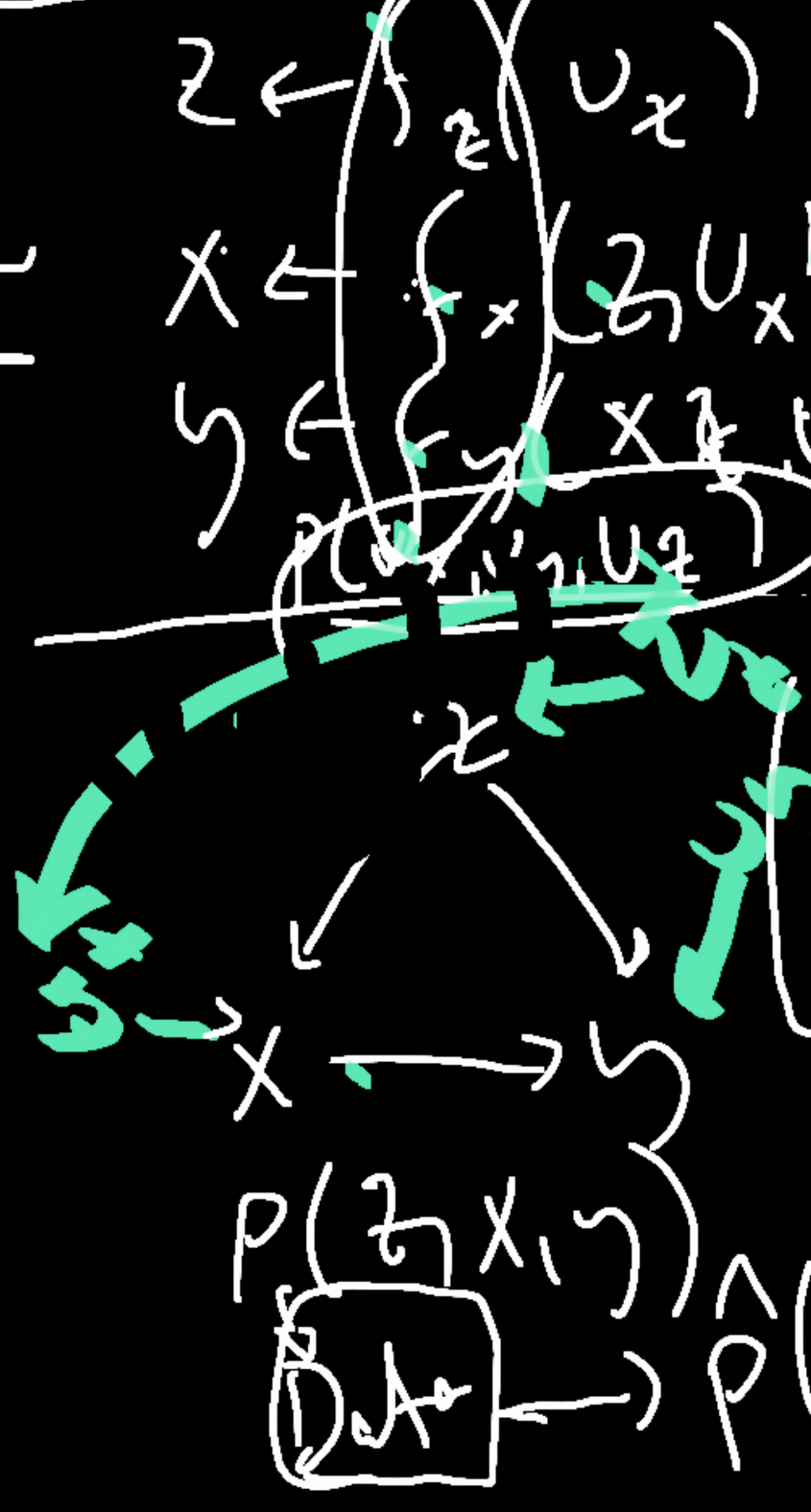
Observational / Natural (truth)

$$\Sigma \quad \begin{aligned} Z &\leftarrow f_Z(U_Z) \\ X &\leftarrow f_X(Z, U_X) \\ Y &\leftarrow f_Y(X, Z, U_Y) \end{aligned}$$

$P(U_X, U_Y, U_Z)$

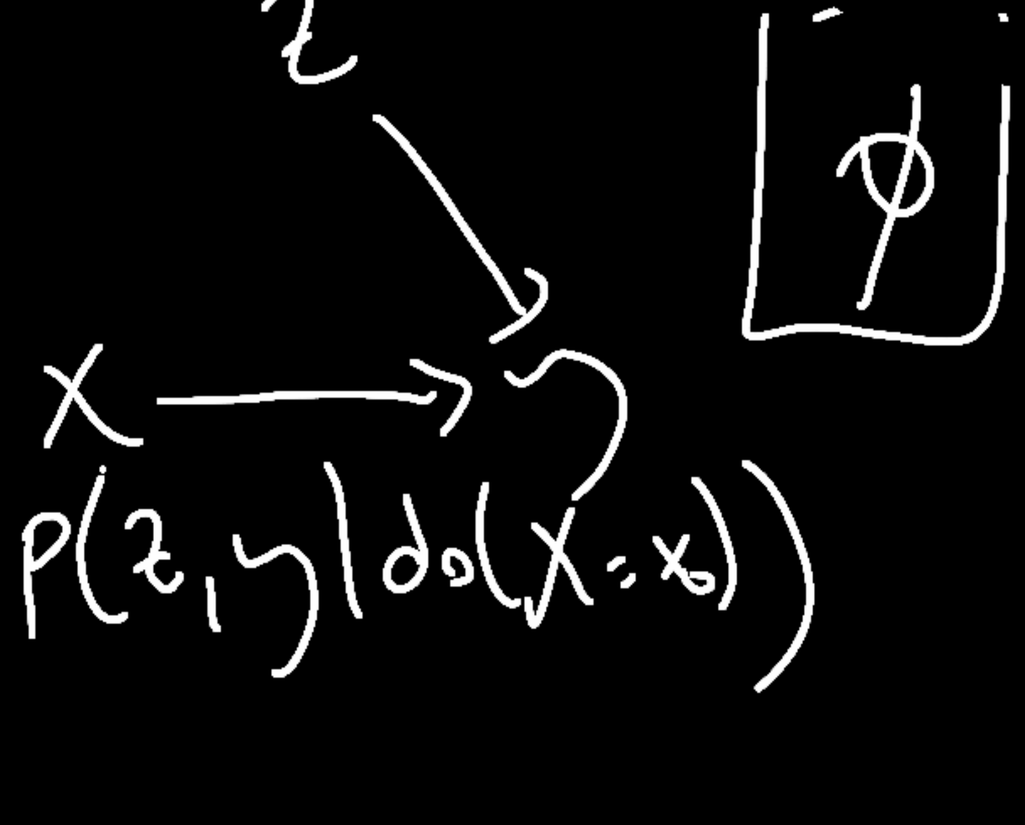


$P(Z, X, Y)$

$\hat{P}(Z, X, Y)$

Intervention / Causal

$$\begin{aligned} Z &\leftarrow f_Z(U_Z) \\ X &\leftarrow x_0 \\ Y &\leftarrow f_Y(X, Z, U_Y) \end{aligned}$$



$P(Z, Y \mid do(X = x_0))$

(ID) of $Q = P(y \mid do(x))$:

Def: Q is ID from $\overbrace{G \text{ and } P(V)}^{\text{Input}}$

iff $\exists\, M_1, M_2$ s.t.

$G = G(M_1) = G(M_2)$,

$P_1(V) = P_2(V) \rightarrow Q_1 \neq Q_2$.

(1) V = set of Endo.

(2) U = set of Exo

(3) F

(4) $P(U)$

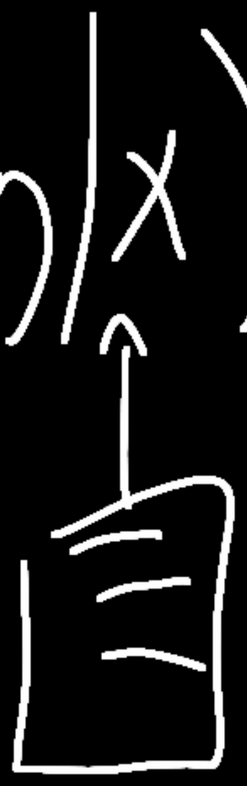
$P(V), G \models P(y \mid do(x))$

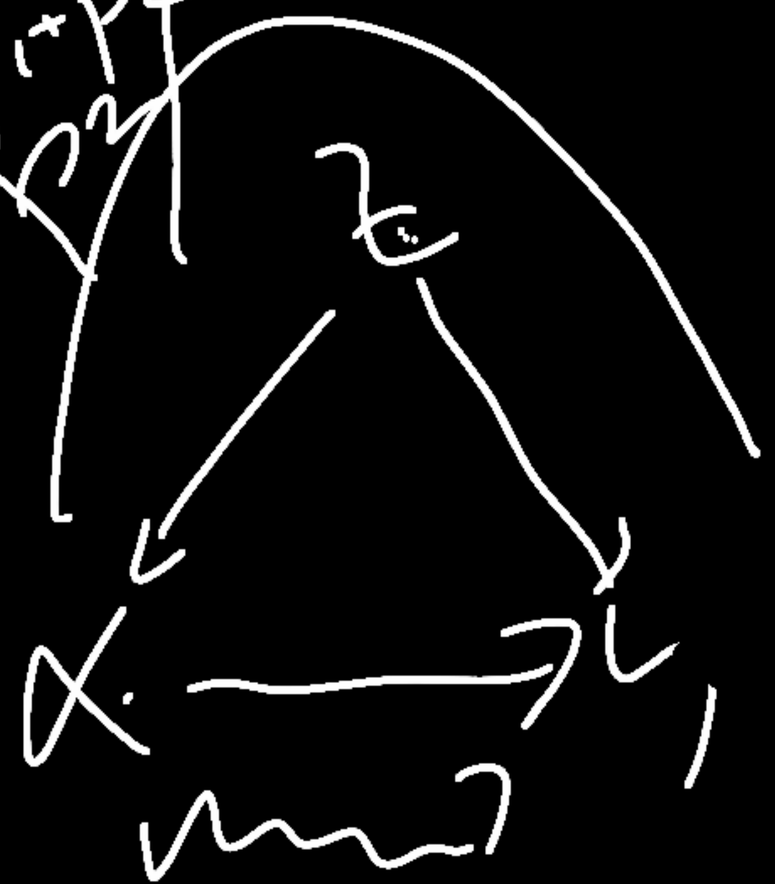
$F, P(U) \models P(y, do(x))$

$\forall v_i \in V,$

$U_i \leftarrow \int_i (pa_i, v_i),$

$P(y|x) = P_1 \cdot P_2$

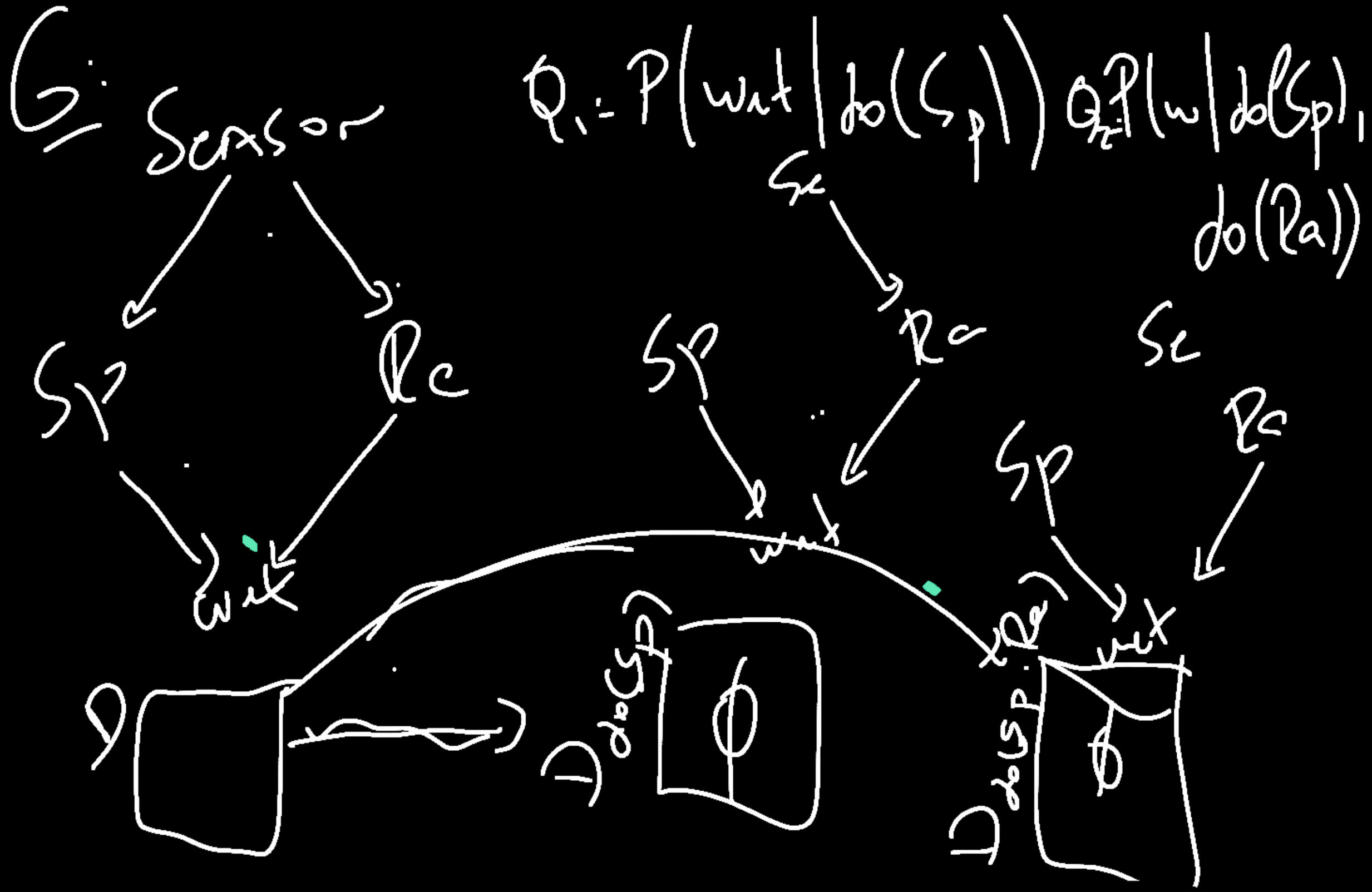


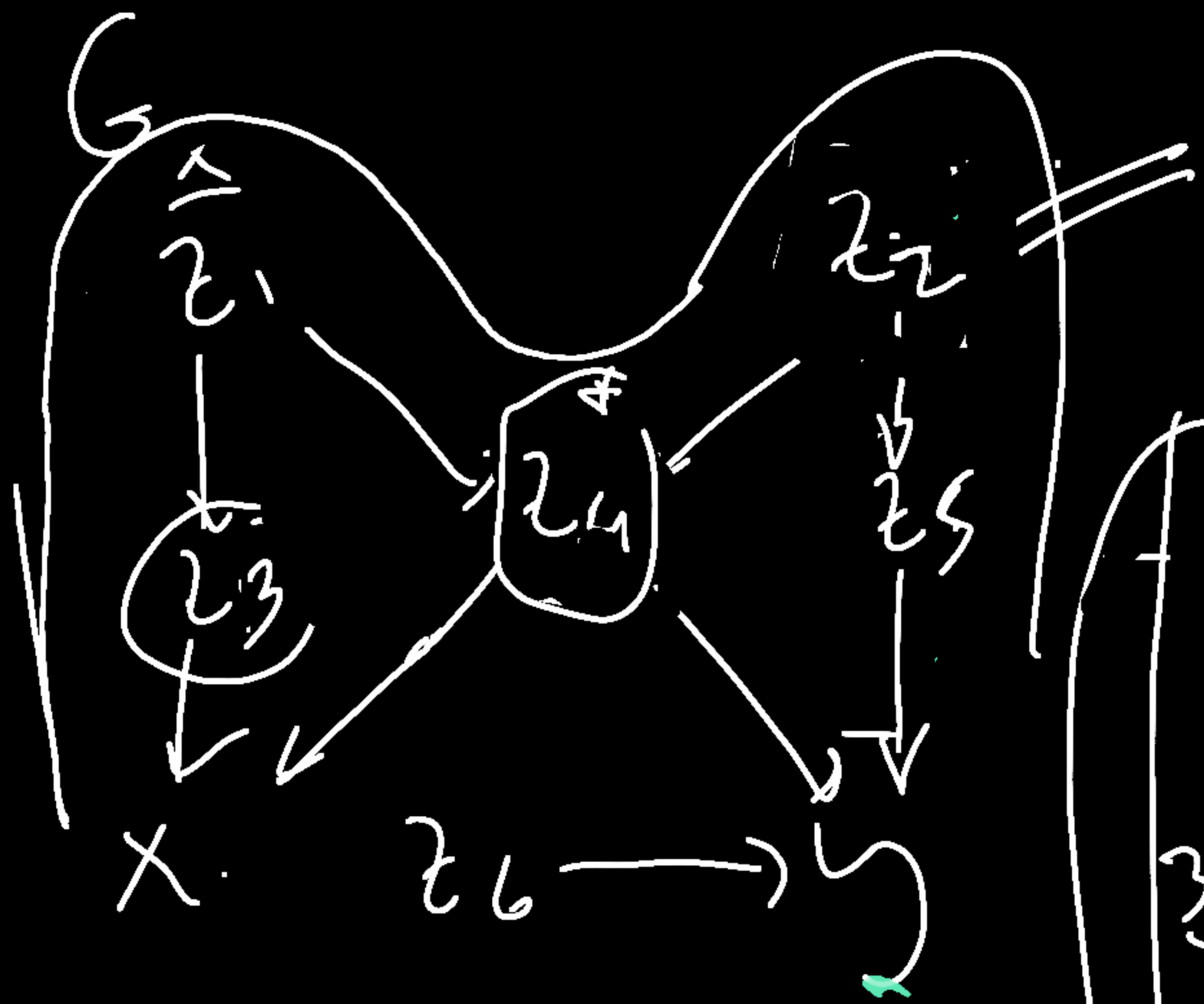


$P_{a_i} \subset V.$

$U_i \subset U.$

$P(y|do(x)) = \sum_{pa_x} P(y|pa_x, x) P(pa_x)$


G:
Season
Sp
Ra
wet
Se
Sp
Ra
wet
Se
Sp
Ra
wet
D
D do(Sp)
D do(Sp, Ra)
Q1 = P(wet | do(Sp))
Q2 = P(w | do(Sp), do(Ra))



$\{z_4, z_3\}$? yes

$\{z_4, z_2\}$? yes

$\{z_4\}$? No!

Normal:

$X \to \boxed{Y} \to Z$

$X \leftarrow \boxed{Y} \leftarrow Z$

$X \leftarrow \boxed{Y} \to Z$

X

3

Abnormal

$X \to \boxed{Y} \leftarrow Z$

$$P(y \mid do(x)) = \sum_z P(y \mid x, z)\, P(z) \quad (1)$$

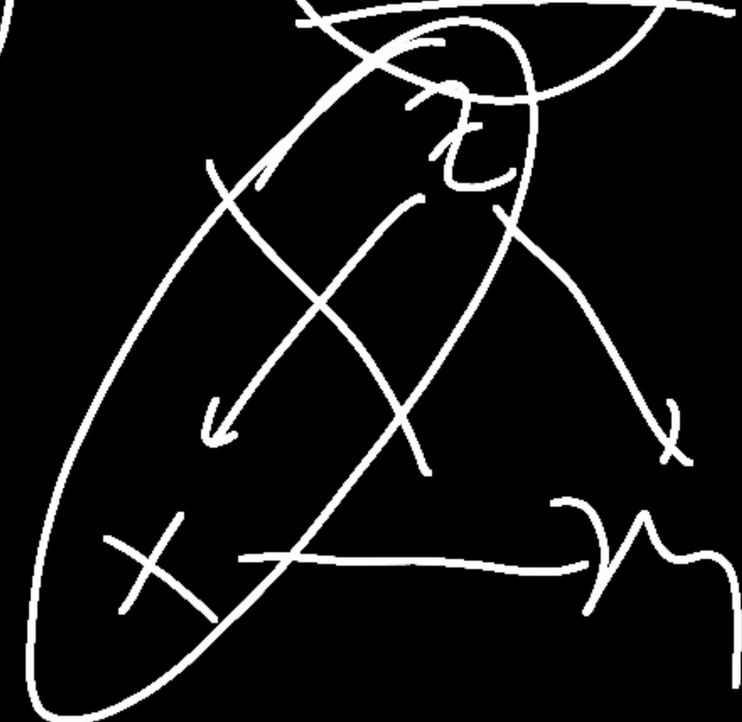
$(P(x,y,z))$ 1993

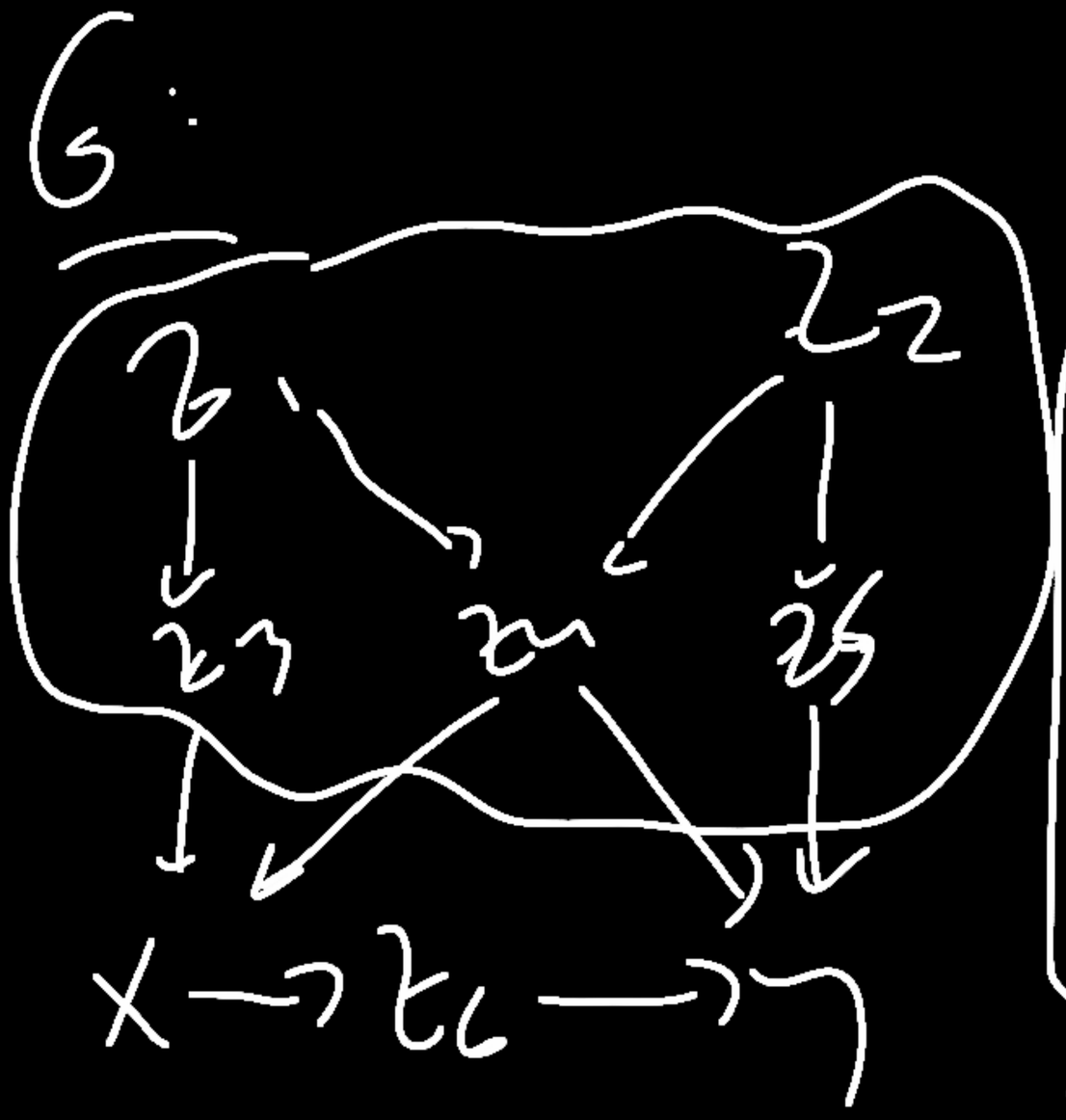
$$P(y \mid X = x) = \sum_z P(y \mid x, z)\, P(z \mid X) \quad (2)$$

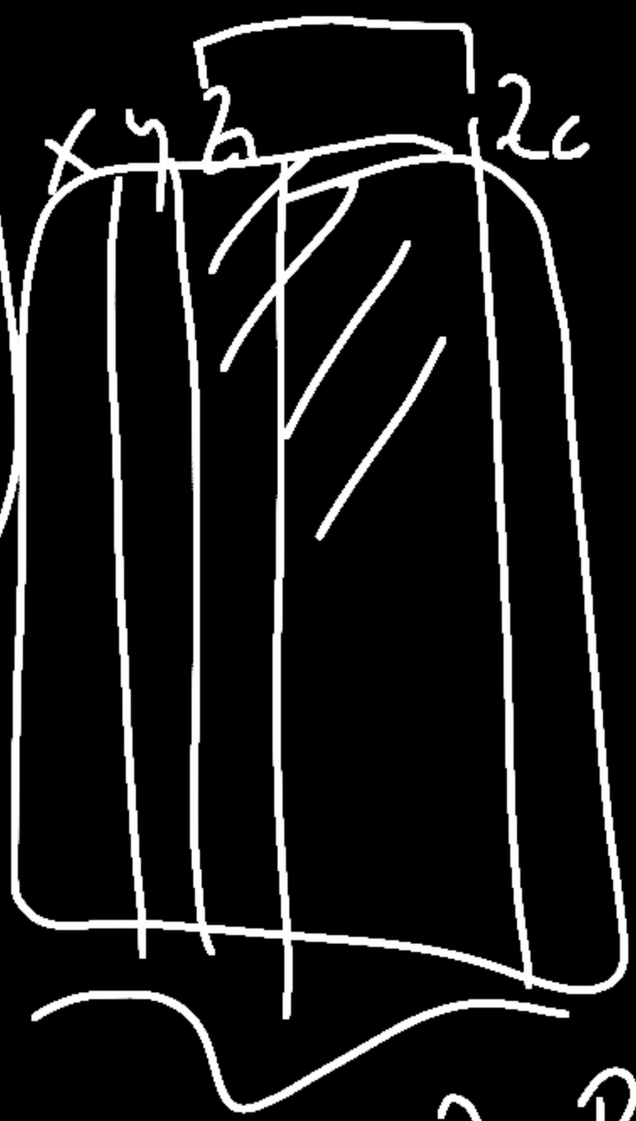
$\#z \mid y$

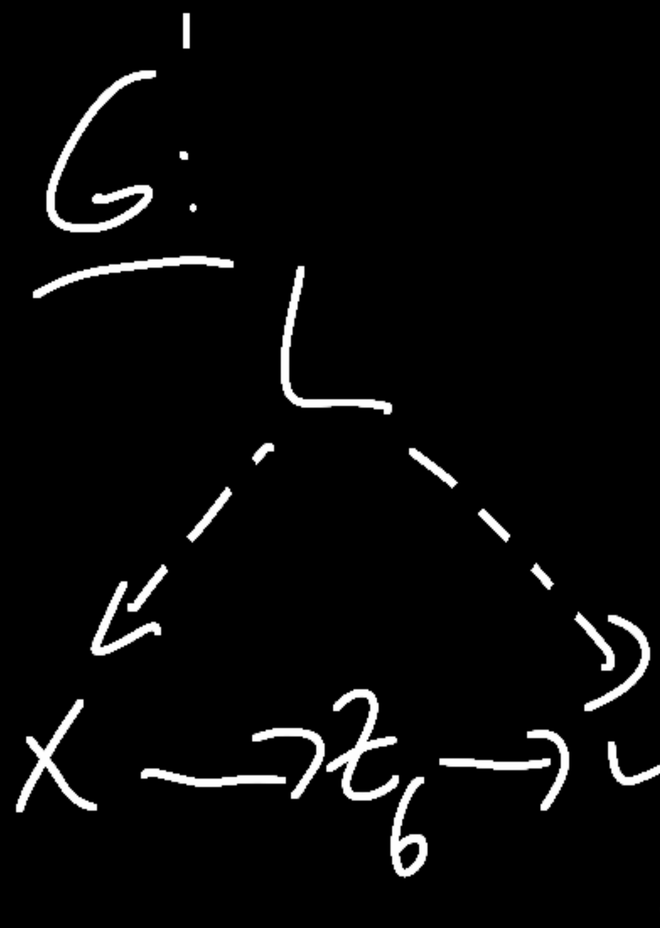
$x \not\#z \mid y$

$x \#z$



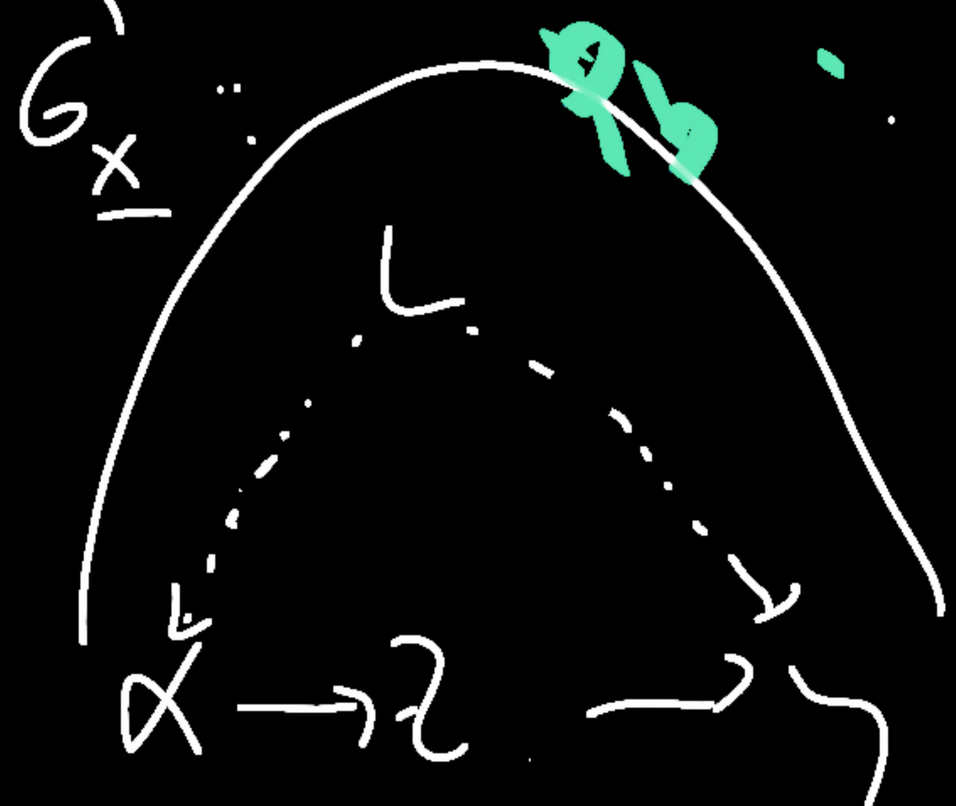






$P(x, z_c) \to P(y|do(x)) \neq$

$\neq P(y$



fd-c theorem:

If z sat fdc then

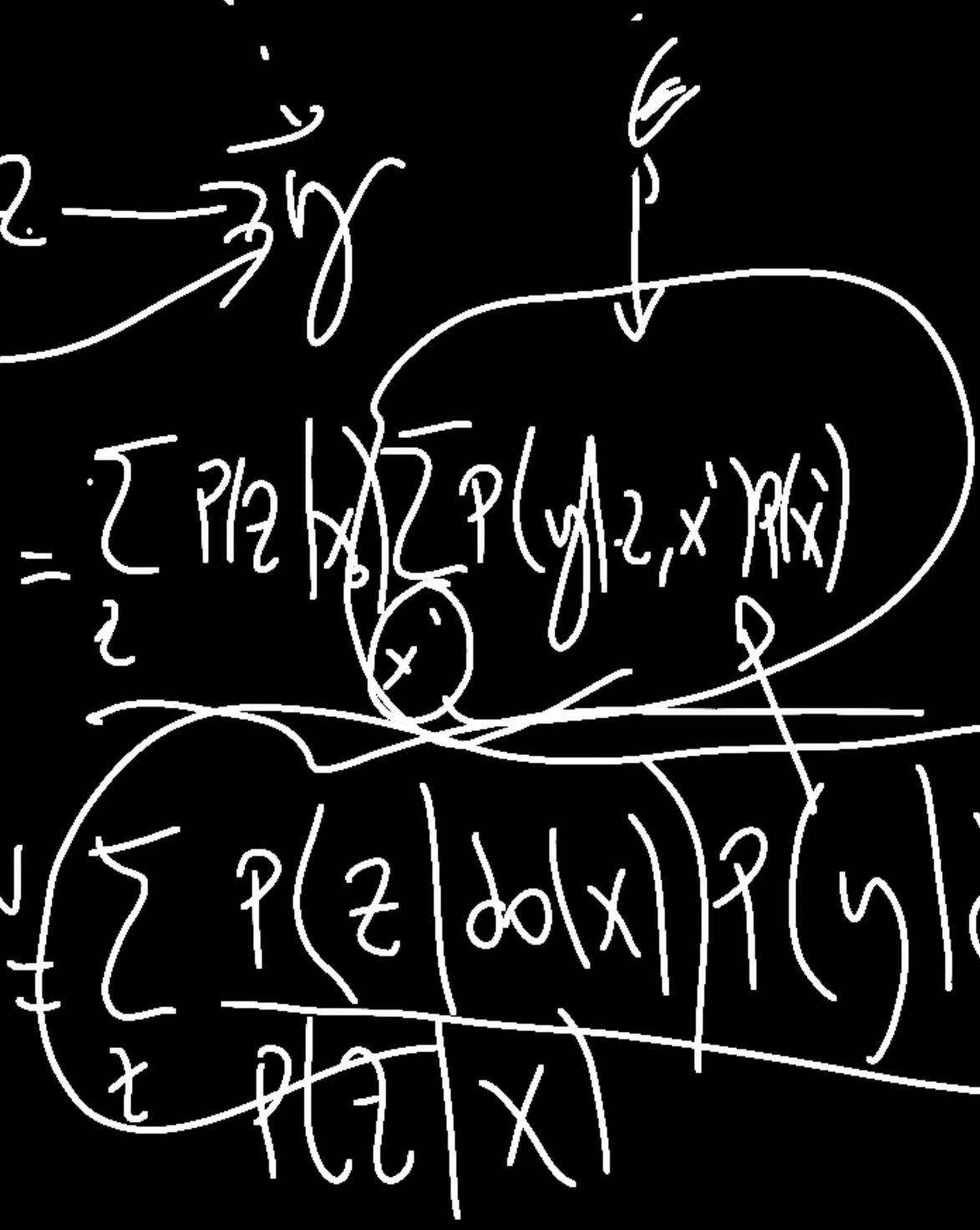
$P(y|do(x))$

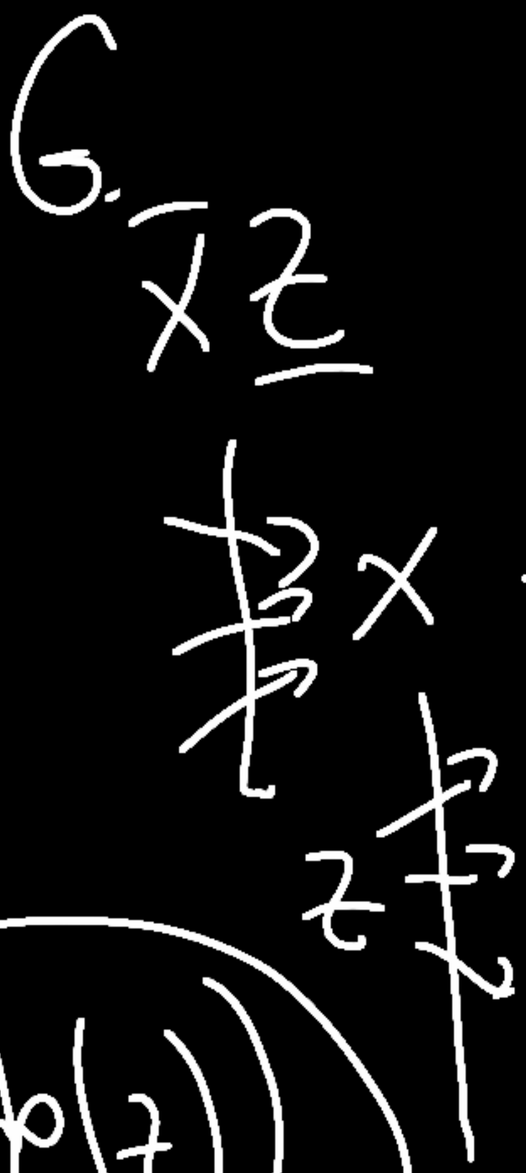
$$= \sum_z P(z|x) \sum_{x'} P(y|z,x') P(x')$$

$$= \sum_z P(z|x) \sum_u P(y|z,u) P(u)$$

$P(y|x,z_c)P(z_c)$

$|x)$





$$P(y|x_0) = \sum_z P(z|x_0) \sum_{x'} P(y|z,x') P(x')$$

$$= \sum_z P(z|do(x)) P(y|do(z))$$

$$\sum_x P(z|x)$$