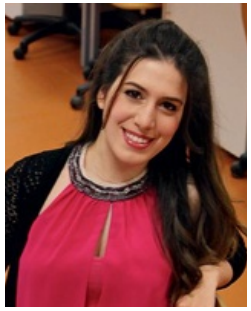
A Tutorial on RL

Benjamin Van Roy
Stanford University and DeepMind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Netflix


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Stanford


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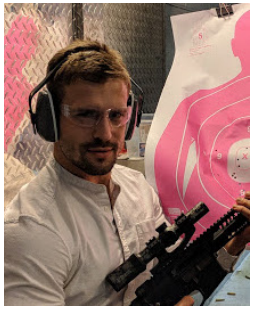
Stanford


Ian Osband
DeepMind


Dan Russo
Columbia


Zheng Wen
DeepMind


Bill Zhu
Facebook

Objectives

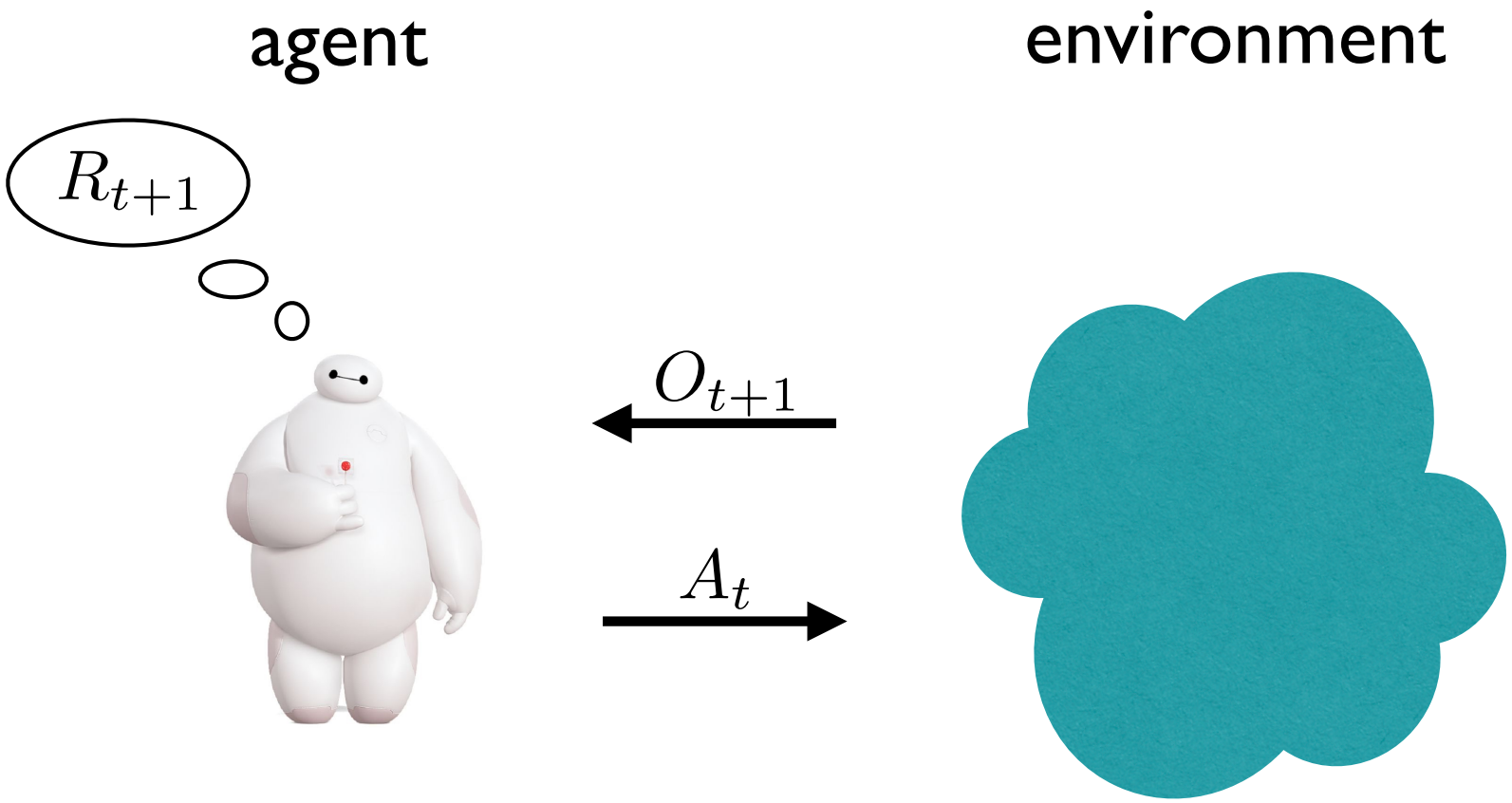
- Introduce you to reinforcement learning
  - Problem, formalism, and examples
  - Algorithms
  - Some current research directions

- Prerequisites
  - Probability / stochastic processes
  - Supervised learning
    - e.g., neural networks, SGD

What is reinforcement learning?

- A problem
- A community who work on the problem
- Methods produced by that community
  - Some of which are also useful for other problems

The Reinforcement Learning Problem



"other learning paradigms are about **minimization**, reinforcement learning is about **maximization**"

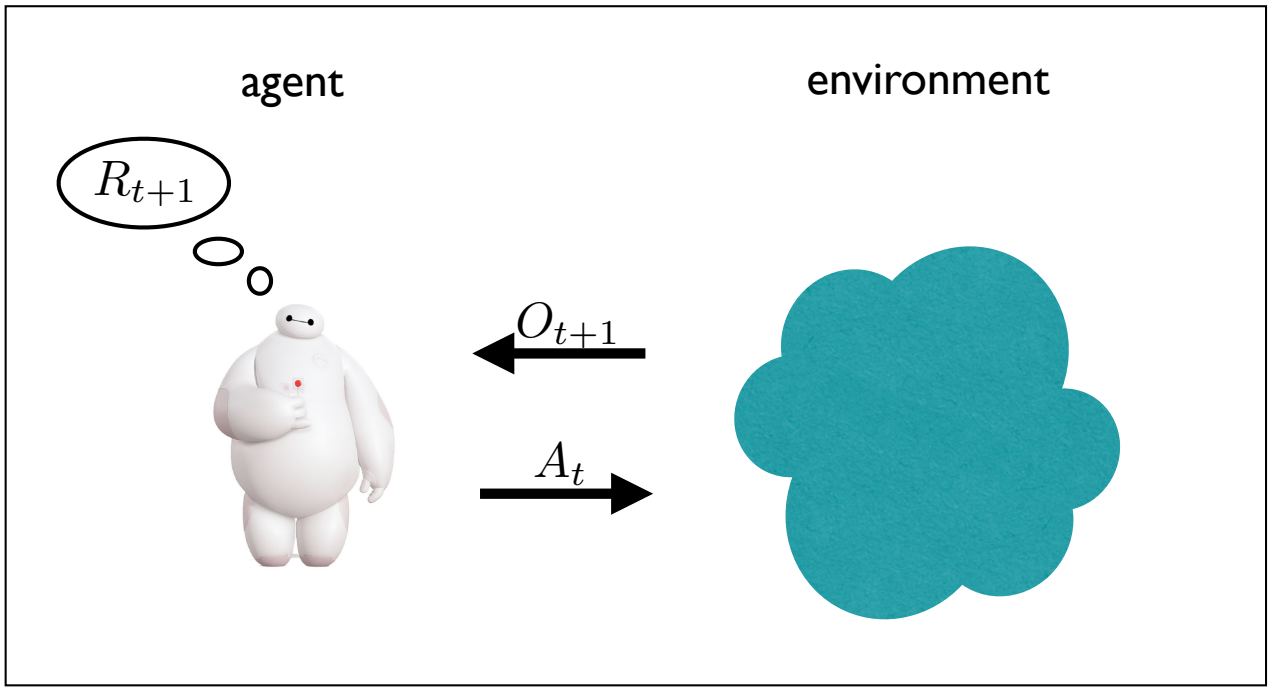
- Klopf

Artificial General Intelligence

- Develop a system that can achieve human-level competence across *all* intellectual tasks

- Can RL achieve AGI?

Episodic Version of the RL Problem



data from episodes 1 through ℓ

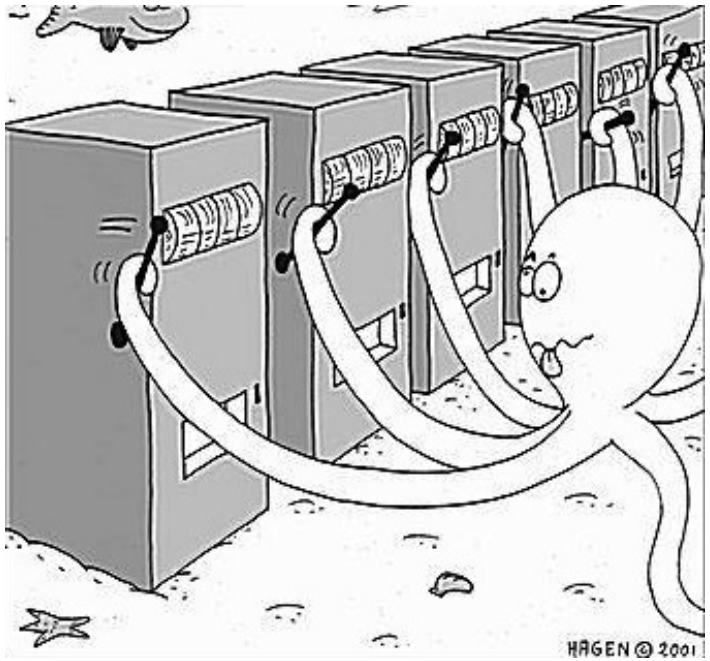
$$O_0^1, A_0^1, O_1^1, \ldots, A_{\tau_1-1}^1, O_{\tau_1}^1$$

$$\vdots$$

$$O_0^\ell, A_0^\ell, O_1^\ell, \ldots, A_{\tau_\ell-1}^\ell, O_{\tau_\ell}^\ell$$

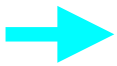
→ **RL algorithm** → policy for next episode π_ℓ

RL versus Bandits



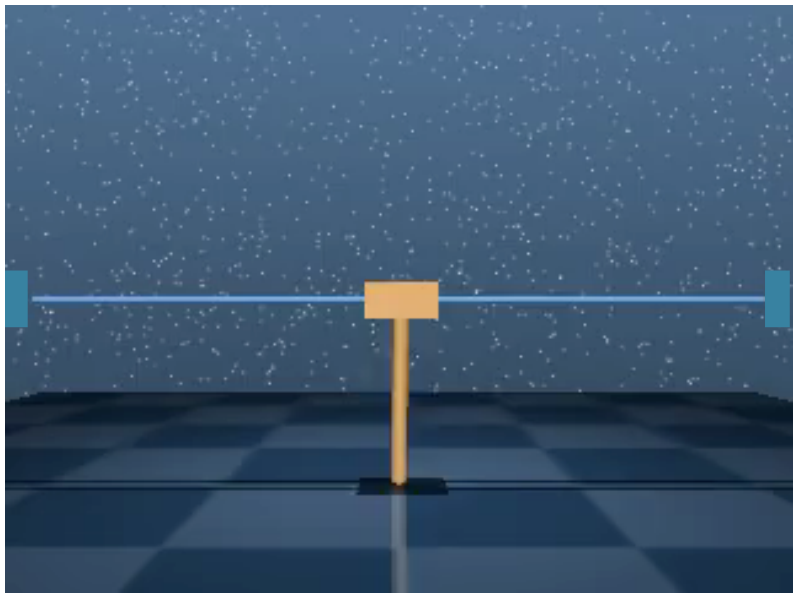
- Bandit problem = RL with episode of duration 1
- Bandit algorithms are RL algorithms applied to bandits
- Bandits much better understood
- RL formulation can be overkill for specific applications
- RL can be critical for delayed consequences
- RL is critical for AGI

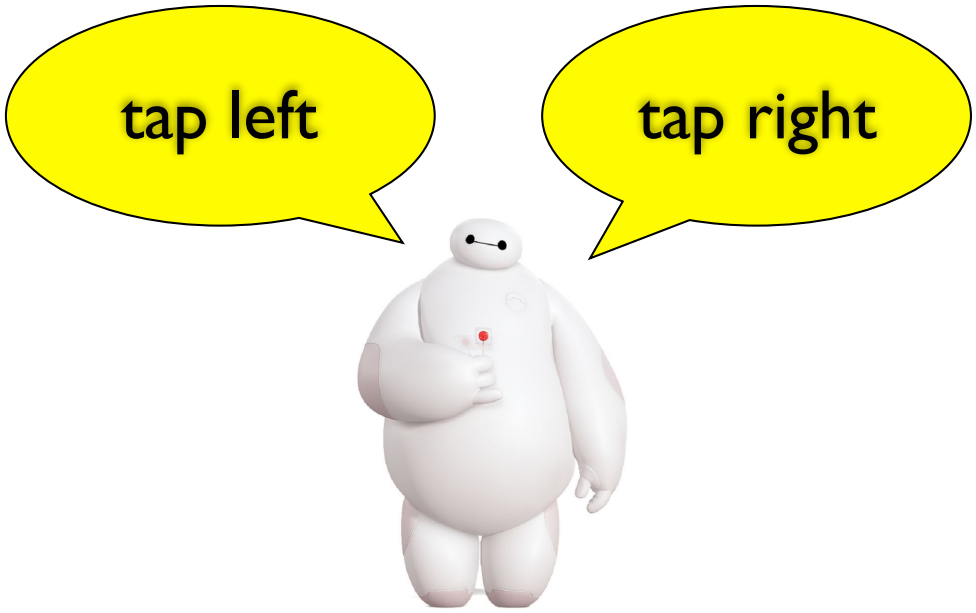
Outline



- Examples
- Mathematical formalism
- Common algorithms
- Challenges of data efficiency

Example: Cart-Pole Swingup

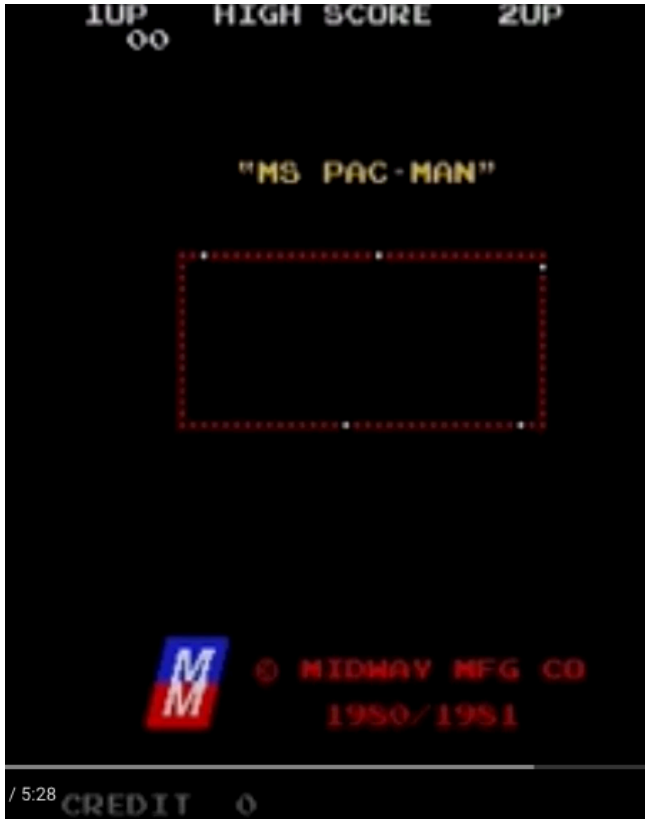


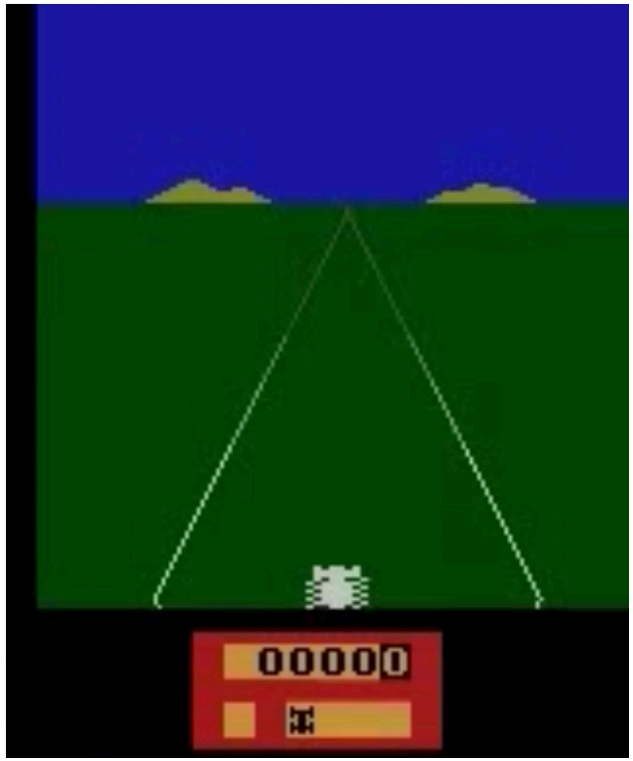


20 seconds per episode

Example: Robotics

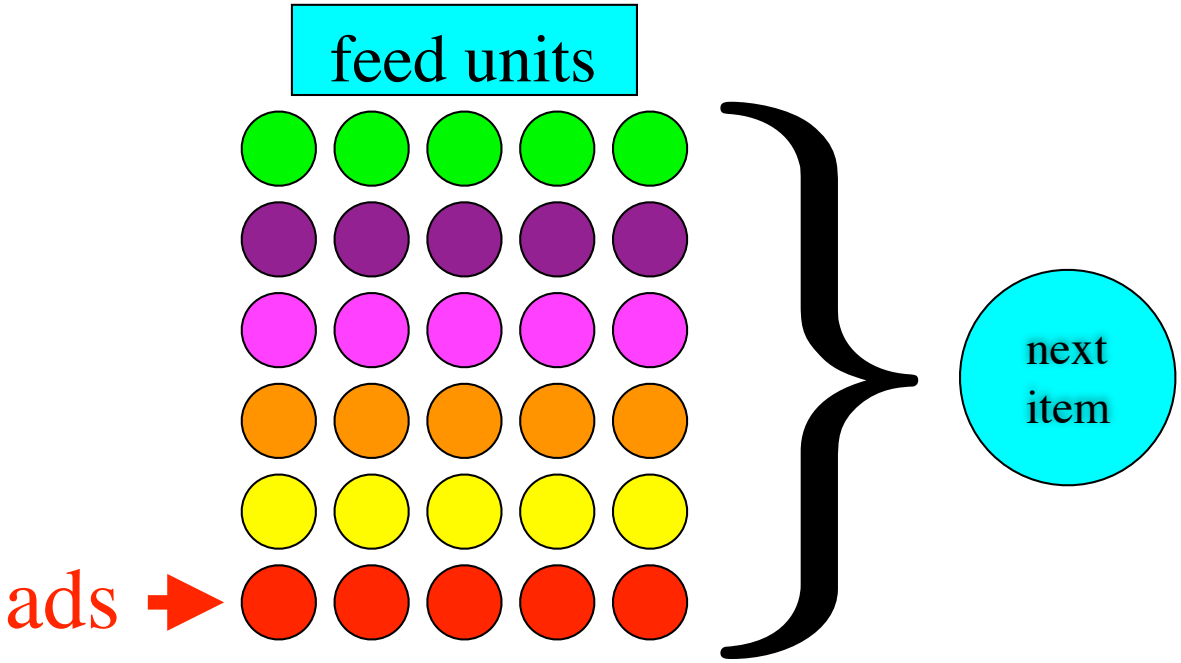
Example: Arcade Games

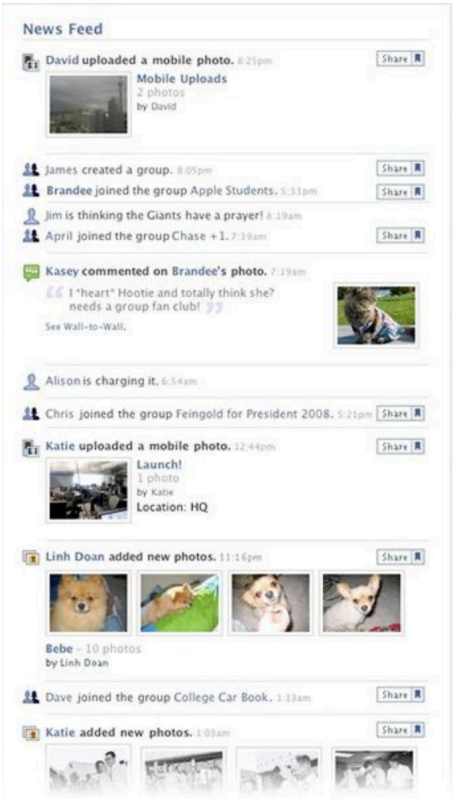




action: joystick actuation
observation: screen shot

Example: News Feed Engagement





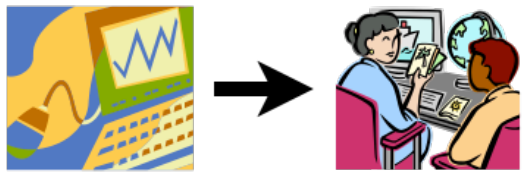
action: next item

observation: click, like, share, comment, scroll, terminate

Example: Revenue Management

seat inventory

Fare Product	Specification	Price
1	Economy SF-LA 8AM 12/11/10 LA-SF 1PM 15/11/10 Nonrefundable ...	$120
2	First SF-Ch 8AM 12/24/10 Ch-NY 4PM 12/24/10 Refundable ...	$2500
...	...	...

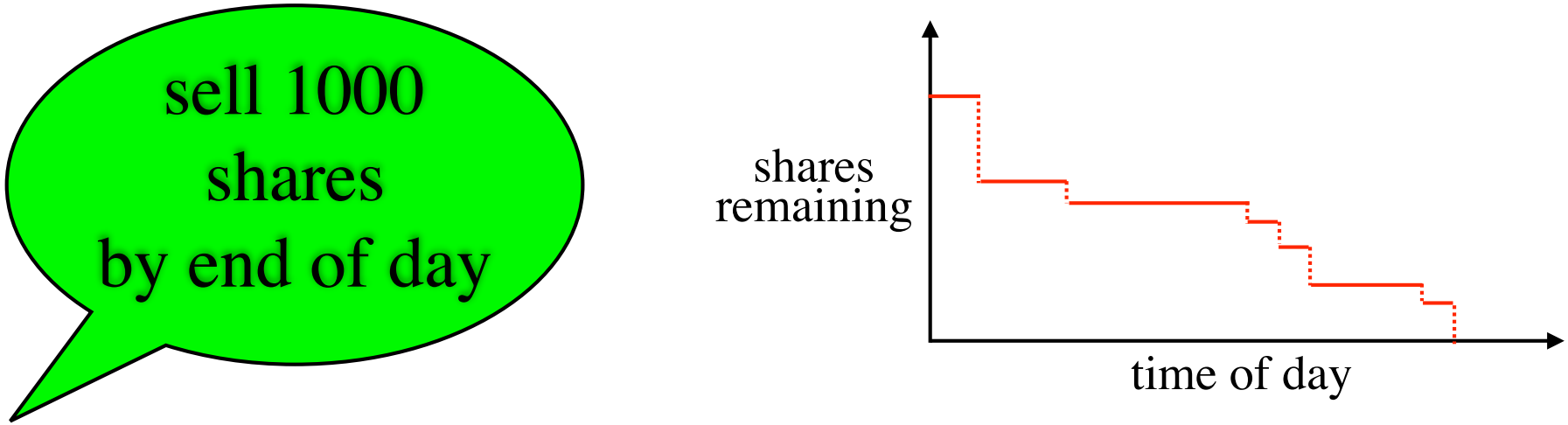


determine
"availability"

action: accept/reject

observation: requests, cancellations, forecasts, etc.

Example: Trade Execution



action: orders to exchange
observation: order book, other market data

Outline

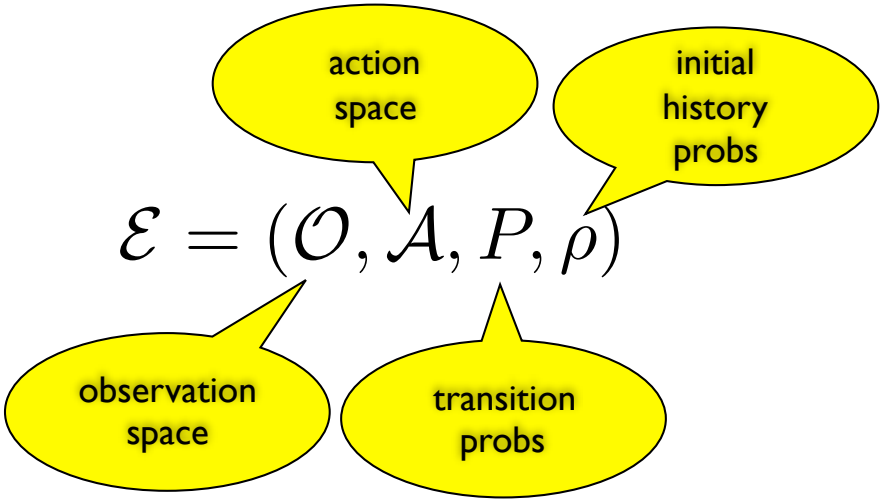
- Examples
- Mathematical formalism
- Common algorithms
- Challenges of data efficiency

The Role of Formalism

- Clarify the class of problems under consideration
- Provide notation for making rigorous statements
- Establish properties that apply across the class
- Design algorithms that apply across the class
- Guide modeling
  - Epistemic versus aleatoric uncertainty

Formalism: Environment

- Environment



- Assume finite
  - observation space
  - action space

- News feed model

$\mathcal{E} = (\mathcal{O}, \mathcal{A}, P, \rho)$

$\mathcal{A} \in \{\text{feed unit}\}$

$$\mathcal{O} \in \left\{ \begin{array}{c} (\text{inventory, user profile}) \\ (\text{like/swipe/quit}) \end{array} \right\}$$

$\rho \rightarrow (\text{inventory, user profile})$

$P \rightarrow (\text{like/swipe/quit})$

Formalism: Dynamics

- History

$$H_0 = (O_o)$$

$$H_t = (O_0, A_0, \ldots, O_t)$$

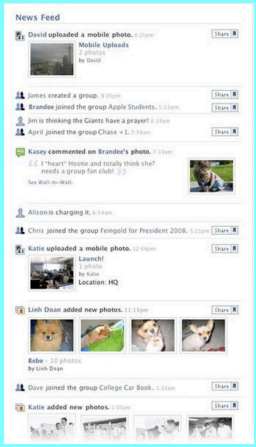
$$H_t \in \mathcal{H} = \mathcal{H}^+ \cup \mathcal{H}^-$$

terminal histories

$$H_0 = ((\text{inventory}, \text{user profile}))$$

$$H_t = (\ldots, \text{feed unit } \#22, (\text{like}, \text{swipe}))$$

$$\mathcal{H}^- = \{(\cdots, (\cdots, \text{quit}))\}$$



- Dynamics

$$\rho_h = \mathbb{P}(H_0 = h | \mathcal{E})$$

$$P_{ahh'} = \mathbb{P}(H_{t+1} = h' | \mathcal{E}, H_t = h, A_t = a)$$

Formalism: Policies

- Policy

$$\pi_h(a) = \mathbb{P}(A_t = a | \mathcal{E}, H_t = h)$$

feed unit $\leftarrow \pi_h$

- Transition probs

$$P_{\pi hh'} = \sum_{a \in \mathcal{A}} P_{ahh'}$$

like/swipe/quit $\leftarrow P_{\pi h}$.

- Finite episode duration τ

total # units displayed

Formalism: Rewards

- Period reward

$$R_{t+1} = r(H_{t+1})$$

$$r(H_{t+1}) = \begin{cases} 1 & \text{if } A_t \in \text{ads} \\ 0 & \text{otherwise} \end{cases}$$

- Episode reward

$$\sum_{t=0}^{\tau-1} R_{t+1}$$

total # ads displayed

How will an optimal policy behave?

Value Functions

- Value functions $V^\pi : \mathcal{H}^+ \to \Re$

$$V^\pi(h) = \mathbb{E}\left[\sum_{k=t}^{\tau-1} R_{k+1} \middle| \mathcal{E}, H_t = h, \pi\right]$$

$$V^*(h) = \sup_\pi V^\pi(h)$$

remaining # ads under π

remaining # ads under π^*

- Value per episode $\overline{V}^\pi \in \Re$

$$\overline{V}^\pi = \sum_{h \in \mathcal{H}^+} \rho_h V^\pi(h)$$

$$\overline{V}^* = \sum_{h \in \mathcal{H}^+} \rho_h V^*(h)$$

total # ads under π

total # ads under π^*

Action Value Functions

- Value conditioned on immediate action $Q^\pi : \mathcal{H}^+ \times \mathcal{A} \to \Re$

$$Q^\pi(h,a) = \mathbb{E}\left[\sum_{k=t}^{\tau-1} R_{k+1} \middle| \mathcal{E}, H_t = h, A_t = a, \pi\right]$$

$$Q^\pi(h,a) = \sum_{h' \in \mathcal{H}} P_{ahh'} r(h') + \sum_{h' \in \mathcal{H}^+} P_{ahh'} V^\pi(h')$$

- Optimal action value function

$$Q^*(h,a) = \sup_\pi Q^\pi(h,a)$$

From Value to Policy

- Greedy policy with respect to Q

$$\pi_h \sim \text{unif}\left(\arg\max_{a \in \mathcal{A}} Q(h, a)\right)$$

- Optimal policy is greedy with respect to Q^*

$$\pi_h^* \sim \text{unif}\left(\arg\max_{a \in \mathcal{A}} Q^*(h, a)\right)$$

- Actually, any distribution over the argmax will work…

Bellman's Equation

- Dynamic programming operators

$$(TV)(h) = \max_{a \in \mathcal{A}} \left(\sum_{h' \in \mathcal{H}} P_{ahh'} r(h') + \sum_{h' \in \mathcal{H}^+} P_{ahh'} V(h') \right)$$

$$(FQ)(h,a) = \sum_{h' \in \mathcal{H}} P_{ahh'} r(h') + \sum_{h' \in \mathcal{H}^+} P_{ahh'} \max_{a' \in \mathcal{A}} Q(h', a')$$

- Bellman's equation

$$TV^* = V^*$$

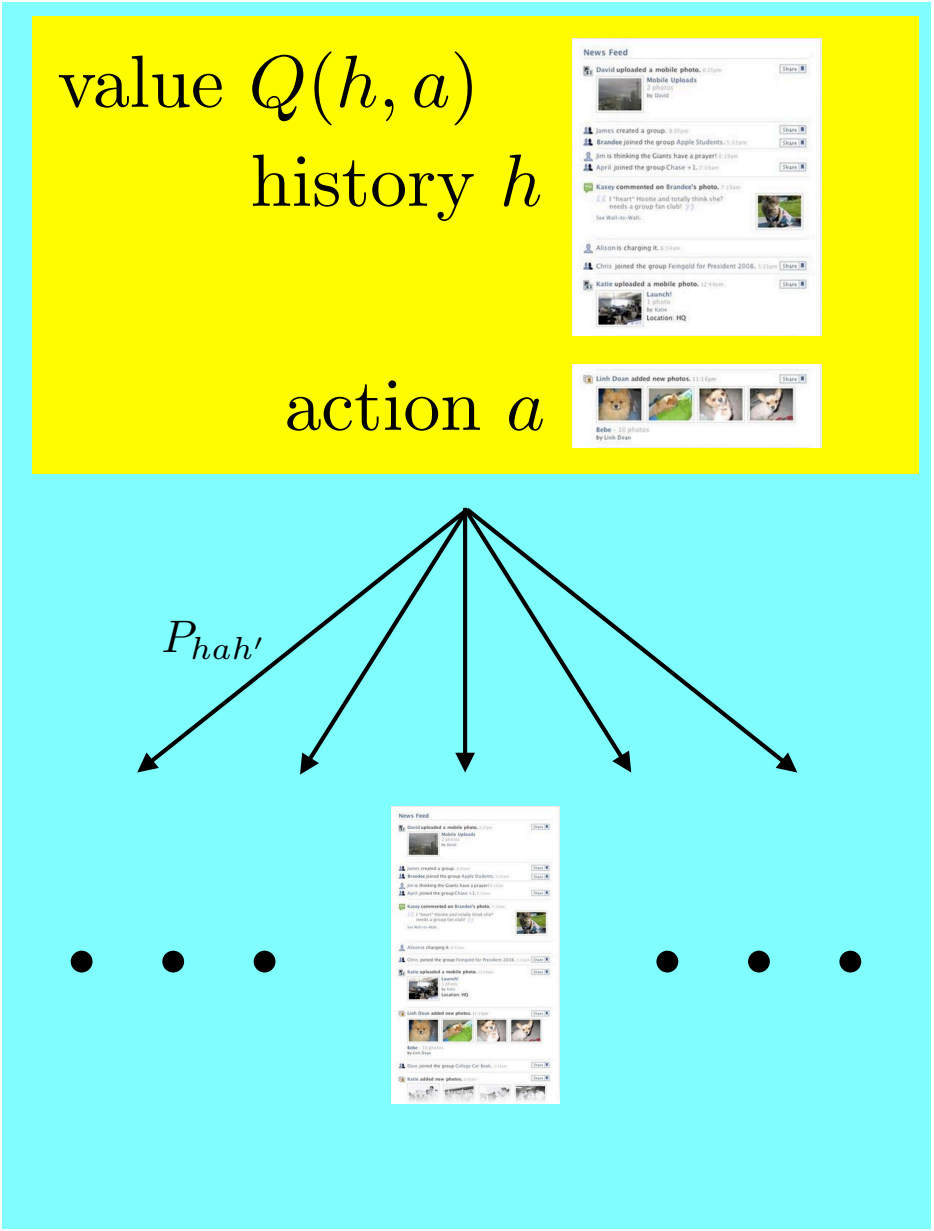
$$FQ^* = Q^*$$

Optimization via Value Iteration

- Computing V^*
  - Repeat $V \leftarrow TV$ forever
- Computing Q^*
  - Repeat $Q \leftarrow FQ$ forever

- VI converges, but
  - Computationally intractable
  - Requires knowledge of P

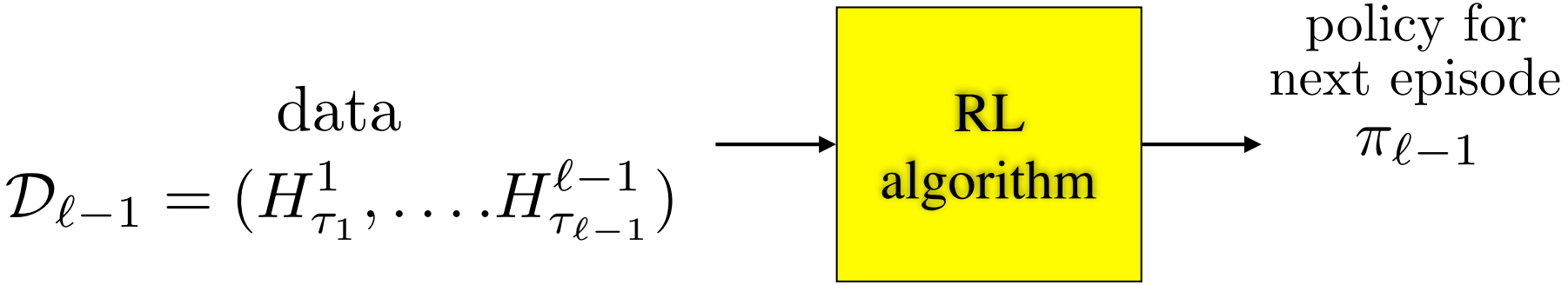
- RL algorithms address these issues



Formalism: Episodic RL

- Environment $\mathcal{E}$ is a random variable
  - In particular, ρ and P are random variables
  - *Epistemic* versus *aleatoric* uncertainty

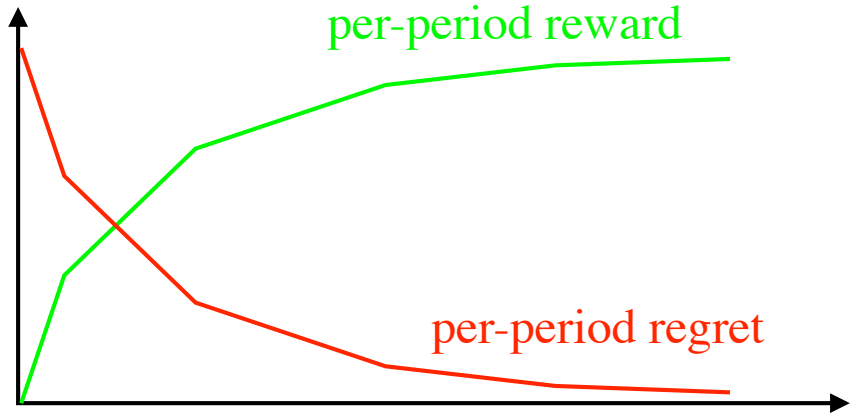
- Episodes $\ell = 1, 2, \ldots, L$



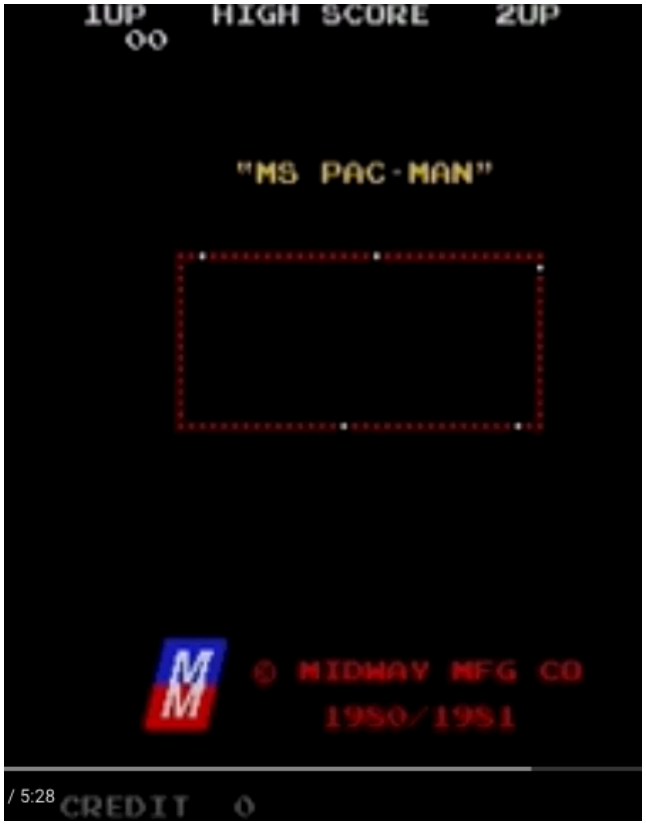
- RL algorithms address
  - Computational issues
  - Statistical issues, like exploration versus exploitation

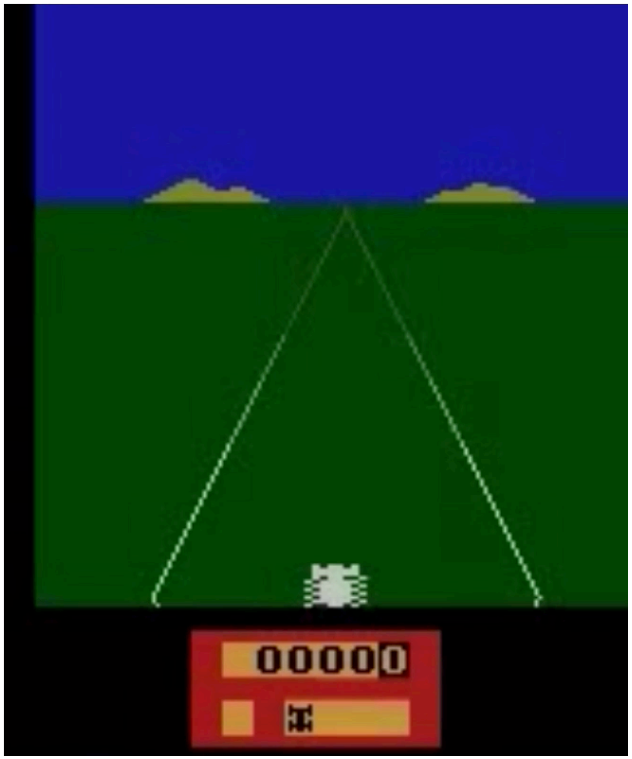
Formalism: Objective

- Expected reward $\quad \mathrm{Reward}(\mathrm{alg}) = \mathbb{E}\left[\sum_{\ell=1}^{L} \overline{V}^{\pi_{\ell-1}}\right]$
- "Bayesian decision problem"
  - RL algorithms can be viewed as heuristics for this problem
- Regret $\quad \mathrm{Regret}(\mathrm{alg}) = \mathbb{E}\left[\sum_{\ell=1}^{L} \left(\overline{V}^{*} - \overline{V}^{\pi_{\ell-1}}\right)\right]$



Atari Arcade Environment





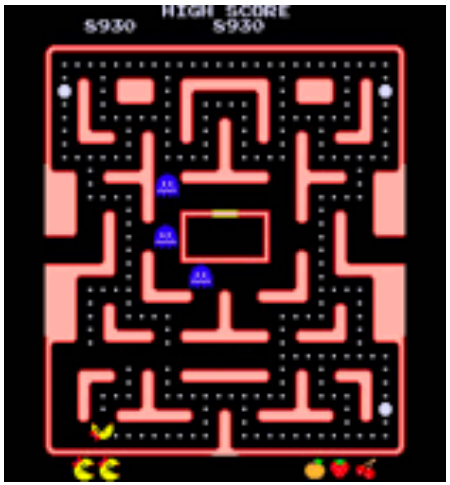
Atari 2600 supports 49 different games

develop an RL algorithm that learns to play

all games of this flavor

Arcade Games: Formulation

- Each game is an environment $\mathcal{E} = (\mathcal{O}, \mathcal{A}, P, \rho)$
  - Time increment: 1/60 of a second

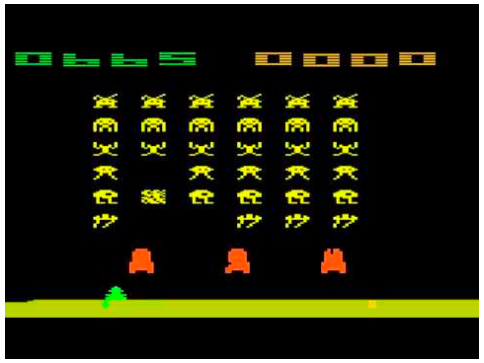


- Observations $O_t \in \mathcal{O}$
  - Video frame
- Actions $A_t \in \mathcal{A}$
  - 18 joystick modes
- Probabilities ρ, P
  - As produced by game

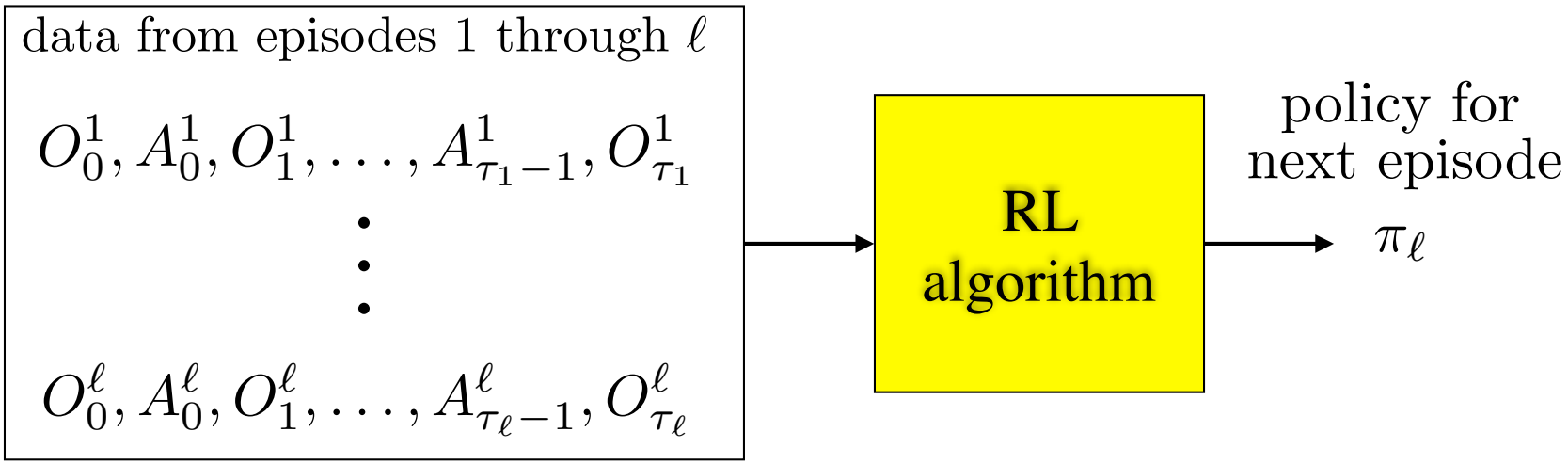
- Reward $R_{t+1} = r(H_{t+1})$
  - -1 if score decreasing
  - +1 if score increasing
  - 0 otherwise

Arcade Games: RL

- Epistemic uncertainty
  - ρ and P are random variables



- RL Algorithm applied over episodes of the same game



Outline

- Examples
- Mathematical formalism
- Common algorithms
- Challenges of data efficiency

Knowledge Representation

- Model learning: learn P
  - Left with a computationally intractable planning problem
  - Exponential explosion of roll-forward errors
- Policy learning: learn π^*
- Value function learning: learn Q^*

Generic RL Loop

Algorithm 1 `live`

Input: `agent`
`environment`

```
for ℓ in (1, 2, ...) do
    agent.learn_from_buffer()
    environment.reset()
    while environment.history is not terminal do
        action ← agent.act(environment.history)
        environment.step(action)
    end while
    agent.update_buffer(environment.history)
end for
```

object	`environment`
attributes	`history`
methods	`reset`
	`step`

object	`agent`
methods	`learn_from_buffer`
	`act`
	`update_buffer`

Agent Methods

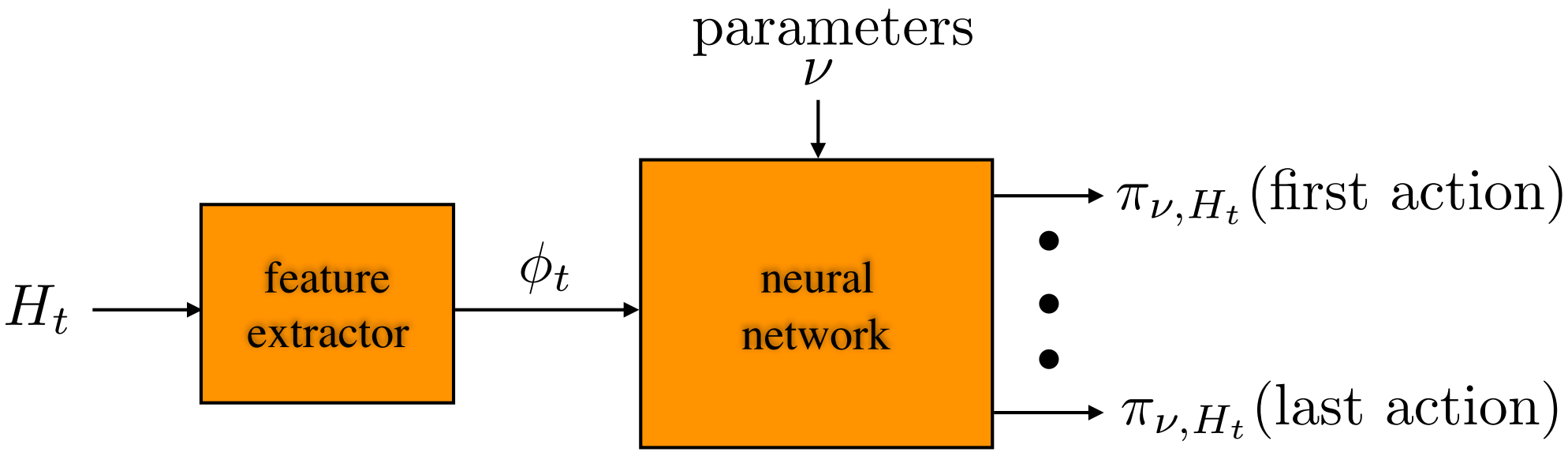
- act
  - policy learning: use stored policy
  - value function learning: greedy
  - possibly dither, e.g., ϵ-greedy

- update_buffer
  - append history
  - possibly subsample or drop very old samples
  - possibly compress to what is needed for learning

- learn_from_buffer
  - update parameters of policy and/or value function
  - sophisticated algorithms
  - we will focus on this part

Policy Representation

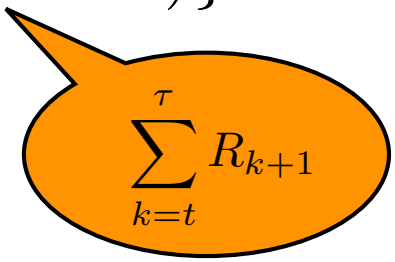
- Policy parameterized by vector ν
- Example
  - neural network with multinomial logistic output



The *Reinforce* Algorithm

$$\texttt{policy} = \tilde{\pi}_{\texttt{parameters}=\cdot,\texttt{features}=\cdot}(\texttt{action} = \cdot)$$

$$\texttt{buffer} = \{(\texttt{features}, \texttt{action}, \texttt{subsequent reward})\}$$



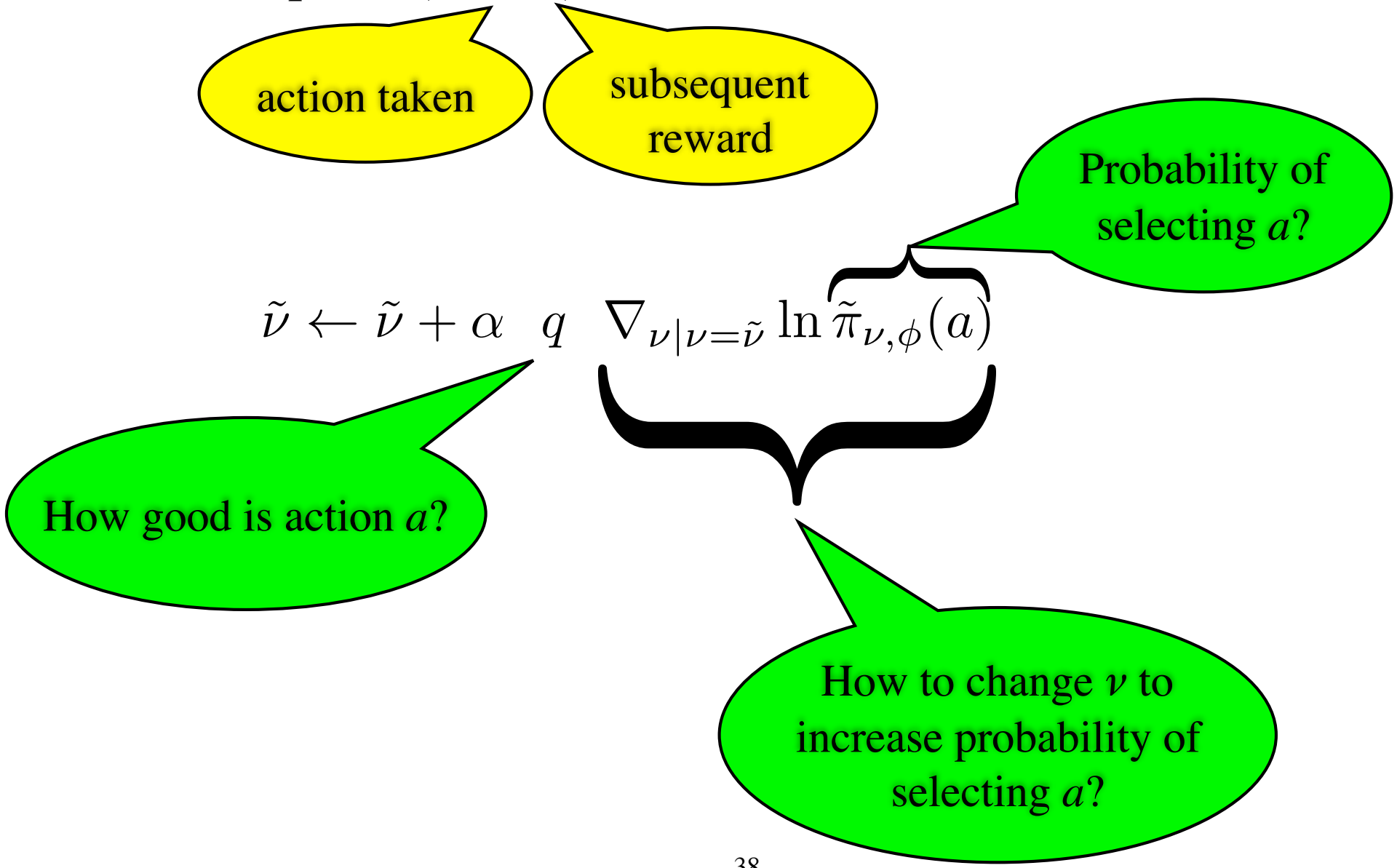
Algorithm 2 `learn_reinforce`

Input: `agent`

$\tilde{\pi} \leftarrow$ `agent.policy`
$\tilde{\nu} \leftarrow$ `agent.parameters`
`buffer` $\leftarrow$ `agent.buffer`
$N \leftarrow$ `agent.num_iters`
$\alpha \leftarrow$ `agent.learning_rate`
for n in $(1, 2, \ldots, N)$ **do**
    $\tilde{\mathcal{D}} \leftarrow$ `buffer.sample_minibatch()`
    $\tilde{\nu} \leftarrow \tilde{\nu} + \alpha \sum_{(\phi,a,q)\in\tilde{\mathcal{D}}} q \nabla_{\nu|\nu=\tilde{\nu}} \ln \tilde{\pi}_{\nu,\phi}(a)$
end for
`agent.parameters` $= \tilde{\nu}$

Intuition about the *Reinforce* Parameter Update

- Consider case with minibatch of size 1
  - Data point (ϕ, a, q)



Reinforce is a *Policy Gradient* Algorithm

- Reinforce is a *policy gradient* method
  - The mean field follows the gradient of $\overline{V}^{\pi_\nu}$
    - Under certain technical conditions…
  - Agent performance improves over episodes
  - Popular due to reliable improvement

- An *on-policy* algorithm
  - Estimates gradient at buffered policy
  - To be effectiveness, only retain recent data
  - Lack of *counterfactual analysis*
    - As opposed to *off-policy* algorithms

Policy Gradient Methods: Obstacles

- SGD can be slow and converge to local minima
  - Use adaptive step size rules as with supervised learning

- Statistically inefficient
  - On-policy learning
  - Naive exploration
  - High variance of gradient estimates
    - Variance reduction techniques
    - Baselining

Variance Reduction Through Baselining

- Thrashing if q is large
  - Reducing step size makes learning too slow when q is small

$$\tilde{\nu} \leftarrow \tilde{\nu} + \alpha \;\; q \;\; \nabla_{\nu|\nu=\tilde{\nu}} \ln \tilde{\pi}_{\nu,\phi}(a)$$

- Baselined update with any $B(\phi)$ retains policy gradient

$$\tilde{\nu} \leftarrow \tilde{\nu} + \alpha \;\; (q - B(\phi)) \;\; \nabla_{\nu|\nu=\tilde{\nu}} \ln \tilde{\pi}_{\nu,\phi}(a)$$

- A good baseline is given by $V^{\tilde{\pi}_{\tilde{\nu}}}$
  - Use approximation $\tilde{V}_\theta(\phi) \approx V^{\tilde{\pi}_{\tilde{\nu}}}$

Learning a Baseline

Algorithm 3 `learn_reinforce_baseline`

Input: `agent`

$\tilde{\pi} \leftarrow$ `agent.policy`
$\tilde{V} \leftarrow$ `agent.value_function`
$(\tilde{\nu}, \tilde{\theta}) \leftarrow$ `agent.parameters`
`buffer` $\leftarrow$ `agent.buffer`
$N \leftarrow$ `agent.num_iters`
$\alpha \leftarrow$ `agent.learning_rate`
for n in $(1, 2, \ldots, N)$ **do**
  $\tilde{\mathcal{D}} \leftarrow$ `buffer.sample_minibatch()`
  $\tilde{\nu} \leftarrow \tilde{\nu} + \alpha \sum_{(\phi,a,q)\in\tilde{\mathcal{D}}} (q - \tilde{V}_{\tilde{\theta}}(\phi)) \nabla_{\nu|\nu=\tilde{\nu}} \ln \tilde{\pi}_{\nu,\phi}(a)$
  $\tilde{\theta} \leftarrow \tilde{\theta} + \alpha \sum_{(\phi,a,q)\in\tilde{\mathcal{D}}} \left(\nabla_{\theta|\theta=\tilde{\theta}} \tilde{V}_{\theta}(\phi)\right) \left(q - \tilde{V}_{\tilde{\theta}}(\phi)\right)$
end for
`agent.parameters` $= (\tilde{\nu}, \tilde{\theta})$

adapt baseline to approximate $V^{\tilde{\pi}_{\tilde{\nu}}}$

- *Actor*: parameterized policy
- *Critic*: parameterized value function
- Broader class of *actor-critic methods*…

Actor-Critic Algorithm

- Actor can learn directly from the action-value critic rather than observed rewards

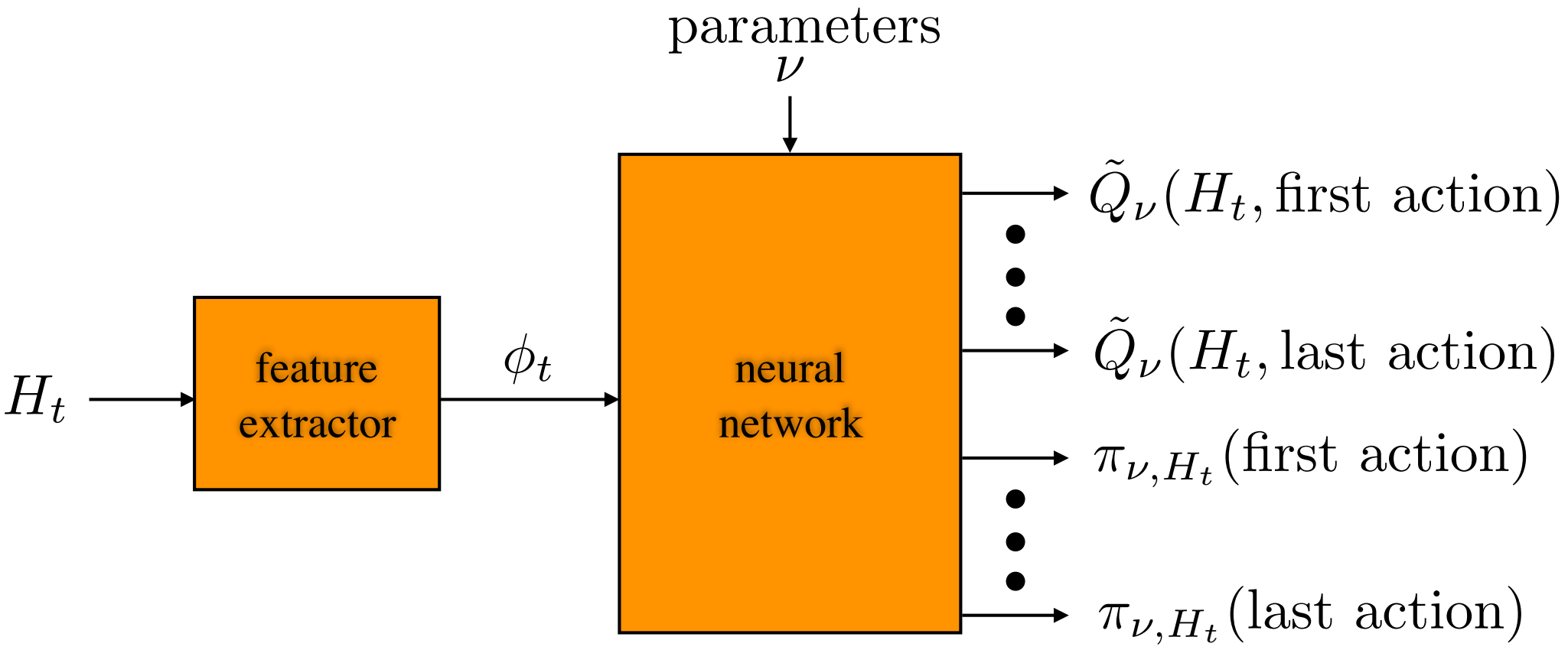
Algorithm 4 `learn_actor_critic`

Input: `agent`

$\tilde{\pi} \leftarrow$ `agent.policy`
$\tilde{Q} \leftarrow$ `agent.value_function`
$(\tilde{\nu}, \tilde{\theta}) \leftarrow$ `agent.parameters`
`buffer` $\leftarrow$ `agent.buffer`
$N \leftarrow$ `agent.num_iters`
$\alpha \leftarrow$ `agent.learning_rate`
for n in $(1, 2, \ldots, N)$ **do**
    $\tilde{\mathcal{D}} \leftarrow$ `buffer.sample_minibatch()`
    $\tilde{\nu} \leftarrow \tilde{\nu} + \alpha \sum_{(\phi,a,q)\in\tilde{\mathcal{D}}} \left(\tilde{Q}_{\tilde{\theta}}(\phi, a) - \sum_{a'\in\mathcal{A}} \tilde{\pi}_{\tilde{\nu},\phi}(a') \tilde{Q}_{\tilde{\theta}}(a') \right) \nabla_{\nu|\nu=\tilde{\nu}} \ln \tilde{\pi}_{\nu,\phi}(a)$
    $\tilde{\theta} \leftarrow \tilde{\theta} + \alpha \nabla_{\theta|\theta=\tilde{\theta}} \sum_{(\phi,a,q)\in\tilde{\mathcal{D}}} \tilde{Q}_{\theta}(\phi, a) \left(q - \tilde{Q}_{\tilde{\theta}}(\phi, a) \right)$
end for
`agent.parameters` $= (\tilde{\nu}, \tilde{\theta})$

Combined Actor-Critic Architecture

- Single neural network
  - Shared features
    - Policy-gradient theorem
  - Shared hidden layers

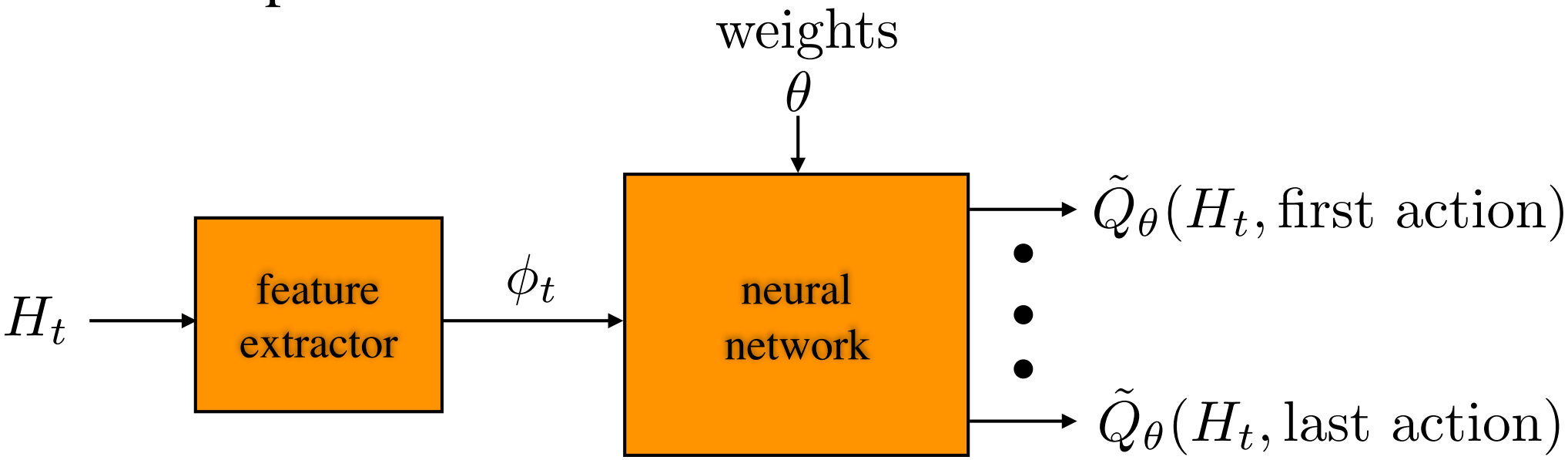


Value Function Learning

- Represent knowledge as parameterized value function
  - Possibly the optimal value function
  - Enables off-policy learning

$$\tilde{Q}_\theta(h, a) \approx Q^*(h, a)$$

- Example



Incremental TD

Algorithm 5 `learn_incremental_td`

Input: `agent`

$\tilde{Q} \leftarrow$ `agent.value_function`
$\tilde{\theta} \leftarrow$ `agent.parameters`
`buffer` $\leftarrow$ `agent.buffer`
$N \leftarrow$ `agent.num_iters`
$\alpha \leftarrow$ `agent.learning_rate`
for n in $(1, 2, \ldots, N)$ **do**
    $\tilde{\mathcal{D}} \leftarrow$ `buffer.sample_minibatch()`
    $\delta \leftarrow$ `buffer.minibatch_size` / `buffer.size`
    $\tilde{\theta} \leftarrow \tilde{\theta} - \alpha \nabla_{\theta|\theta=\tilde{\theta}} \left(\mathcal{L}(\theta, \tilde{\theta}, \tilde{\mathcal{D}}) + \delta \mathcal{R}(\theta) \right)$ ▷ TD step
end for

TD loss $$\mathcal{L}(\theta, \theta^-, \mathcal{D}) = \frac{1}{\sigma^2_{\text{outcome}}} \sum_{(\phi,a,r,\phi') \in \mathcal{D}} \left(r + \max_{a' \in \mathcal{A}} \tilde{Q}_{\theta^-}(\phi', a') - \tilde{Q}_\theta(\phi, a) \right)$$

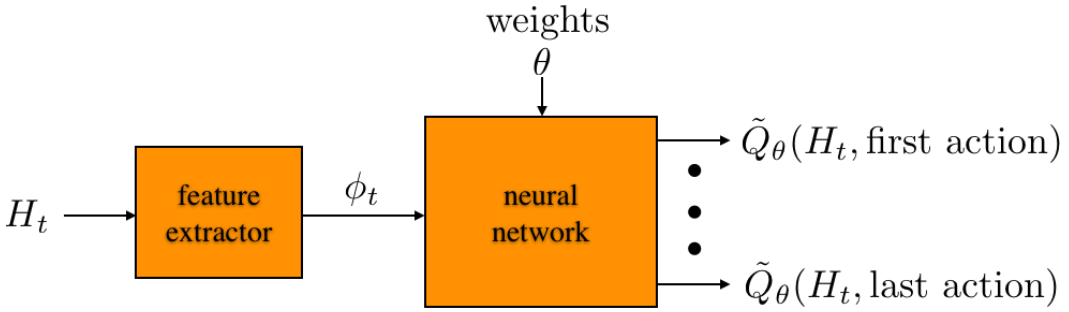
regularizor $$\mathcal{R}(\theta) = \frac{1}{\sigma^2_{\text{prior}}} \|\theta\|_2^2$$

Off-Policy Learning

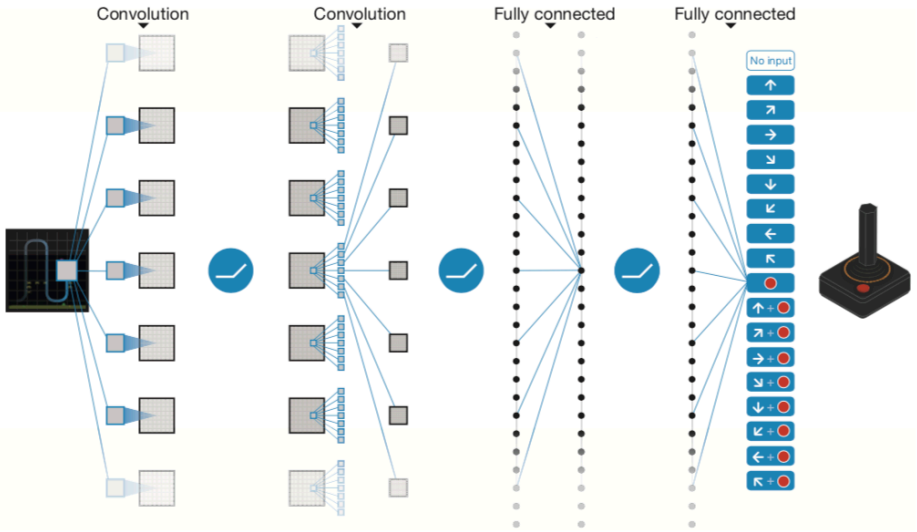
- TD error aligns prediction with max at next step
  - Not total reward under current policy

$$\text{TD loss} \qquad \mathcal{L}(\theta, \theta^-, \mathcal{D}) = \frac{1}{\sigma^2_{\text{outcome}}} \sum_{(\phi,a,r,\phi') \in \mathcal{D}} \left(r + \max_{a' \in \mathcal{A}} \tilde{Q}_{\theta^-}(\phi', a') - \tilde{Q}_\theta(\phi, a) \right)$$

Representation for Arcade RL Algorithm

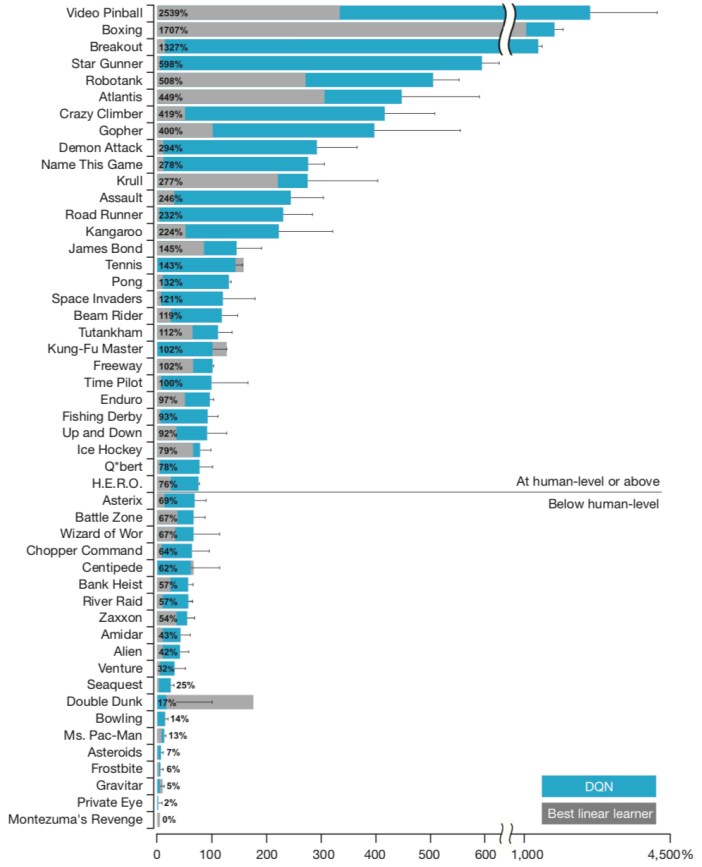


- Features
  - Last four frames, with some preprocessing
- Neural network architecture
  - Convolutional, RELUs, one linear output node per action



Performance in Atari Arcade Environment

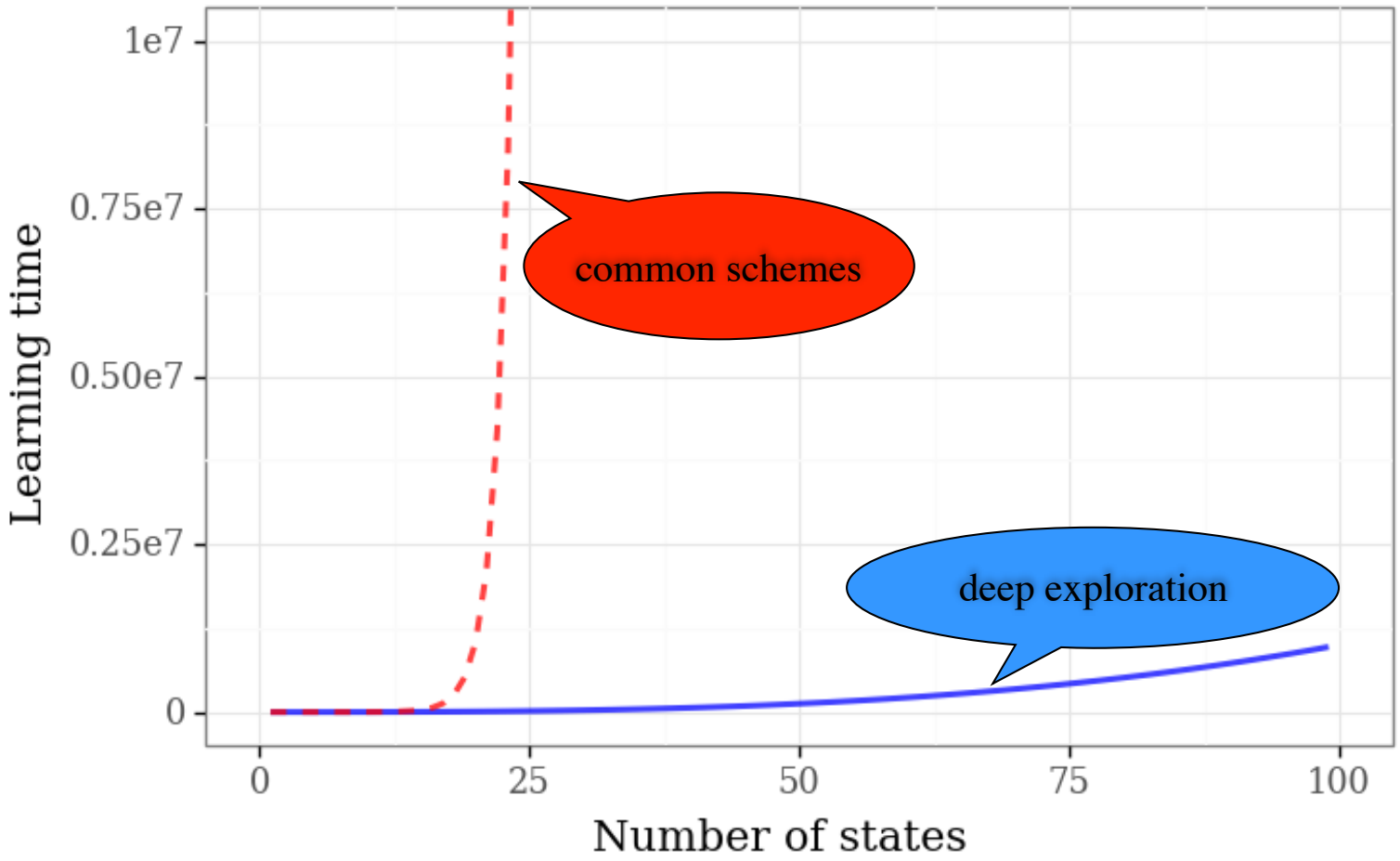
- DQN (incremental TD)
  - outperforms humans in most games
- Policy gradients approaches
  - perform even better
  - require much more data



Outline

- Examples
- Mathematical formalism
- Common algorithms
- Challenges of data efficiency

Big Data vs Smart Data

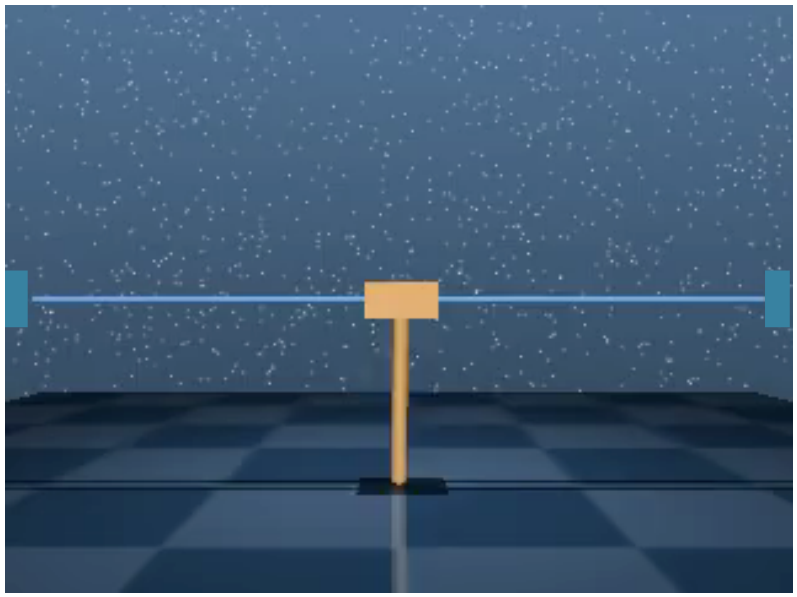


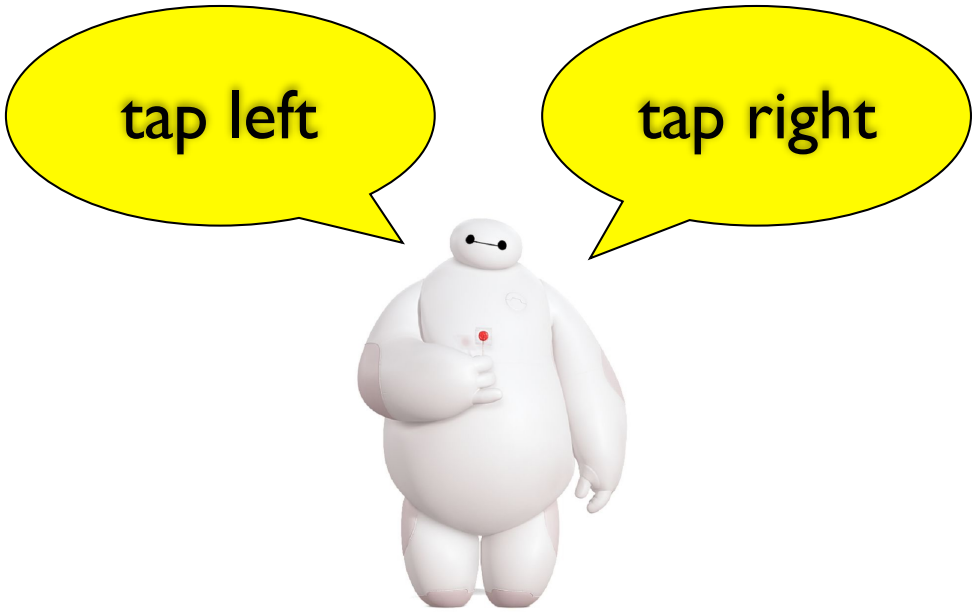
common exploration schemes can require exponentially large data sets even for simple reinforcement learning tasks

Representing and Resolving Uncertainty

- Many common RL methods learn point estimates
- Efficient exploration requires uncertainty representation
- How to learn neural networks that do this

Cart-Pole Problem

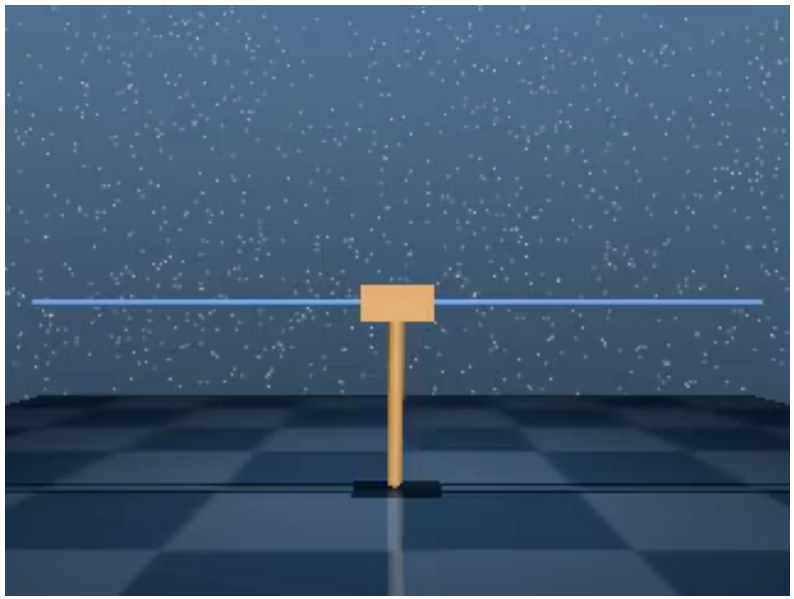




20 seconds per episode

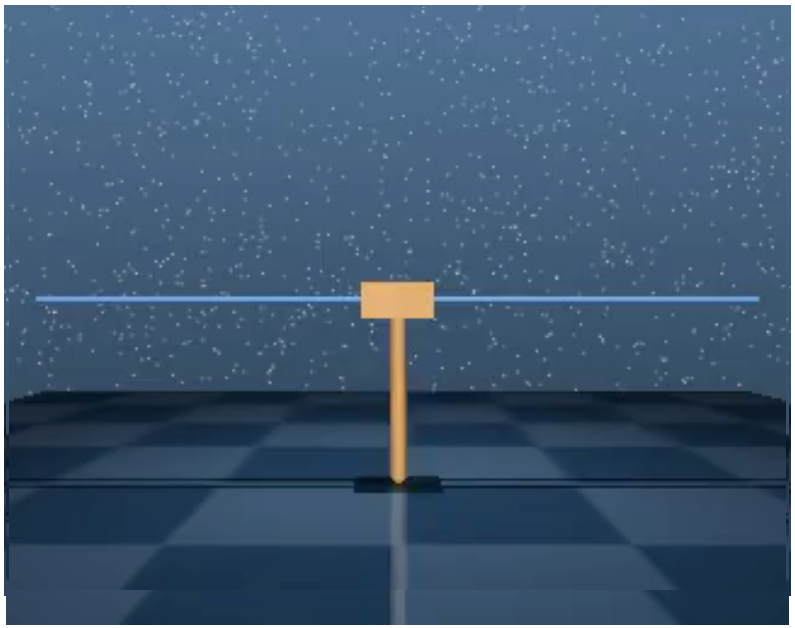
Cart-Pole Swing-Up

ε-greedy



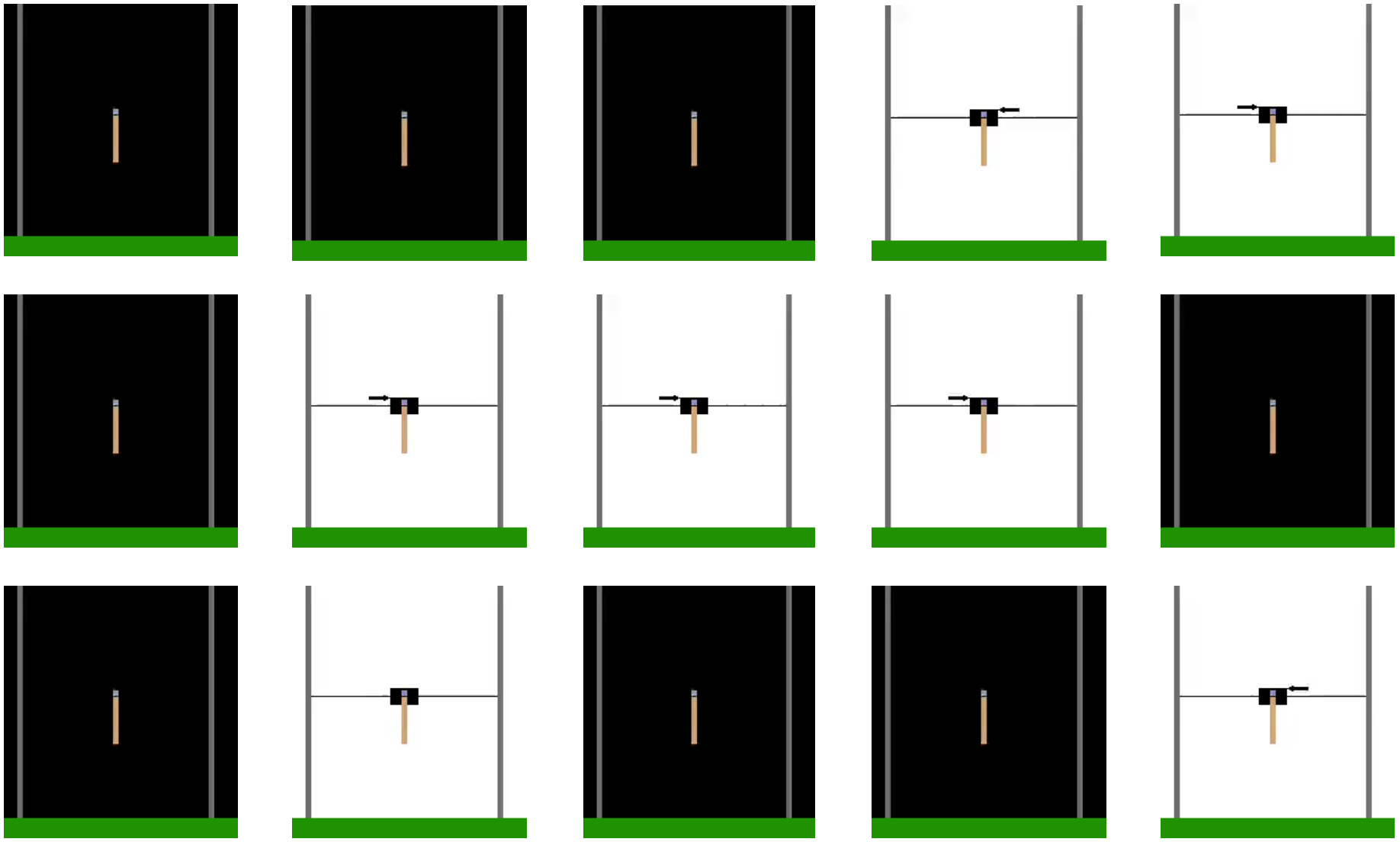
after 1000 episodes

deep exploration



after 1000 episodes

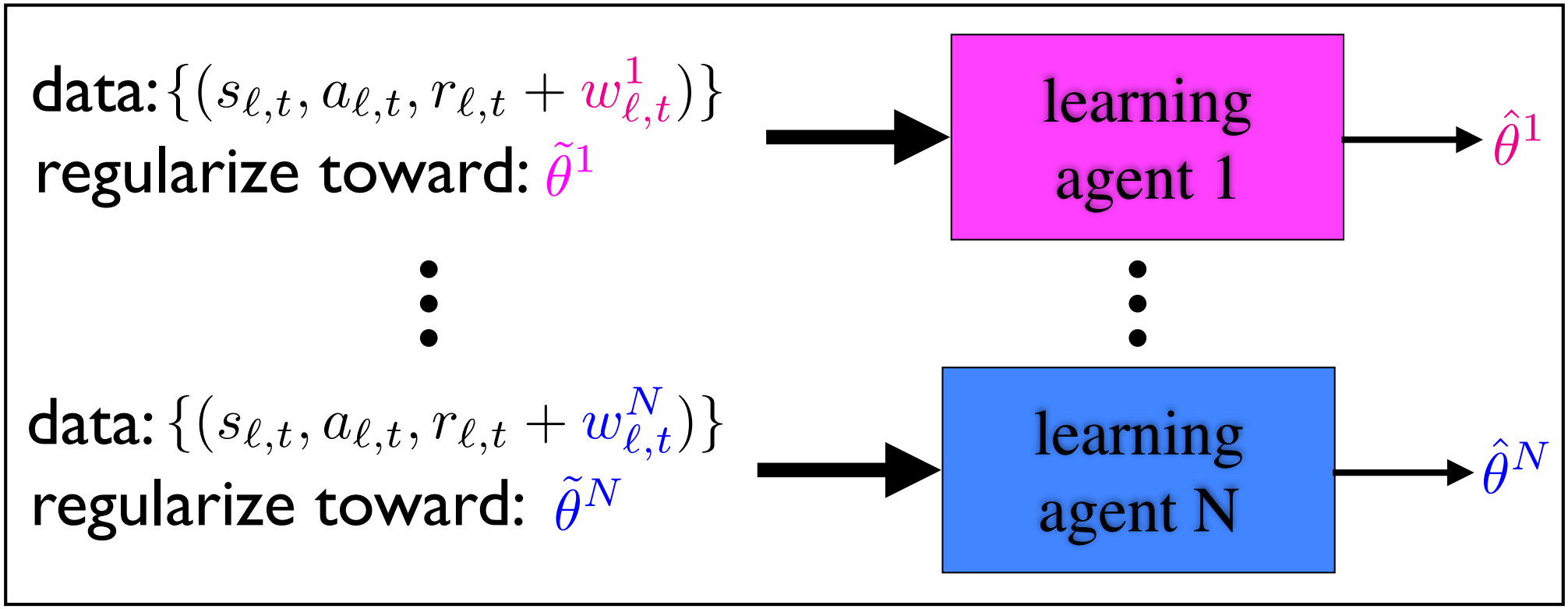
Information Sharing and Coordination



100 agents total

Ensemble Sampling

- Learn ensemble of N value functions, streaming data



- Before each episode, sample one uniformly, then argmax
- Other incremental approaches…

Hierarchical Representations

- Specific model classes, like linear, are too restrictive
- Can we do better than "flat" neural networks?
- CNNs induce useful inductive biases for vision
- What inductive biases are right for RL?
- HRL: effective policies decompose hierarchically

Leveraging Rich Observations

- Common methods often focus on predicting and maximizing future reward
- Richer observations can dramatically accelerate learning
- Don’t learn to optimize GDP based only on GDP
- Robots should learn from outcomes, not only rewards


Hello again,
and thank you!