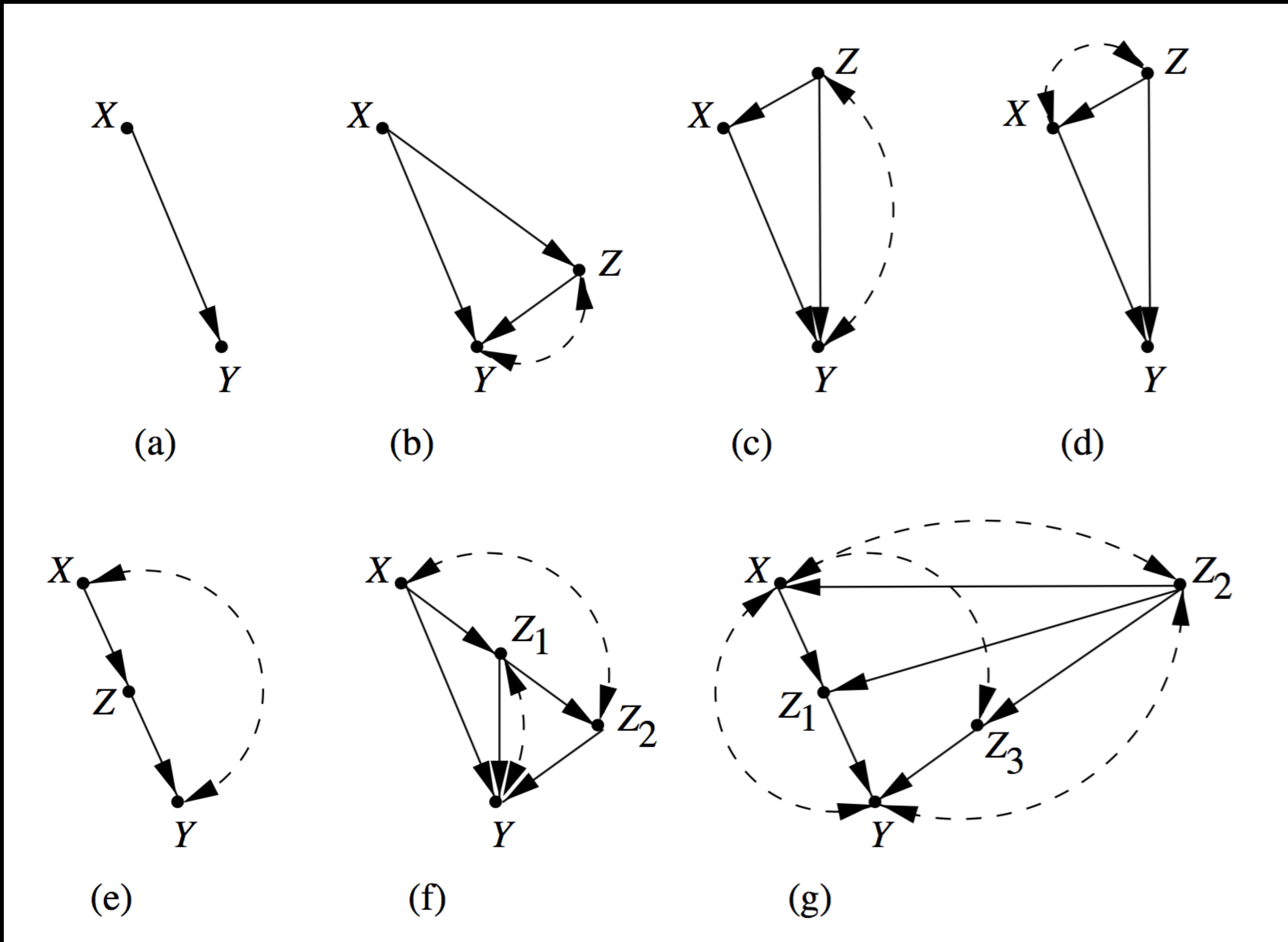

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$$= \sum_z P(y \mid do(X = g(z)), z) \; P(z)$$

$$= E_z[P(y \mid do(x), z) \mid_{x = g(z)}]$$

(Conditioning on Z might create dependencies that will prevent the successful reduction of $P(y \mid do(x), z)$ to a do-free expression.)

IDENTIFYING THE EFFECT OF STOCHASTIC POLICIES

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Given $Z = z$, the intervention $do(X = x)$ will occur with probability $P^*(x \mid z)$ and will produce a causal effect given by $P(y \mid do(x), z)$. Averaging over x and z gives

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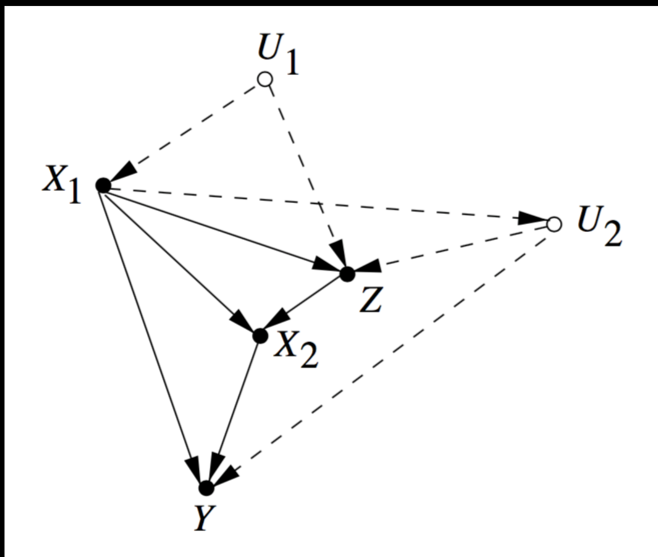
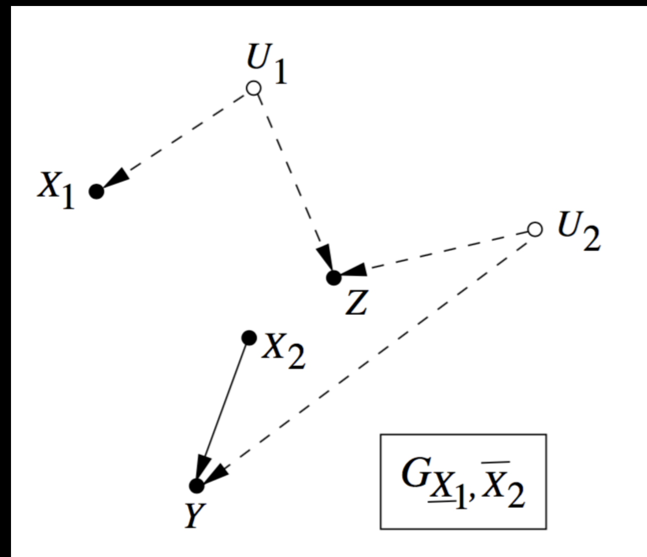
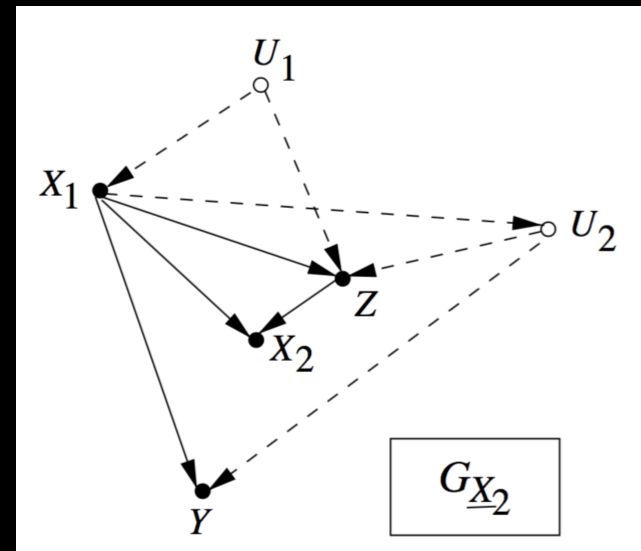
P*(x | z) is specified externally. Therefore, the identifiability of P(y | do(x), z) is a necessary and sufficient condition for the identifiability of any stochastic policy that shapes the distribution of X by the outcome Z.

IDENTIFYING THE EFFECT OF PLANS







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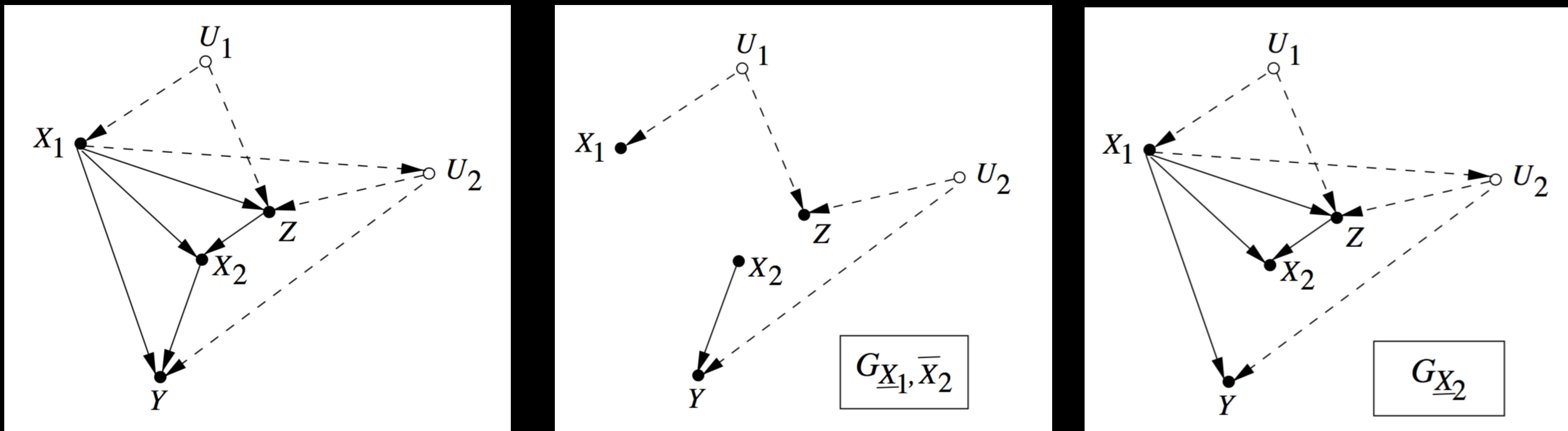

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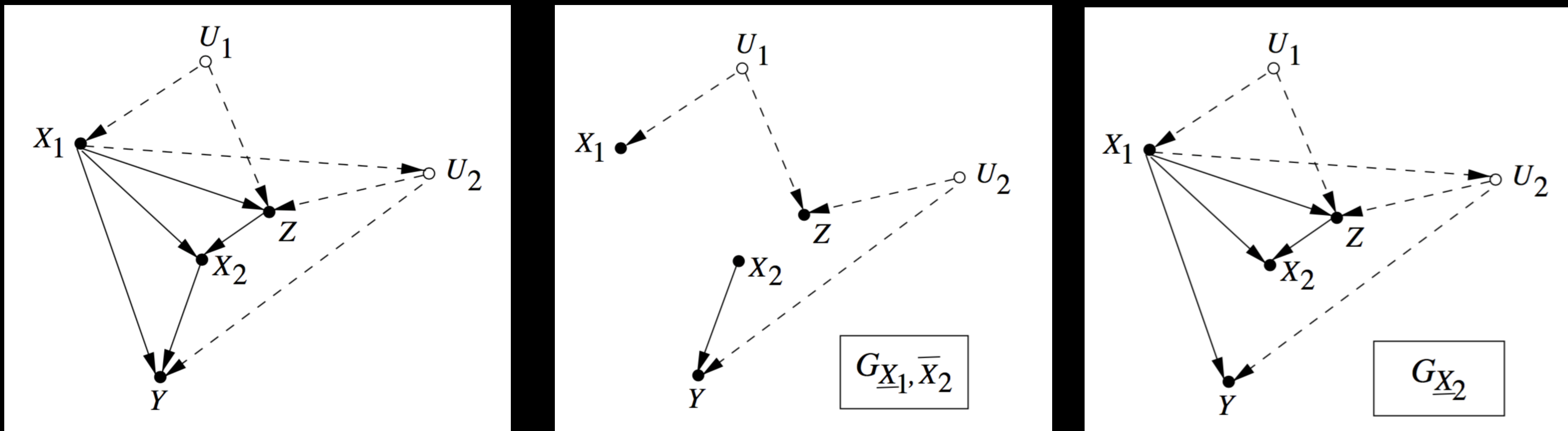

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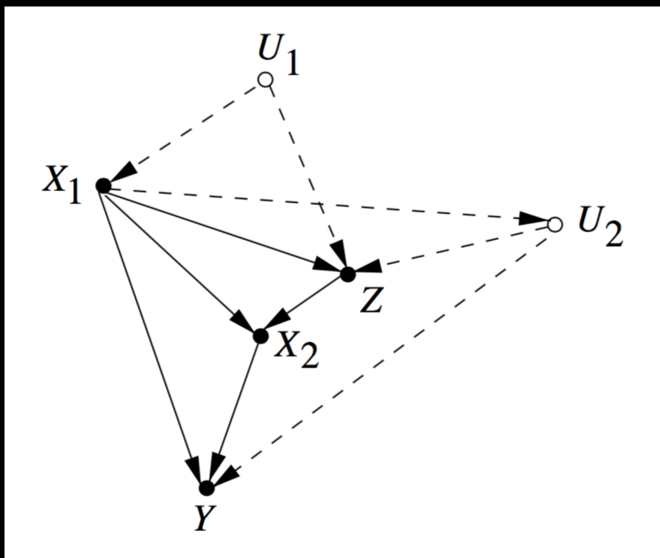

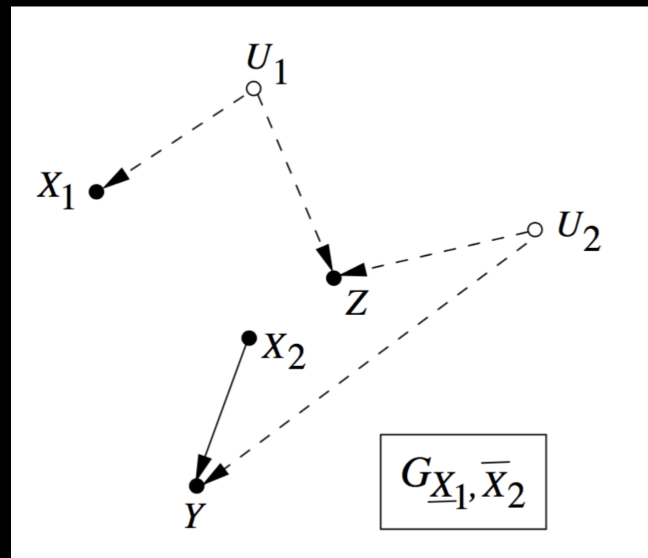

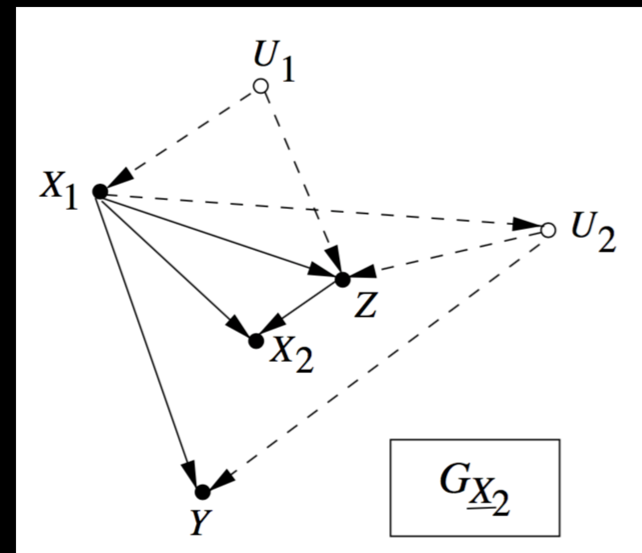

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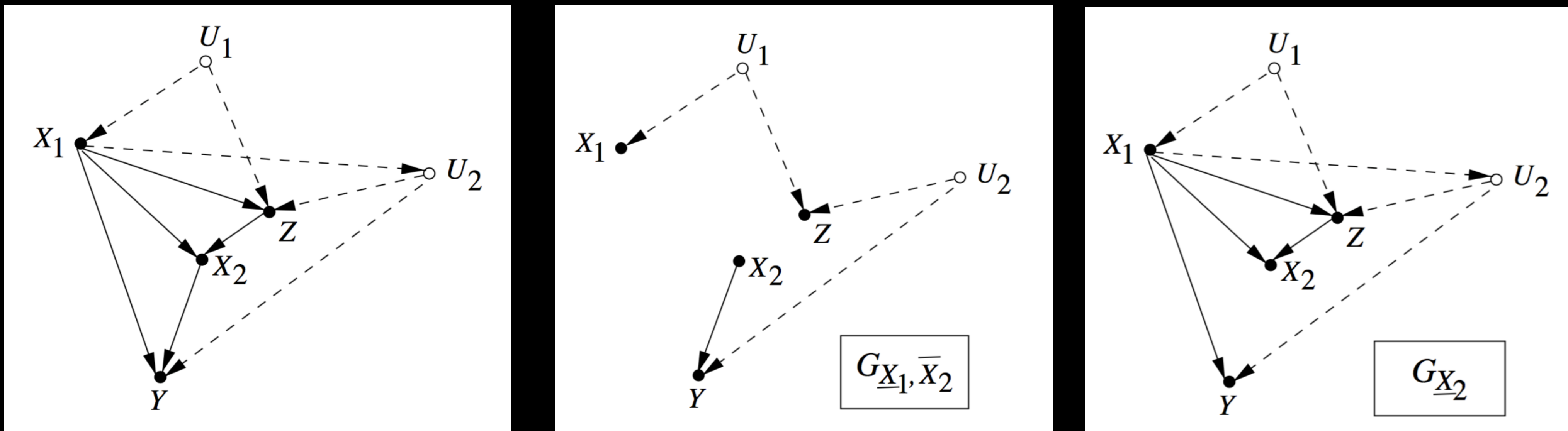

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Simpson’s Paradox
(Pearson, 1899, Yule, 1903, Simpson 1951)

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- Pearson's shock: "spurious correlation"

We are thus forced to the conclusion that a mixture of heterogenous groups, each of which exhibits in itself no organic correlation, will exhibit a greater or less amount of correlation. This correlation may properly be called spurious, yet as it is almost impossible to guarantee the absolute homogeneity of any community, our results for correlation are always liable to an error, the amount of which cannot be foretold. To those who persist on looking upon all correlation as cause and effect, the fact that correlation can be produced between two quite uncorrelated characters A and B by taking an artificial mixture of two closely allied races, must come as rather a shock.

[Pearson, Lee, Brandy-Moore (1899)]

- Causation = perfect correlation
- "Not all correlations are causal" (Aldrich, 1994)

Simpson's Paradox
(Pearson, 1899, Yule, 1903, Simpson 1951)

- e.g., UC Berkeley's alleged sex bias in graduate admission (Science, 1975)

Overall, data showed a higher rate of admission among male applicants, but, broken by departments, data showed a slight bias in favor of admitting female applicants.

- e.g., "reverse regression" (1970-1980)

Should one, in salary discrimination cases, compare salaries of equally qualified men and women, or, instead, compare qualifications of equally paid men and woman. (Opposite conclusions.)

Simpson's Paradox

(Robot: seeing and making sense)

	surv. (Y)	not-surv. (¬Y)		Recovery Rate
drug (X)	20	20	40	50%
no-drug (¬X)	16	24	40	40%
	36	44		

$P(Y \mid F, X) < P(Y \mid F, \neg X)$

$P(Y \mid \neg F, X) < P(Y \mid \neg F, \neg X)$

but

$P(Y \mid X) > P(Y \mid \neg X)$!!

male (¬F)	surv. (Y)	not-surv.		Recovery Rate
drug (X)	18	12	30	60%
no-drug (¬X)	7	3	10	70%
	25	15	40	

female (F)	surv. (Y)	not-surv. (¬Y)		Recovery Rate
drug (X)	2	8	10	20%
no-drug (¬X)	9	21	30	30%
	11	29	40	

Simpson's paradox

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- It will not be possible to obtain:

$$P(Y \mid do(X)) > P(Y \mid do(\neg X))$$

Proof

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- $P(Y \mid do(X)) = P(Y \mid do(X), F)\, P(F \mid do(X))$
  $+ P(Y \mid do(X), \neg F)\, P(\neg F \mid do(X))$

Proof

- $P(Y \mid do(X)) = P(Y \mid do(X), F)\, P(F \mid do(X))$

$$+ P(Y \mid do(X), \neg F)\, P(\neg F \mid do(X))$$

$$= P(Y \mid do(X), F)\, P(F) + P(Y \mid do(X), \neg F)\, P(\neg F)$$

Proof

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Proof

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$$P(Y \mid do(X)) < P(Y \mid do(\neg X))$$

WOULD DECISION-MAKING BE DIFFERENT WITH A DIFFERENT STORY?

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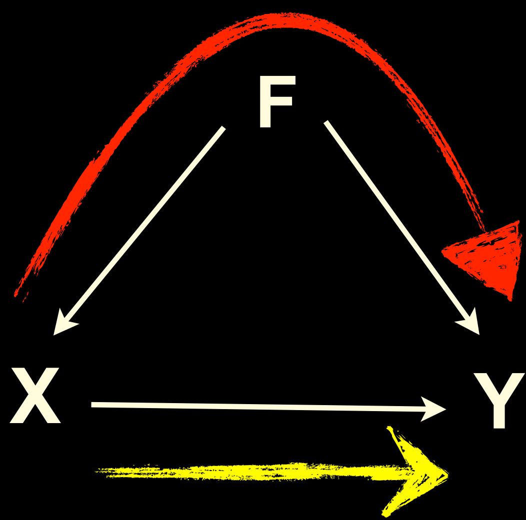
G_1:



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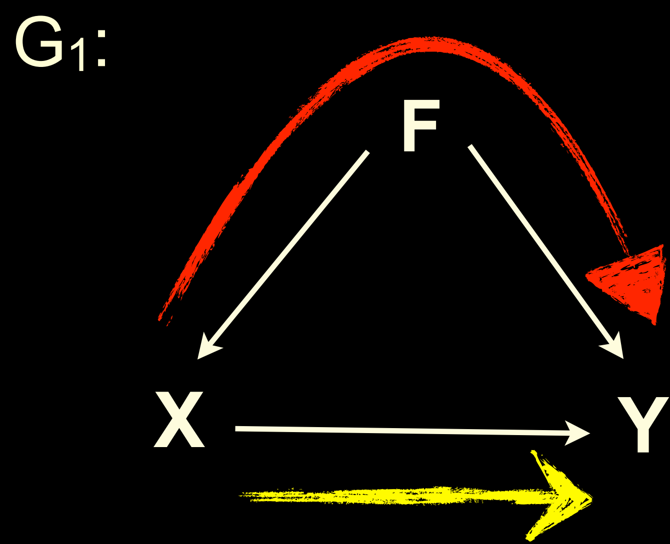

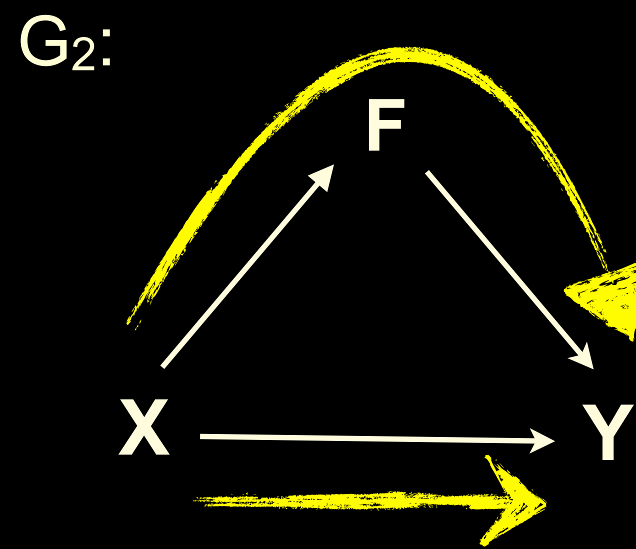

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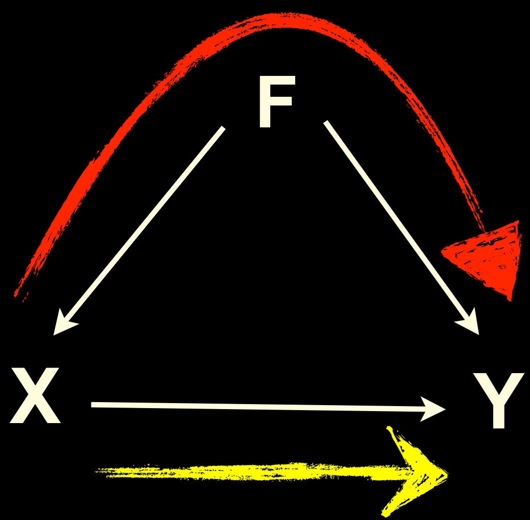

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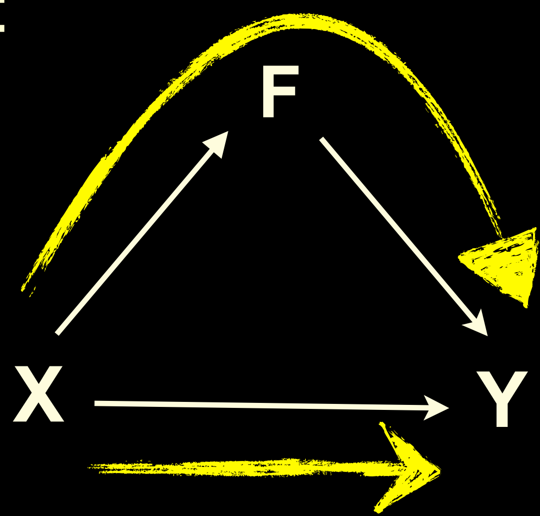

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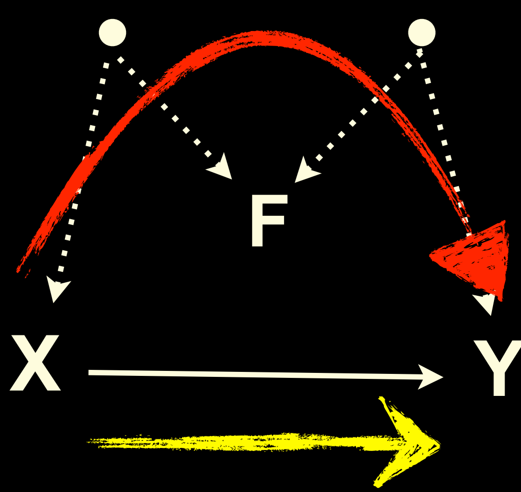

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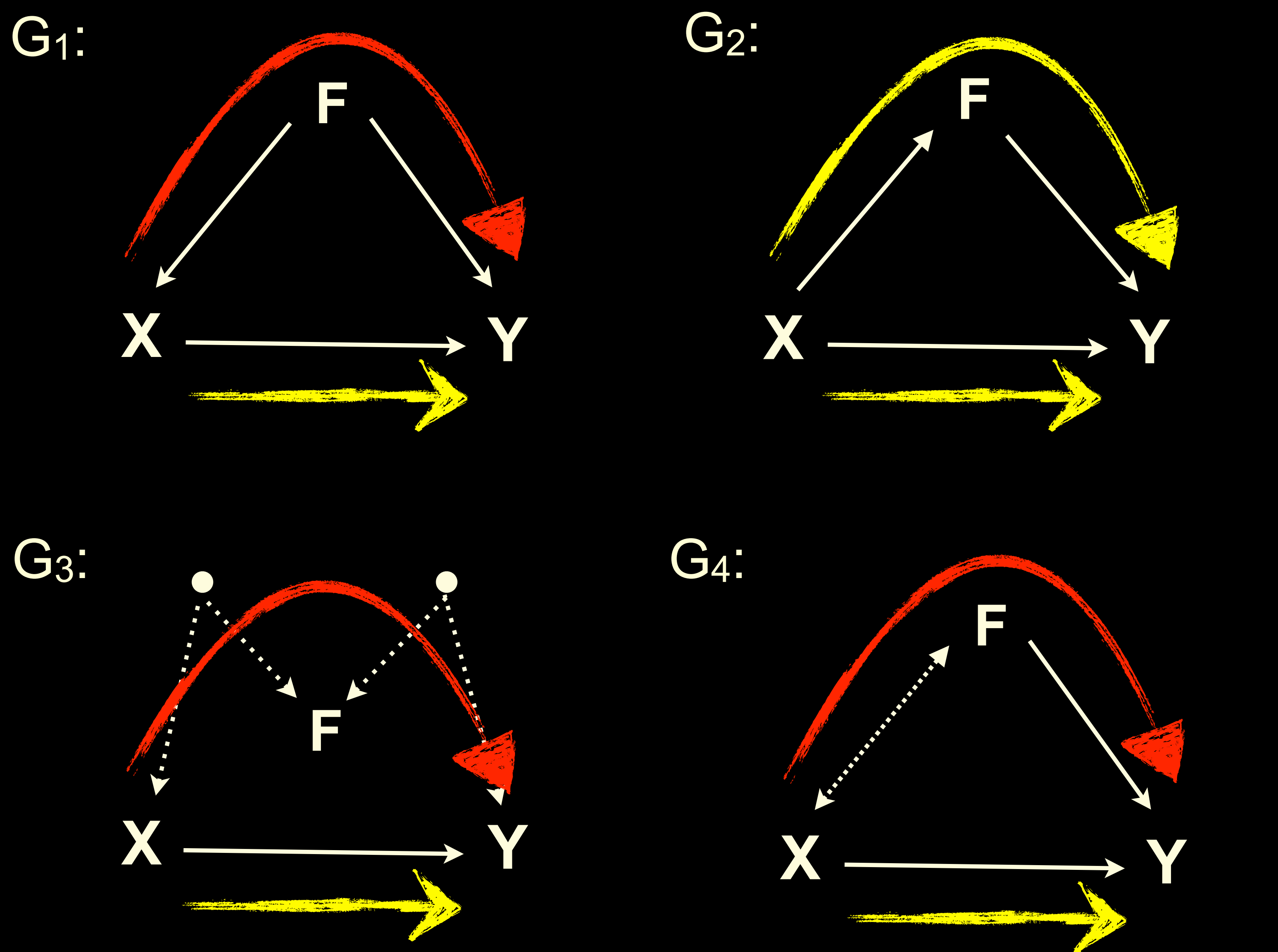

THE SURE THING PRINCIPLE
(formalizing Savage, 1972)

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Theorem 6.1.1. An action X that increases the probability of an event Y in each subpopulation must also increase the probability of Y in the population as a whole, provided that the action does not change the distribution of the sub-populations.

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Summary.

- **Any** statistical relationship can be reversed by including additional factors in the analysis
- “Reversal” is possible in the calculus of proportions, but impossible in the calculus of causes
- People thinks causes, not proportions

SUMMARY

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- The do-calculus provides a syntactical characterization to the problem of policy evaluation for static and dynamic interventions.
- The problem of confounding and identification is essentially solved, non-parametrically.
- Simpson’s Paradox is mathematized and dissolved.
- Applications are pervasive in the social and health sciences as well as in statistics, machine learning, and artificial intelligence.

OUTLINE

- Introduction
  * Data, data, data…
  * Basic definitions
- Technical Results
  * The truncated product formula
  * The back-door adjustment formula
  * The front-door adjustment formula
  * The do-calculus
- Tasks
  * Policy evaluation
  * Generalizability and Robustness
  * Decision-making & RL

OUTLINE

SUMMARY - BIG PICTURE

1. query

2. model

3. data

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$Q = P^*(y \mid do(x))$

2 model

3 data

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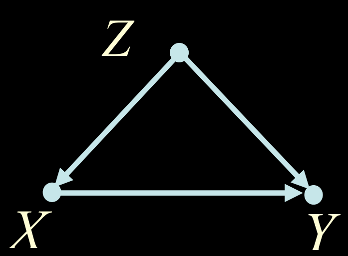

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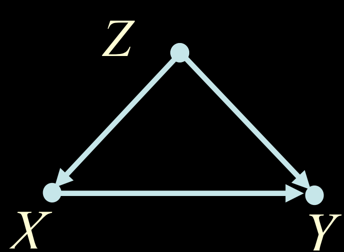

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P(x, y, z)

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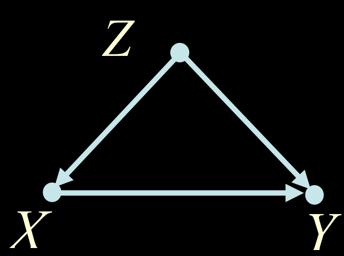

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P(x, y, z)

inference engine

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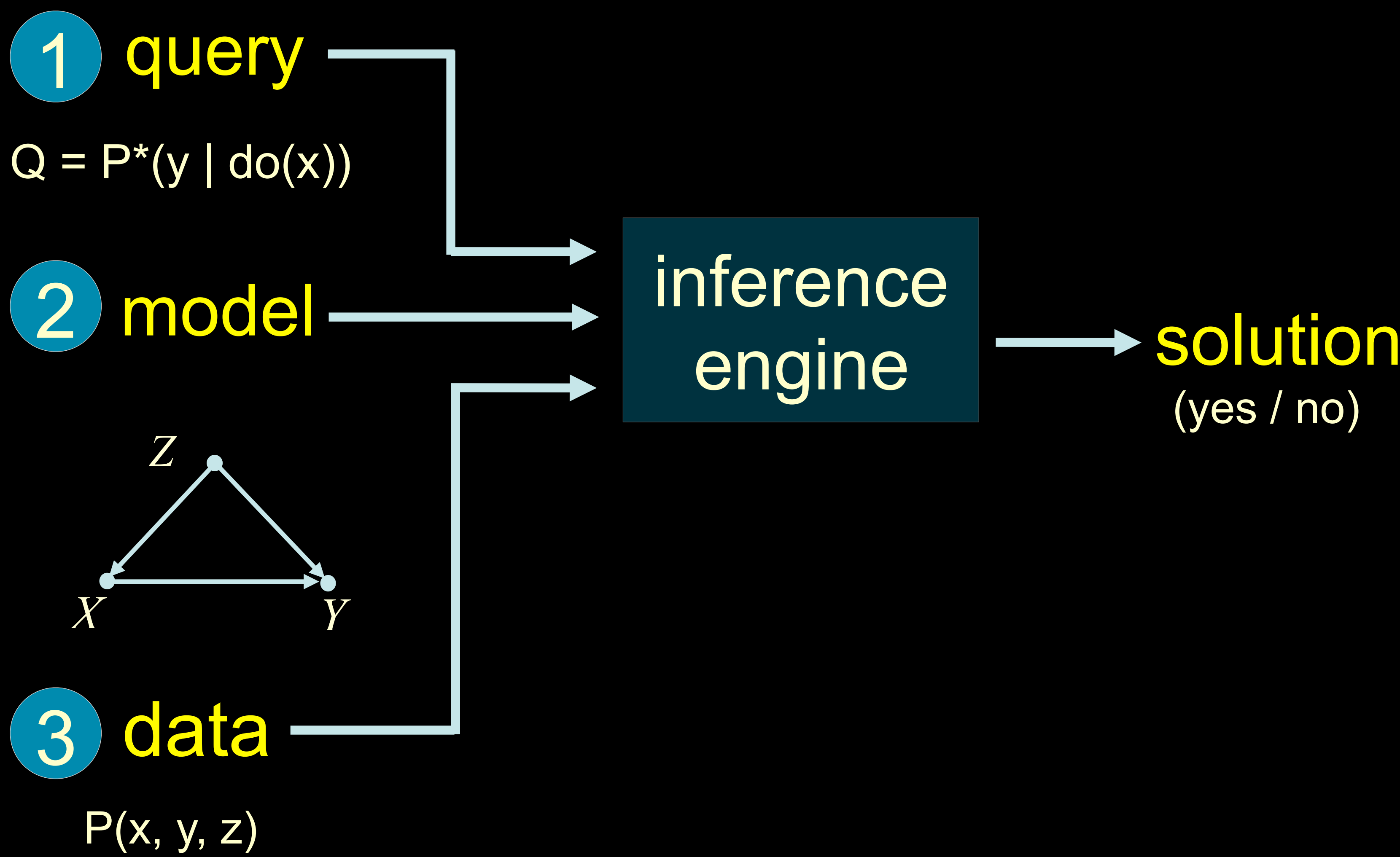

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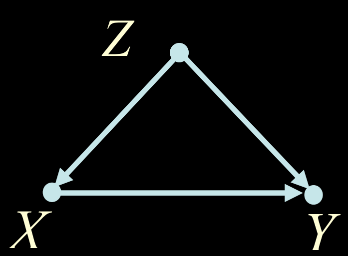

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(yes / no)

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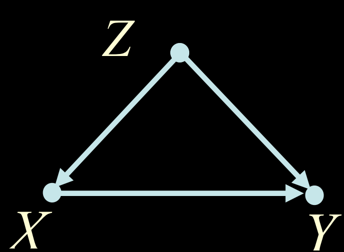

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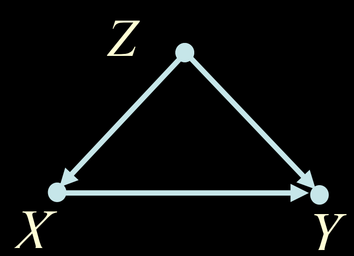

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$$P(y \mid do(x)) = \sum_Z P(y \mid x, z)\, P(z)$$

MOTIVATION FOR DATA-FUSION

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Target population Π^*

Query of interest $Q = P^*(y \mid do(x))$

(a) **US**

Census data available

MOTIVATION FOR DATA-FUSION

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Census data available	Survey data resembling target

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(a) **US**	(b) **New York**	(c) **Los Angeles**
Census data available	Survey data resembling target	Survey data younger population

MOTIVATION FOR DATA-FUSION

Target population Π^*

Query of interest $Q = P^*(y \mid do(x))$

(a) **US** Census data available	(b) **New York** Survey data resembling target	(c) **Los Angeles** Survey data younger population
(d) **Boston** Age not recorded Mostly successful lawyers	(e) **San Francisco** High post-treatment blood pressure	(f) **Texas** Mostly Spanish subjects High attrition
(g) **Arkansas** Randomized trial College students	(h) **Utah** RCT, paid volunteers, mainly unemployed	(i) **Wyoming** Natural experiment young athletes

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(“All data is not created equal”)

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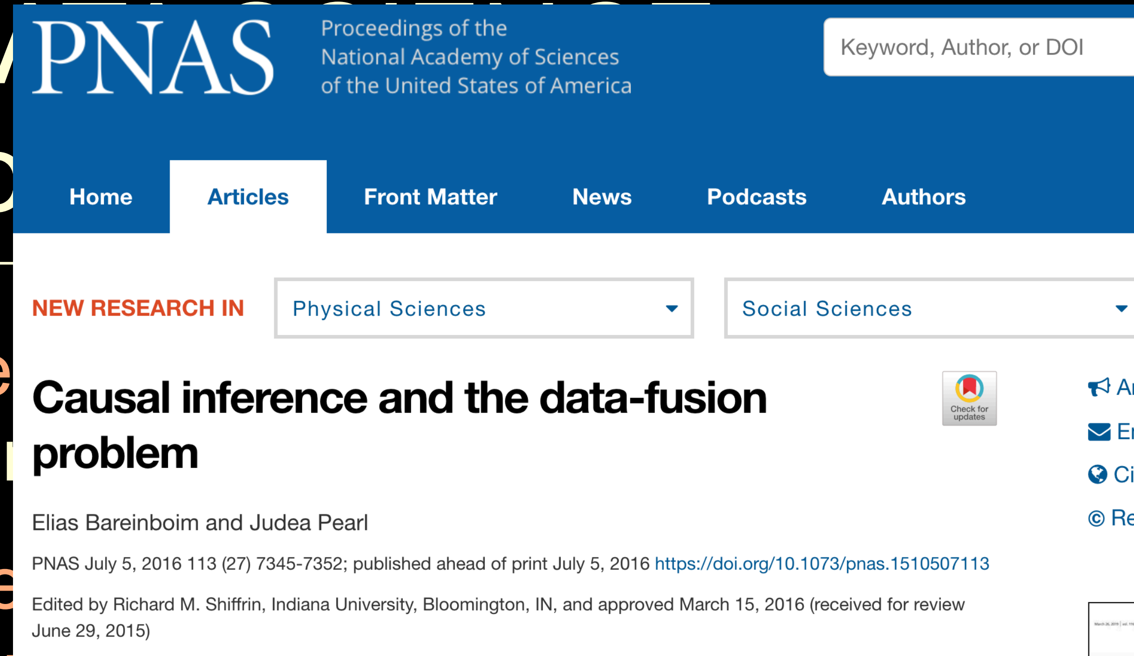

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Causal inference and the data-fusion problem

Elias Bareinboim and Judea Pearl

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Edited by Richard M. Shiffrin, Indiana University, Bloomington, IN, and approved March 15, 2016 (received for review June 29, 2015)

CAUSAL DATA SCIENCE (“All data is not created equal”)

- Heterogenous datasets are pervasive in the empirical sciences since the data is collected:

  - under different experimental conditions,
  - the underlying populations are different,
  - the sampling procedure is not random,
  - the treatment assignment is not random,
  - many variables are not measured.

- All these dimensions are now formalized.

- And there are conditions and algorithms to decide what is “entailed” from a certain data collection.

HETEROGENOUS DATASETS

Target $Q = P^*(y \mid do(x))$	Dataset 1	Dataset 2	…	Dataset n

Population	Los Angeles	New York	Texas
Obs. / Exp.	Experimental	Observational	Experimental
Treat. Assign.	Randomized Z_1	-	Randomized Z_2
Sampling	Selection on Age	Selection on SES	-
Measured	X_1, Z_1, W, M, Y_1	X_1, X_2, Z_1, N, Y_2	X_2, Z_1, W, L, M, Y_1

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d_3	Sampling	Selection on Age	Selection on SES	-
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DATA-FUSION CHALLENGES

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Description of each dataset: tuple (d_1, d_2, d_3, d_4)
(population, obs./exp., sampling, measure.)

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   $(\text{Bonobos}, d_2, d_3, d_4) \rightarrow (\text{Humans}, d_2, d_3, d_4)$

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Data-fusion:
$\{ (d_1, d_2, d_3, d_4) \} \rightarrow (d'_1, d'_2, d'_3, d'_4)$

DATA-FUSION IN 3-D

1. Experimental conditions
   - Preliminary. Observational to Experimental regime
2. Environmental conditions
3. Sampling conditions

DATA-FUSION IN 3-D

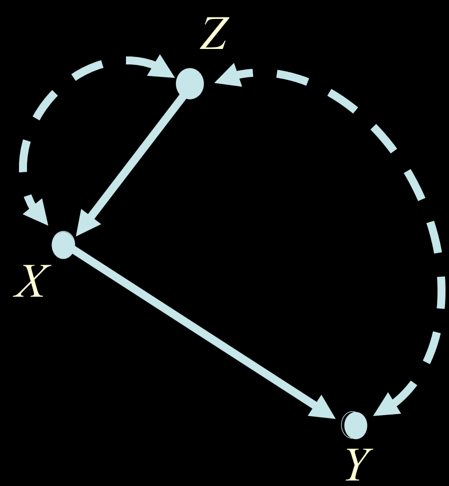
THE DATA-FUSION PROBLEM

EXPERIMENTAL CONDITIONS (D = 1) —
GENERAL IDENTIFIABILITY

THE CHALLENGE OF EXPERIMENTAL IDENTIFIABILITY

(also called z-ID)

G:



Z: Diet
X: Cholesterol level
Y: Heart Attack

Measured:

Observational study: $P(x, y, z)$

Needed: $Q = P(y \mid do(x)) = ?$

WHAT CAN EXPERIMENTS ON DIET REVEAL ABOUT THE EFFECT OF CHOLESTEROL ON HEART ATTACK?

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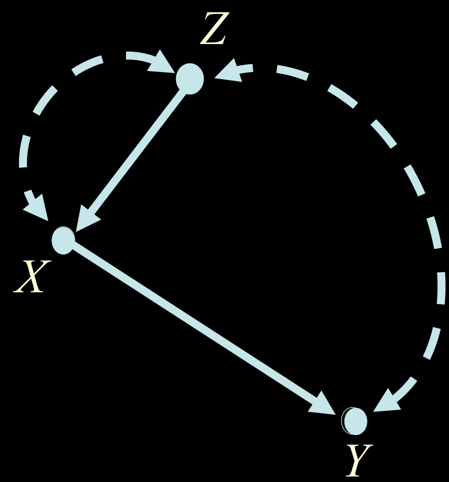

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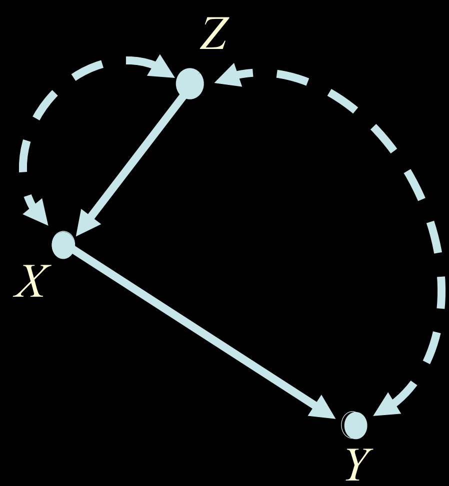

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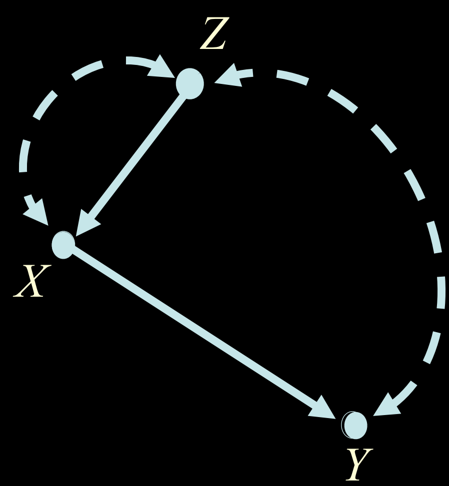

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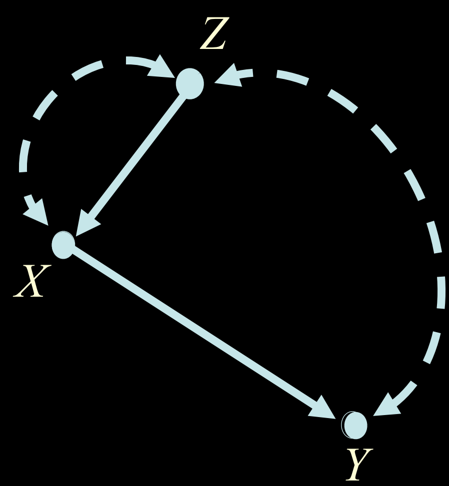

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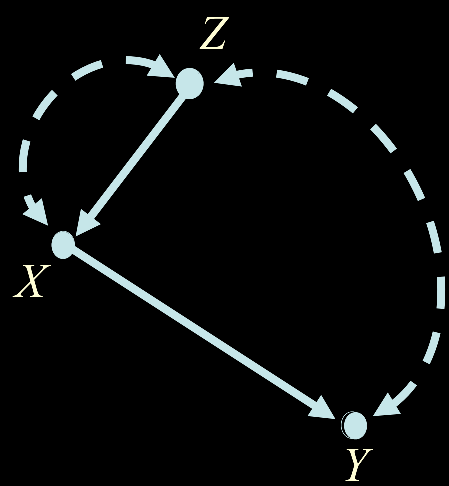

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Observational study: *P(x, y, z)*
Experimental study: *P(x, y | do(z))*

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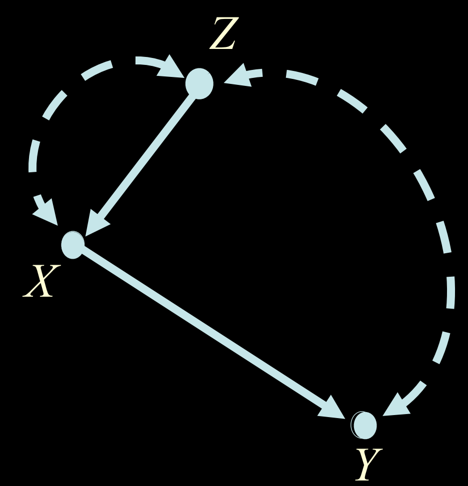

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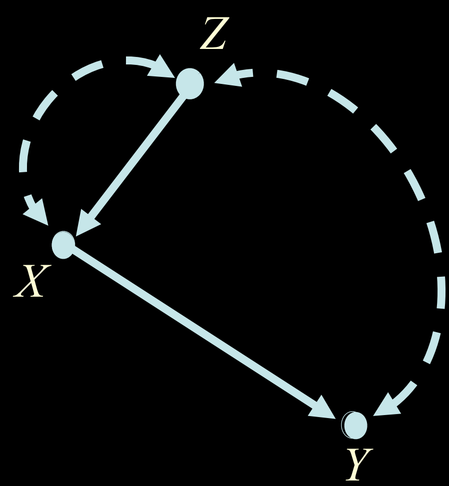

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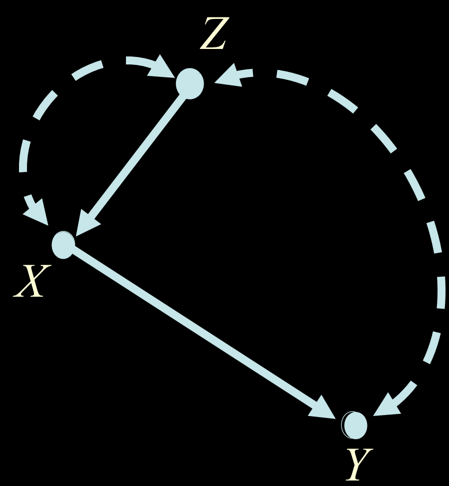

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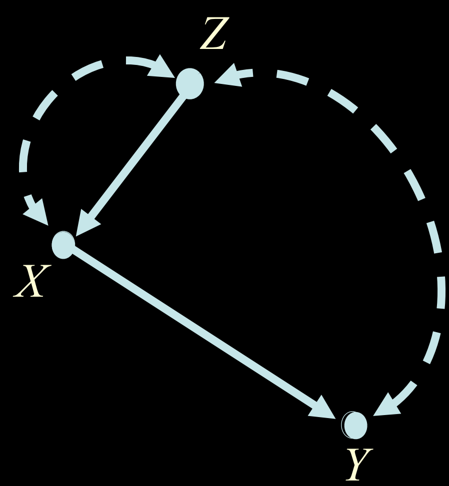

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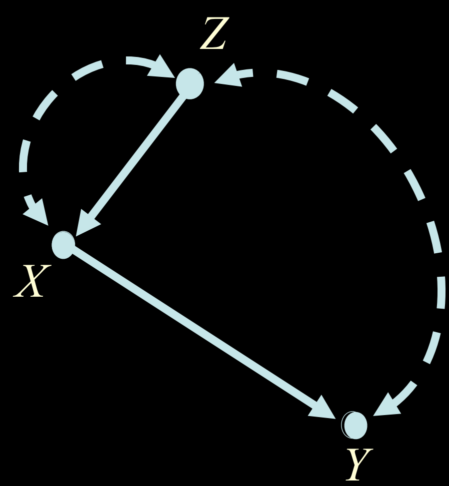

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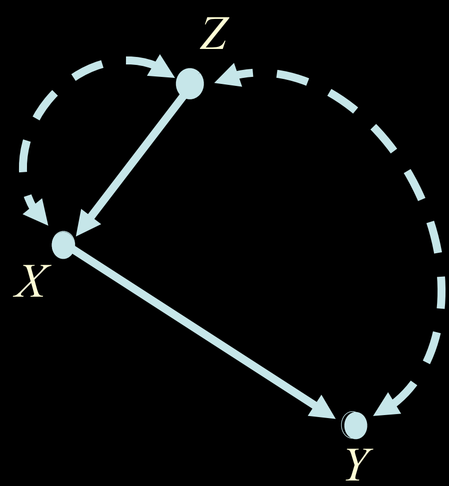

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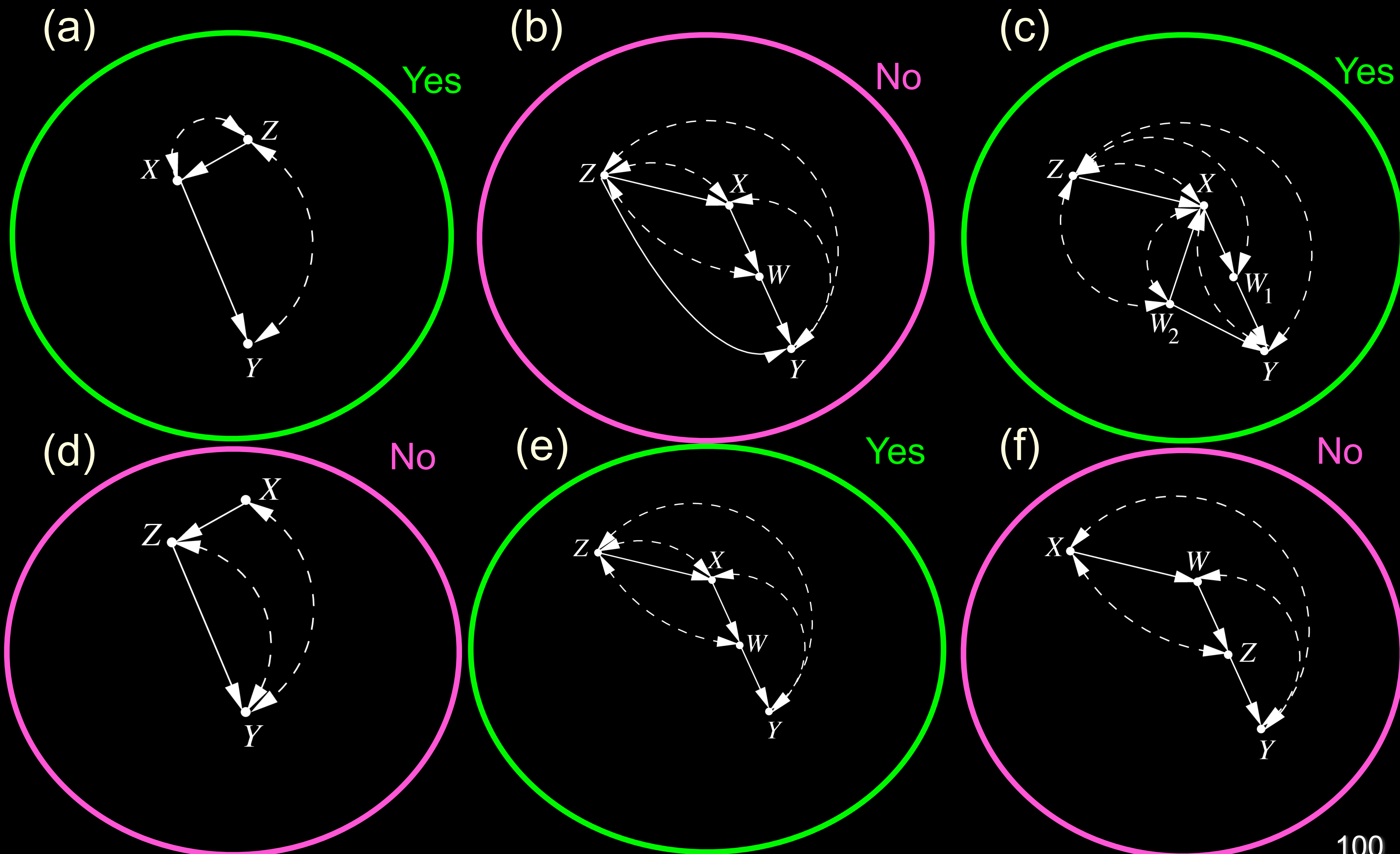
(i.e., $\exists\, f$, $f : P(v), P(v \mid do(z)) \rightarrow P(y \mid do(x))$)

WHICH MODEL LICENSES THE z-IDENTIFICATION OF THE CAUSAL EFFECT $X \rightarrow Y$?



SUMMARY D=1: POLICY EVALUATION RESULTS

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- The estimability of any expression of the form

  $Q = P(y_1, y_2, \ldots, y_n \mid do(x_1, x_2, \ldots, x_m), z_1, z_2, \ldots, z_k)$

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- The causal calculus is complete for this task.
- The algorithm is complete.
- References

  Pearl (1993, 1995) – Back-door criterion & *do*-calculus
  Tian & Pearl (2002) – General ID algorithm
  Shpitser & Pearl (2006) – Completeness (observational)
  Bareinboim & Pearl (2012) – Completeness (obs. + experimental)
  Lee, Correa & Bareinboim (2019) – Completeness (arbitrary exp.)

THE DATA-FUSION PROBLEM

ENVIRONMENTAL CONDITIONS (D = 2) —
TRANSPORTABILITY, EXTERNAL VALIDITY, META-ANALYSIS

THE TRANSPORTABILITY PROBLEM

Question:

Is it possible to predict the effect of X on Y in a target population $\prod^*$ (where no experiments are feasible), using experimental findings from a different population $\prod$?

Answer: Sometimes yes.

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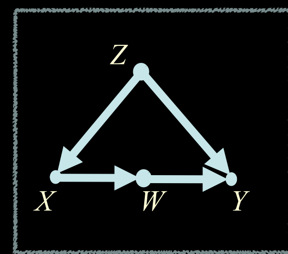
Answer: Sometimes yes.

Our goal is to formally characterize when and how.

MOVING FROM THE “LAB” TO THE “REAL WORLD”...

Lab



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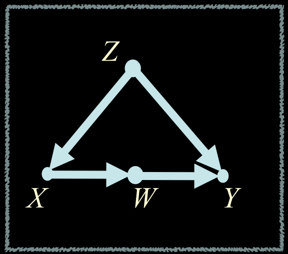

* The lab stands for any environment, population, domain, setting.

MOVING FROM THE "LAB" TO THE "REAL WORLD"...

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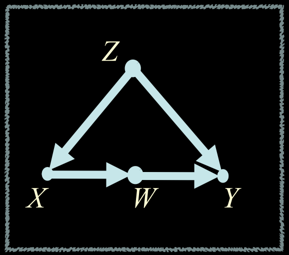

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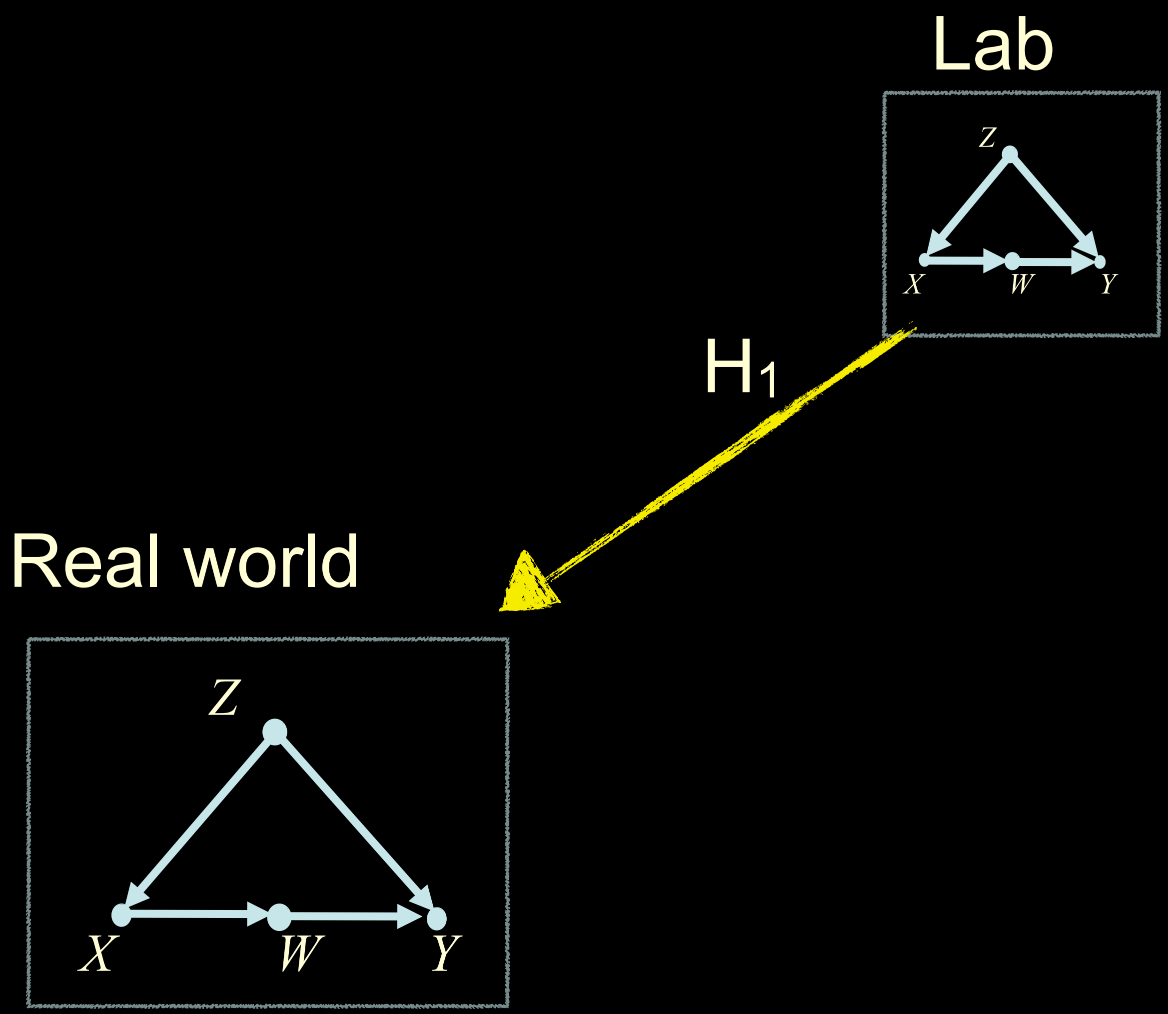

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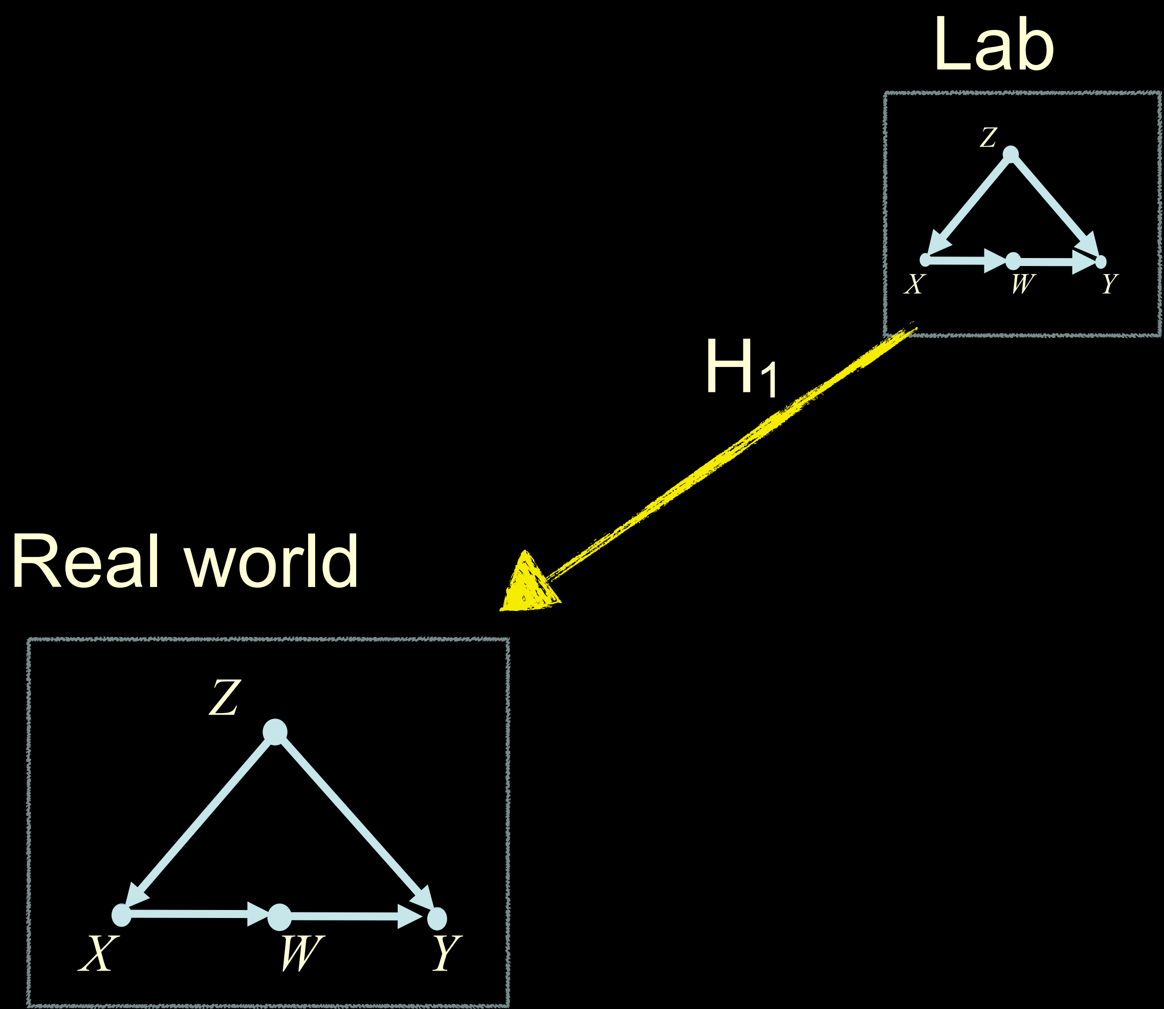

Everything is assumed to be
the same, trivially transportable!

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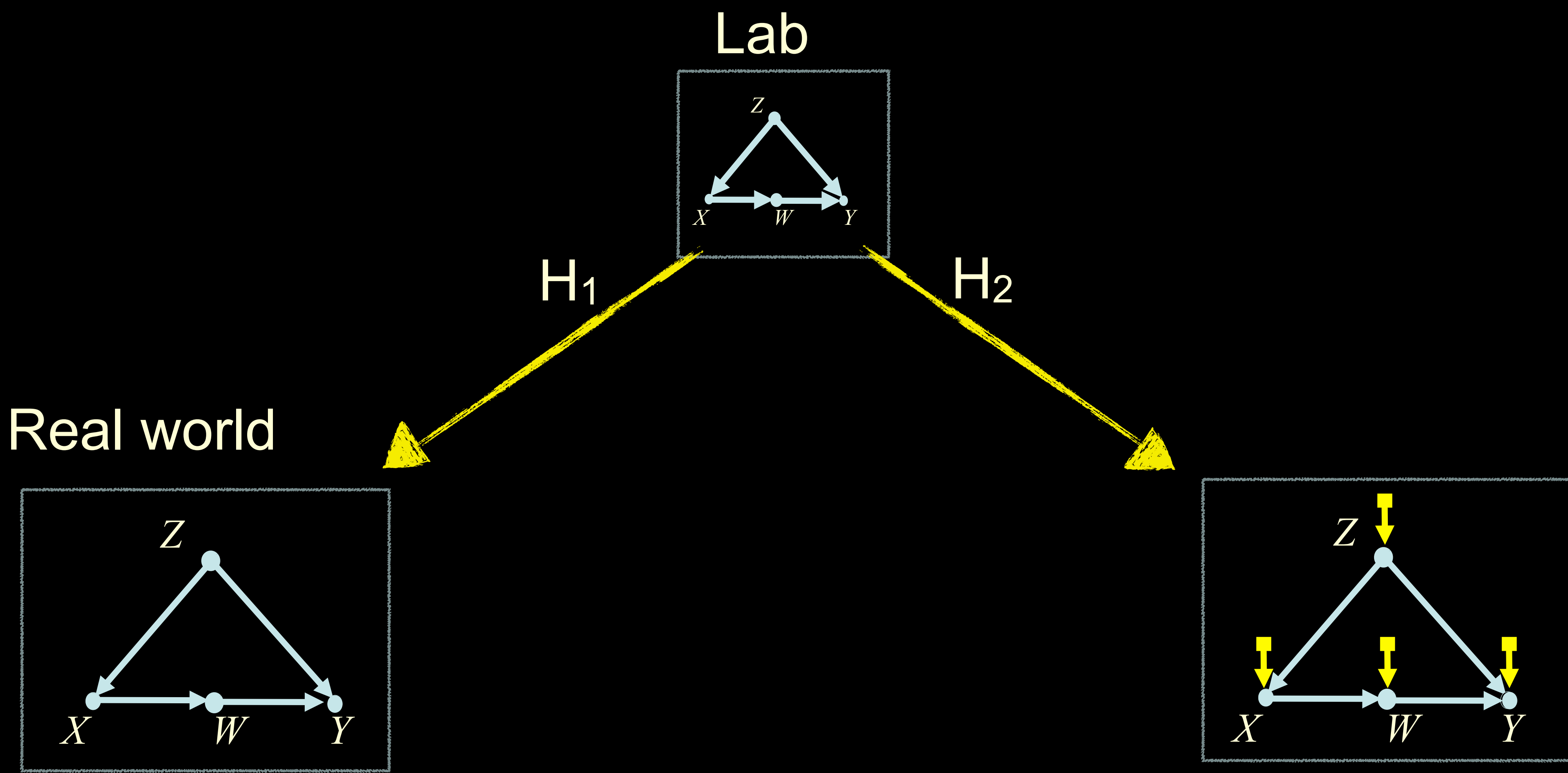

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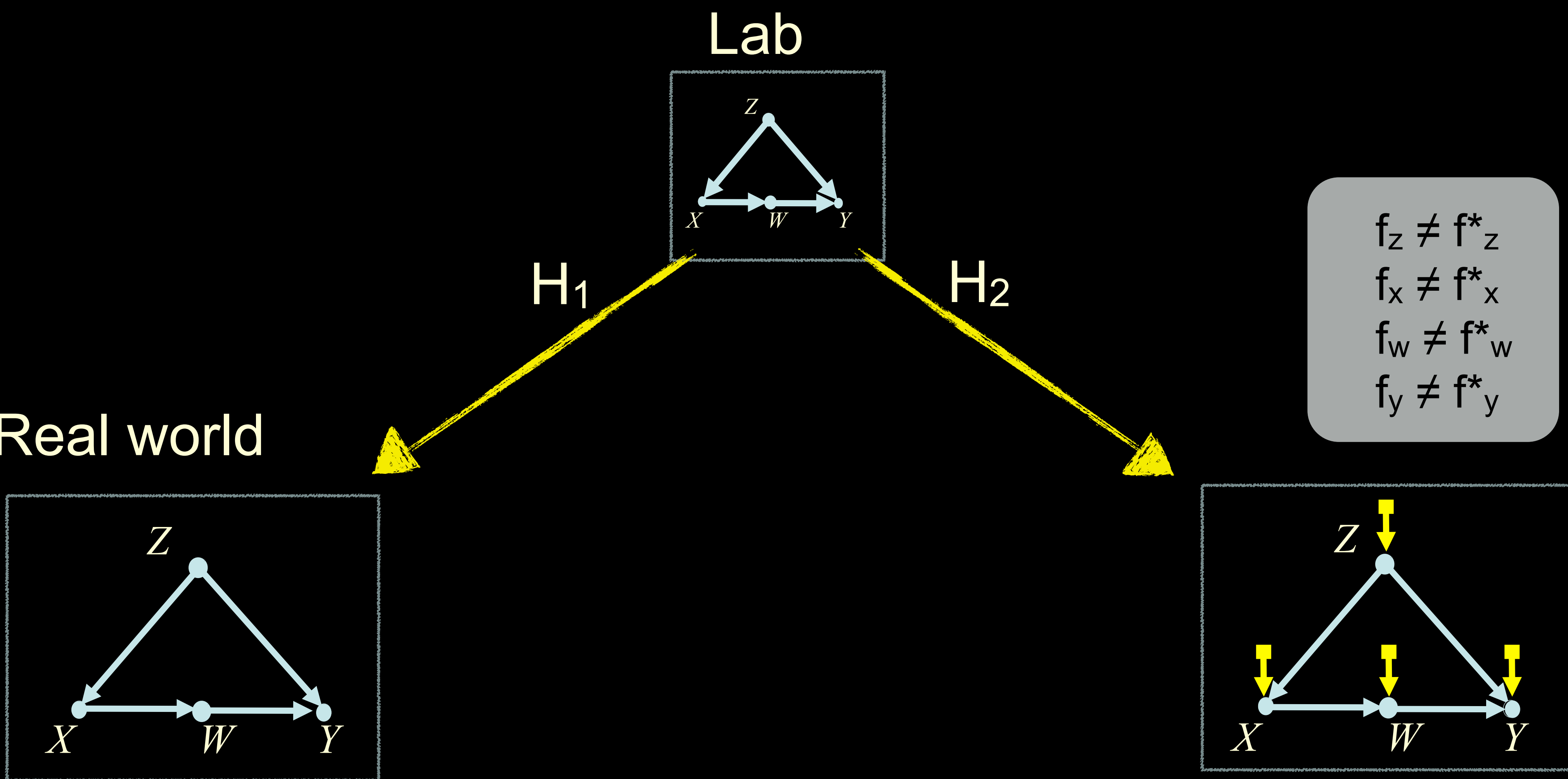

$f_Z \neq f^*_Z$
$f_X \neq f^*_X$
$f_W \neq f^*_W$
$f_Y \neq f^*_Y$

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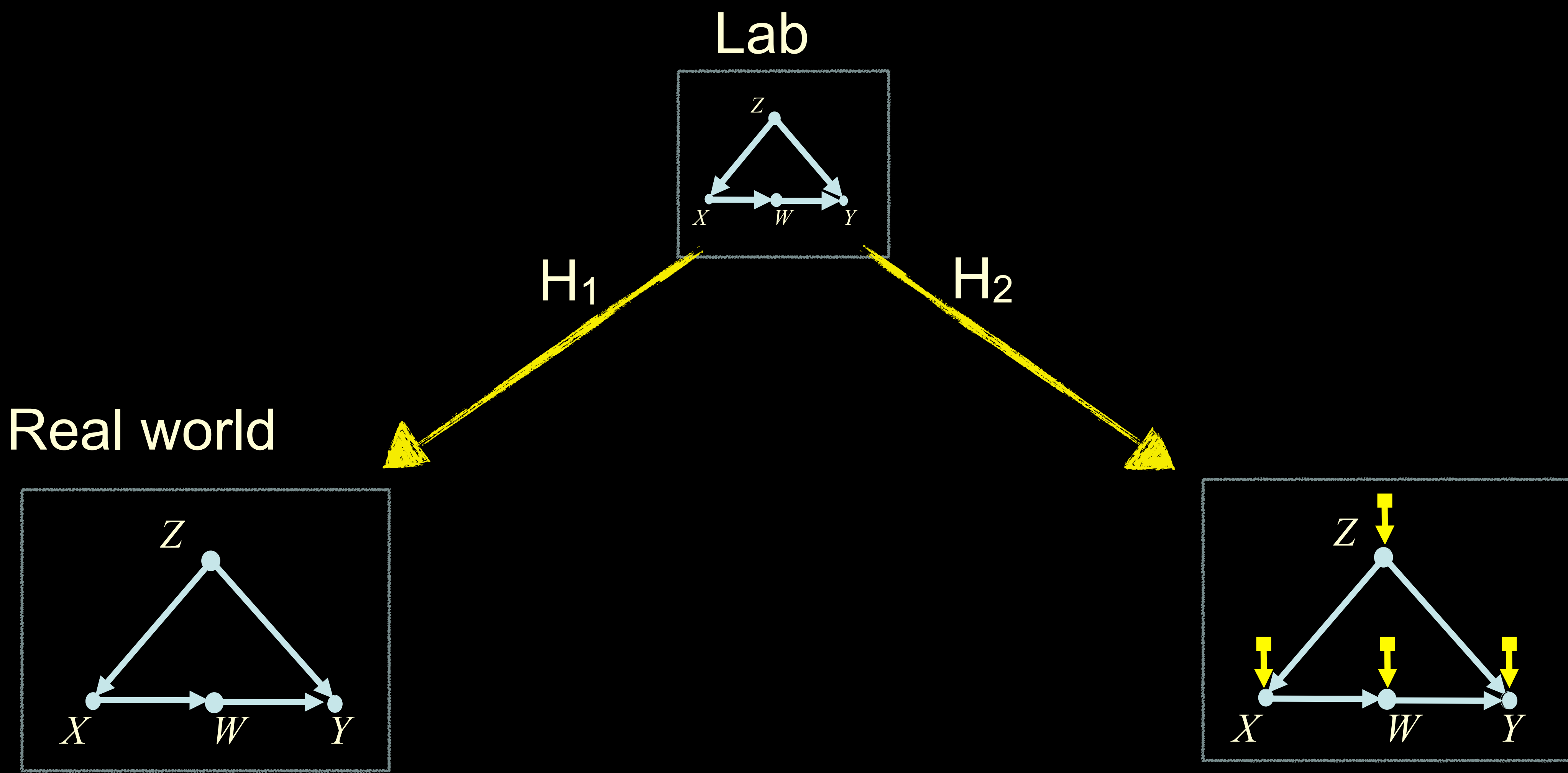

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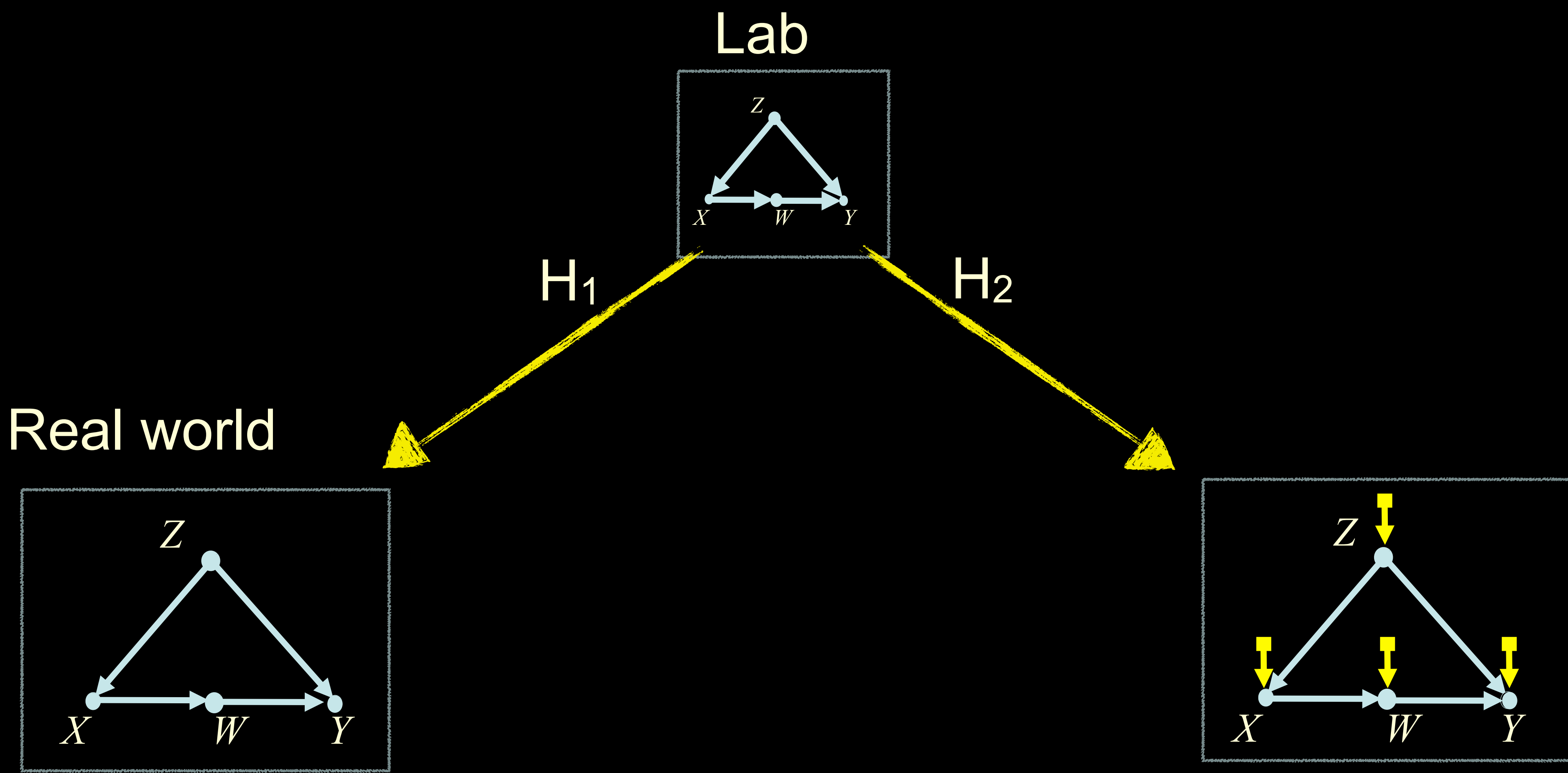

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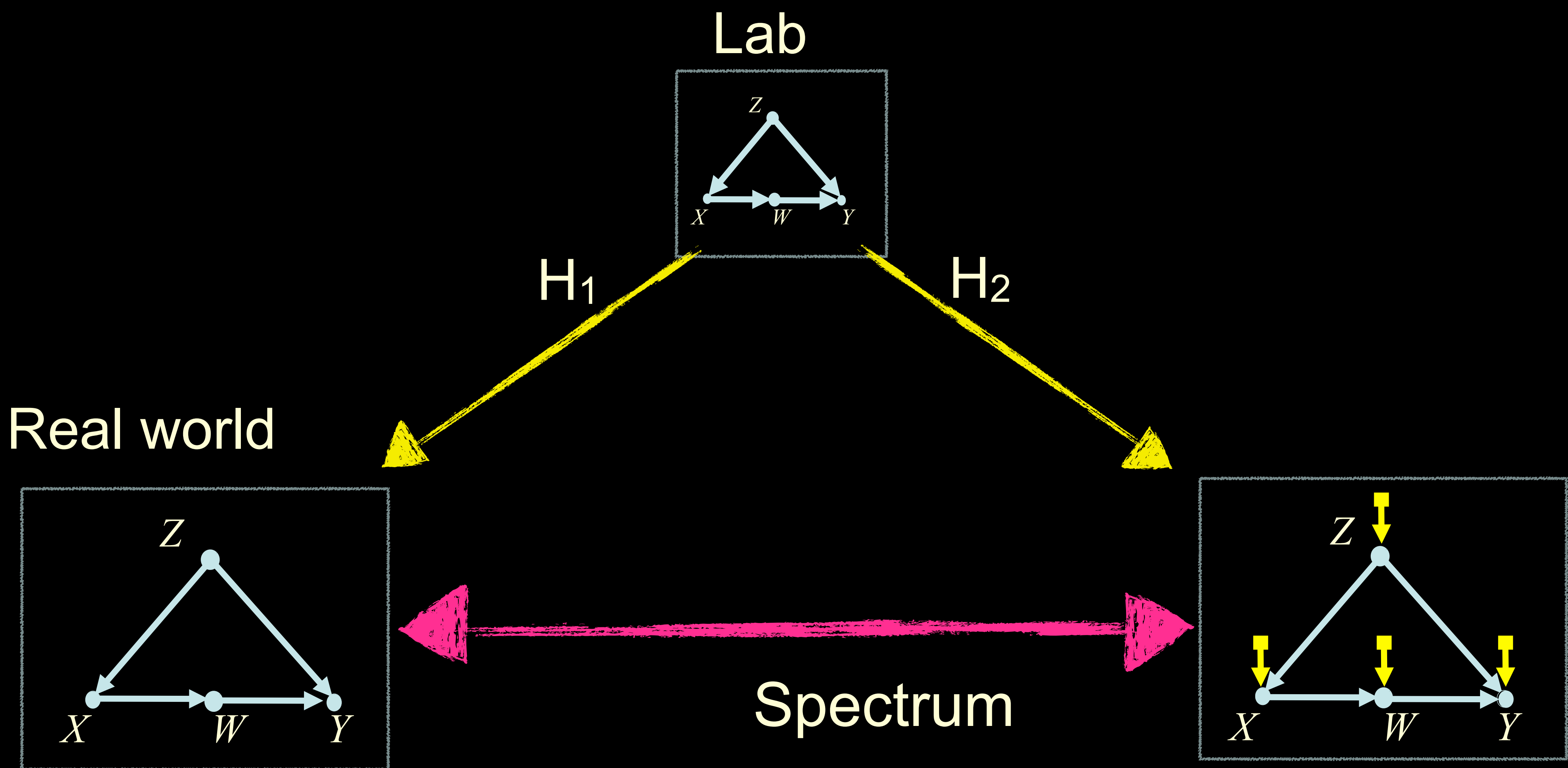
Everything is assumed to be different, not transportable...

MOVING FROM THE "LAB" TO THE "REAL WORLD"...



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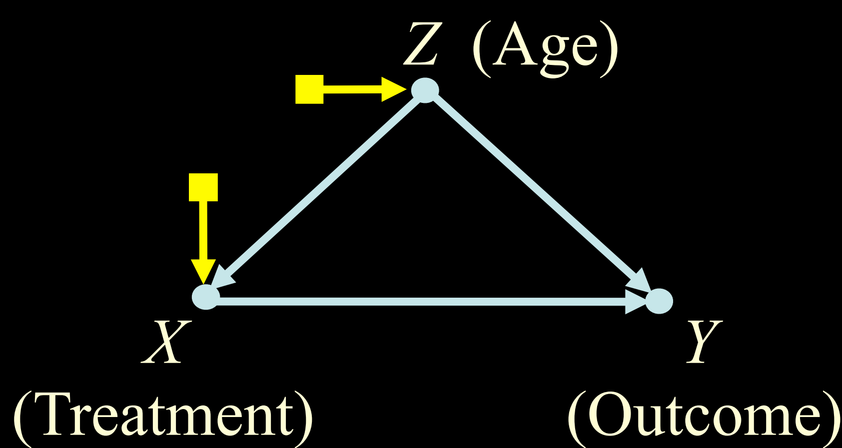
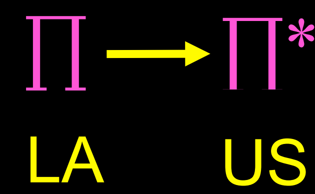
MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL US ABOUT US?





MOTIVATION

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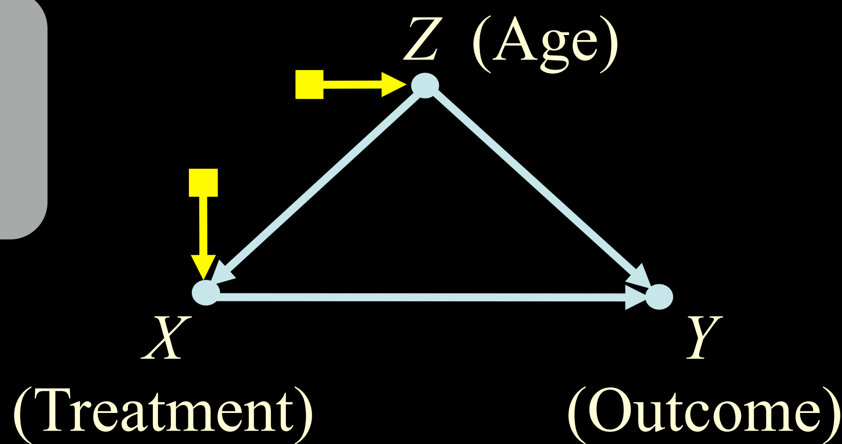
$f_z \neq f^*_z$
$f_x \neq f^*_x$
$f_y = f^*_y$

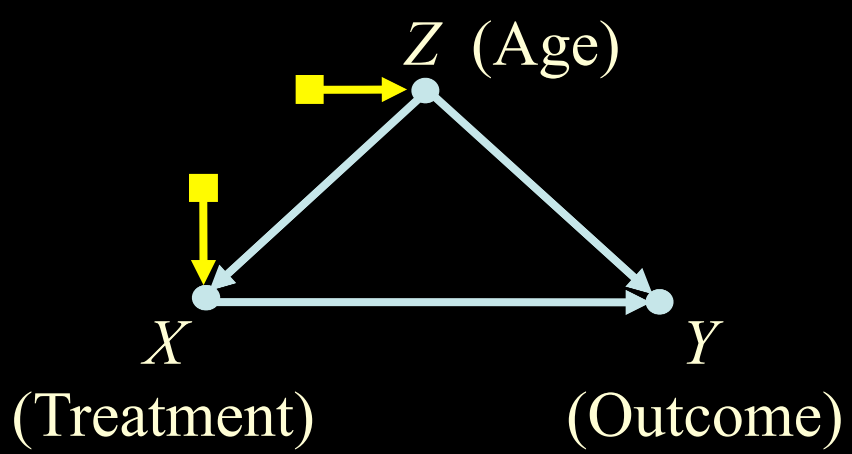


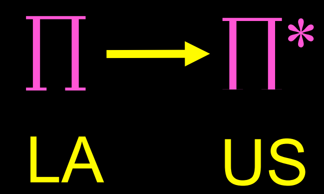
$\prod \longrightarrow \prod^*$

LA US

MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL US ABOUT US?

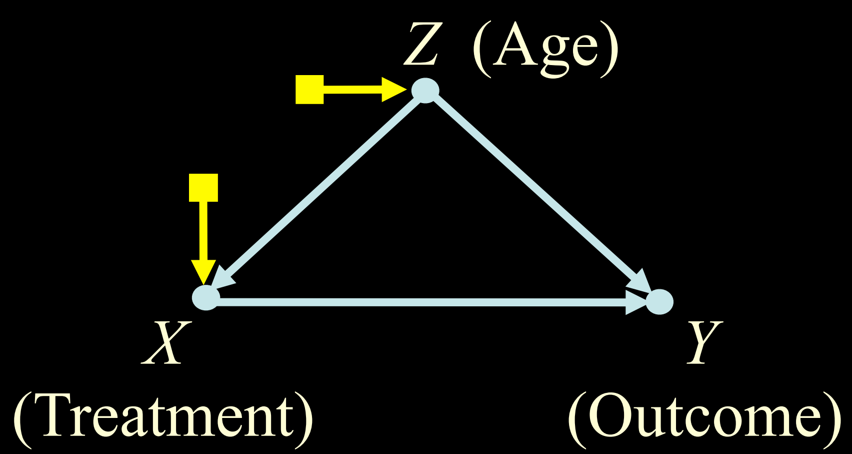




MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL US ABOUT US?

Input



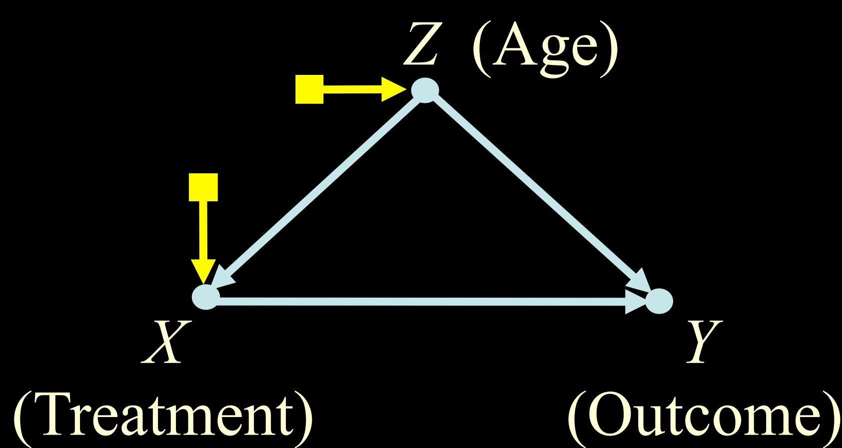
$\prod \longrightarrow \prod^*$

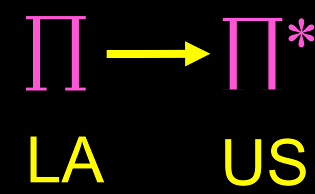
LA US

MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL US ABOUT US?

Input





Experimental study in LA

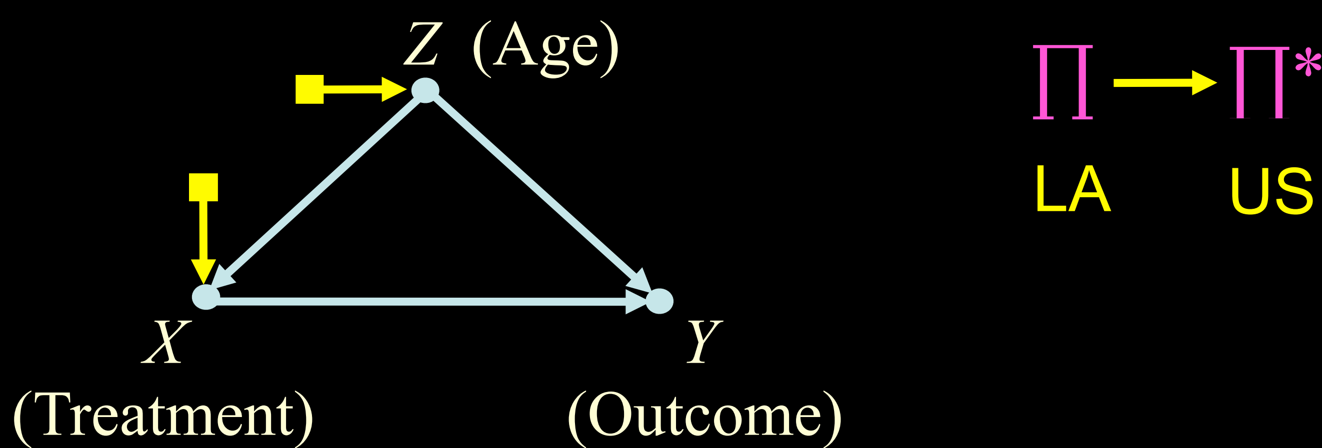
Measured: $P(x, y, z)$

$P(y \mid do(x), z)$

MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL US ABOUT US?

Input



Experimental study in LA

Measured: $P(x, y, z)$
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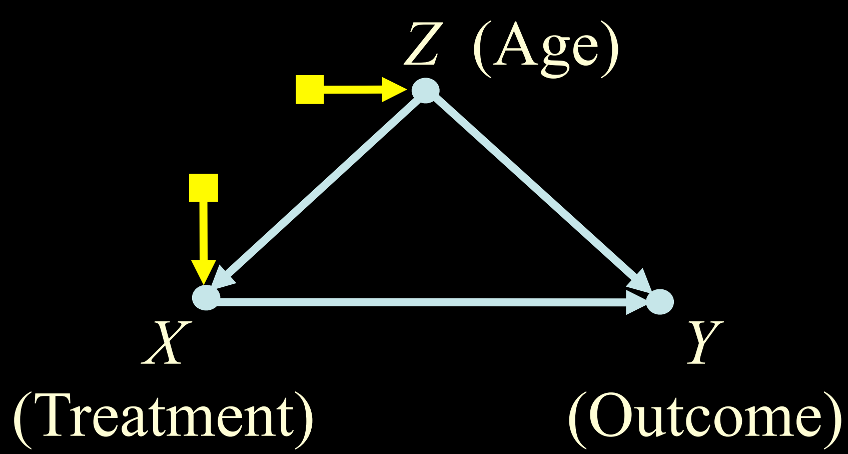
Observational study in US

Measured: $P^*(x, y, z)$
$[\ P^*(z) \neq P(z)\ ]$

MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL US ABOUT US?

Input



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$P(y \mid do(x), z)$

Observational study in US

Measured: $P^*(x, y, z)$
$[\ P^*(z) \neq P(z)\ ]$

Needed: $Q = P^*(y \mid do(x)) = ?$

MOTIVATION

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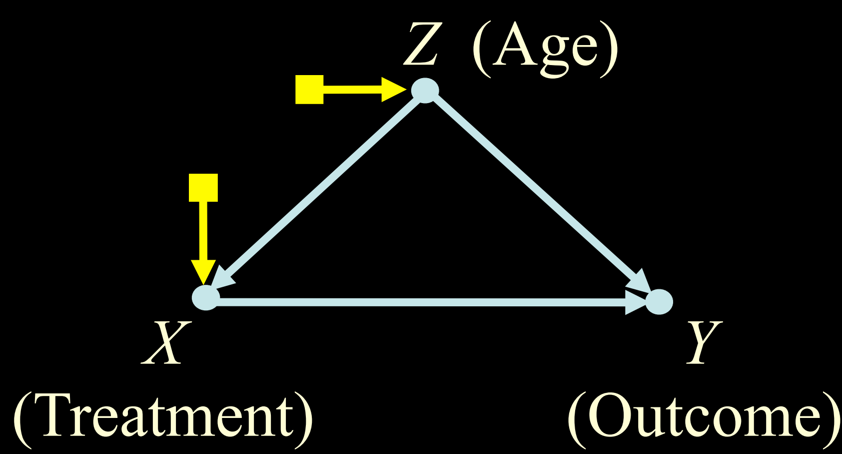
Input



$\prod \longrightarrow \prod^*$

LA US

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MOTIVATION

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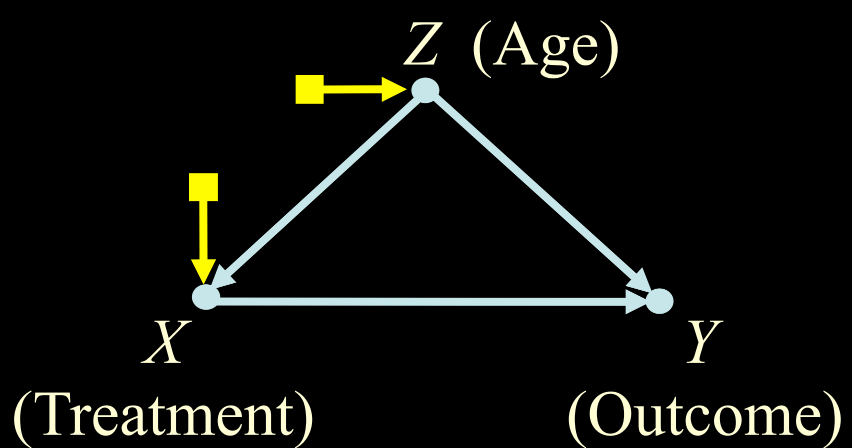

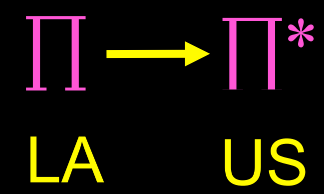

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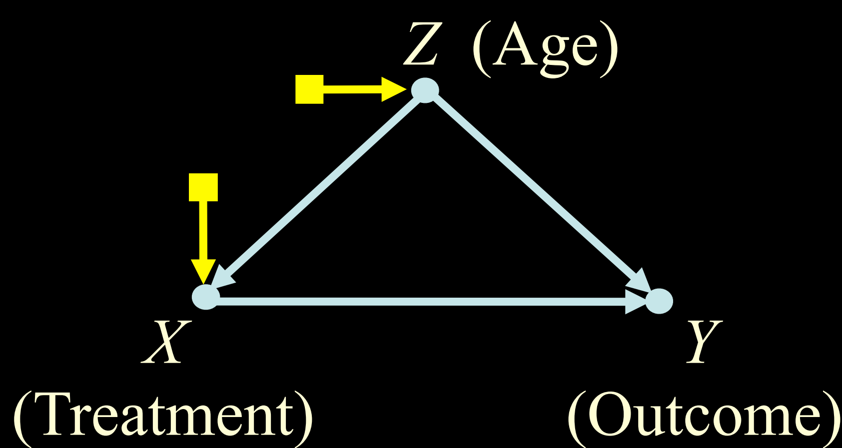


$\prod \longrightarrow \prod^*$

LA US

Experimental study in LA

Measured: $P(x, y, z)$
$P(y \mid do(x), z)$

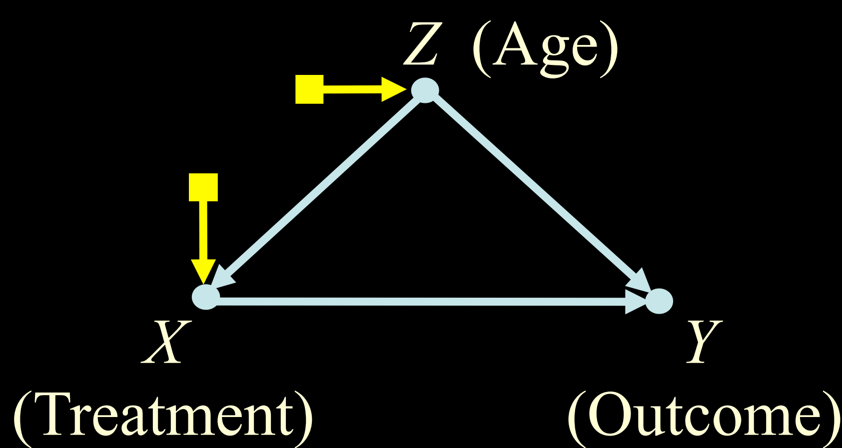
Observational study in US

Measured: $P^*(x, y, z)$
$[\, P^*(z) \neq P(z) \,]$

Needed: $Q = P^*(y \mid do(x)) = \sum_Z P(y \mid do(x), z)\, P^*(z)$

MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL US ABOUT US?



$\prod \longrightarrow \prod^*$

LA US

Experimental study in LA

Measured: $P(x, y, z)$

$P(y \mid do(x), z)$

Observational study in US

Measured: $P^*(x, y, z)$

$[\, P^*(z) \neq P(z) \,]$

Needed: $Q = P^*(y \mid do(x)) = \sum_z \underbrace{P(y \mid do(x), z)}_{\text{LA}} \underbrace{P^*(z)}_{\text{US}}$

MOTIVATION

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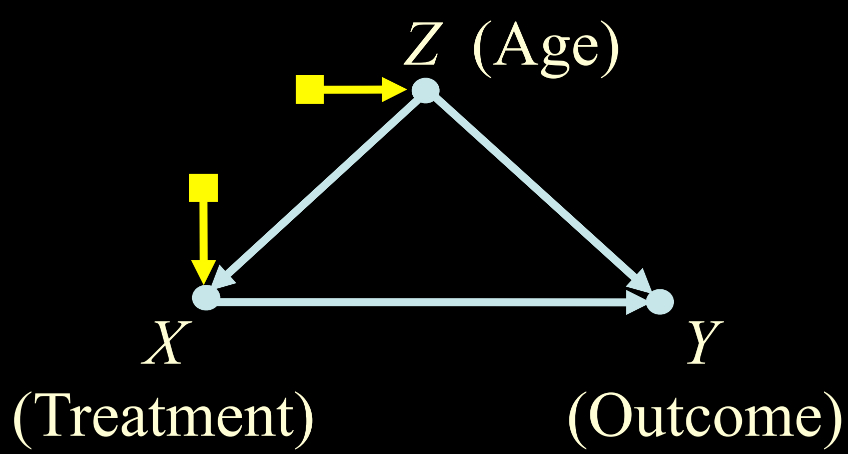


$\Pi \longrightarrow \Pi^*$

LA US

Experimental study in LA

Measured: $P(x, y, z)$
$P(y \mid do(x), z)$

Observational study in US

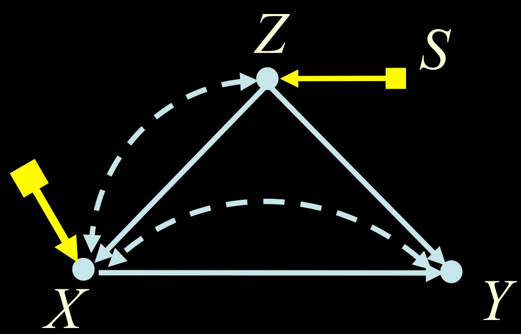
Measured: $P^*(x, y, z)$
$[\, P^*(z) \neq P(z) \,]$

Needed: $Q = P^*(y \mid do(x)) = \sum_z \underbrace{P(y \mid do(x), z)}_{\text{LA}} \; \underbrace{P^*(z)}_{\text{US}}$

Transport Formula (recalibration): $Q = F(P, P_{do}, P^*)$

TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY

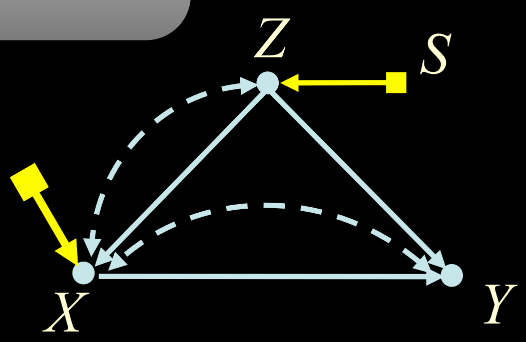
TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY



(a)

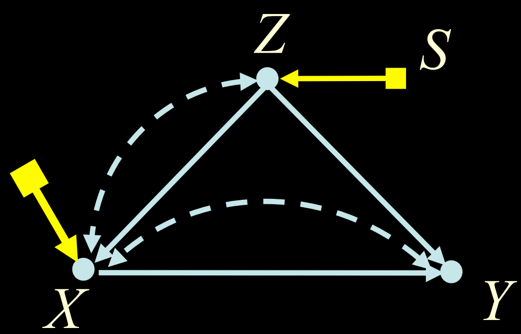
TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY

$z \leftarrow f_z(u_z, u_{xz})$
$x \leftarrow f_x(x, u_x, u_{xz}, u_{xy})$
$y \leftarrow f_y(x, z, u_y, u_{xy})$



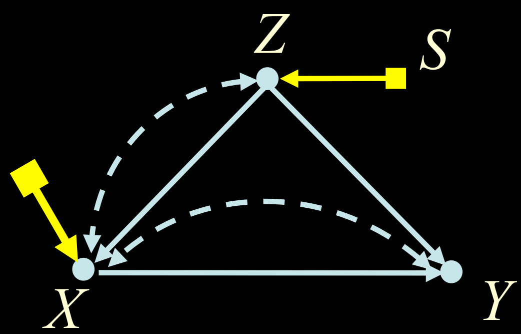
(a)

TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY



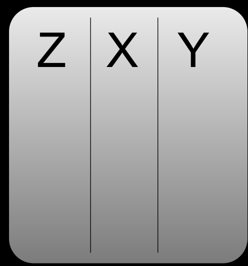
(a)

TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY

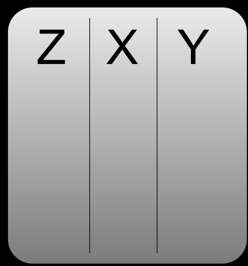


(a)

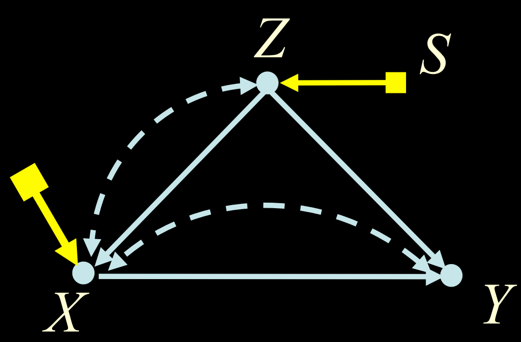
LA: P(y | do(x),z)



US: P*(x,y,z)

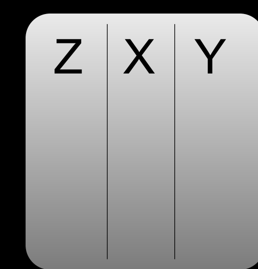


TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY

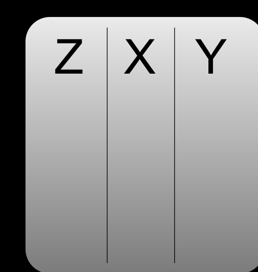


(a)

LA: P(y | do(x),z)



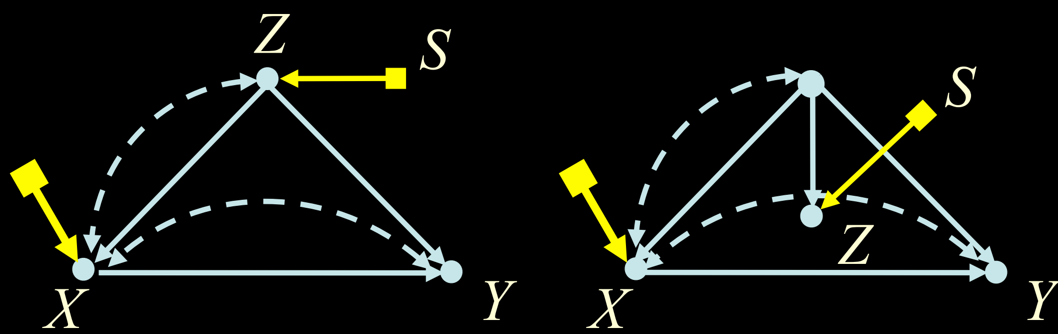
US: P*(x,y,z)



a) ***Z*** represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY



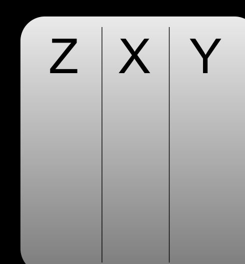
(a) (b)

a) ***Z*** represents age

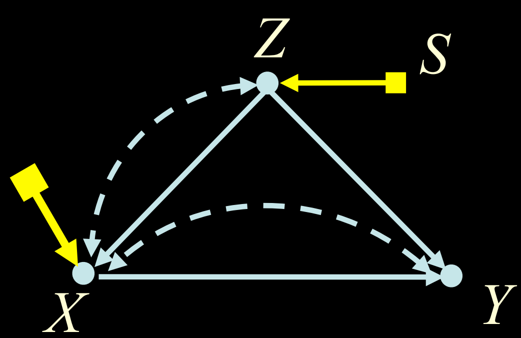
$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

LA: P(y | do(x),z)

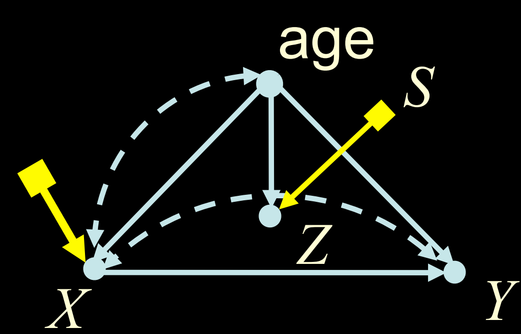
US: P*(x,y,z)



TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY



(a)



(b)

a) $\boldsymbol{Z}$ represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

b) $\boldsymbol{Z}$ represents language skill

$$P^*(y \mid do(x)) = \; ?$$

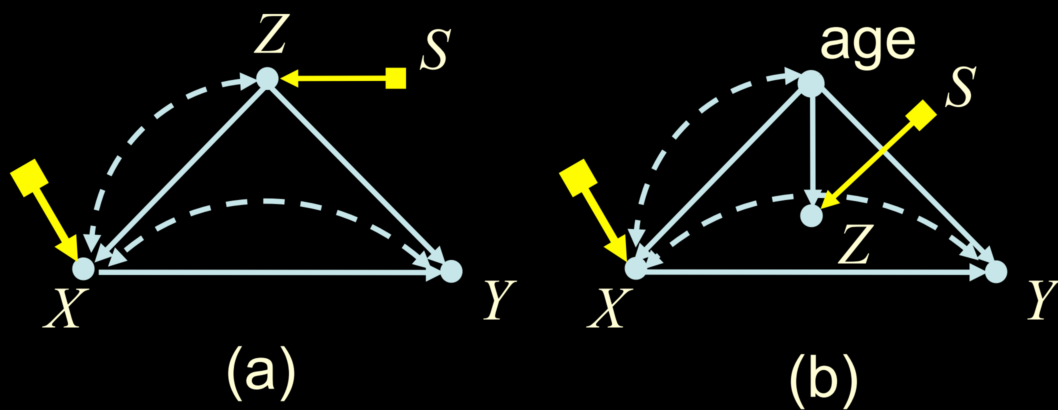
LA: P(y | do(x),z)

Z	X	Y

US: P*(x,y,z)

Z	X	Y

TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY



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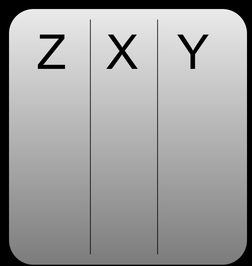
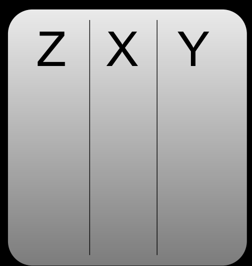


US: P*(x,y,z)



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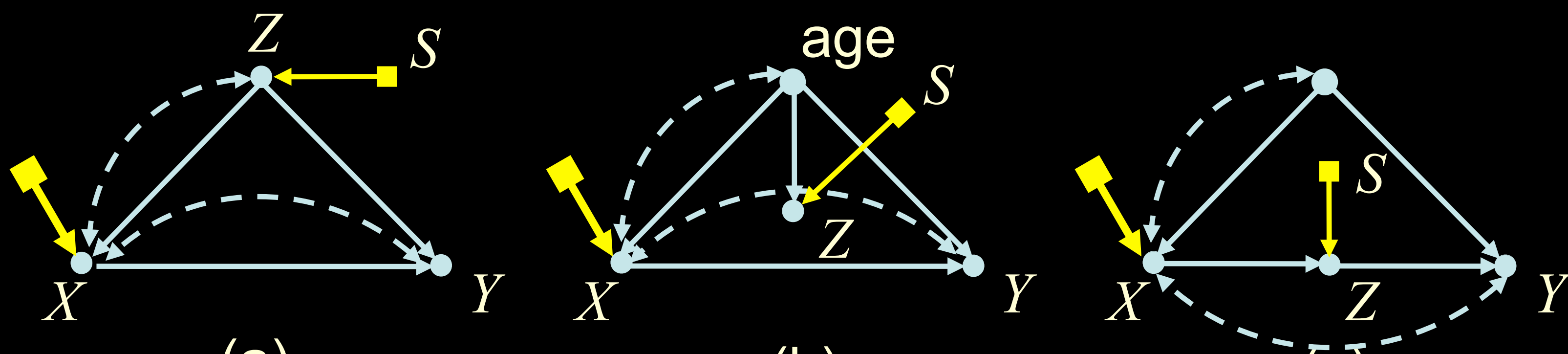


(a) (b) (c)

LA: P(y | do(x),z)

Z	X	Y

US: P*(x,y,z)

Z	X	Y

a) ***Z*** represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

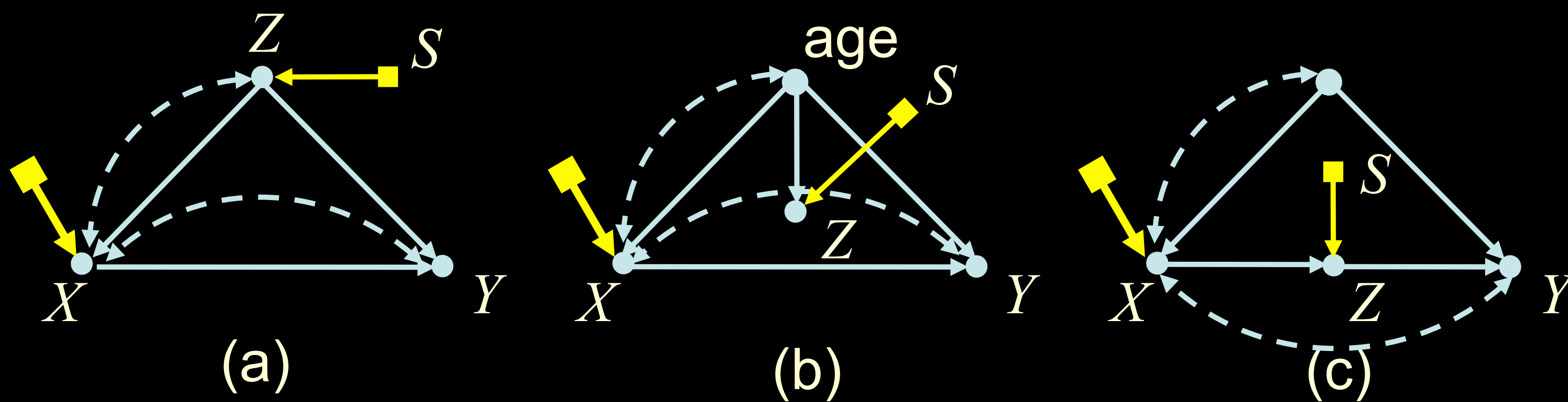
b) ***Z*** represents language skill

$$P^*(y \mid do(x)) = P(y \mid do(x))$$

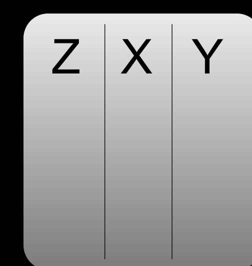
c) ***Z*** represents a bio-marker

$$P^*(y \mid do(x)) = \ ?$$

TRANSPORT FORMULAS DEPEND ON THE CAUSAL STORY



LA: P(y | do(x),z)



US: P*(x,y,z)

a) Z represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

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$$P^*(y \mid do(x)) = P(y \mid do(x))$$

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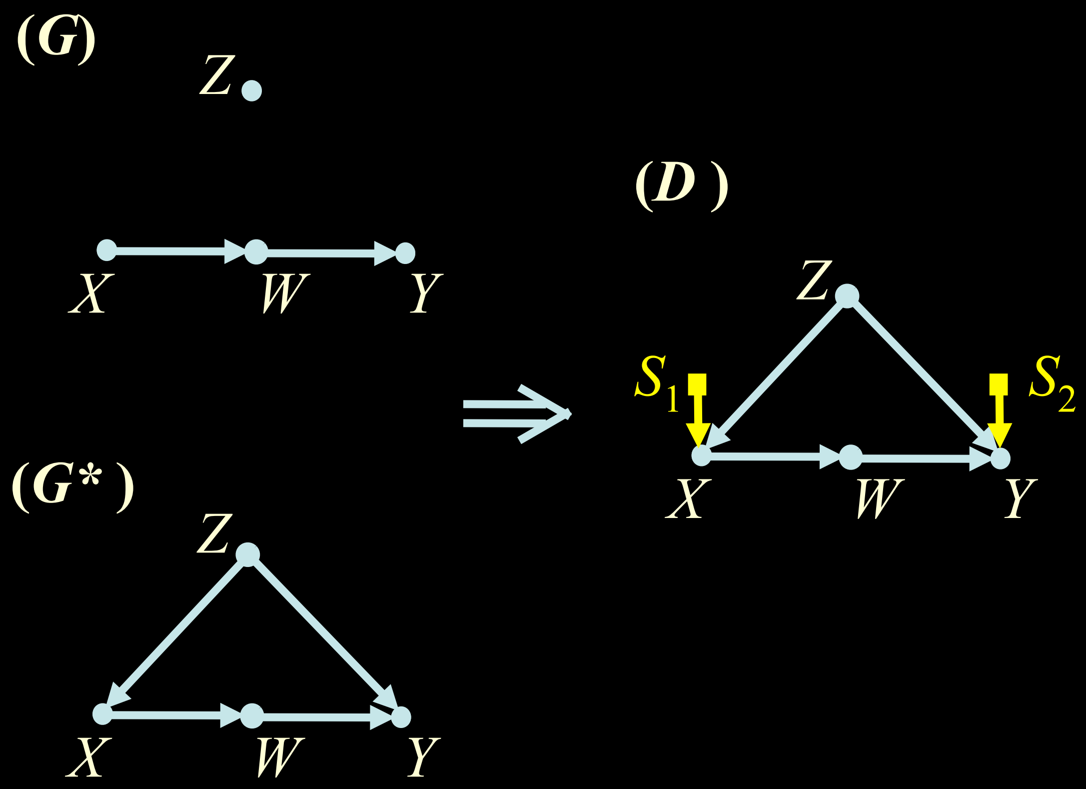
$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z \mid x)$$

SELECTION DIAGRAMS

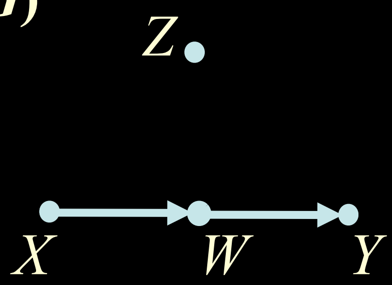
- How to encode disparities and commonalities about domains?



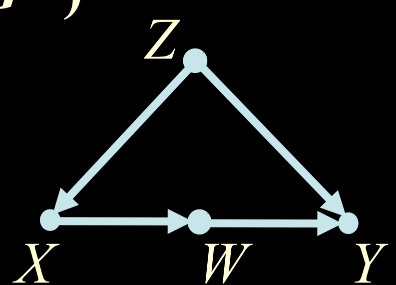
SELECTION DIAGRAMS

- How to encode disparities and commonalities about domains?

(*G*)

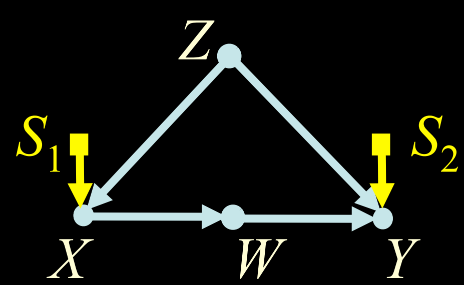


(*G)**



$\Longrightarrow$

(*D*)



$=$

$f_Z(u_Z) = f^*_Z(u_Z)$
$f_W(x, u_w) = f^*_W(x, u_w)$
$f_X(z, u_x) \neq f^*_W(x, u_x)$
$f_Y(w, z, u_y) \neq f^*_Y(w, z, u_y)$

RESULT 1: TRANSPORTABILITY REDUCED TO CALCULUS

Theorem:

A causal relation Q is transportable from Π to Π^* iff $Q(\Pi^*)$ is reducible, using the rules of causal calculus, to an expression in which S is separated from *do()*.

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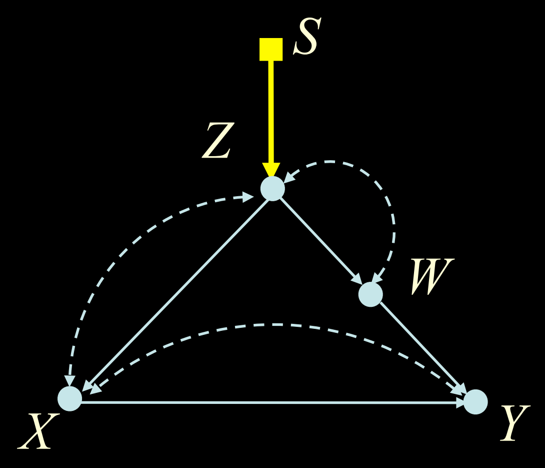
Ex:

$$
\begin{aligned}
Q = P^*(y \mid do(x)) &= P(y \mid do(x), s) \\
&= \sum_w P(y \mid do(x), s, w)\, P(w \mid do(x), s) \\
&= \sum_w P(y \mid do(x), w)\, P(w \mid do(x), s) \\
&= \sum_w P(y \mid do(x), w)\, P(w \mid s) \\
&= \sum_w P(y \mid do(x), w)\, P^*(w)
\end{aligned}
$$



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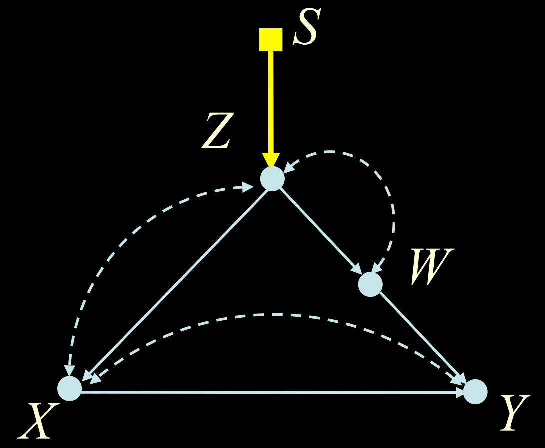

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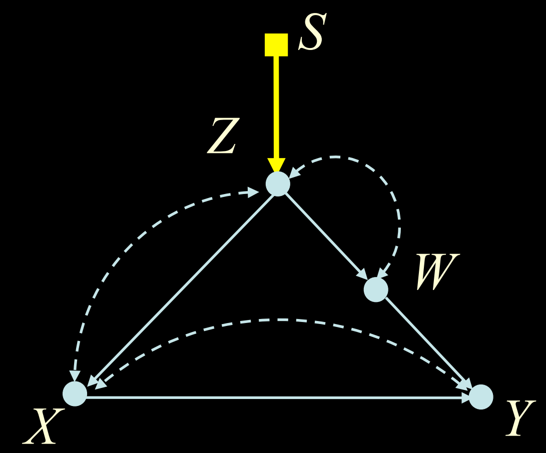
$$

data



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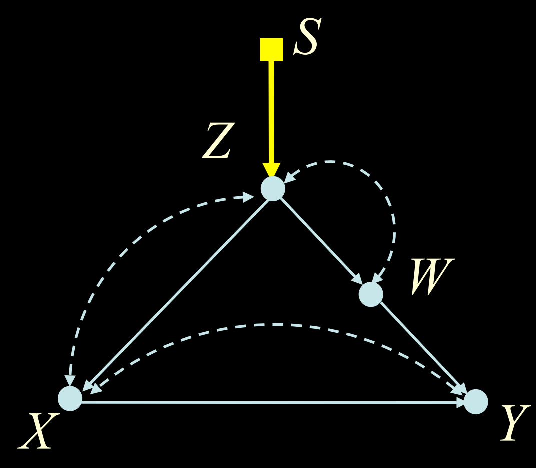
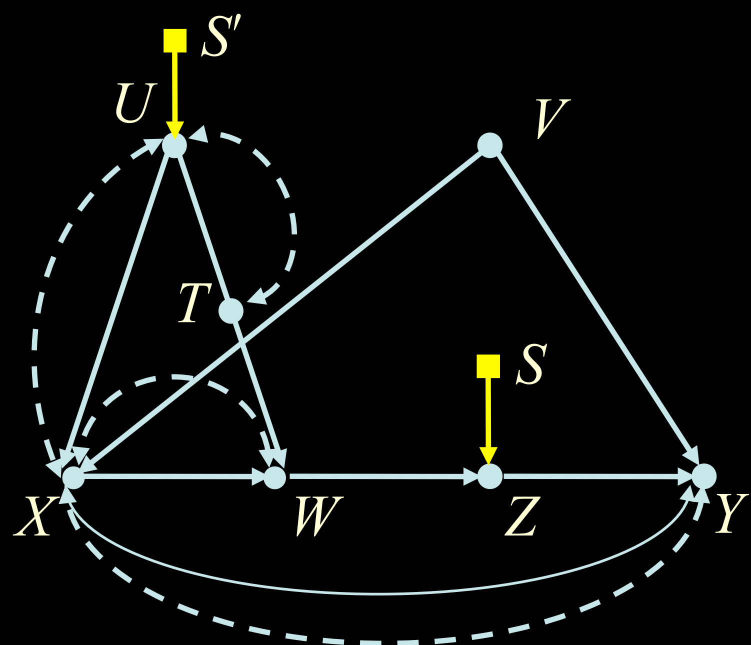
$$

data



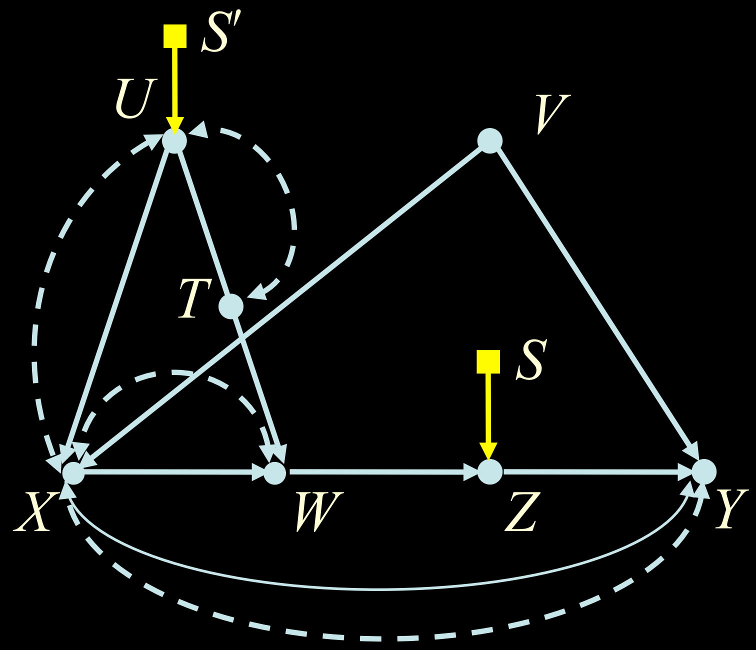
* Deterministic sep.: (Dawid, 1979, 2002; Constantinou & Dawid, 2016)

RESULT 2: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE



RESULT 2: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE

INPUT: Annotated Causal Graph



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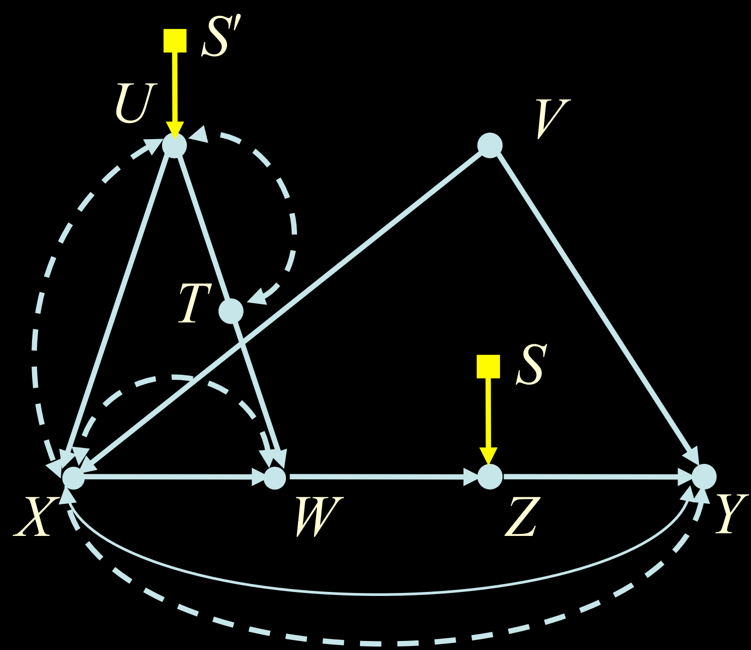
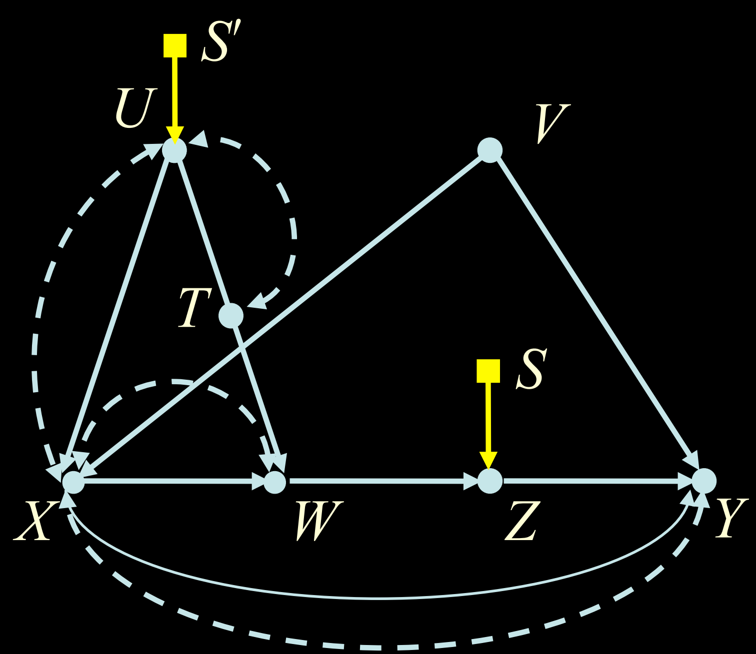


INPUT: Annotated Causal Graph

S ⟼ Factors creating differences

RESULT 2: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE

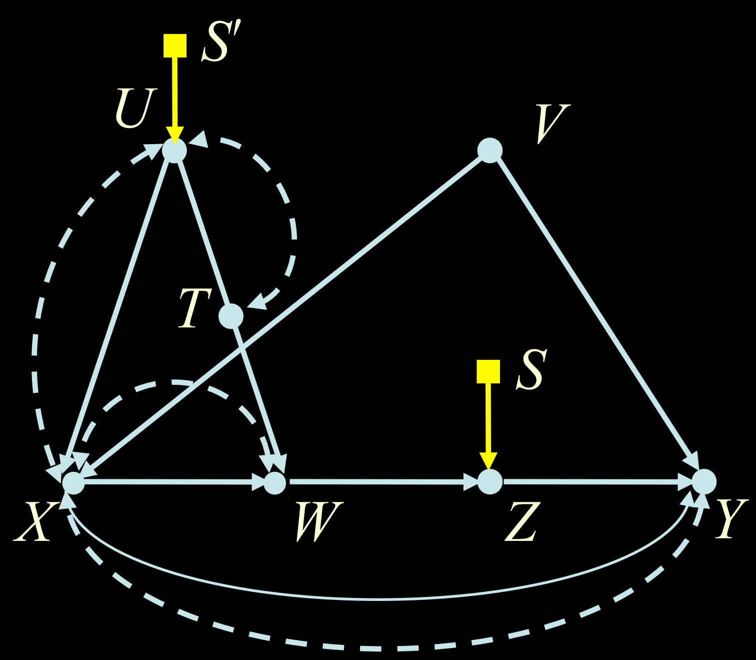


INPUT: Annotated Causal Graph

S ⟼ Factors creating differences

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) \sum_w P^*(z \mid w) \sum_t P(w \mid do(x), t)\, P^*(t)$$

RESULT 2: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE



INPUT: Annotated Causal Graph

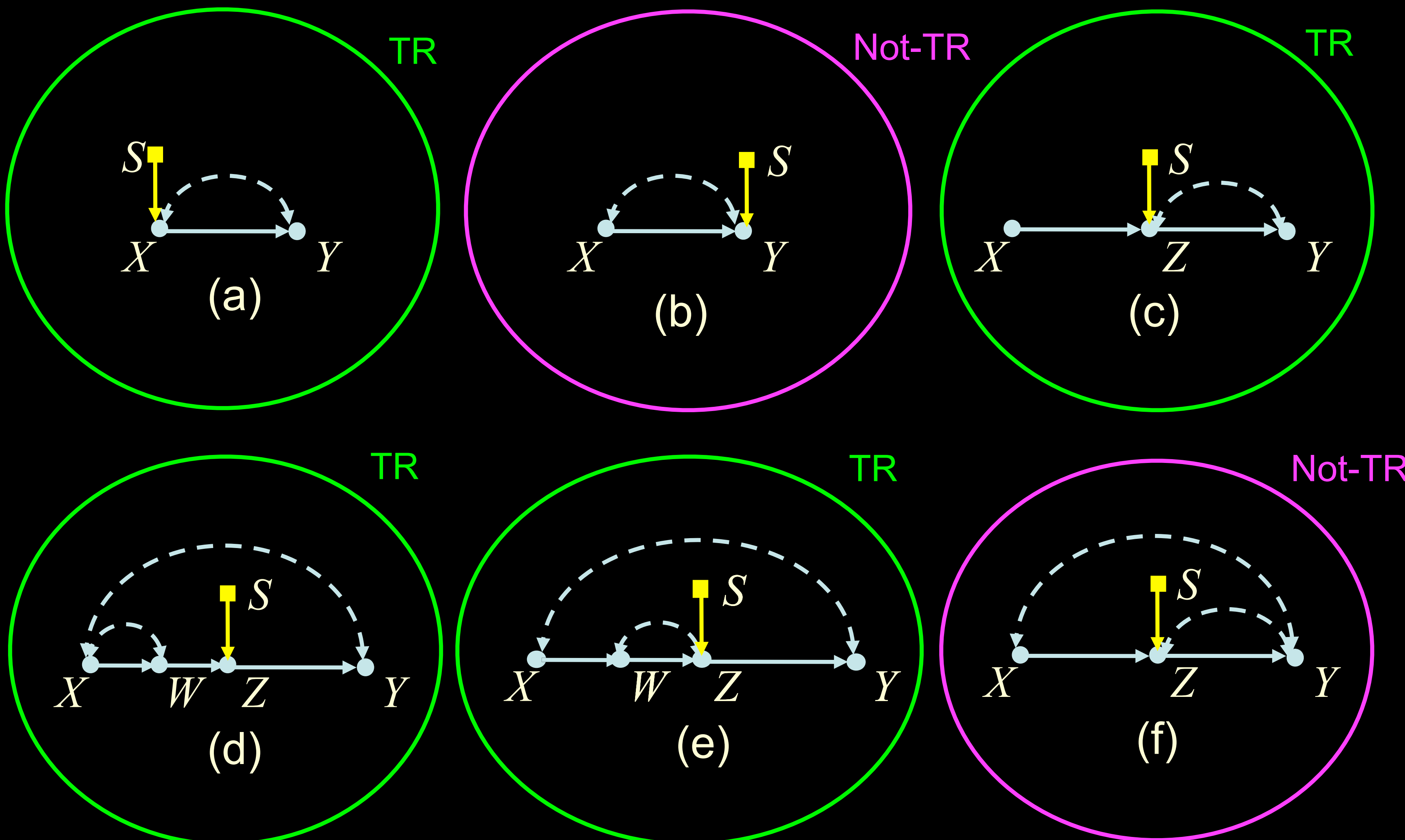
S ⟼ Factors creating differences

OUTPUT:

1. Transportable or not?
2. Yes = Transport formula
    1. Measurements to be taken in the experimental domain
    2. Measurements to be taken in the target domain
3. No = Not possible

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) \sum_w P^*(z \mid w) \sum_t P(w \mid do(x), t) P^*(t)$$

RESULT 3: GRAPHICAL CONDITION FOR TRANSPORTABILITY



THE CHALLENGE OF GENERAL TRANSPORTABILITY

Assume we want to transport $Q = P^*(y \mid do(x))$ from $\{ \prod^a, \prod^b \}$ with experimental information $\{X, Z\}$ in $\prod^a$, $\{X, Z\}$ in $\prod^b$.

LA:
$P(y, z \mid do(x))$
$P(y, x \mid do(z))$

:NY
$P(y, z \mid do(x))$
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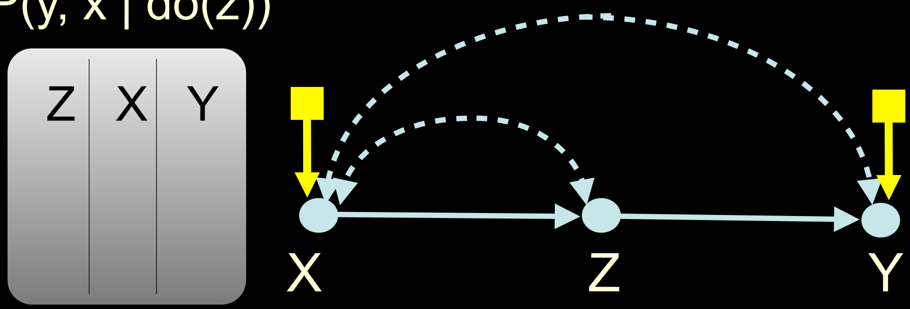
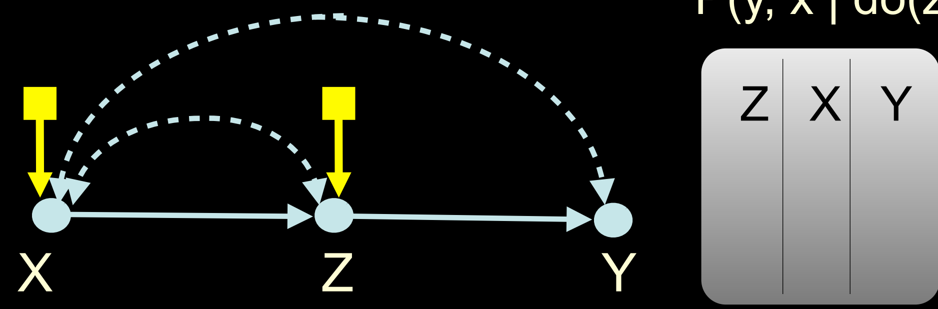


$\prod^a$ (=LA)



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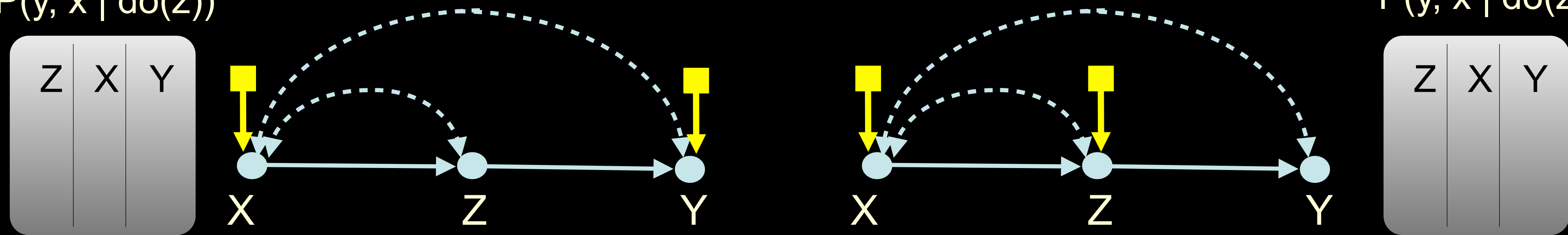
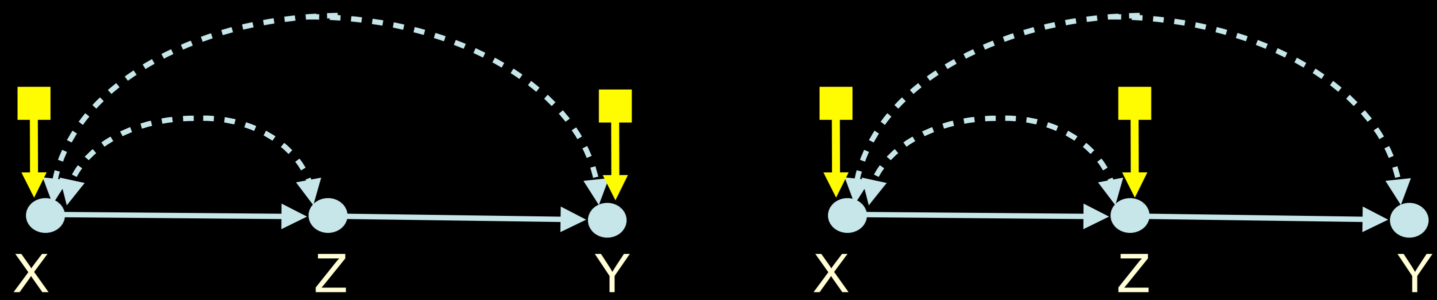


$\prod^a$ (=LA) $\prod^b$ (=NYC)

If we apply the result from limited transportability (1D), Q is (provably) not transportable from $\prod^a$ and $\prod^b$ separately.

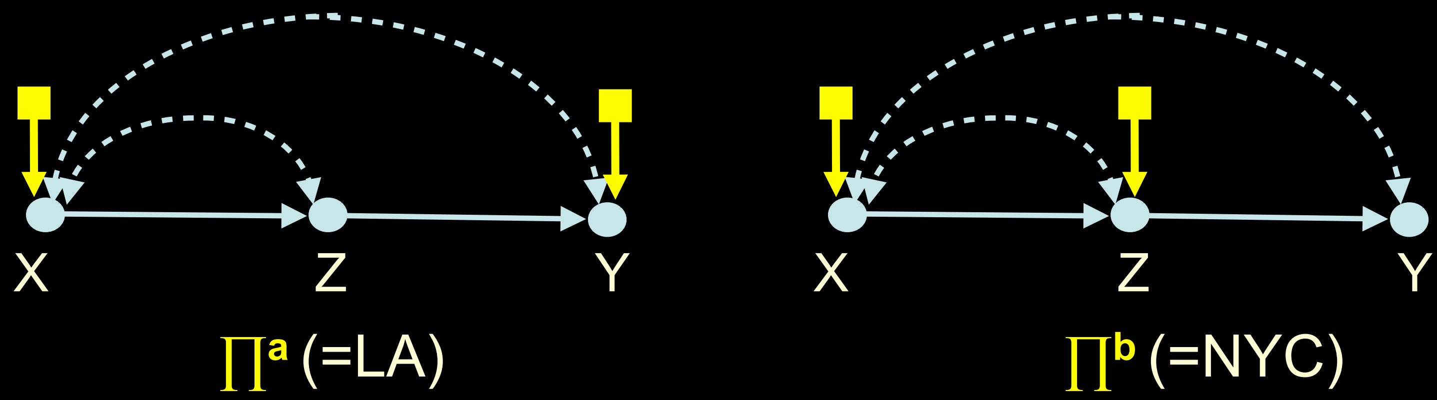
DERIVATION OF GENERAL TRANSPORTABILITY



$\prod^{a}$ (=LA) $\qquad$ $\prod^{b}$ (=NYC)

$Q = P^{*}(y \mid do(x)) = P(y \mid do(x), s^{a}, s^{b})$	Meta-TR Def.
$= \sum_{z} P(y \mid do(x), z, s^{a}, s^{b})\, P(z \mid do(x), s^{a}, s^{b})$	Probability Axioms
$= \sum_{z} P(y \mid do(x), do(z), s^{a}, s^{b})\, P(z \mid do(x), s^{a},$	Rule 2
$= \sum_{z} P(y \mid do(z), s^{a}, s^{b})\, P(z \mid do(x), s^{a}, s^{b})$	Rule 3
$= \sum_{z} P(y \mid do(z), s^{a})\, P(z \mid do(x), s^{a}, s^{b})$	Rule 1* in D_b
$= \sum_{z} P(y \mid do(z), s^{a})\, P(z \mid do(x), s^{b})$	Rule 1* in D_a
$= \sum_{z} P^{(b)}(y \mid do(z))\, P^{(a)}(z \mid do(x))$	Meta-TR Def.

DERIVATION OF GENERAL TRANSPORTABILITY



query

$Q = P^*(y \mid do(x)) = P(y \mid do(x), s^a, s^b)$	Meta-TR Def.
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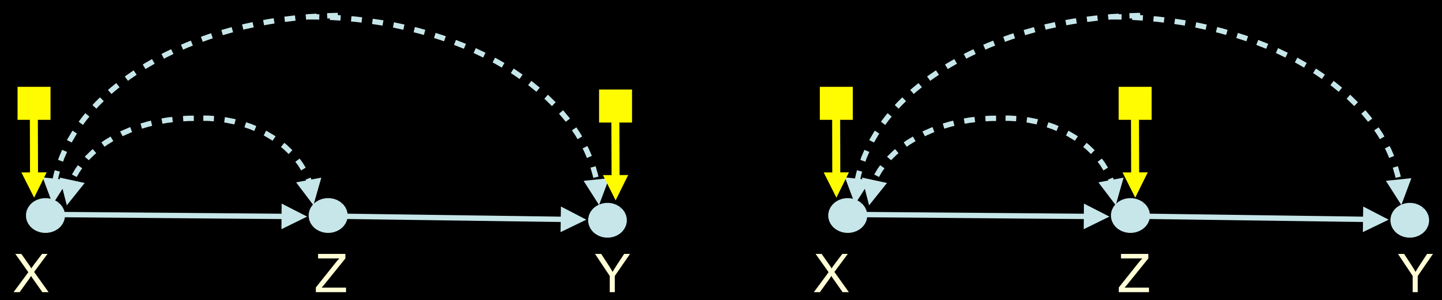

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data $\qquad$ NYC $\qquad$ LA

A MORE CHALLENGING INSTANCE …

Assume we want to transport $Q = P^*(y \mid do(x))$ from $\{ \prod^a, \prod^b \}$ by intervening over $\{Z_1\}$ in $\prod^*$, $\{Z_2\}$ in $\prod^a$, $\{Z_1\}$ in $\prod^b$.

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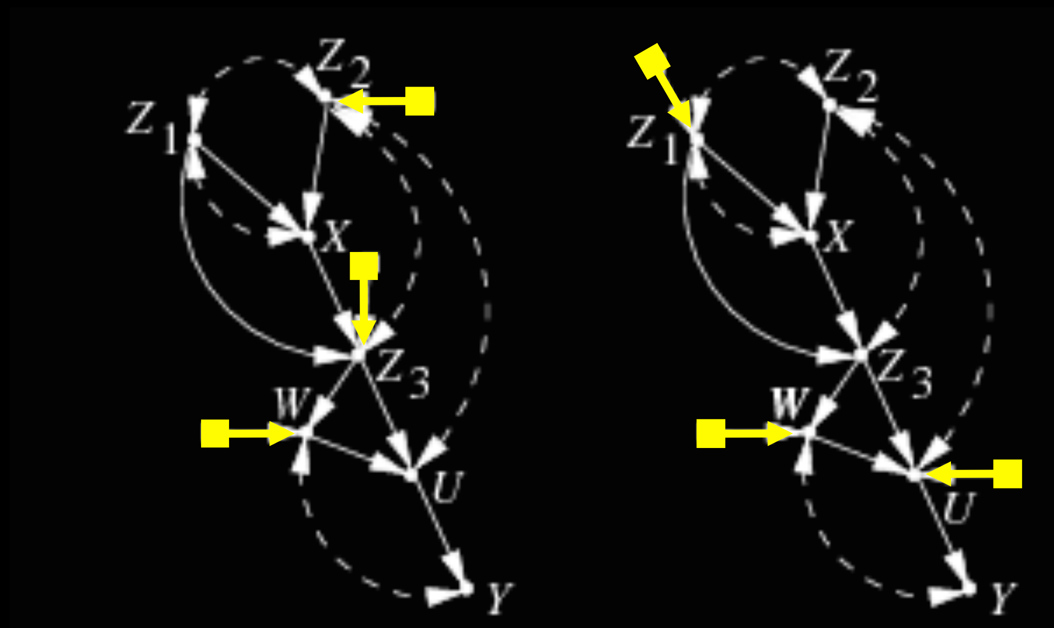

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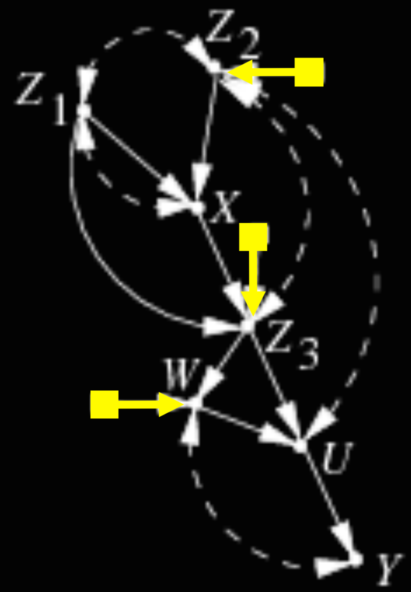

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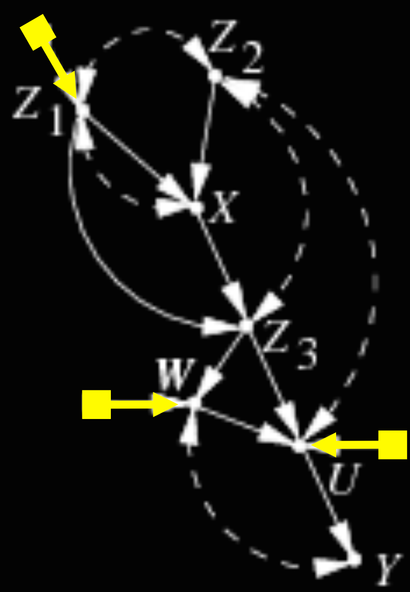


$\prod^b$ (=NYC)

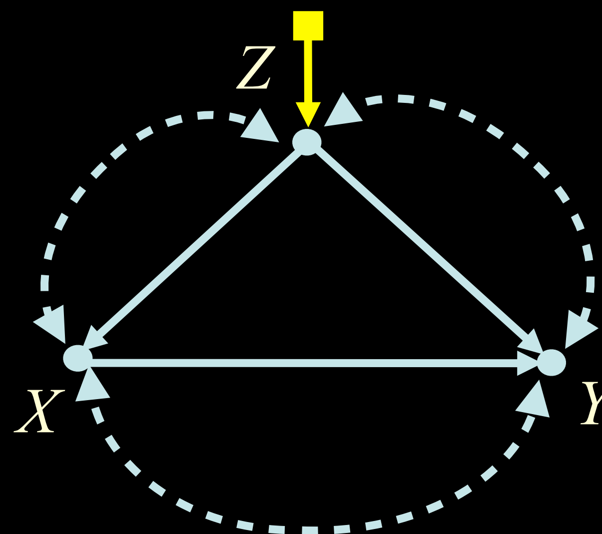
$$P^*_x(y) = \sum_{z_1,z_3,w,u} P^*(z_1) \Big(w_1^{(1)} \sum_{Z_{2'}} P^*_{z_1}(z_3|x, Z_2')P^*_{z_1}(Z_2') + w_2^{(1)} \sum_{Z_{2'}} P^{(b)}_{z_1}(z_3|x,Z_2')\, P^{(b)}_{z_1}(Z_2') \Big)$$

$$\Big(w_1^{(2)} \Big(\sum_{Z_{2'}} P^*_{z_1}(u|w, z_3, x, Z_2')\, P^*_{z_1}(z_3|x,Z_2')\, P^*_{z_1}(Z_2')\Big) \Big/ \Big(\sum_{Z_{2''}} P^*_{z_1}(z_3|x,Z_2'')\, P^*_{z_1}(Z_2'') \Big)$$

$$+ w_2^{(2)} P^{(a)}_{z_2}(u|w, z_3, x, Z_2') \Big)\, P^*(w|x,z_1,z_2,z_3)\, P^*(y|x,z_1,z_2,z_3,w, u)$$

IS THE GOLD STANDARD SO GOLD?
(GENERALIZABILITY FROM EXPERIMENTS)

Before randomization



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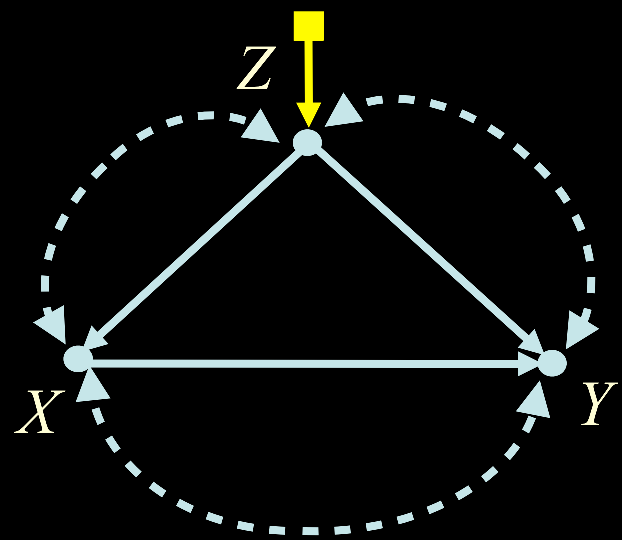
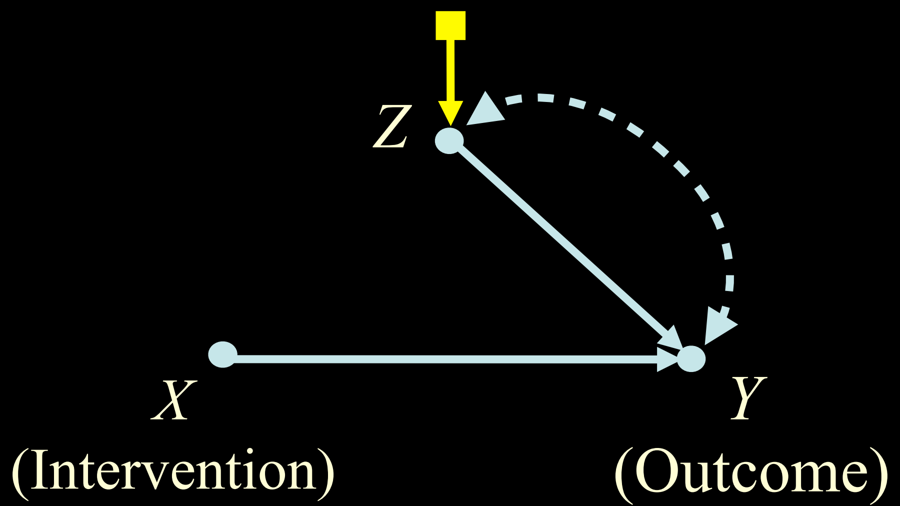


After randomization



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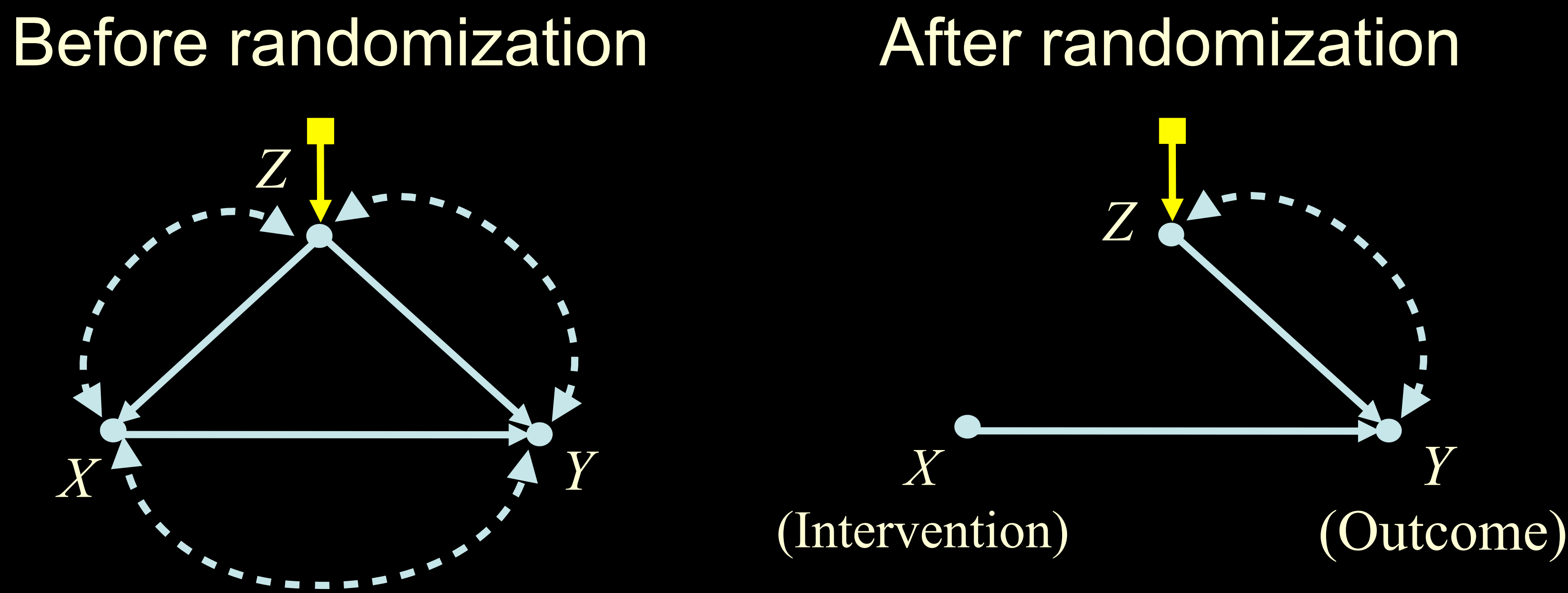

The gold standard is not absolute, and strong assumptions are required to validate claims from trials.

SUMMARY D=2: POLICY GENERALIZATION

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- References:

Pearl & Bareinboim (2011) – semantics for transportability
Bareinboim & Pearl (2012) – completeness of limited TR
Bareinboim et al (2013) – multiple & limited + extensions
Bareinboim & Pearl (2014) – completeness of general TR

THE DATA-FUSION PROBLEM

SAMPLING CONDITIONS (D = 3)

THE SELECTION BIAS PROBLEM

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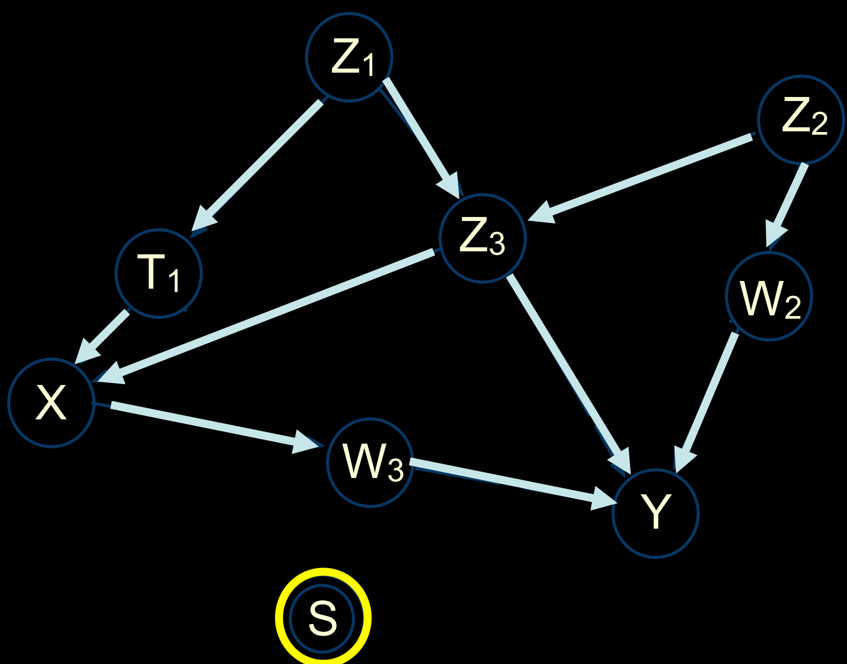

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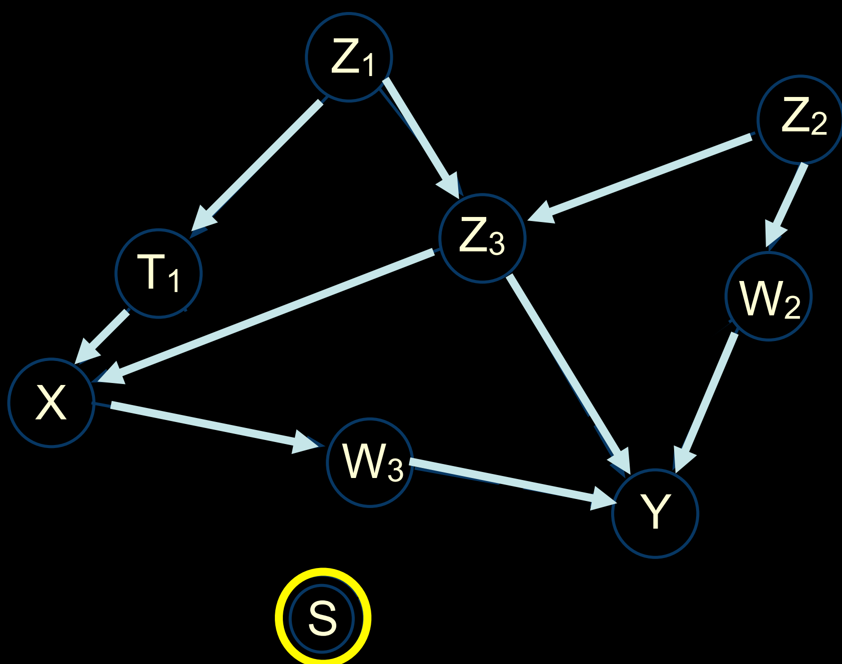

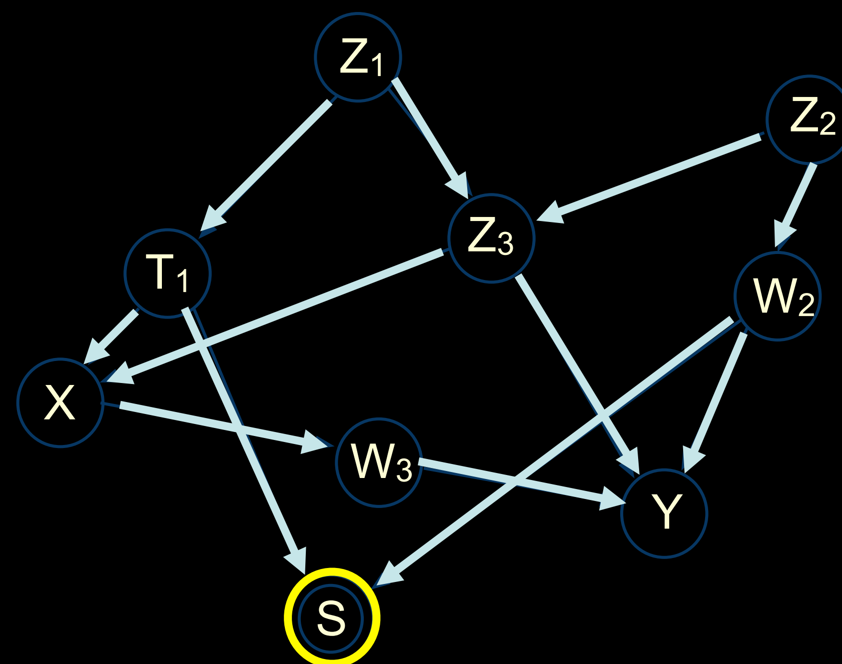

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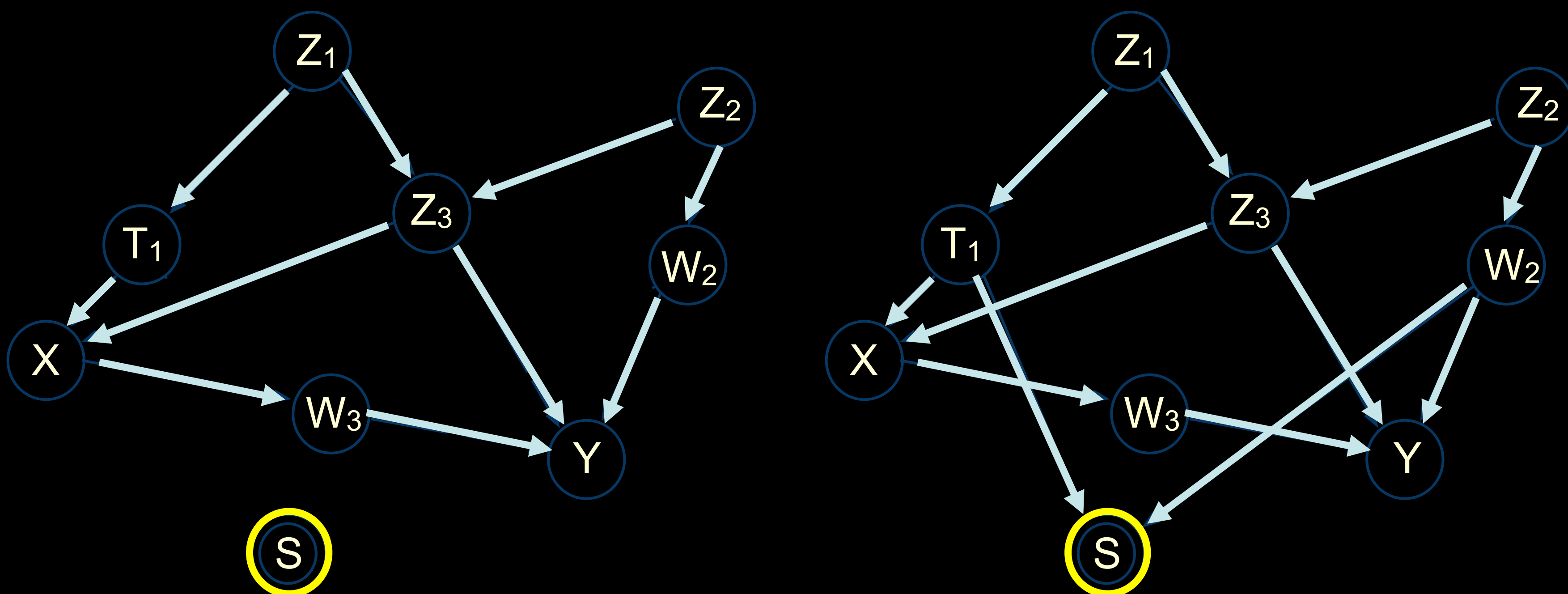
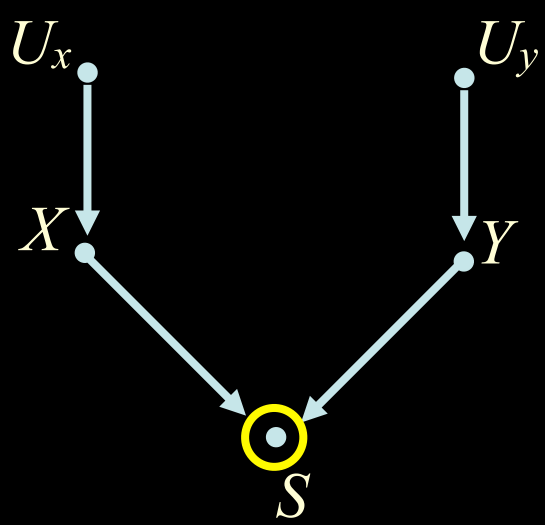


- Under what conditions can we estimate the query (e.g., $P(y \mid x)$) from $P(v \mid S = 1)$.

THE ORIGIN OF SELECTION BIAS

G:



Under selection (S=1):

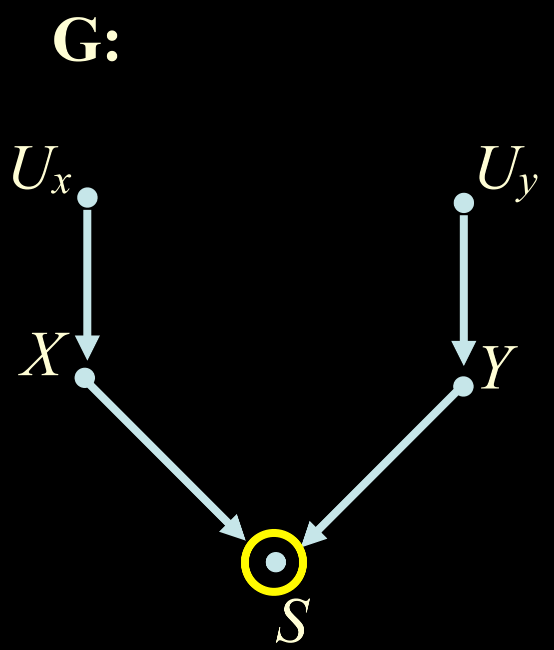
$P(X=1, Y=1 \mid S=1) = 1/2$
$P(X=0, Y=0 \mid S=1) = 1/2$
(correlation=1)

$P(Y=y|X=1,S=1) -$
$P(Y=y|X=0,S=1) = 1$

M:
$X = U_x$
$Y = U_y$
$S = \neg(X \oplus Y)$

$P(U_x,U_y) = 1/4$

THE ORIGIN OF SELECTION BIAS



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$P(Y=y|X=1,S=1) -$
$P(Y=y|X=0,S=1) = 1$

but

$P(Y = y, X = x) = 1/4$

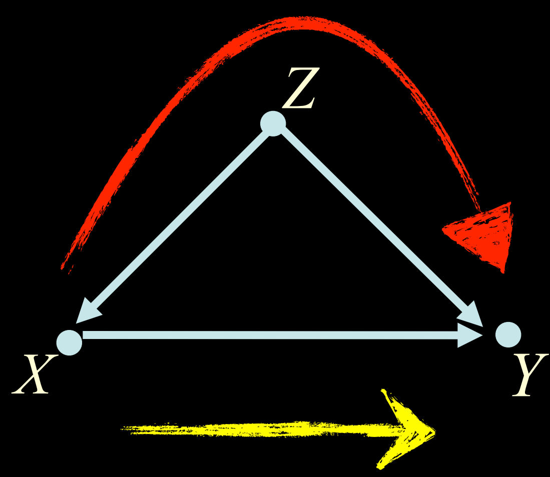
$P(Y=y \mid X=1) -$
$P(Y=y \mid X=0) = 0$

M:
$X = U_x$
$Y = U_y$
$S = \neg(X \oplus Y)$

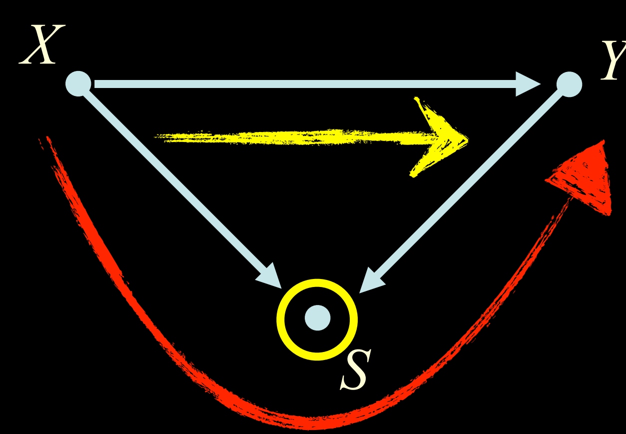
$P(U_x, U_y) = 1/4$

CONFOUNDING BIAS vs SELECTION BIAS

- Unblockable "flow" of information between treatment and outcome — spurious correlation.



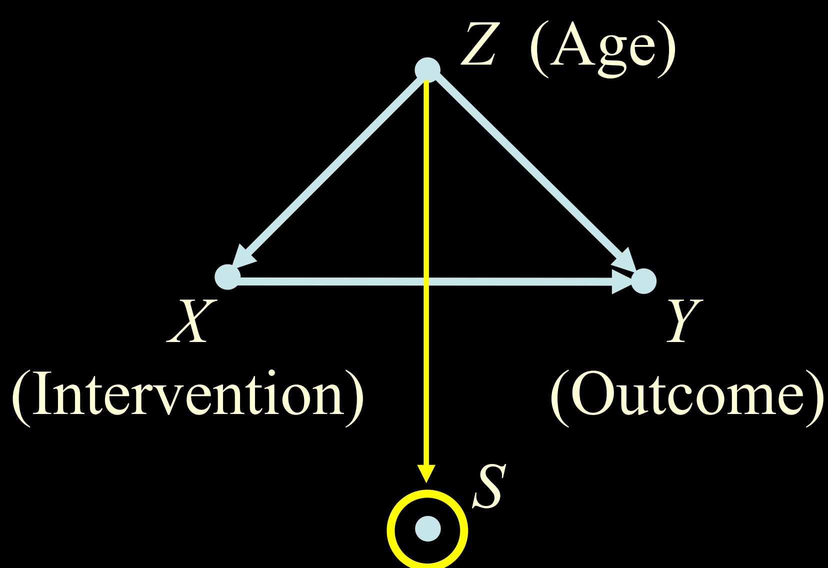
G_1



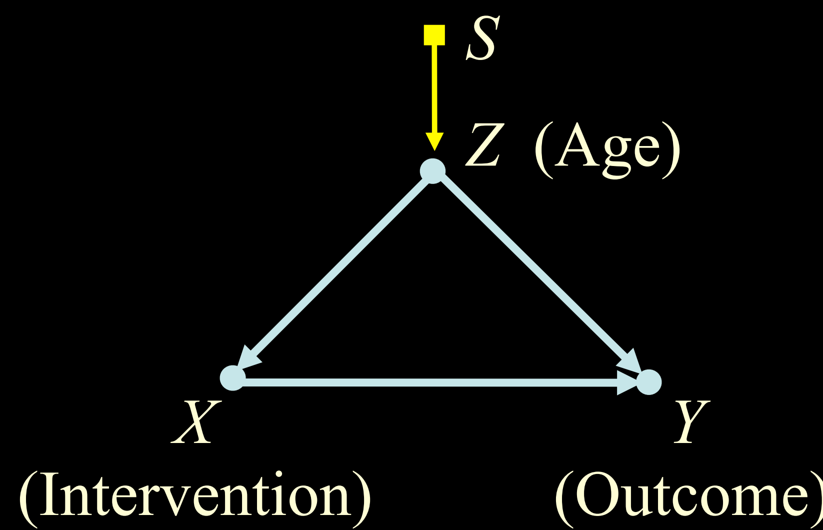
G_2

SELECTION VERSUS TRANSPORTABILITY (MAN-MADE VERSUS NATURE-MADE)

Selection Bias



Transportability



Z = sample-producing factors
S = selection indicator

Z = disparities across populations
S = difference-producing factors

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   [Elkan'01, Zadrozny'04, Cortes'08, Storkey'09]

SELECTION BIAS -- VARIANTS

- **Selection bias without external data (Odds Ratio):**
  Input: $P(v \mid S = 1)$;
  Output: $OR(y,x)$ recoverable? if so, how?

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- **Selection bias in causal inference:**
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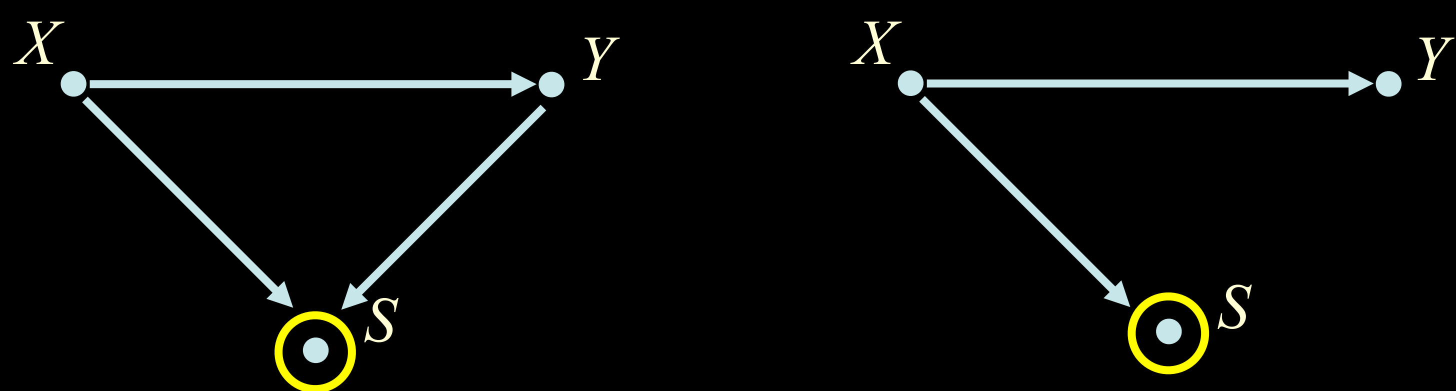
  Output: $P(y \mid do(x))$ recoverable? if so, how?

- **Selection bias with instrumental variables:**
  Input: $P(v \mid S = 1) + P(t)$, $T \subseteq V$;
  Output: Bounds for $P(y \mid do(x))$ recoverable? if so, how?

SELECTION BIAS WITHOUT EXTERNAL INFORMATION

Question. Under what conditions can we estimate the distribution $P(y \mid x)$ from $P(v \mid S = 1)$.

Theorem. $Q = P(y \mid x)$ is recoverable from selection biased data if and only if $(S \perp\!\!\!\perp Y \mid X)$.

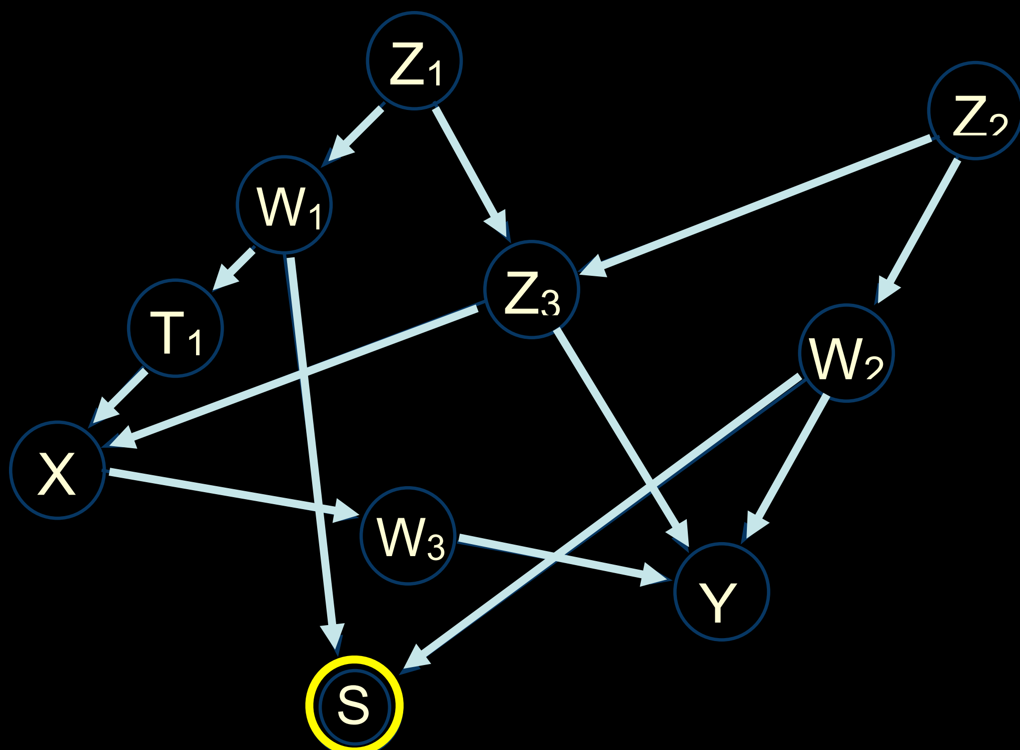


P(y | x) is not recoverable.

P(y | x) is recoverable.

SELECTION WITH EXTERNAL INFORMATION

Assume we want to estimate $Q = P(y \mid x)$ from selection…



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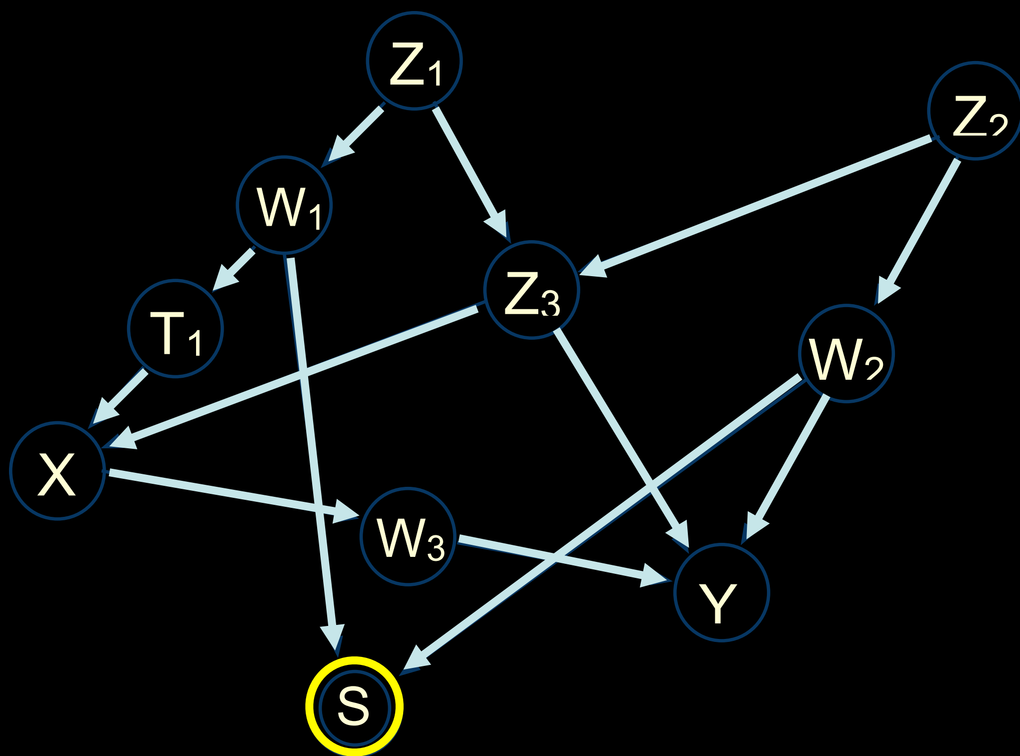

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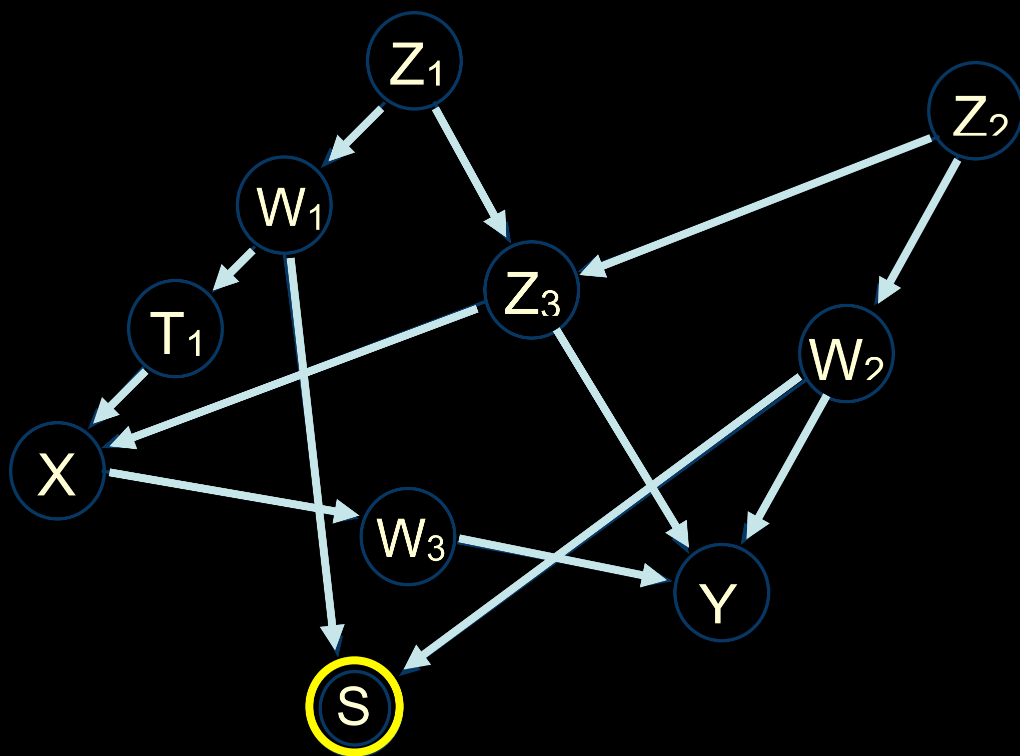
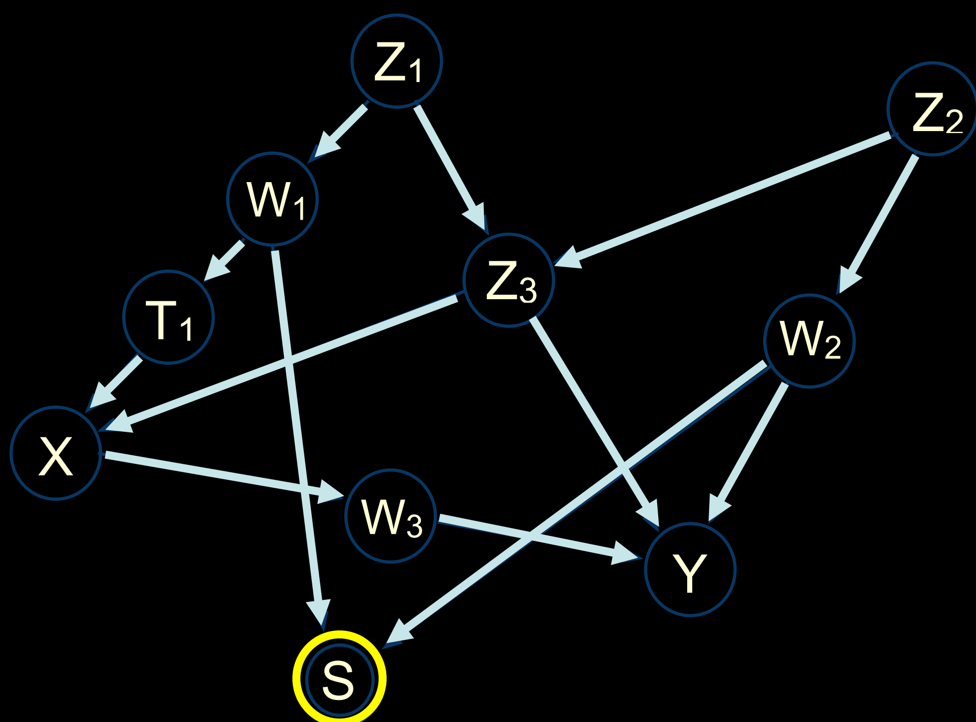


Q is not recoverable by the previous theorem…
but what if $P(W_1, W_2)$ is available?

RECOVERABILITY WITH EXTERNAL INFORMATION

Theorem. $P(y \mid x)$ is recoverable if there is a set C such that $(Y \perp\!\!\!\perp S \mid C, X)$ holds in G and $P(C, X)$ is estimable.

Moreover, $$P(y \mid x) = \sum_{c} P(y \mid x, c, S = 1)\, P(c \mid x)$$



$$C = \{W_1, W_2\}?$$
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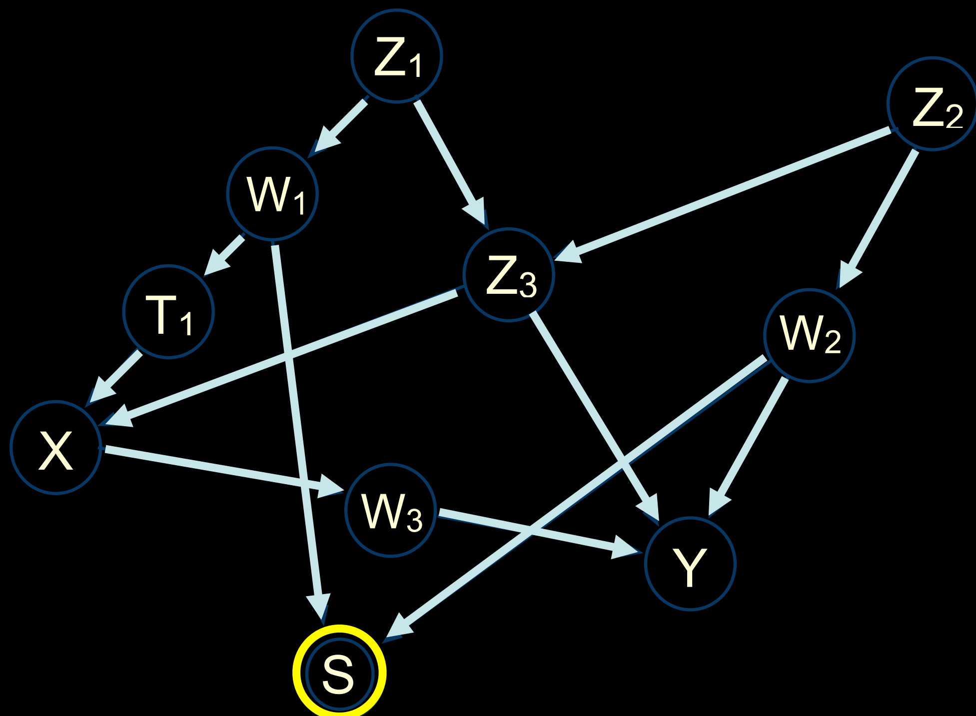

$C = \{W_1, W_2\}$? yes

$= \{W_2, Z_1, Z_2\}$?

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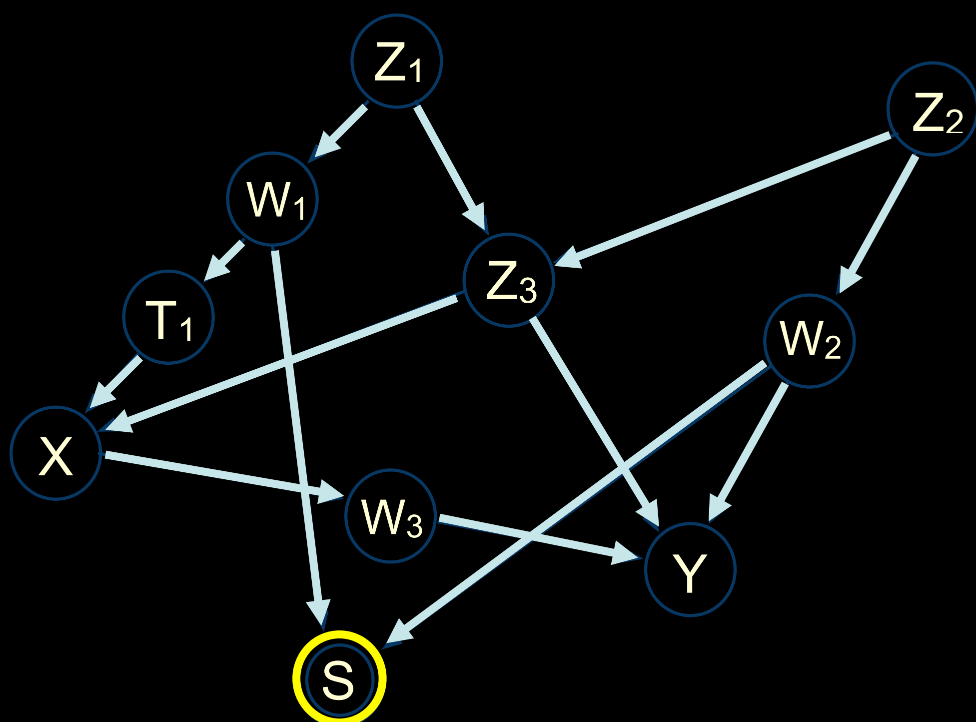

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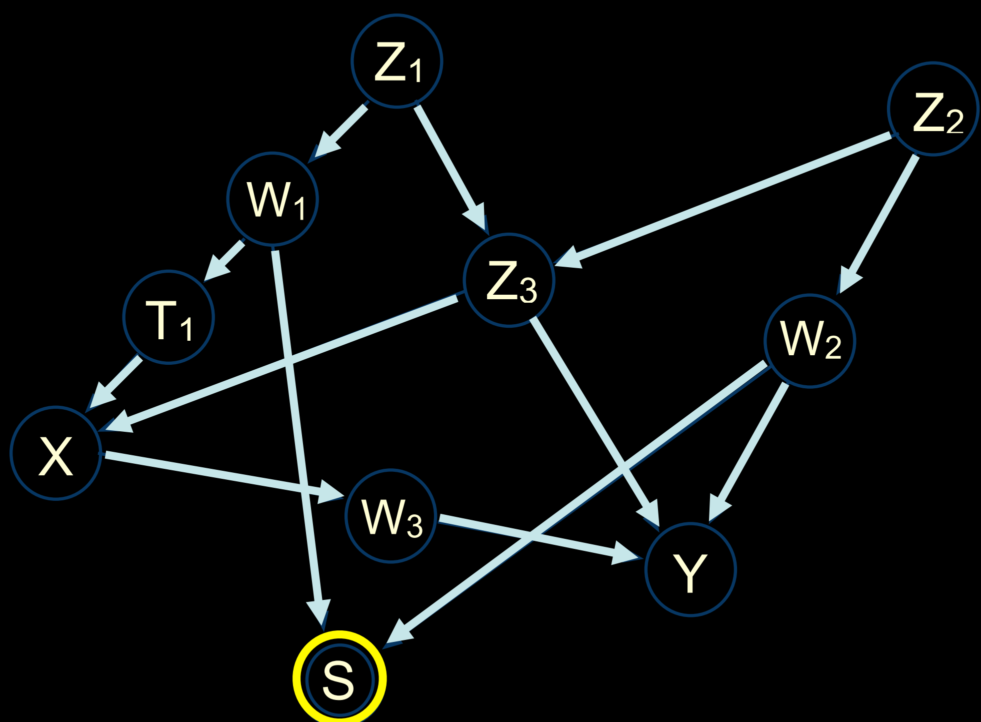


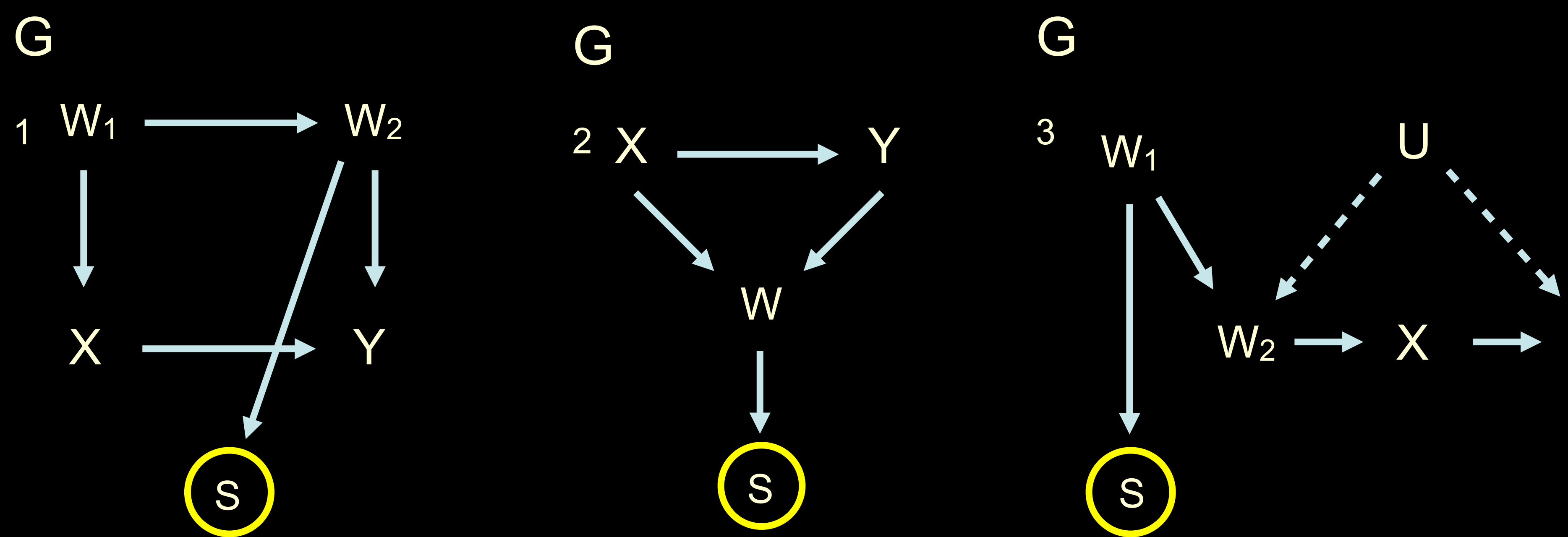
$C = \{W_1, W_2\}$? yes

$= \{W_2, Z_1, Z_2\}$? no

$= \{W_2, Z_3\}$? yes

SELECTION BIAS IN CAUSAL INFERENCE

We want to recover $Q = P(y \mid do(x))$.



$C = \{W_2\}$ recoverable
$= \{W_1\}$ not recoverable

$C = \{\ \}$ not recoverable
$= \{W\}$ not recoverable
$= \{W, X\}$ recoverable

recoverable
$(S \perp\!\!\!\perp Y \mid X, W_2)$
does not hold

SELECTION BIAS IN CAUSAL INFERENCE

Def. Let a set Z be partitioned into Z^+ and Z^- such that Z^+ contains all non-descendants of X and Z^- the descendants of X. Z is said to satisfy the s-backdoor criterion relative to an ordered pair of variables (X, Y) and an ordered pair (M, T) in a graph G_s if Z^+ and Z^- satisfy the following conditions:

(i) Z^+ blocks all backdoor paths from X to Y;
(ii) X and Z^+ block all paths between Z^- and Y;
(iii) X and Z block all paths between S and Y;
(iv) $Z \cup \{X, Y\} \subseteq M$, $Z \subseteq T$.

Thm. If a set Z satisfies the s-backdoor relative to the pairs (X, Y) and (M, T), then the effect of X on Y is identifiable and recoverable and is given by the formula

$$P(y \mid do(x)) = \sum_{z} P(y \mid x, z, S = 1)\, P(z)$$

SUMMARY (D=3): SELECTION BIAS

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- References:

Bareinboim, Pearl (2012) – OR completeness & IVs
Bareinboim, Tian, Pearl (2014) – Markovian completeness
Bareinboim, Tian (2015) – Causal queries
Correa, Bareinboim (2017) – Complete adjustment

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- Data-fusion requires encoding of structural features of the data-generating model.
- Both population-level and individual-level causal inference and fusion can be performed once the proper formal machinery is employed; algorithms and conditions are available.
- Principled framework for data-fusion — pooling and aggregating observational, experimental, and counterfactual information spread throughout heterogenous domains for population- and individual-level causal inference (understanding) and decision-making.

"Development of Western Science is based on two great achievements, the invention of the formal logical system (in Euclidean geometry) by the Greek philosophers, and the discovery of the possibility to find out causal relationships by systematic experiment (during the Renaissance)". Albert Einstein.

"Imagine how much harder physics would be if electrons had feelings!". Richard Feynman.

THANK YOU!