

Privacy-preserving machine learning



@MLRS
Bangkok, Thailand
Aug 11, 2019
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Privacy & your data

- Increasingly more and more devices collect & stream your data

Internet & emails



[Lookeen]

Home appliances



[LG newsroom]

Drones



[Intogagets]

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Home appliances

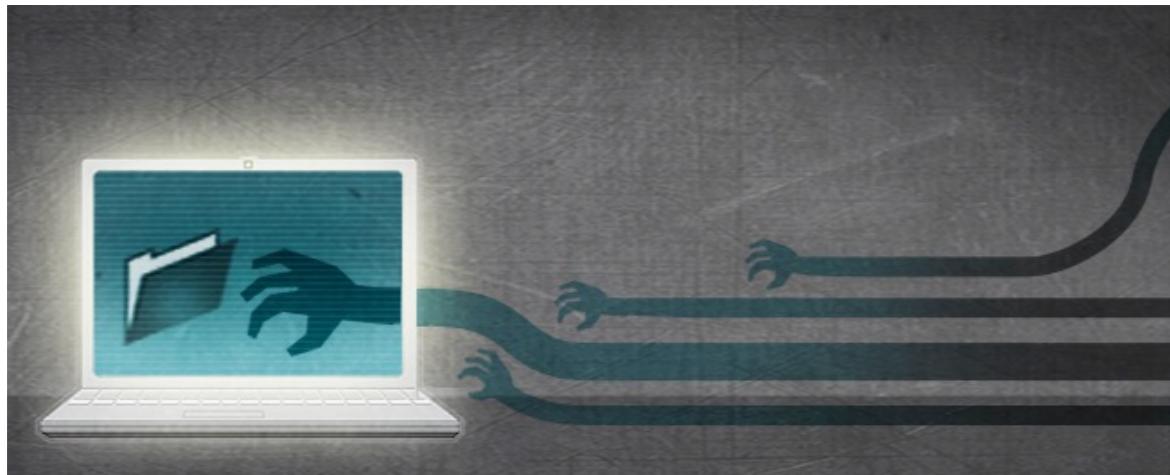


[LG newsroom]

Drones



[Intogagets]



[\[Electronic Frontier Foundation\]](#) via Wikimedia Commons]

Worry:
A few corporations
own your data
& might abuse them!

My name is Cayla!



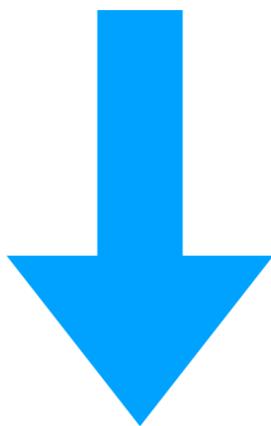
- Cayla is a children's smart toy, that can have interactive **conversations** with kids

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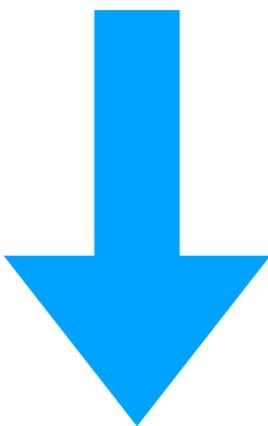


- Cayla is a children's smart toy, that can have interactive **conversations** with kids
- Issue 1: Conversations **streamed to company's server**
- Issue 2: anybody who has a bluetooth connection can **hack this doll.**

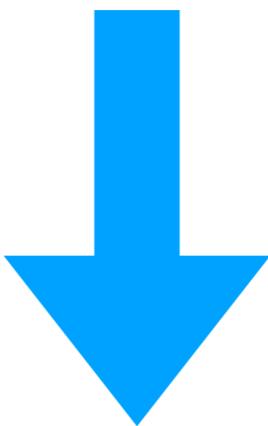
Threat!



hmm, maybe we should do
something for privacy!

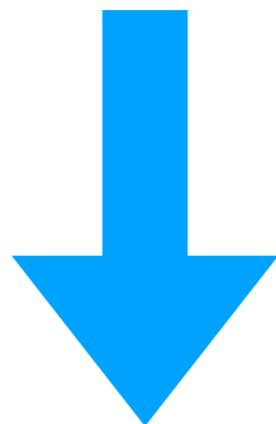


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Philanthropic reason!



hmm, maybe we should do
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Mallory Freeman | TED@UPS

Your company's data could help end world hunger

so the first thing they can do
is start donating that data.

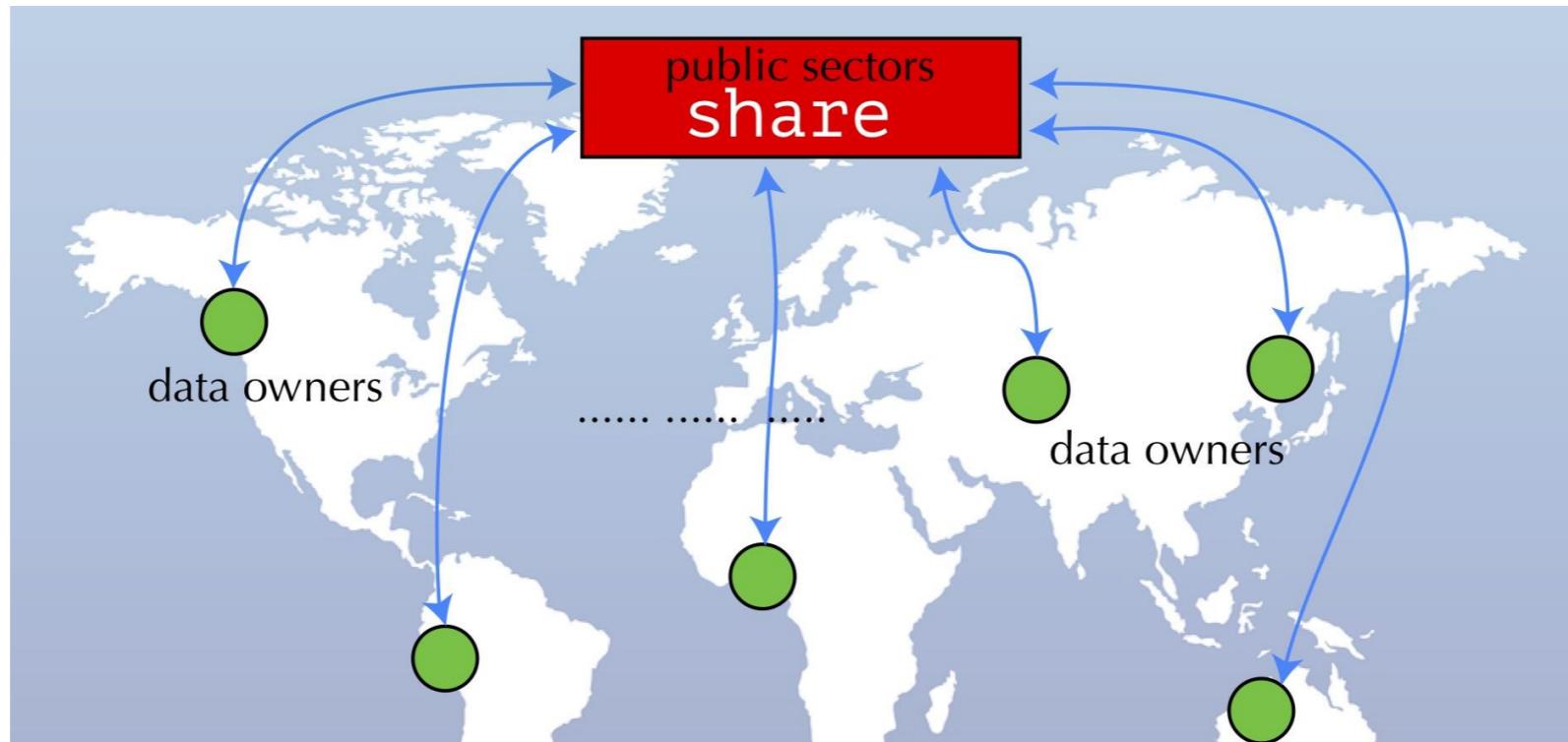


6:25

- Emphasising the importance of sharing data for public good

Data philanthropy

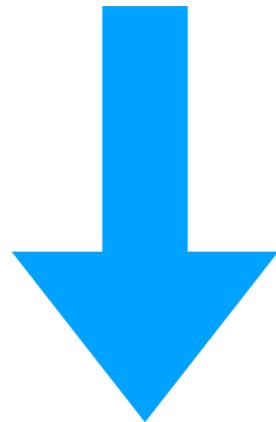
[United Nations Global Pulse]



“--- think of big data as a new kind of natural resource – infinitely renewable, increasingly ubiquitous – but … has fallen into the hands of… industry …Data has a social opportunity – and we have a social responsibility – …data reaches the people who need it most.”

[Director of UN Global Pulse]

Threat! Philanthropic reason!



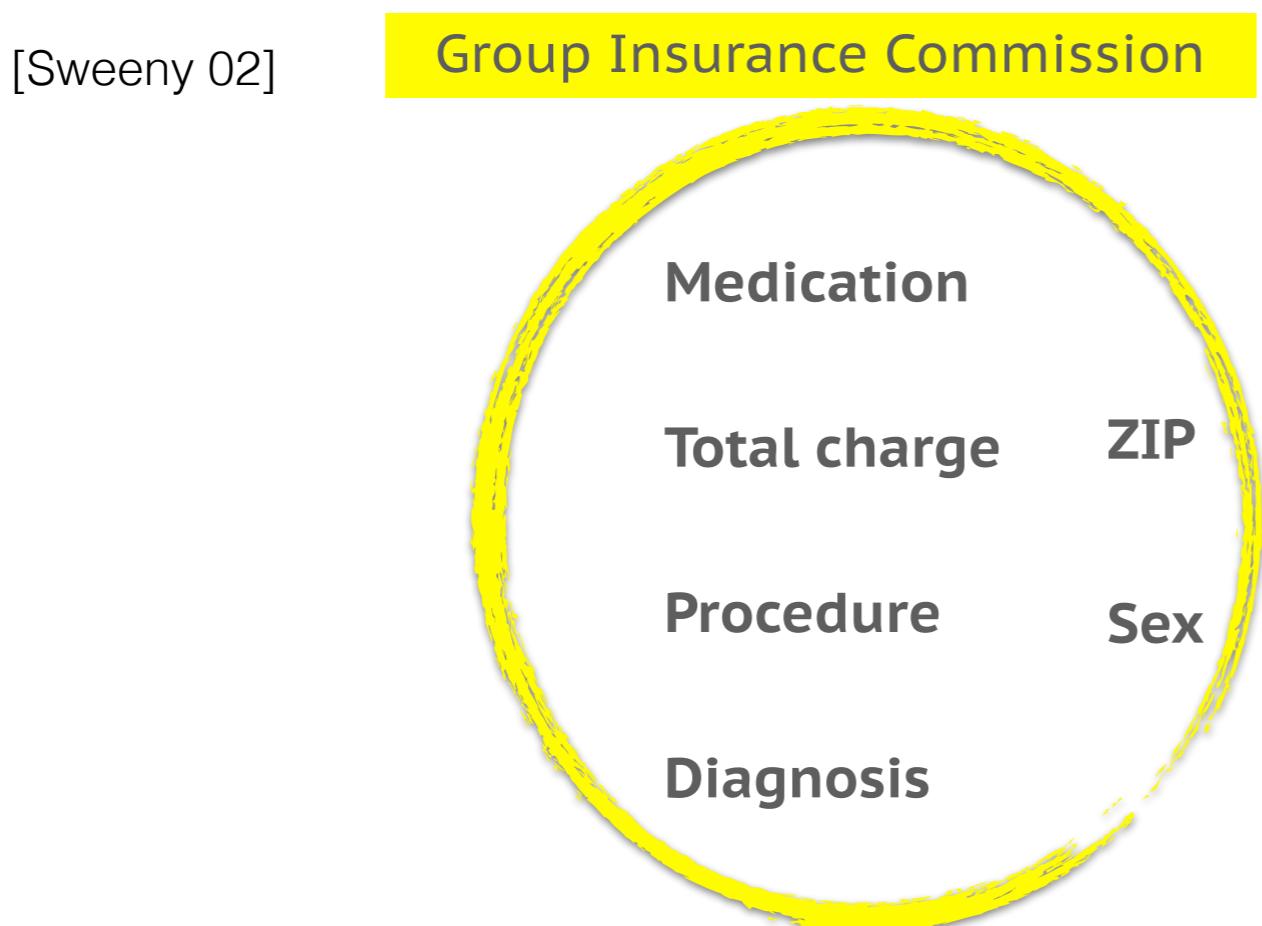
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Let's anonymise data!

- Isn't anonymising, removing obviously identifiable information, enough?

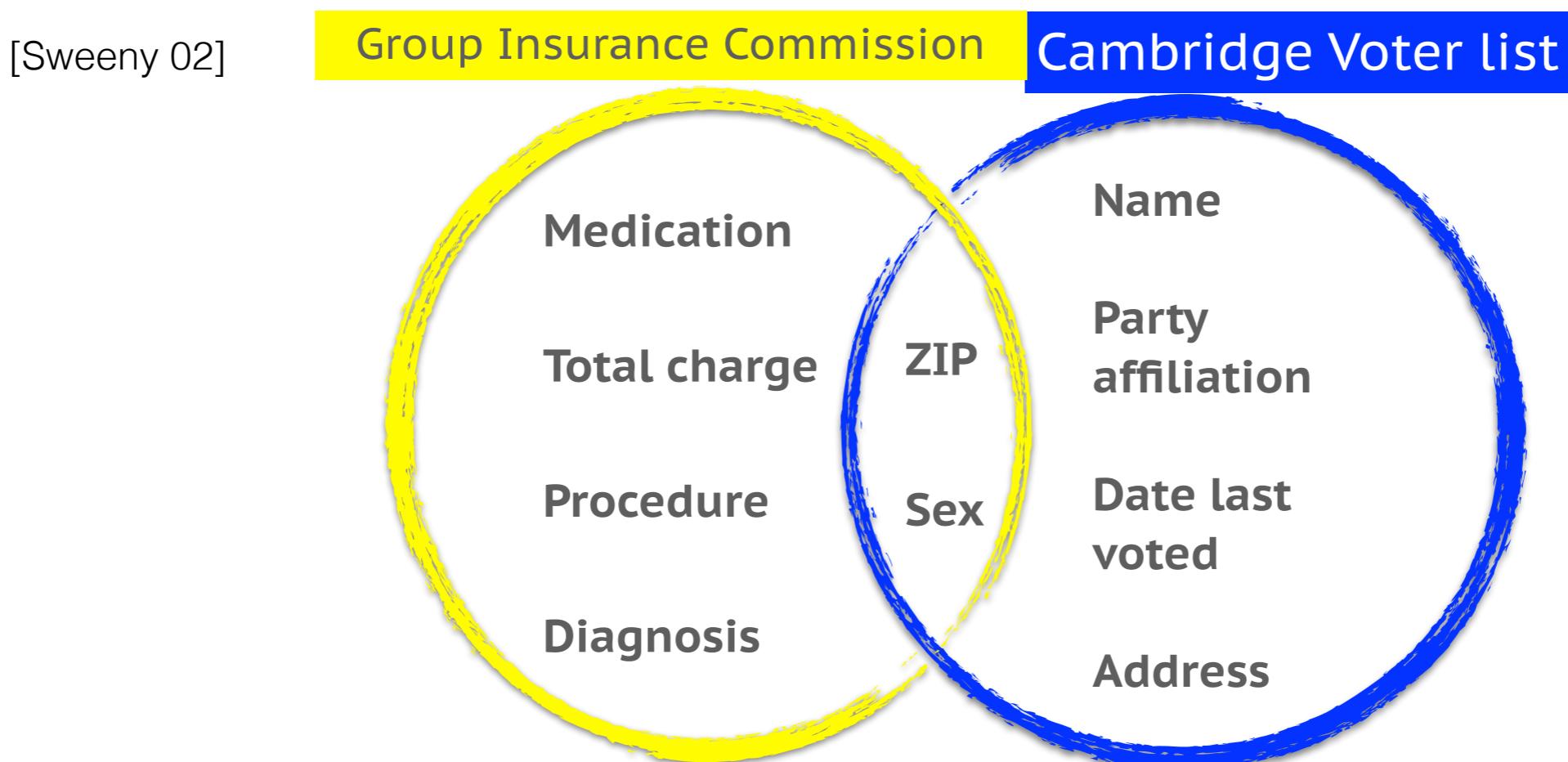
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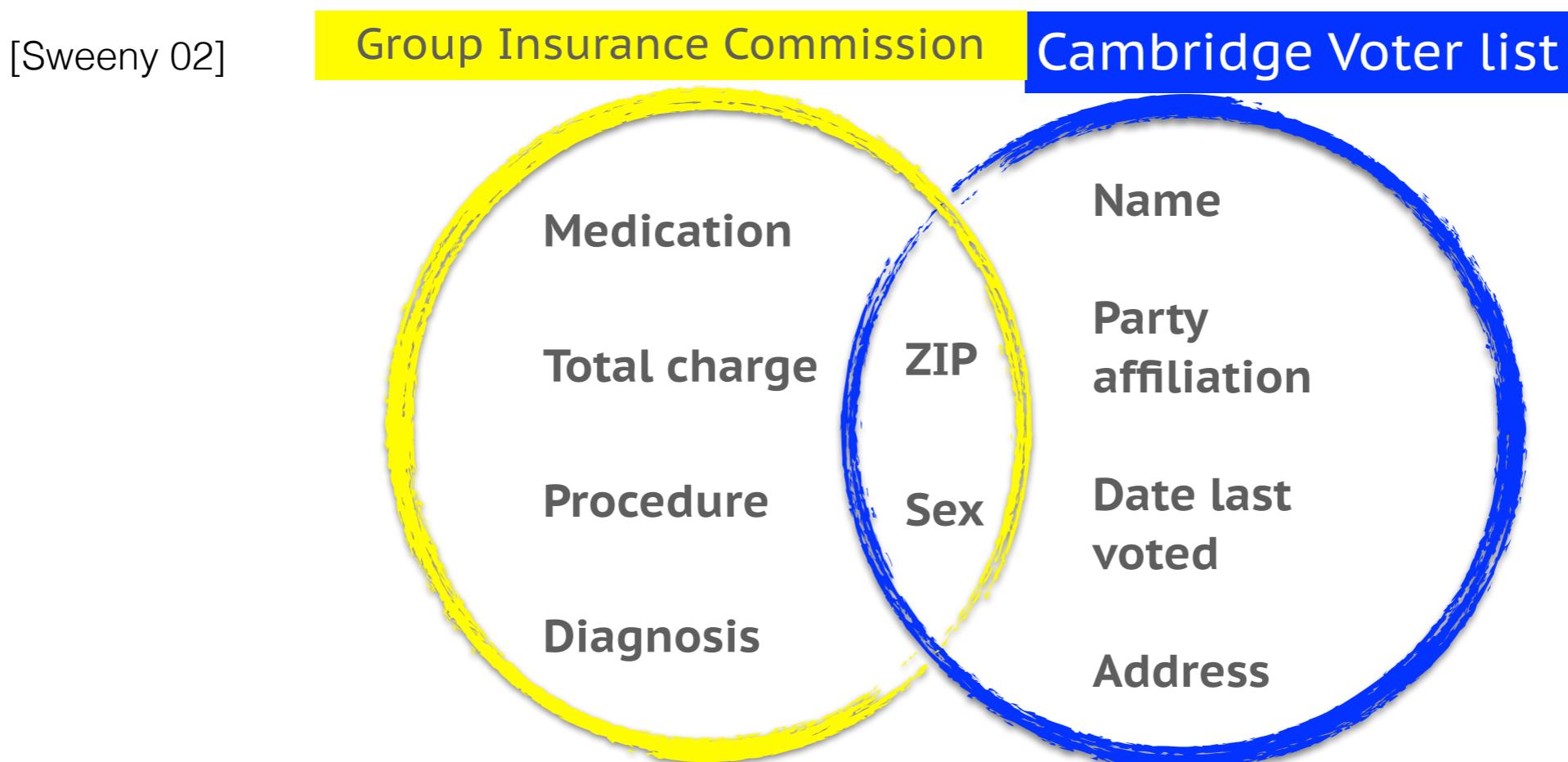
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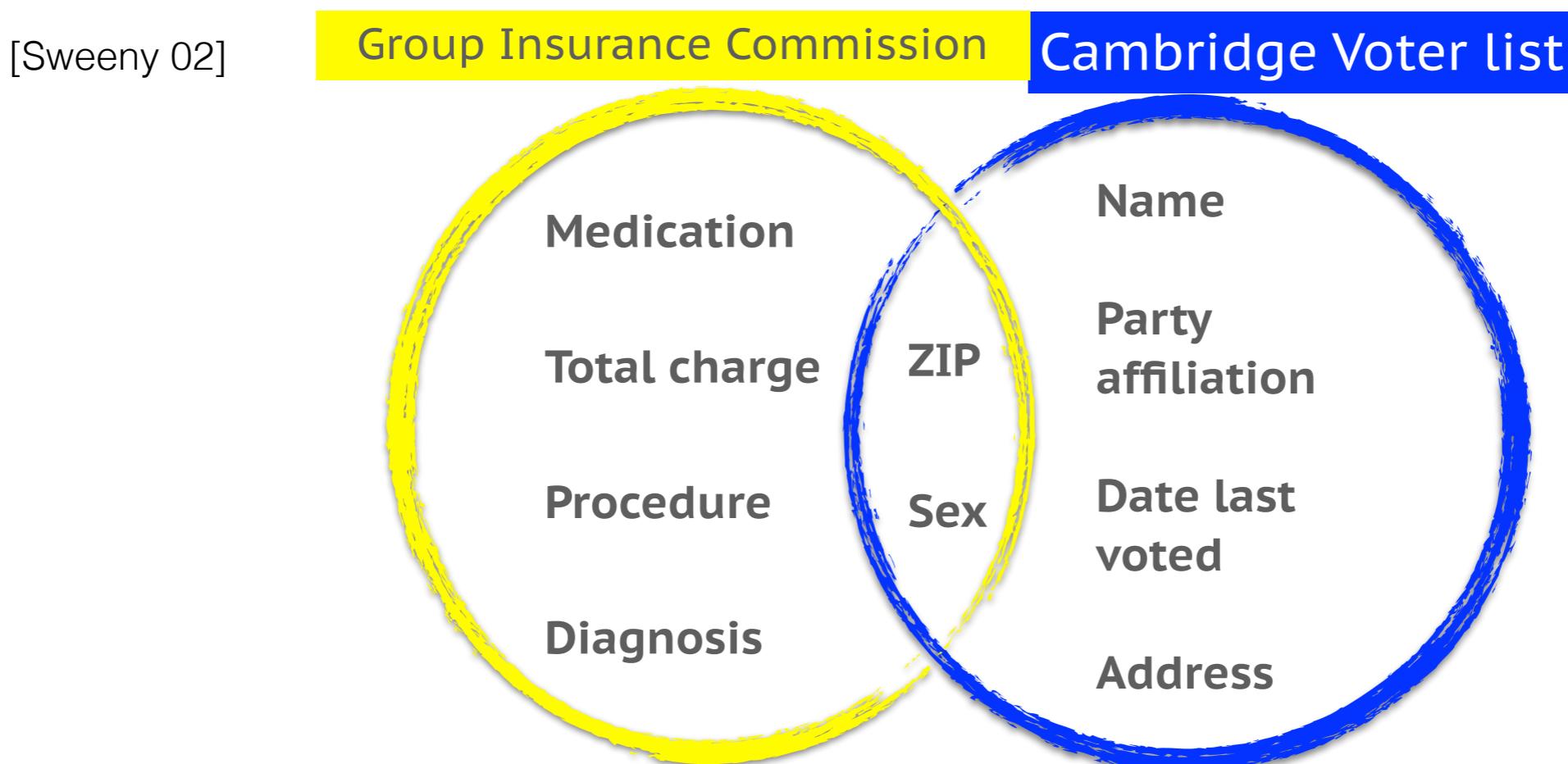
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Six people listed, three men, but only one living in that particular zip-code

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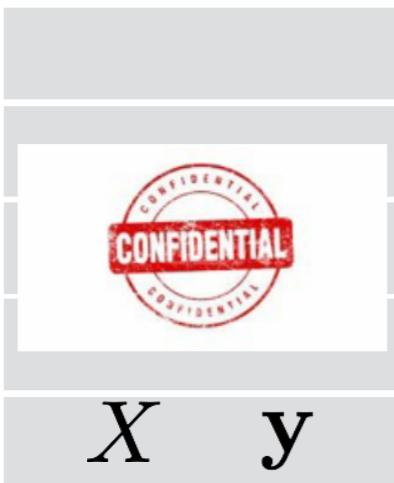


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Governor William Weld!

What about releasing statistics?

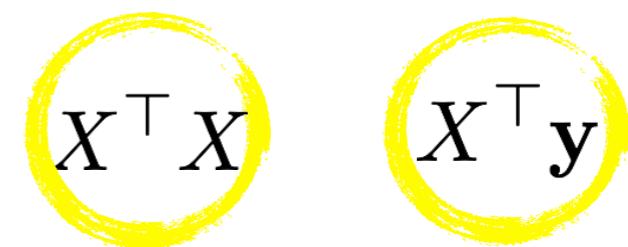
Sensitive Data



• Privacy Wall •
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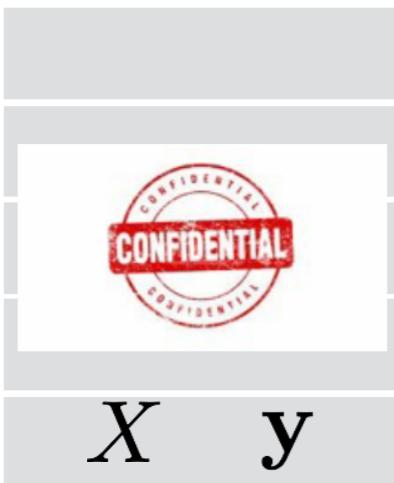
Public



first & second moments

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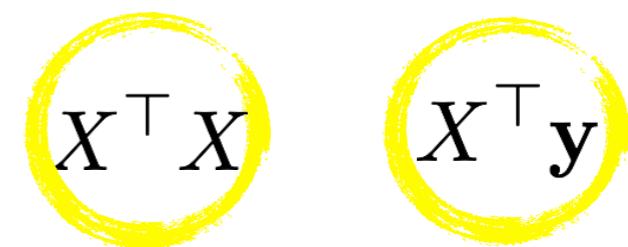
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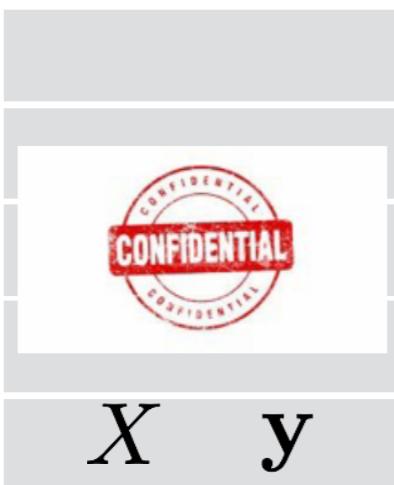


first & second moments

- Advanced attacking methods (e.g., model inversion attacks)!

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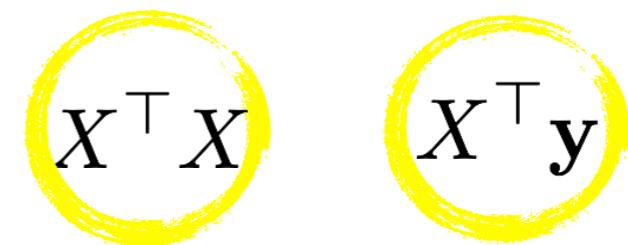
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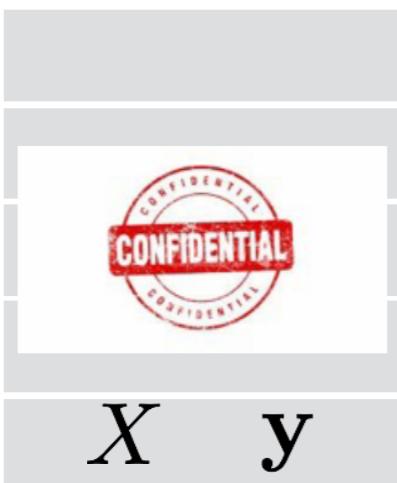
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prior knowledge

y_n \mathbf{x}_n
[known;
unknown]

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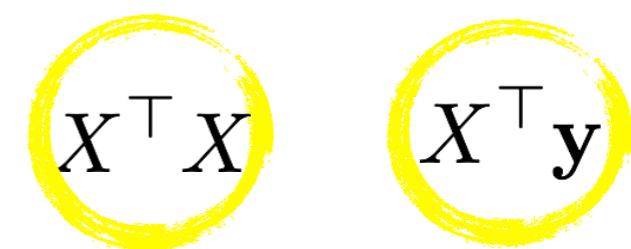
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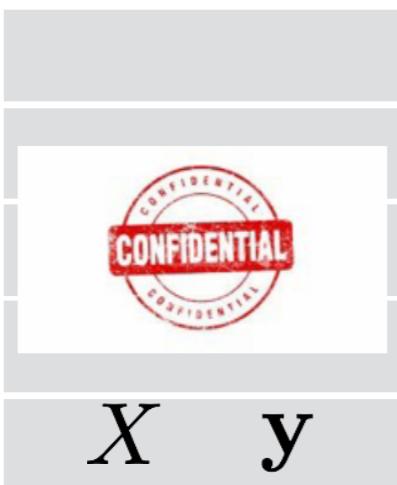
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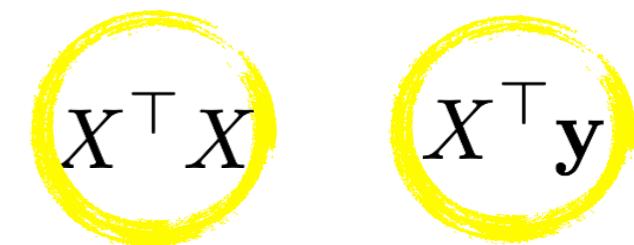
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$$y_n \approx \hat{\theta}_{ls}^\top \mathbf{x}_n$$

Statistically sound
prediction of
sensitive attributes

Simply anonymising data is unsafe!

Releasing statistics of data is unsafe!

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We need a provably strong privacy notion!

Privacy definition

Differential privacy!

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[Dwork 06]

Differential privacy is the statistical science of trying to learn as much as possible about a group while learning as little as possible about any individual in it.

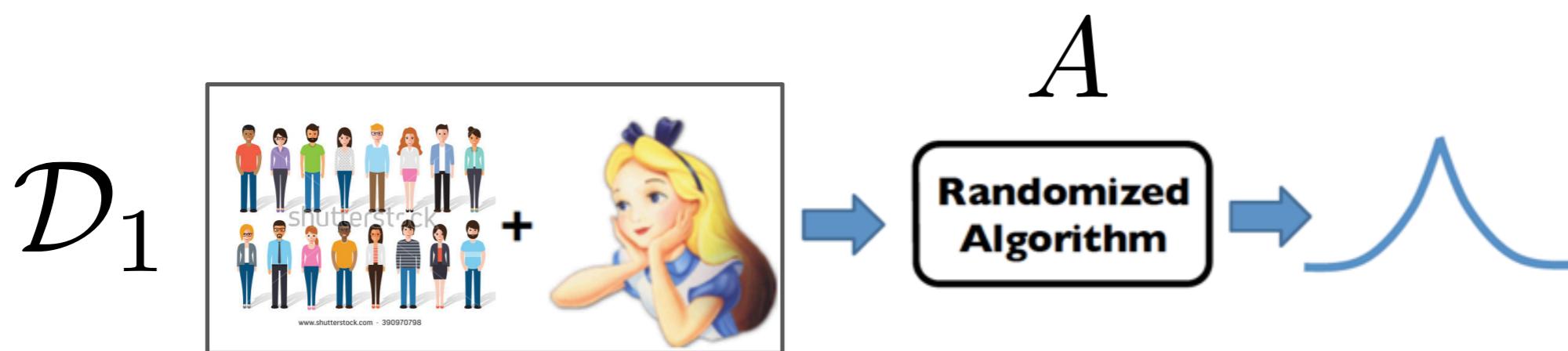
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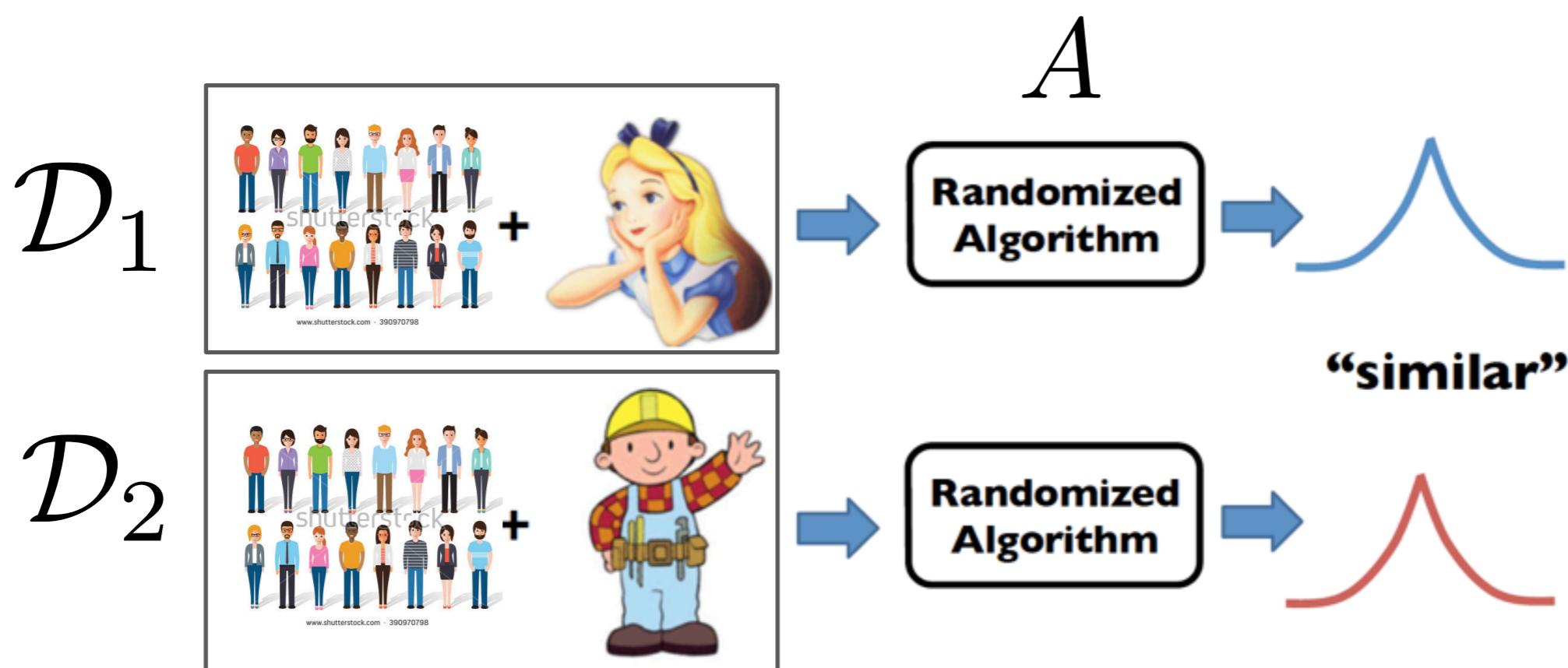


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for all o and all pairs of datasets

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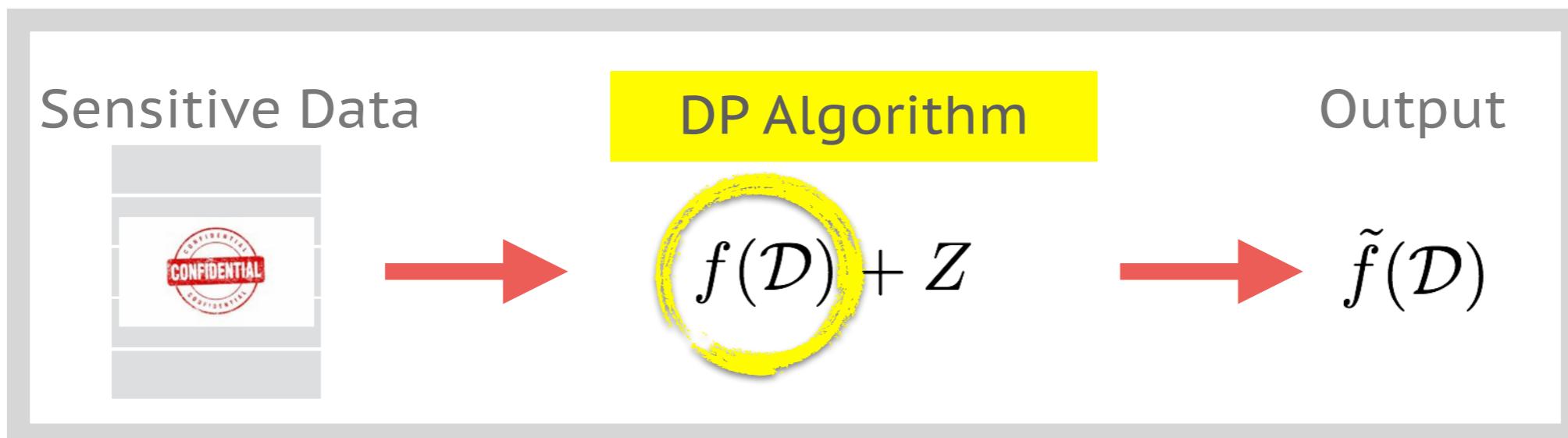
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- Epsilon quantitatively measures **how much the risk** to an individual privacy may increase **due to that individual's data inclusion** in the inputs to the algorithm.

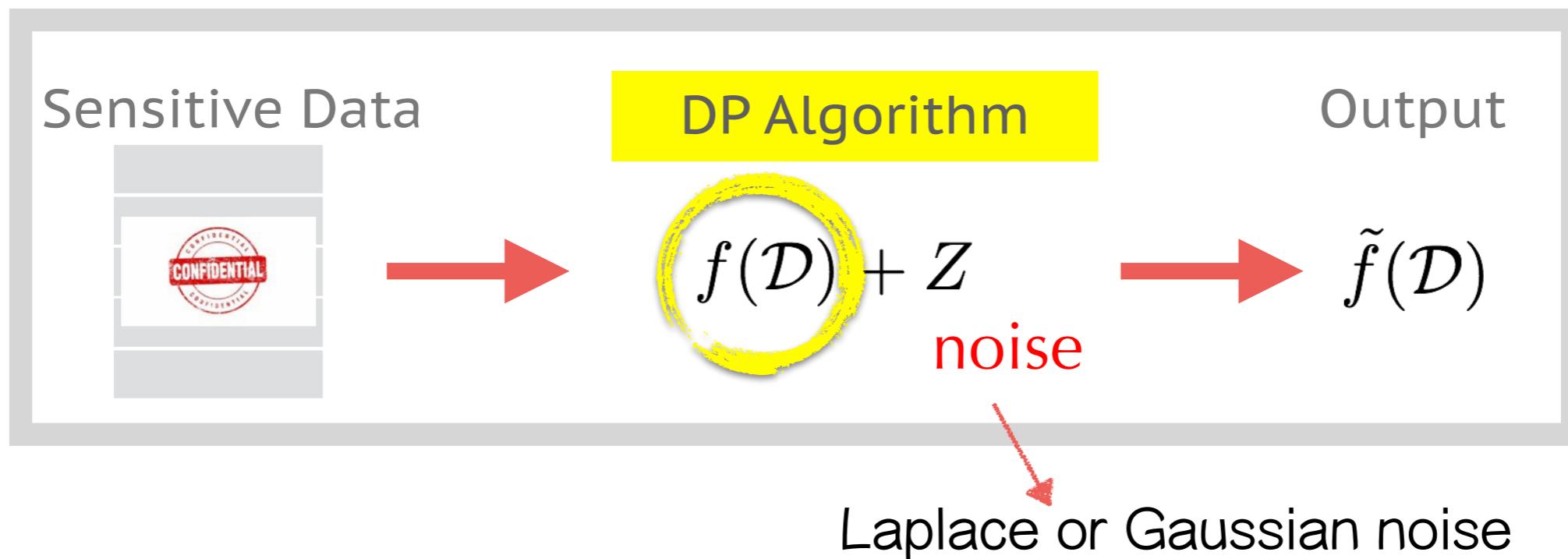
How to develop a DP Algorithm?

[Global Sensitivity Method, DMNS06]



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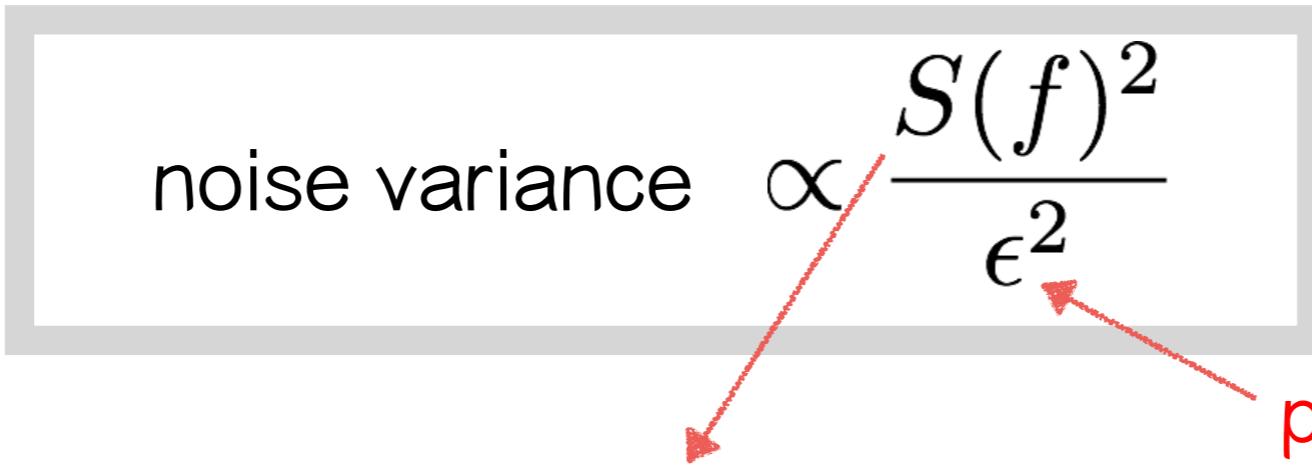
privacy loss

How much noise to add?

$$\text{noise variance} \propto \frac{S(f)^2}{\epsilon^2}$$

Sensitivity

privacy loss

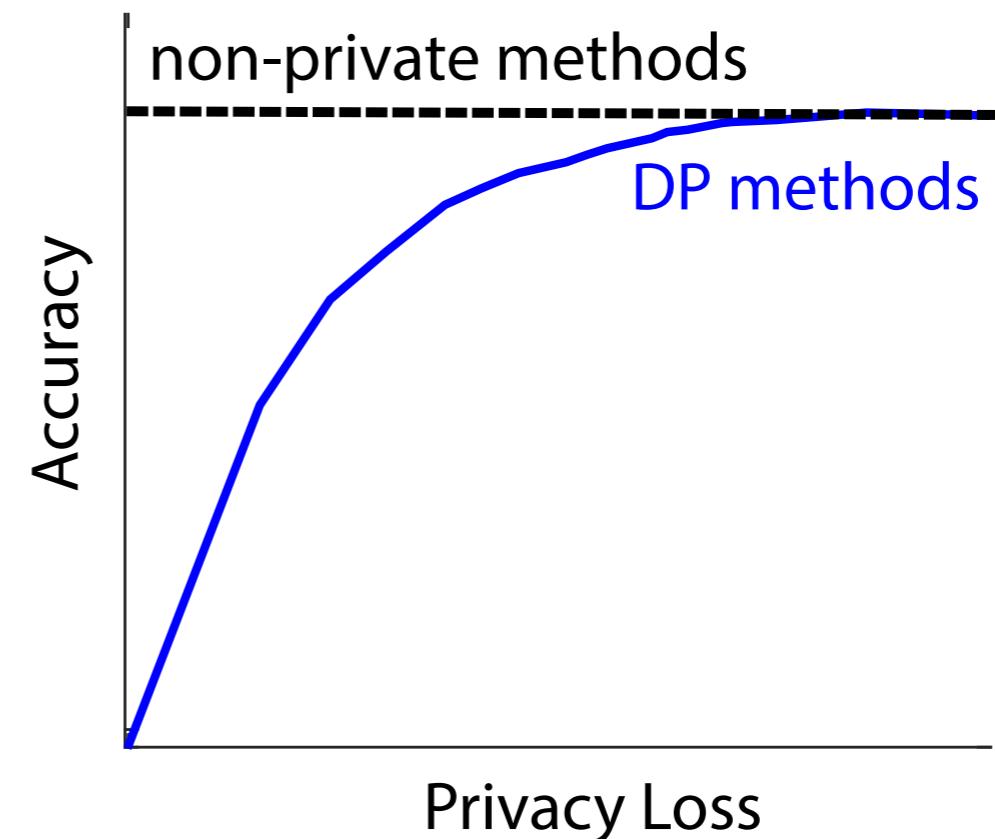


$$S(f) = \max_{D(\mathcal{D}, \mathcal{D}')=1} |f(\mathcal{D}) - f(\mathcal{D}')|$$

maximum over all pairs of datasets

Privacy & Accuracy Trade-off

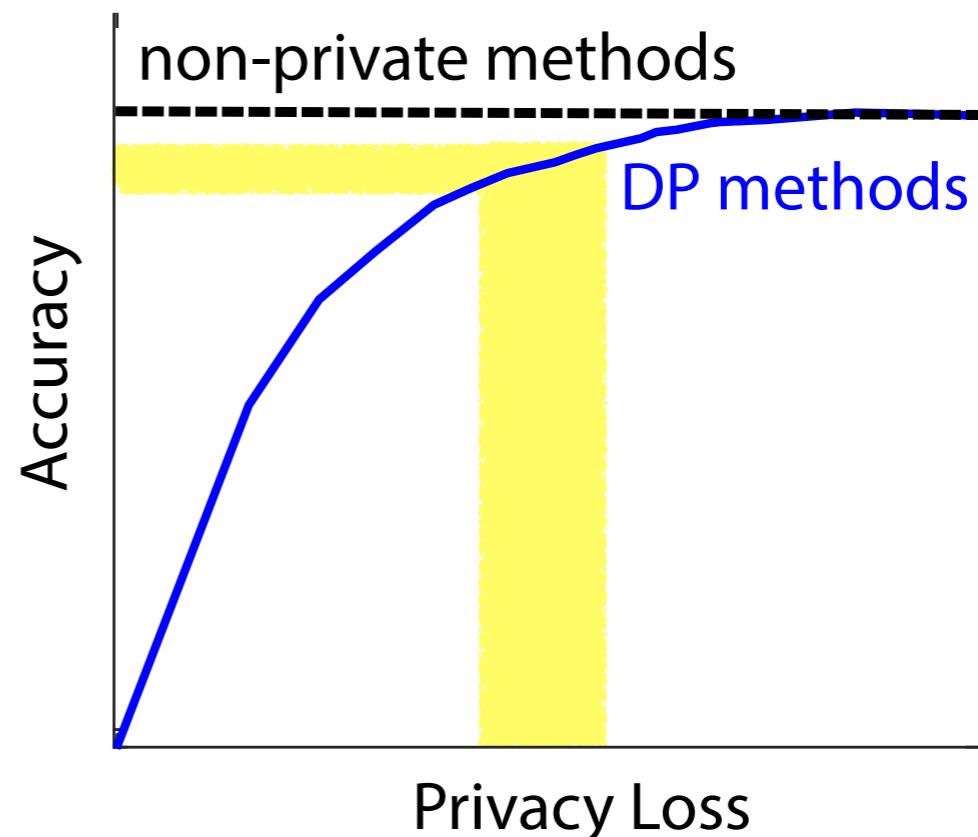
$$\text{noise} \propto \frac{\text{sensitivity}}{\text{privacy loss}}$$



Privacy & Accuracy Trade-off

$$\text{noise} \propto \frac{\text{sensitivity}}{\text{privacy loss}}$$

*Good DP-ML algorithms:
provide accurate
& differentially private outputs



Example DP mechanisms

Laplace Mechanism

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Input: Monthly salary x_1, x_2, \dots, x_N

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Differentially private average:

$$\bar{x} + Z$$

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$$\boxed{\bar{x} + Z} \longrightarrow Z \sim \text{Laplace}\left(\frac{S(\bar{x})}{\epsilon}\right)$$

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Differentially private average:

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$$Z \sim \text{Laplace}\left(\frac{S(\bar{x})}{\epsilon}\right)$$

Sensitivity: $S(\bar{x}) = \max_{\mathcal{D}, \mathcal{D}'} |\bar{x} - \bar{x}'|$

Laplace mechanism is DP

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Gaussian Mechanism

Add Gaussian noise

$$f(\mathcal{D}) + Z$$

$$Z \sim \mathcal{N}(0, \sigma^2)$$

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Privacy loss

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Approximate DP: $P(L^{(o)} > \epsilon) \leq \delta$

Privacy loss

$$(\epsilon, \delta) - \text{DP}$$

Pure DP holds w.p. at least $1 - \delta$

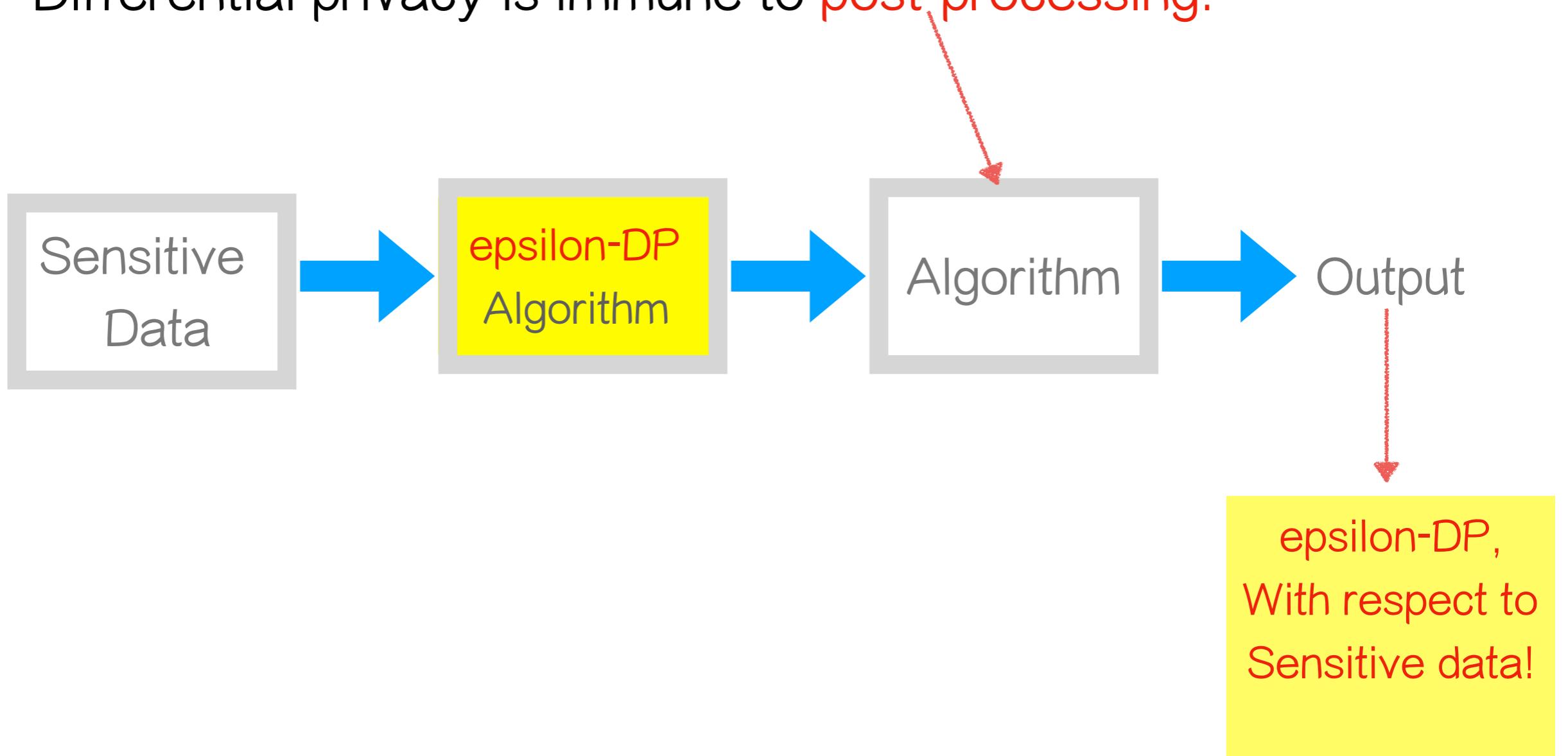
DP Mechanisms

- Laplace mechanism: provides a pure DP guarantee
- Gaussian mechanism: provides an approximate DP guarantee
- There are more mechanisms. See [Algorithmic foundations of differential privacy, Dwork & Roth 2014].

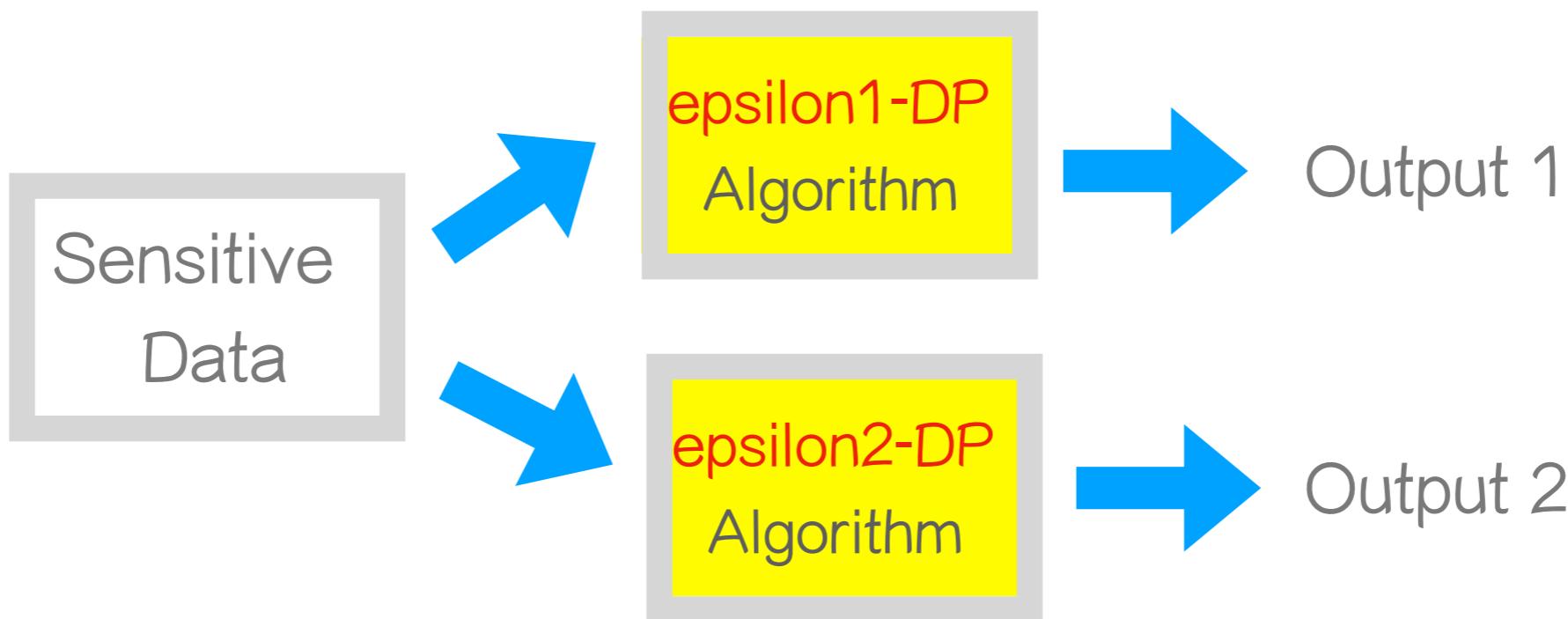
Properties of DP

1. Postprocessing invariance

- Differential privacy is immune to **post-processing**:



2. Composition



- Union of output 1 (from epsilon1-DP algorithm) & output 2 (from epsilon2-DP algorithm) is **(epsilon1+epsilon2)-DP!**
- More refined composition methods do exist! e.g., [Abadi et al,16]

Key properties of DP

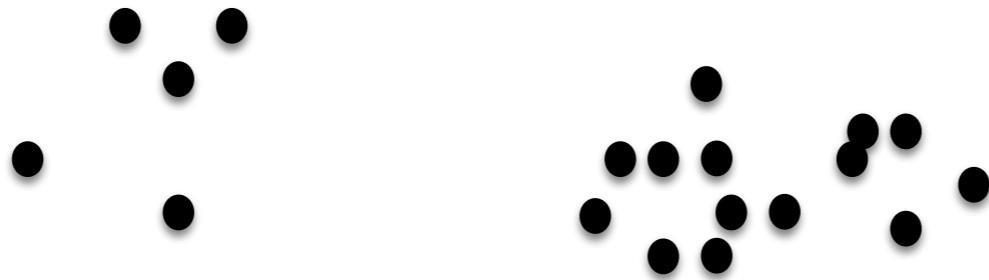
- Post-processing invariance
- Composition : introduces challenges to make iterative algorithms in ML differentially private!

Examples of DP-ML algorithms

Clustering & density estimation

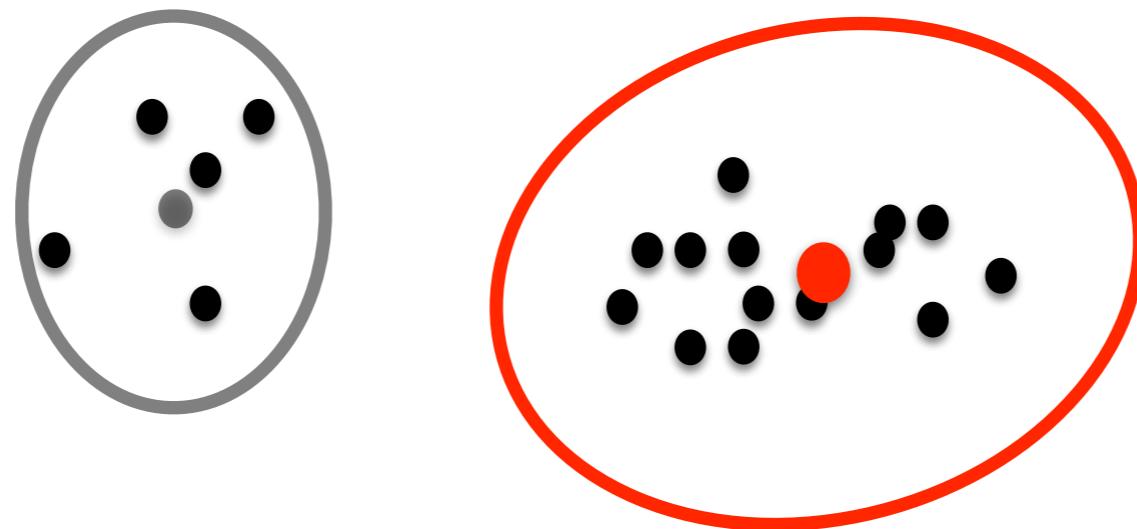
Mixture of Gaussians

For clustering & density estimation



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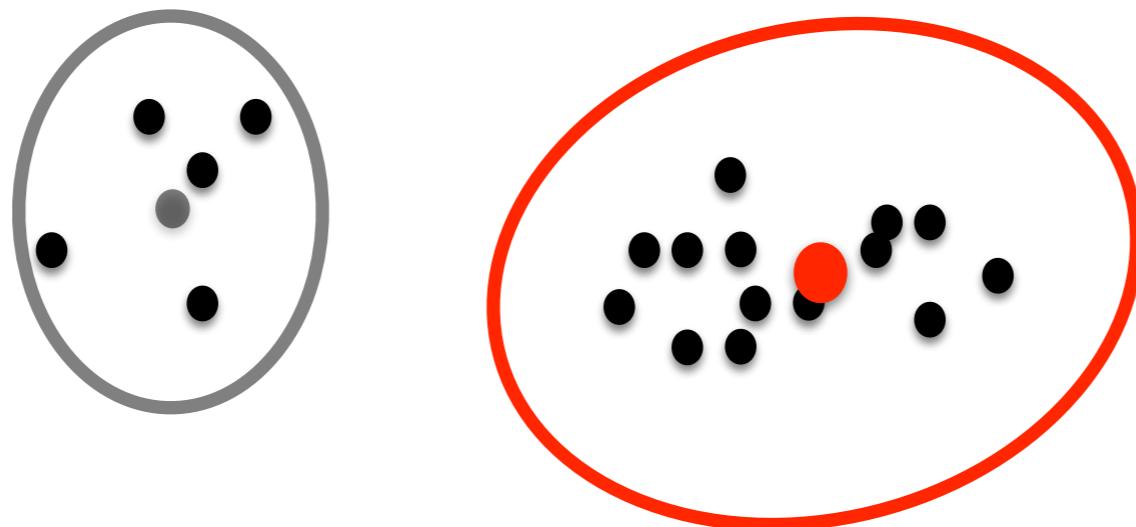
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- Assign a Gaussian to learn each cluster

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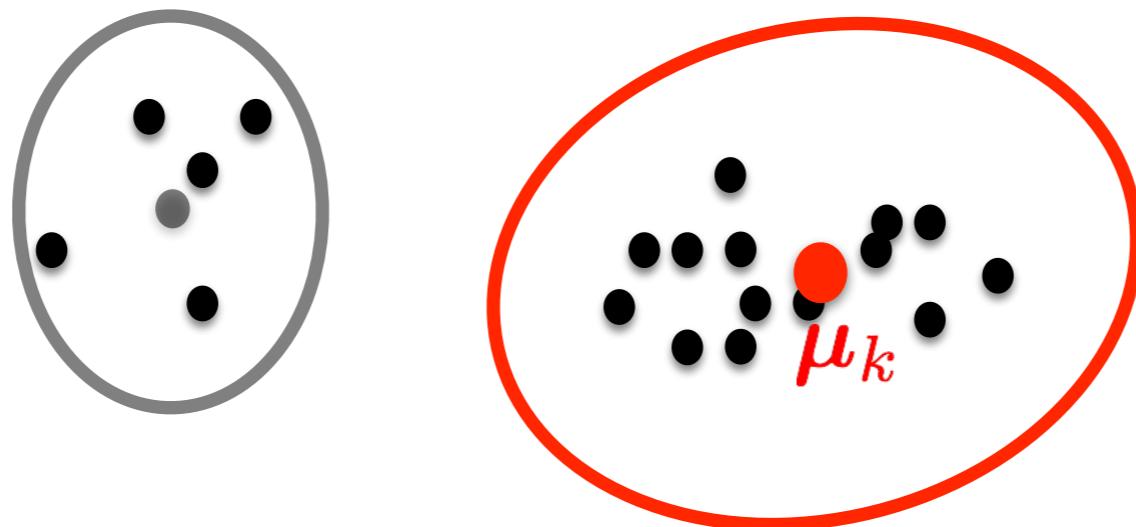
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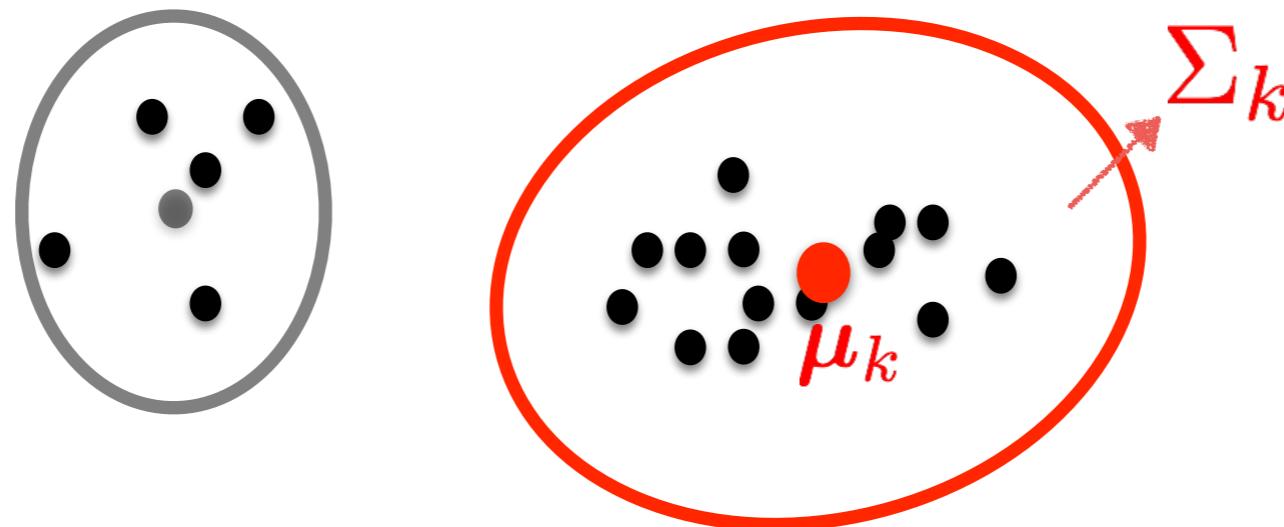
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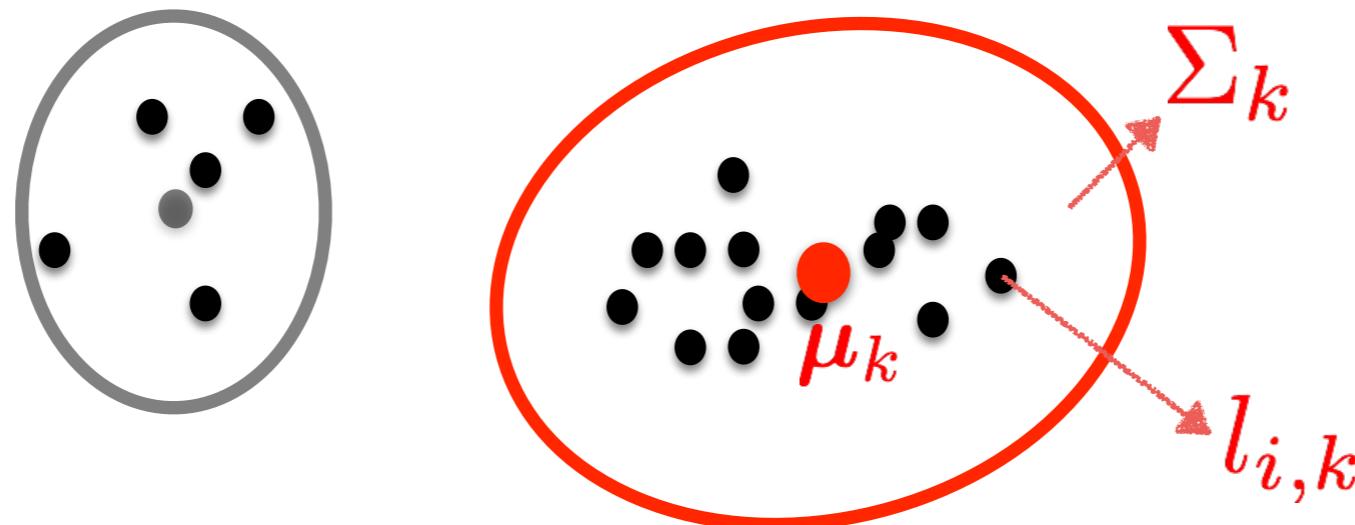
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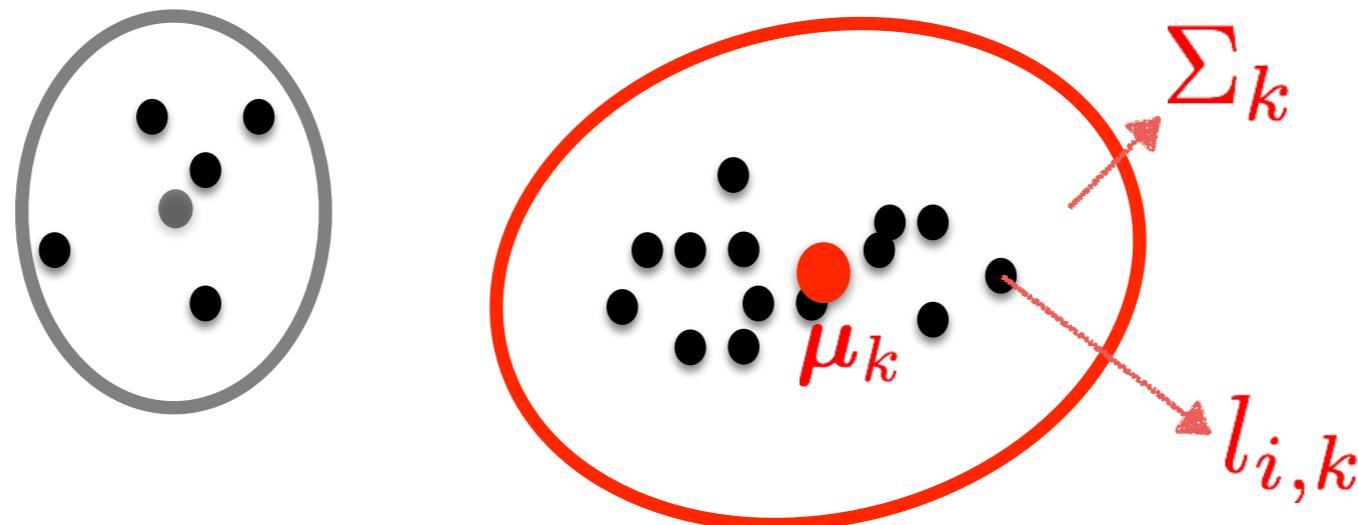
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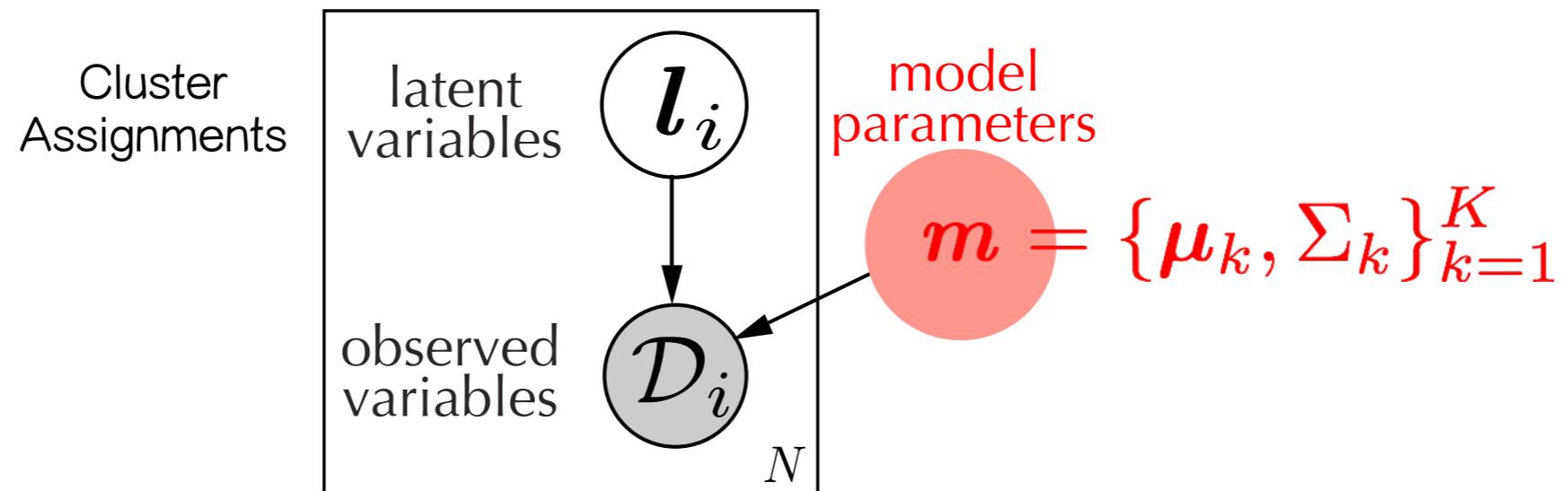
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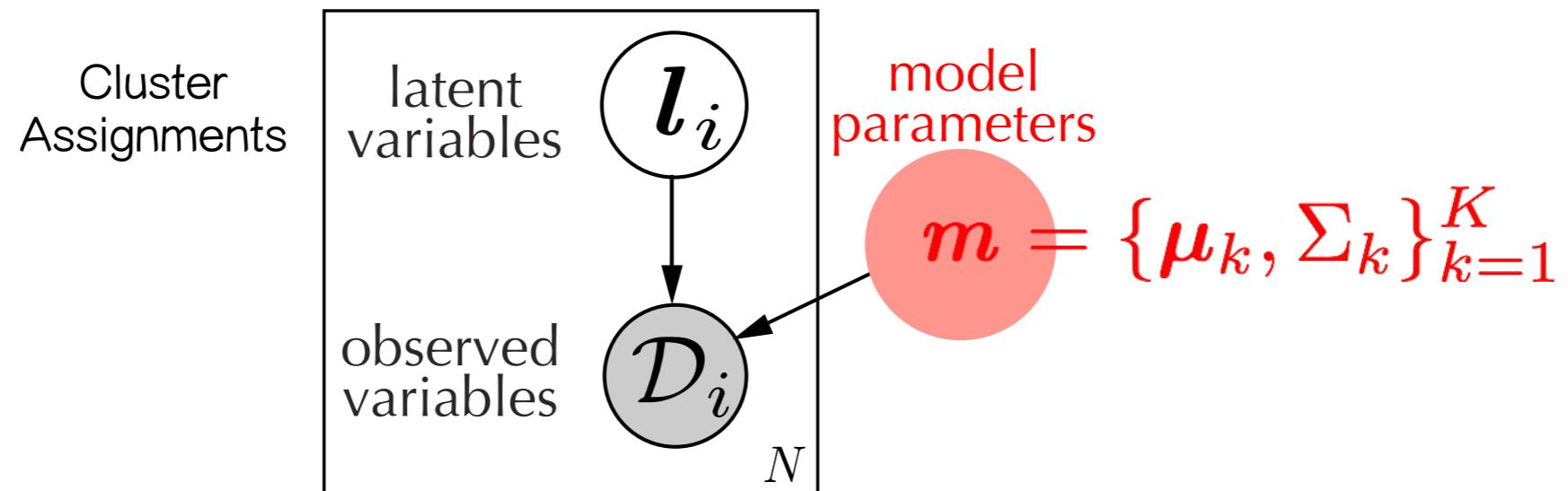


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- **Goal:** estimate the parameters

Expectation Maximization



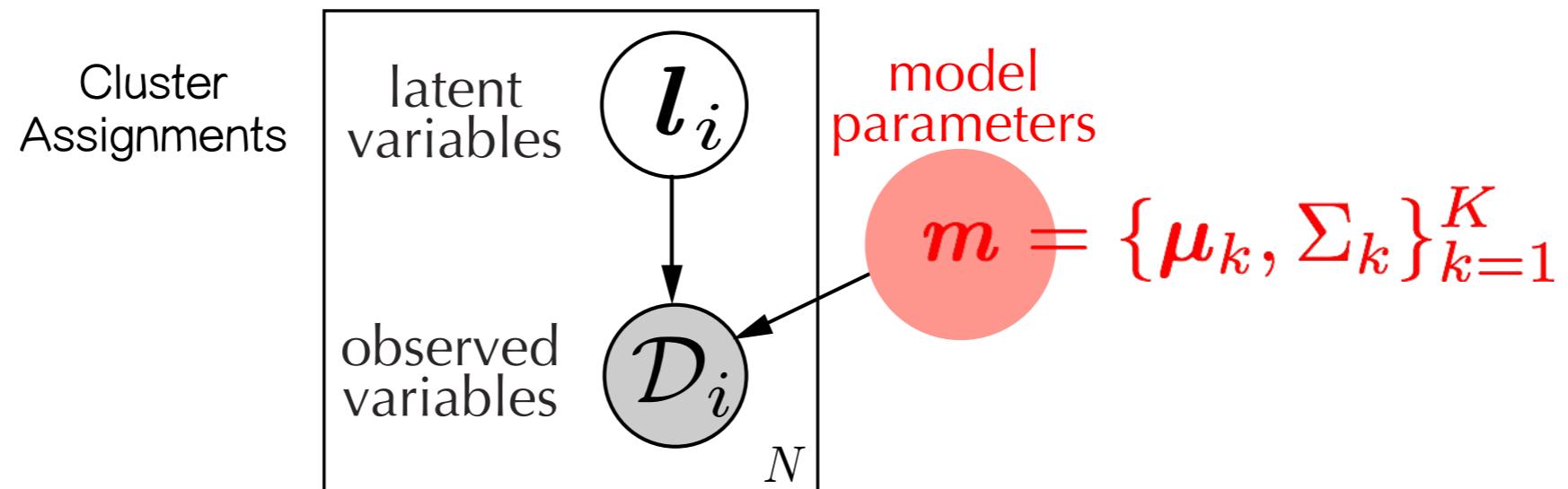
Expectation Maximization



log-likelihood

$$\log p(\mathcal{D}|\mathbf{m}) = \log \int d\mathbf{l} p(\mathcal{D}, \mathbf{l}|\mathbf{m})$$

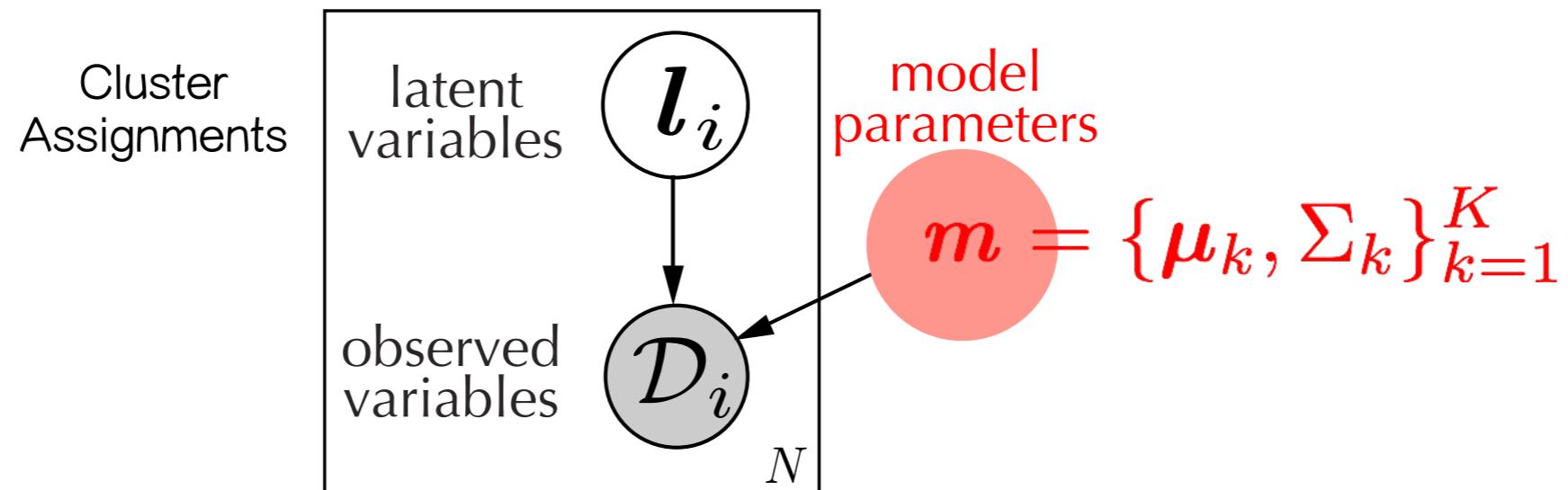
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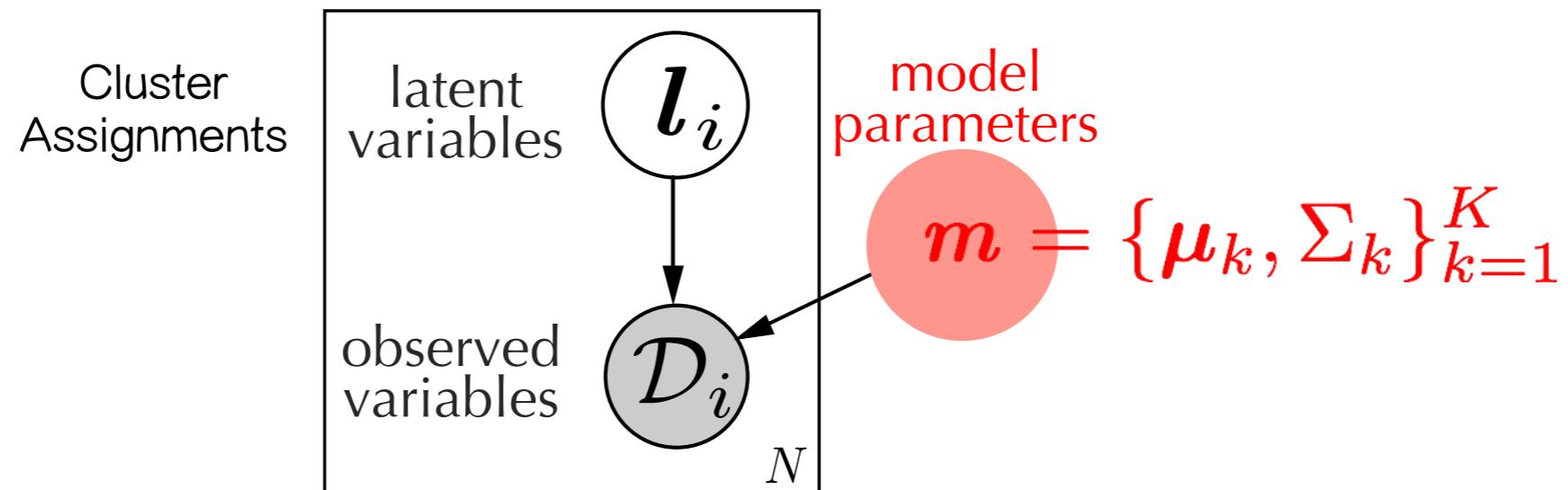


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$\mathcal{F}(q, \mathbf{m})$ Free-energy

Expectation Maximization



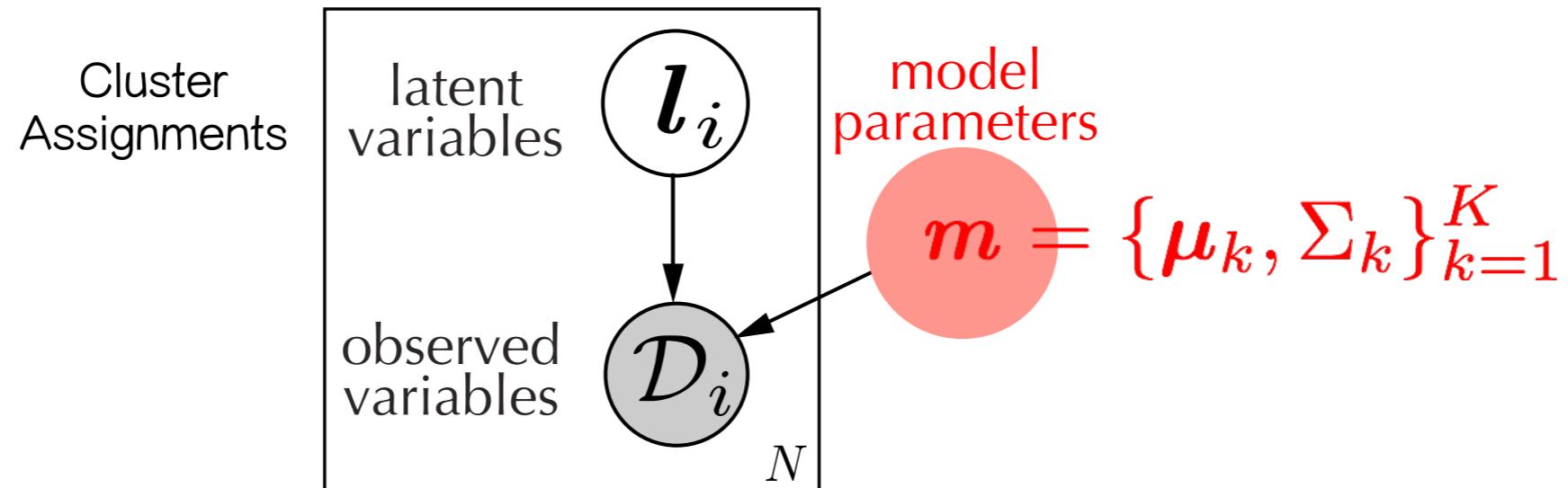
log-likelihood

$$\log p(\mathcal{D}|\mathbf{m}) = \log \int d\mathbf{l} p(\mathcal{D}, \mathbf{l}|\mathbf{m}) \geq \int d\mathbf{l} q(\mathbf{l}) \log \frac{p(\mathcal{D}, \mathbf{l}|\mathbf{m})}{q(\mathbf{l})}$$

$\mathcal{F}(q, \mathbf{m})$ Free-energy

$$\text{E-step: } q^{(t)}(\mathbf{l}) = \arg \max_{q(\mathbf{l})} \mathcal{F}(q(\mathbf{l}), \mathbf{m}^{(t-1)})$$

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DP-EM

[Park et al, 2017]

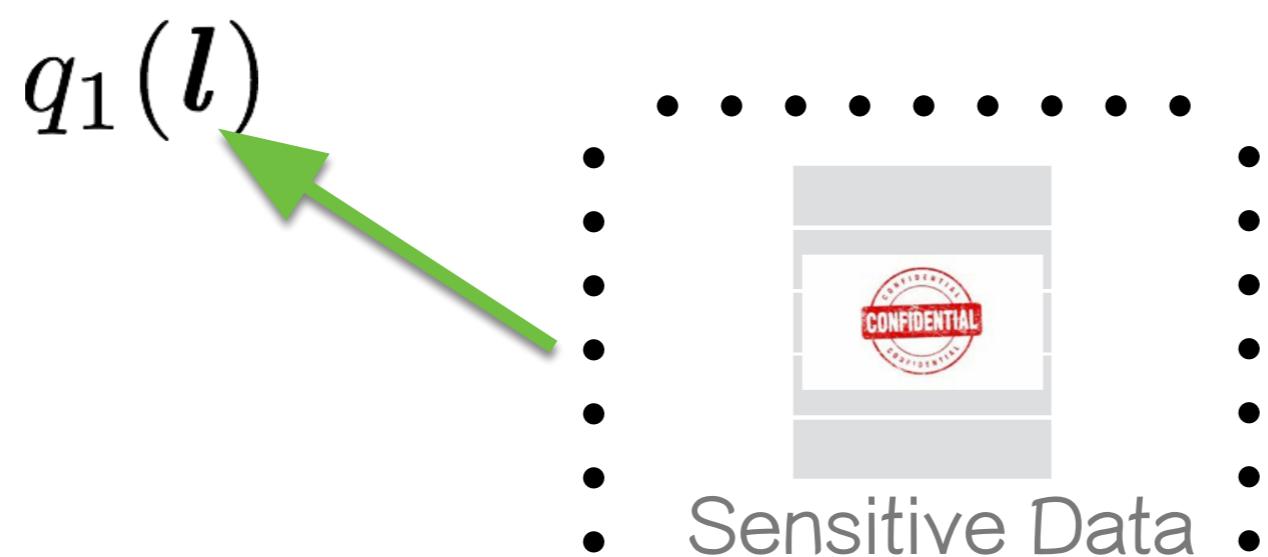
DP-EM

[Park et al, 2017]



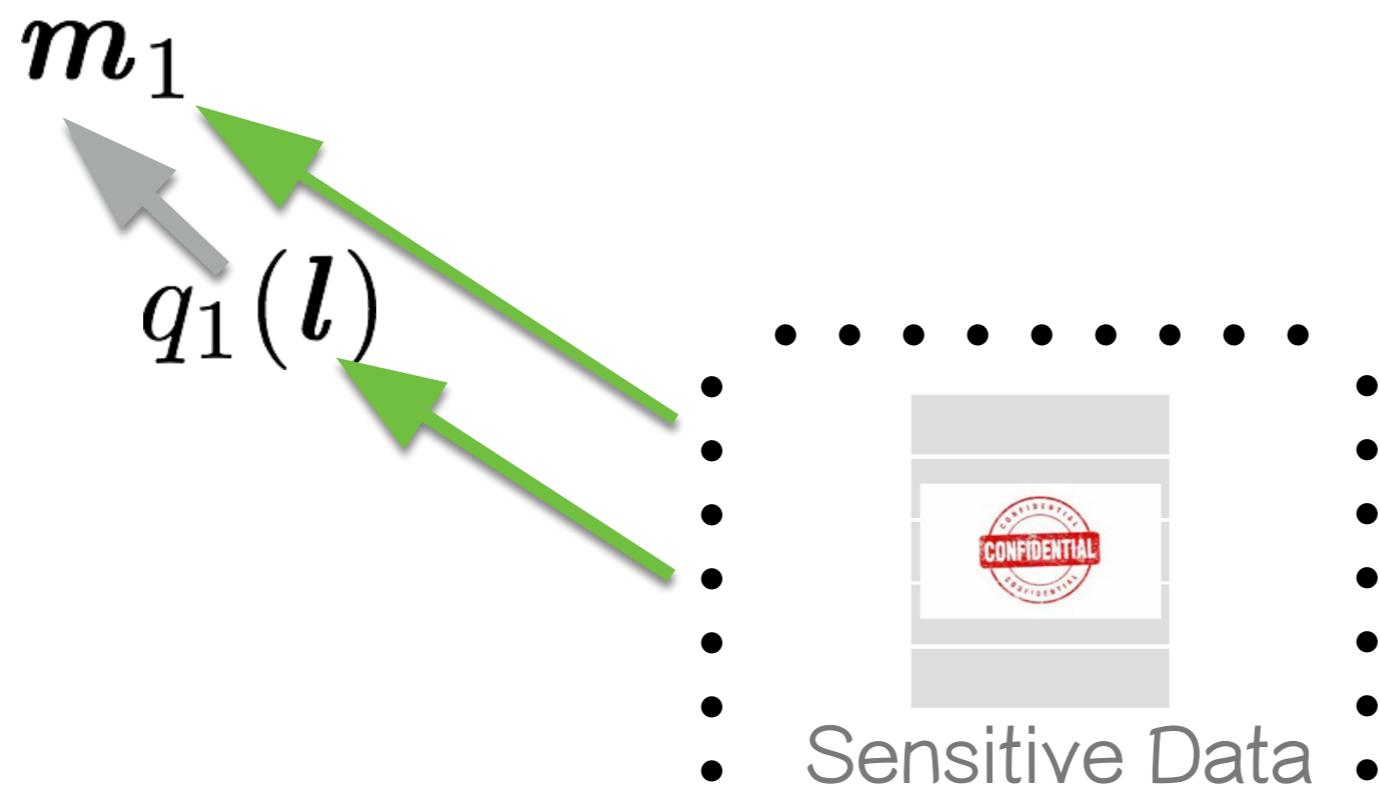
DP-EM

[Park et al, 2017]



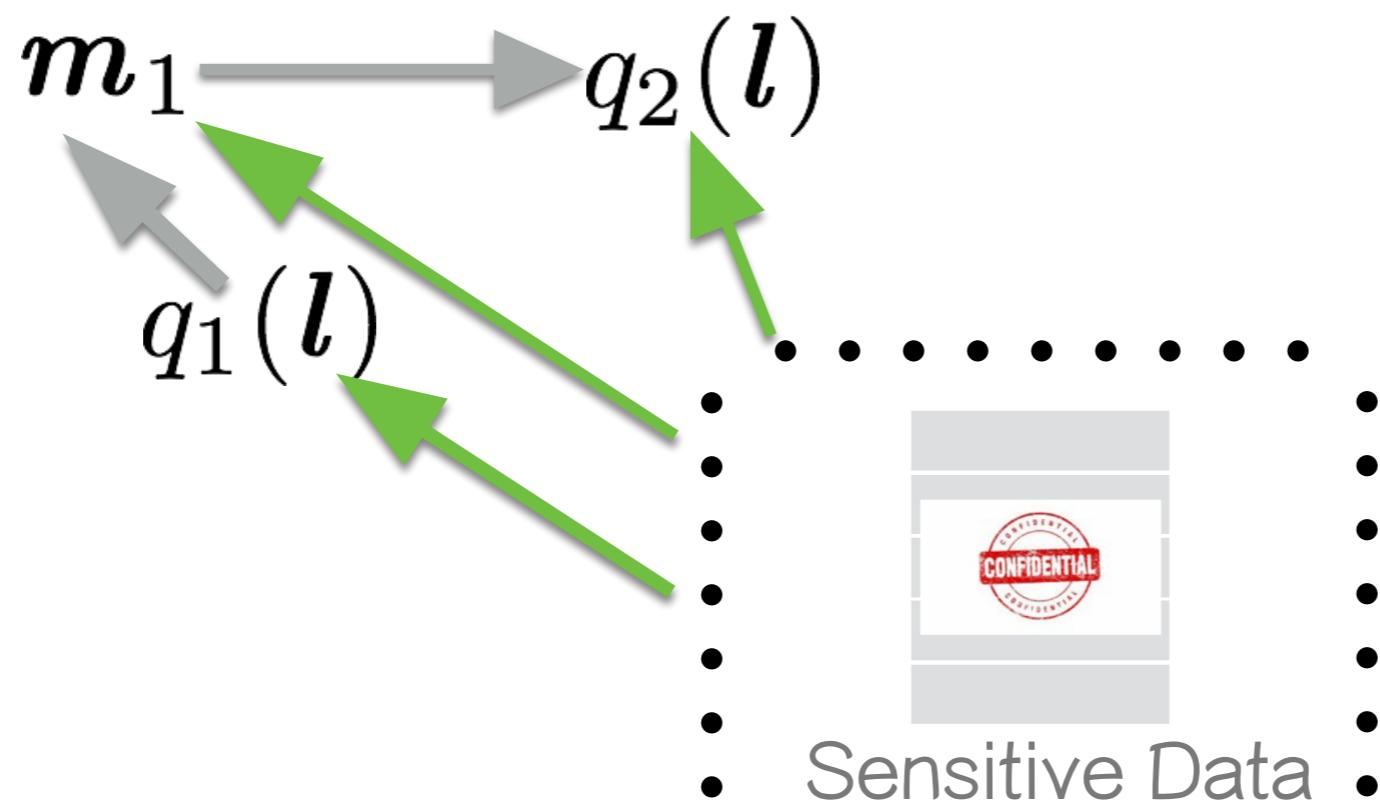
DP-EM

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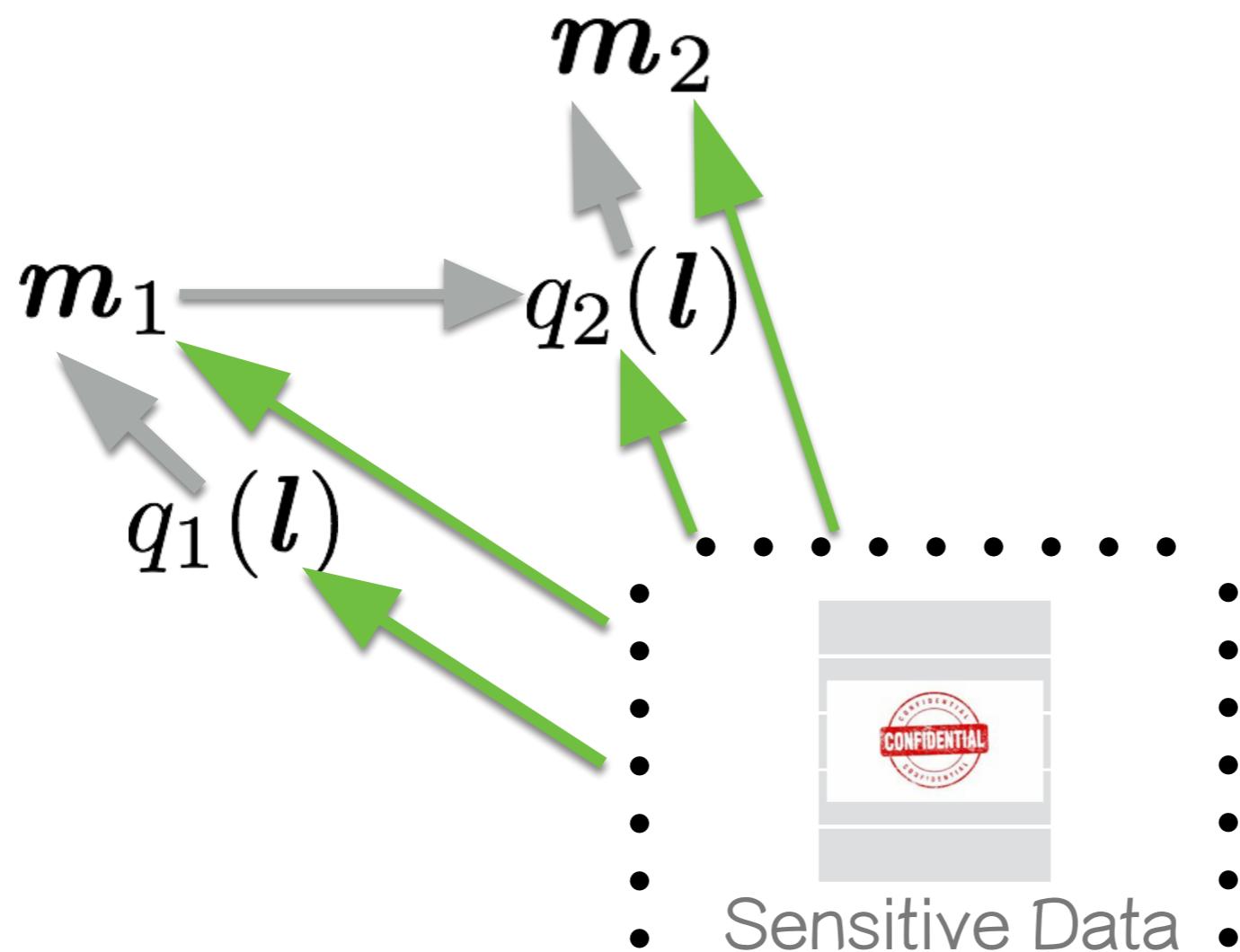
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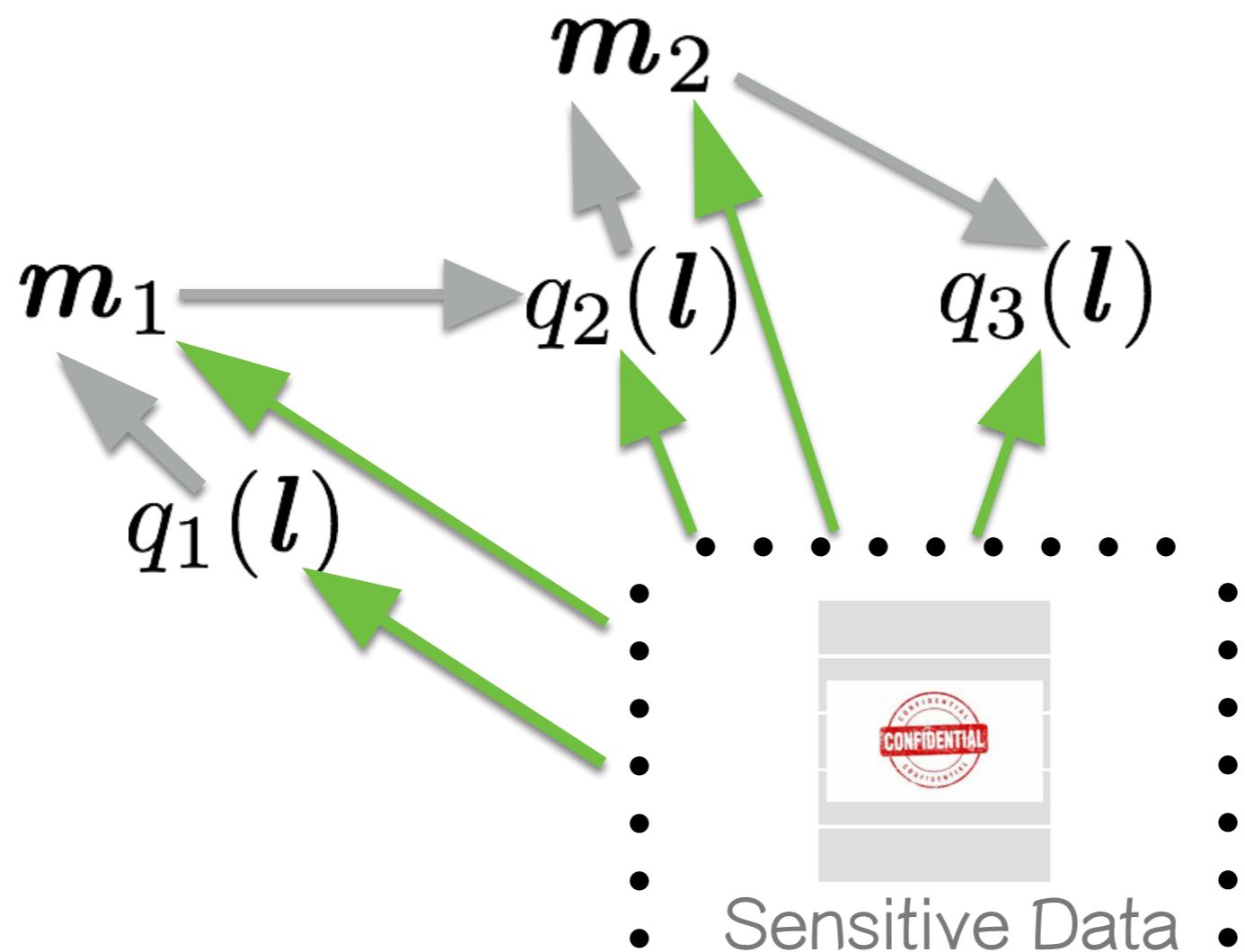
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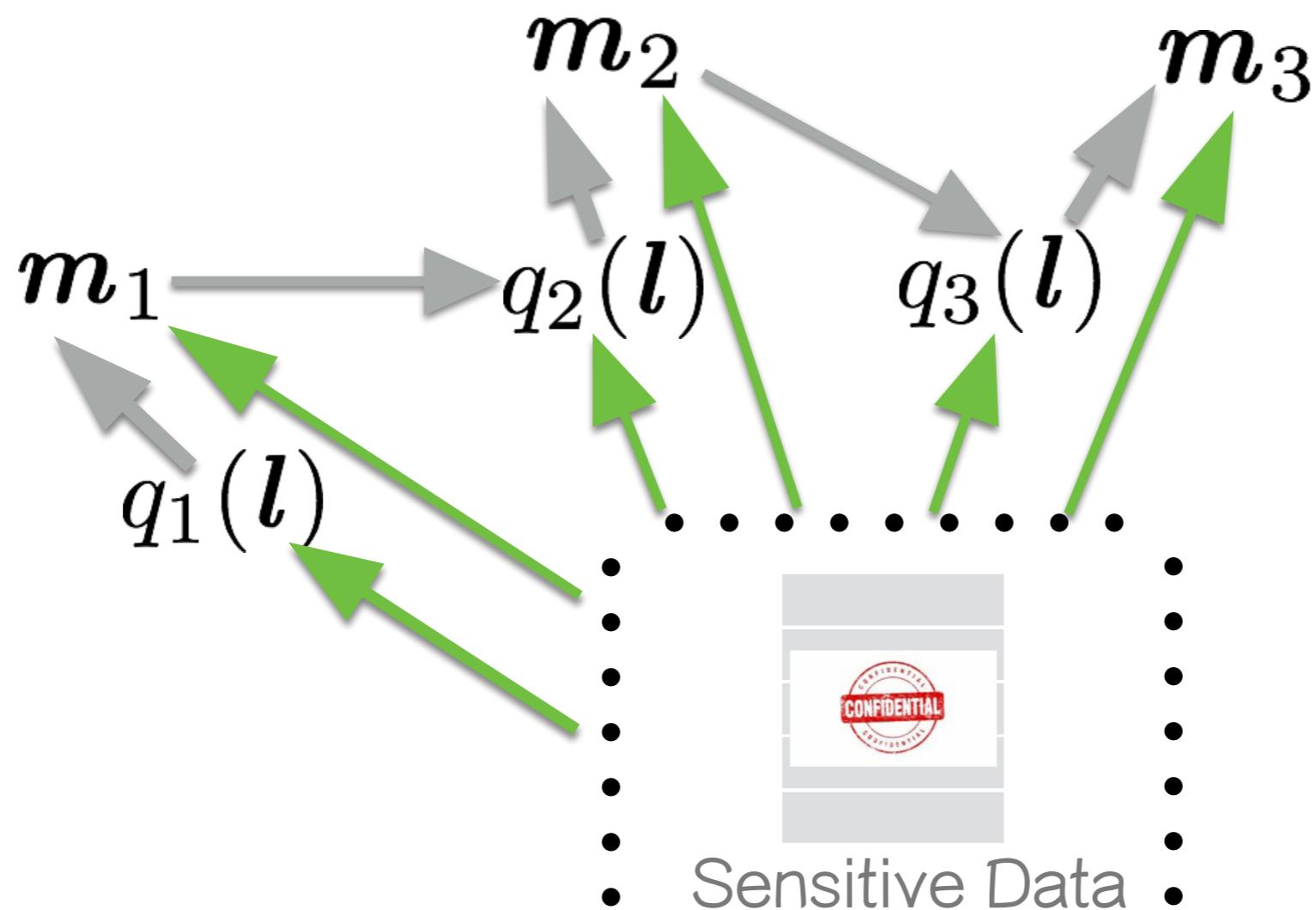
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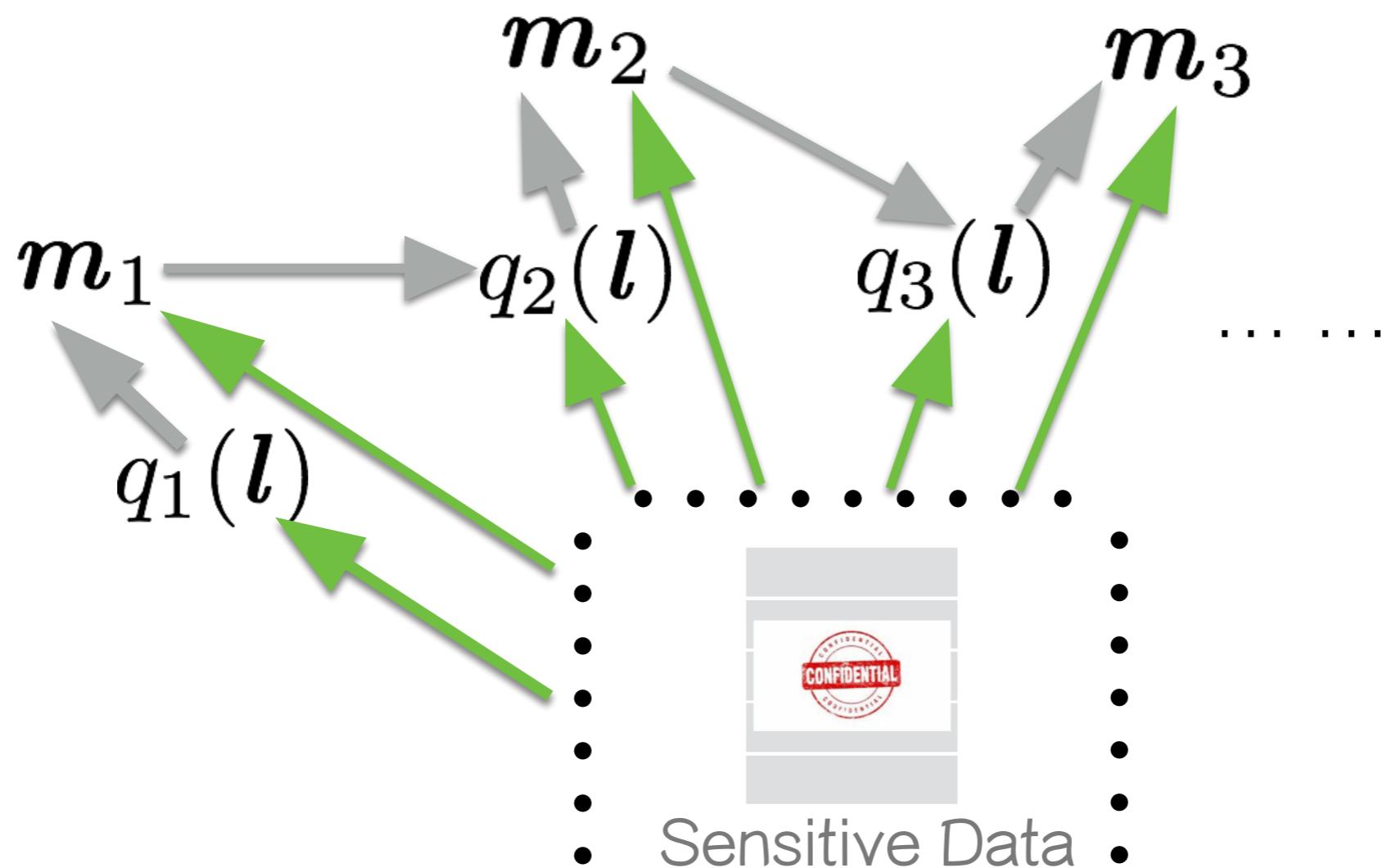
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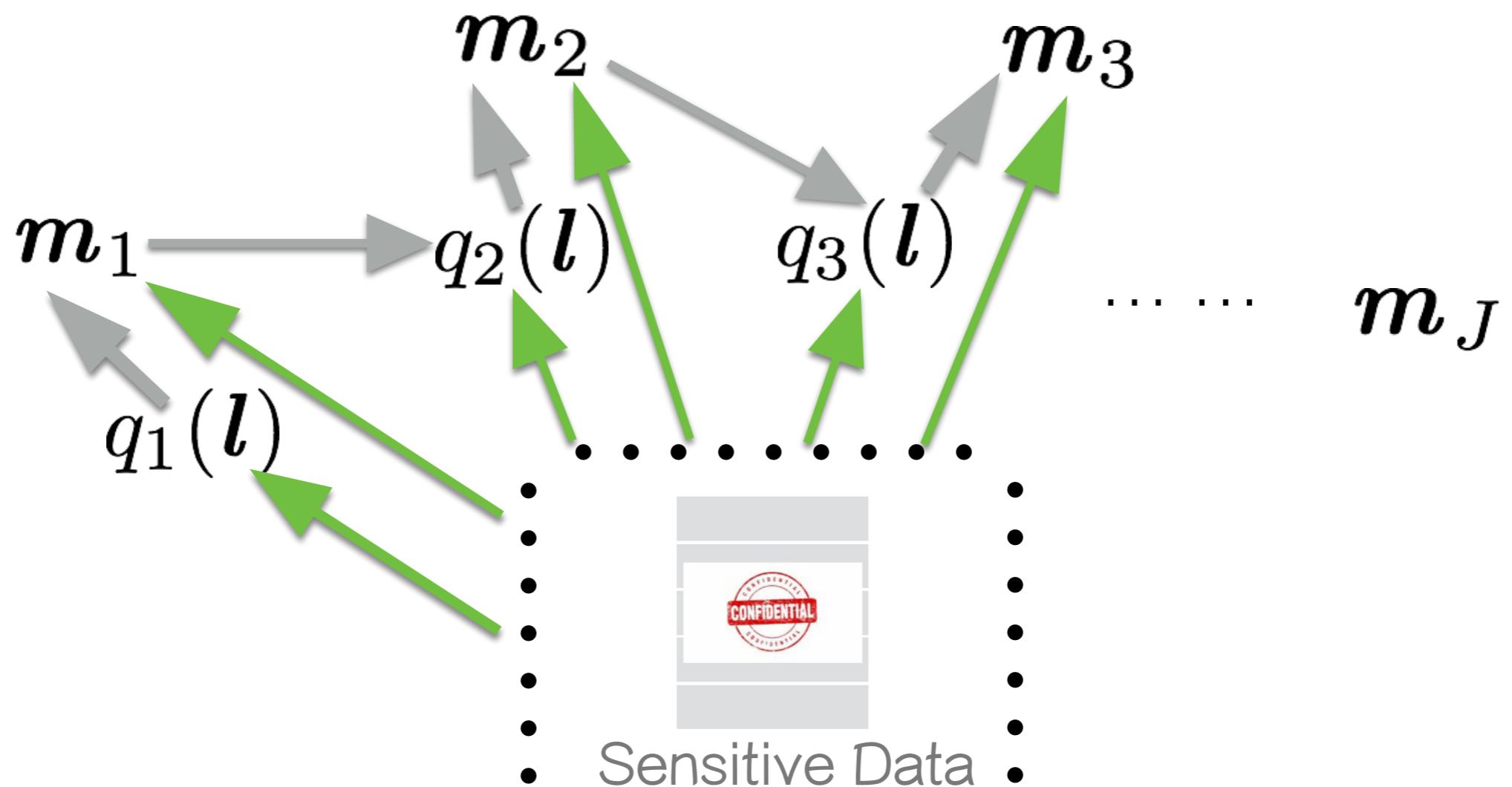
DP-EM

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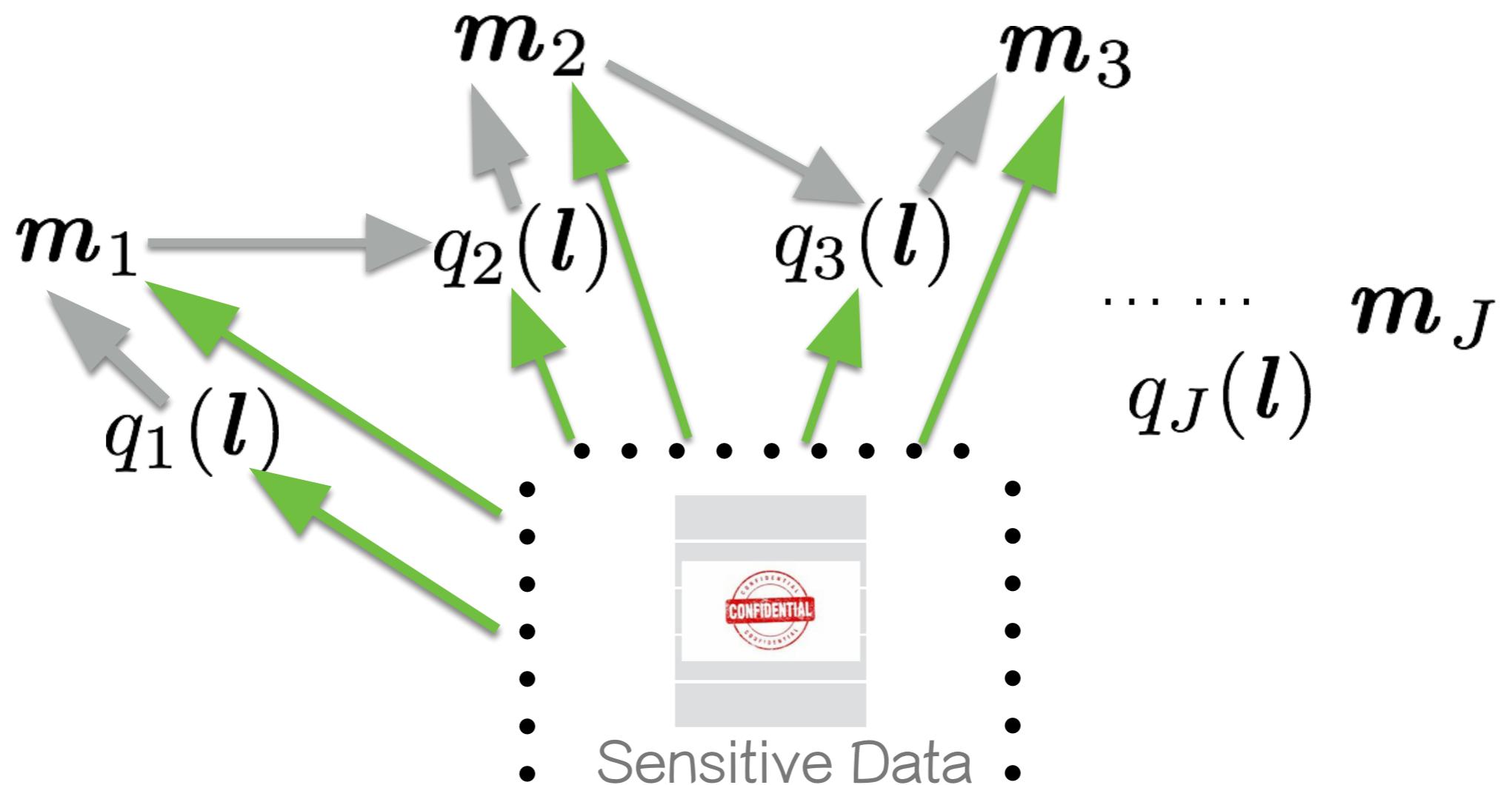
DP-EM

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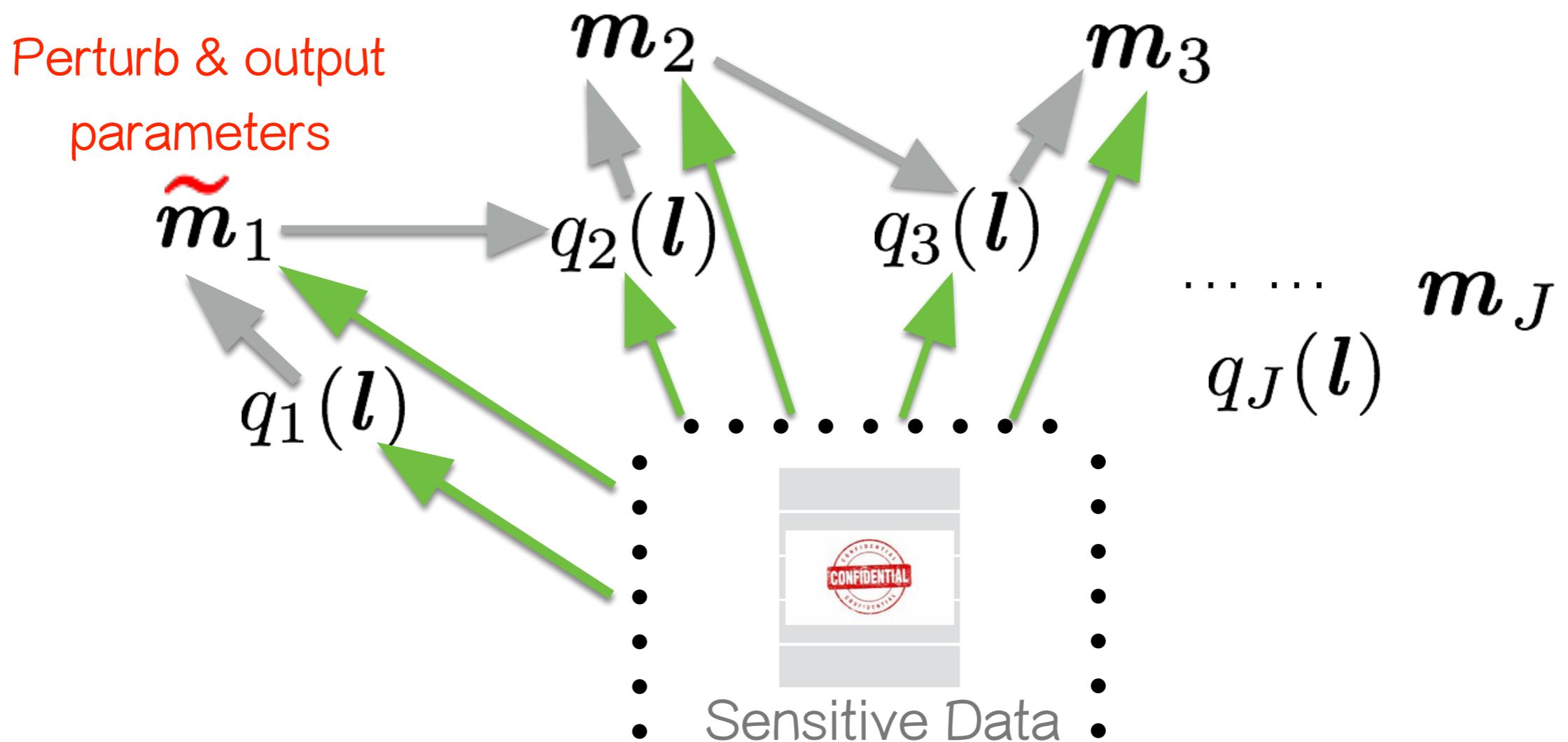
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[Park et al, 2017]



DP-EM

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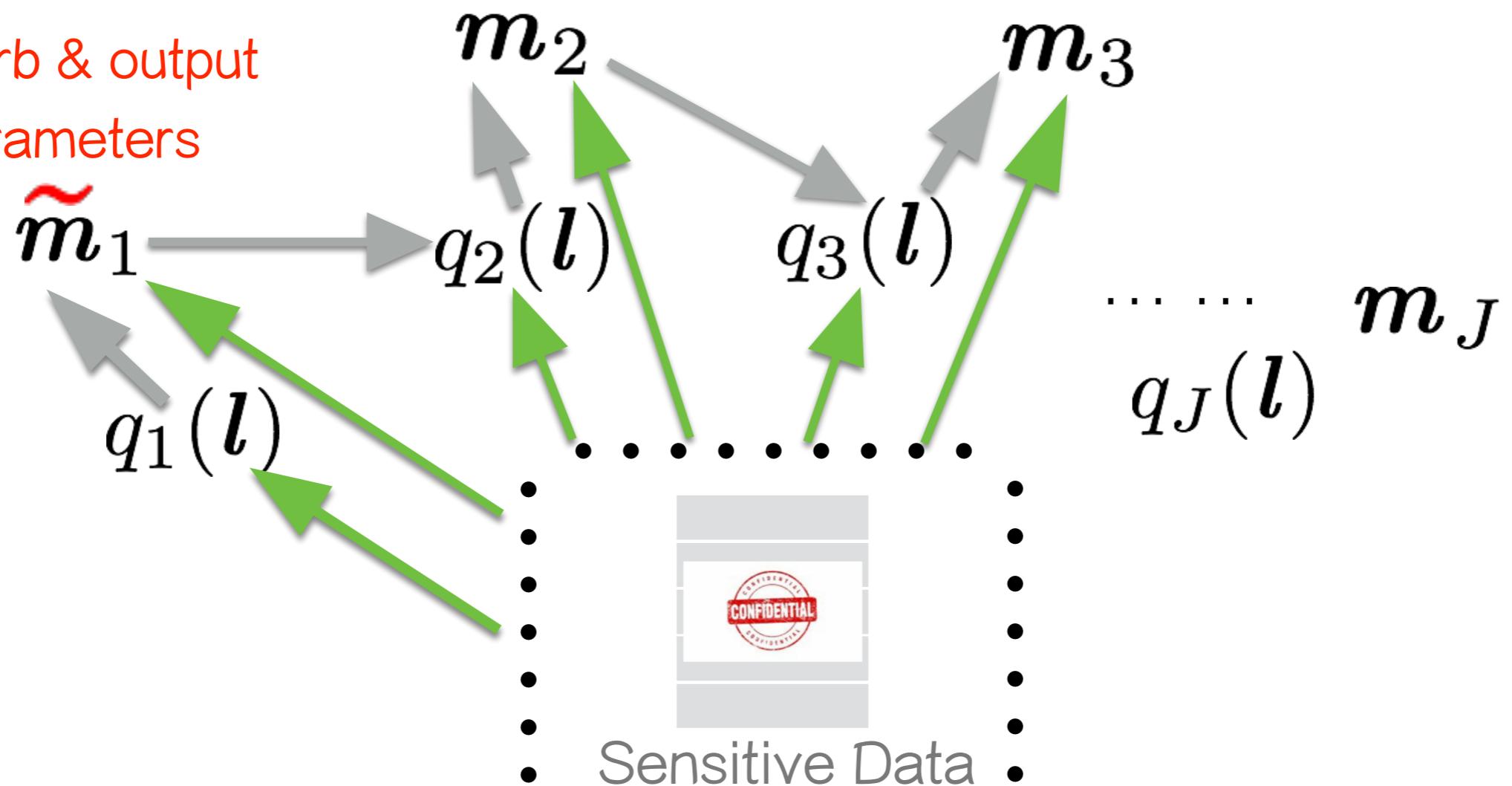


DP-EM

Sensitivity: one-d datapoint's worth
directly from dataset & indirectly through $q(\mathbf{l})$

[Park et al, 2017]

Perturb & output parameters

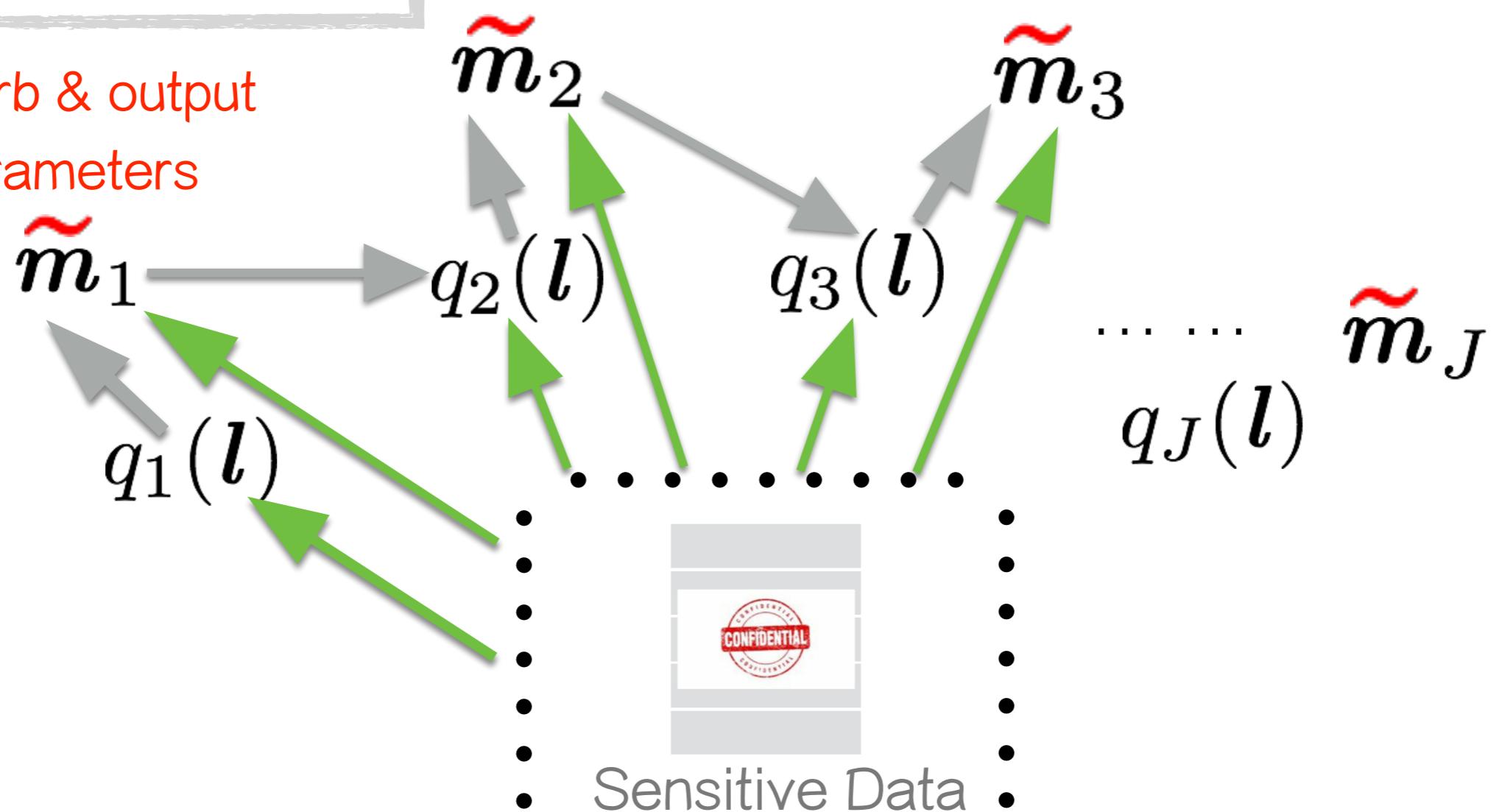


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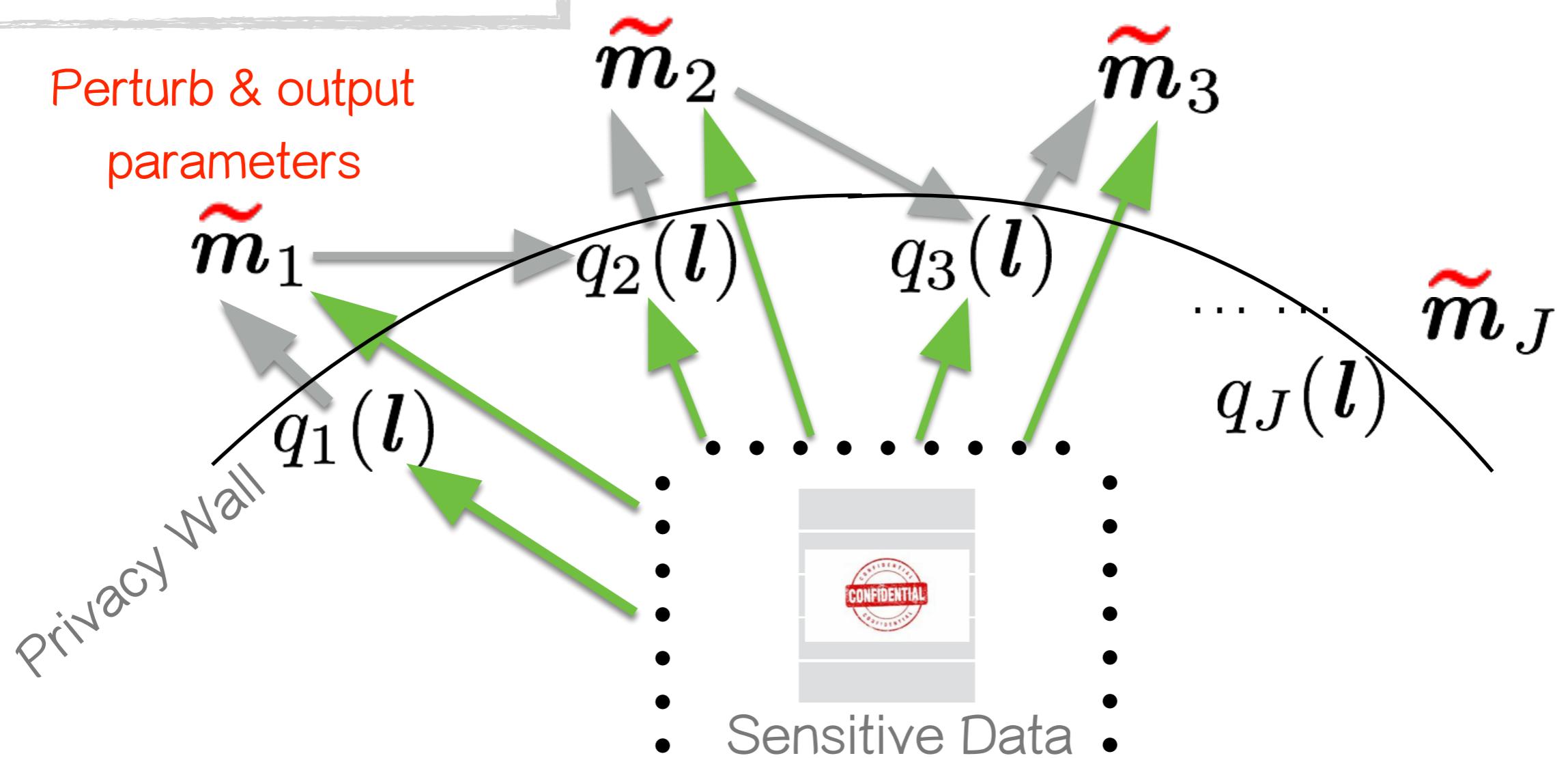
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Repetitive access to data, large privacy loss

Repetitive access to data, large privacy loss

Moments accountant

[Abadi et al, 2016]

Repetitive access to data, large privacy loss

Moments accountant

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- Treat privacy loss (PL) as

$$L^{(o)} = \log \frac{p(A(\mathcal{D}) = o)}{p(A(\mathcal{D}') = o)}$$

random variable

Repetitive access to data, large privacy loss

Moments accountant

[Abadi et al, 2016]

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random variable

keep track of MGF of L ,
**convert moment bound
to tail bound**

→ **strong privacy guarantee**

Repetitive access to data, large privacy loss

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[Abadi et al, 2016]

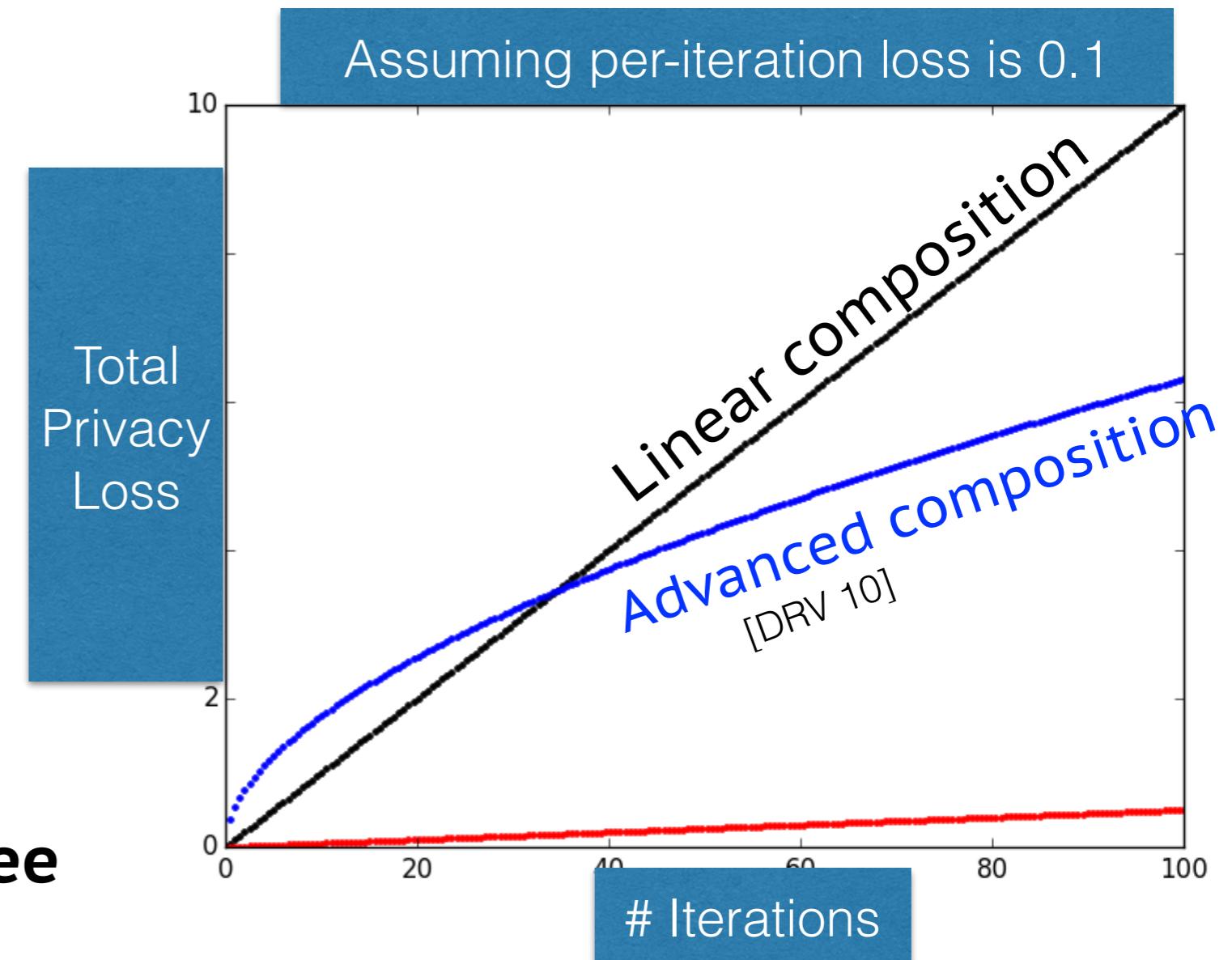
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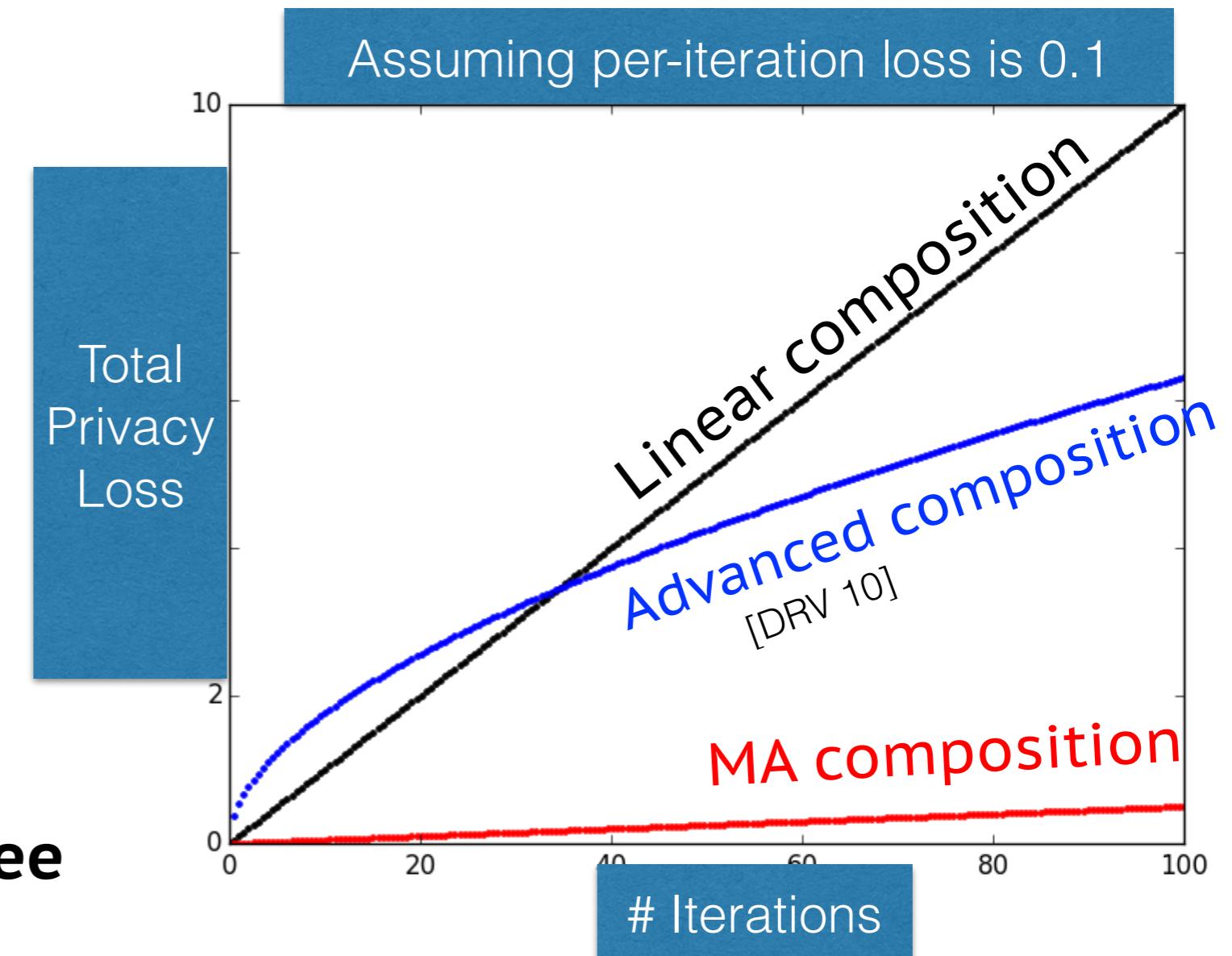
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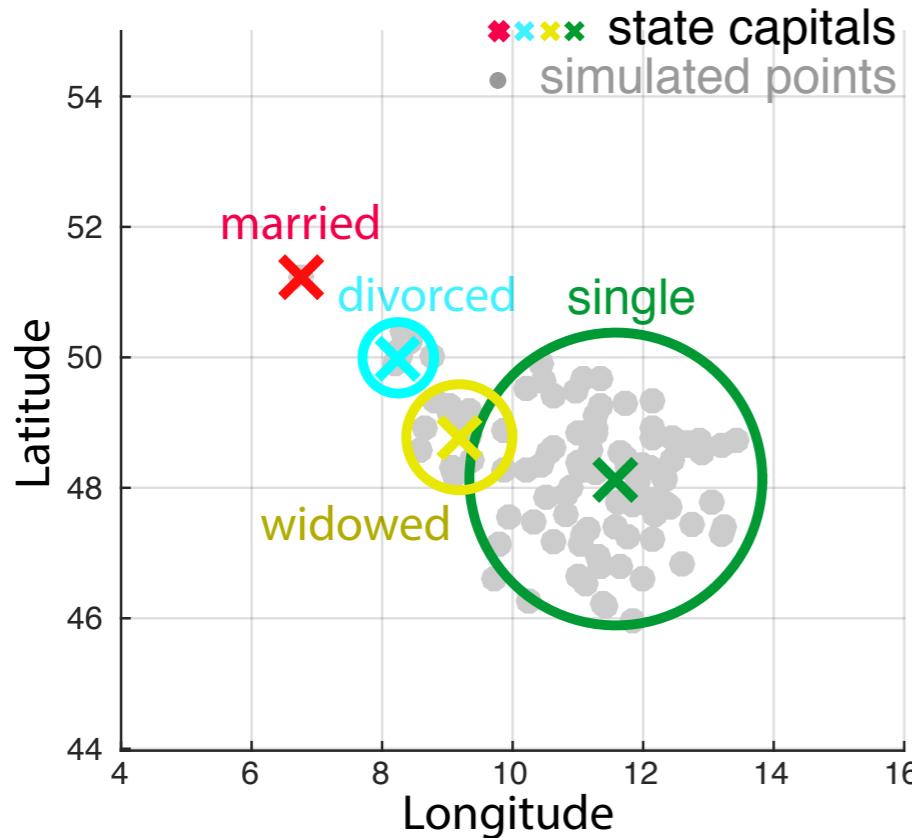
→ **strong privacy guarantee**



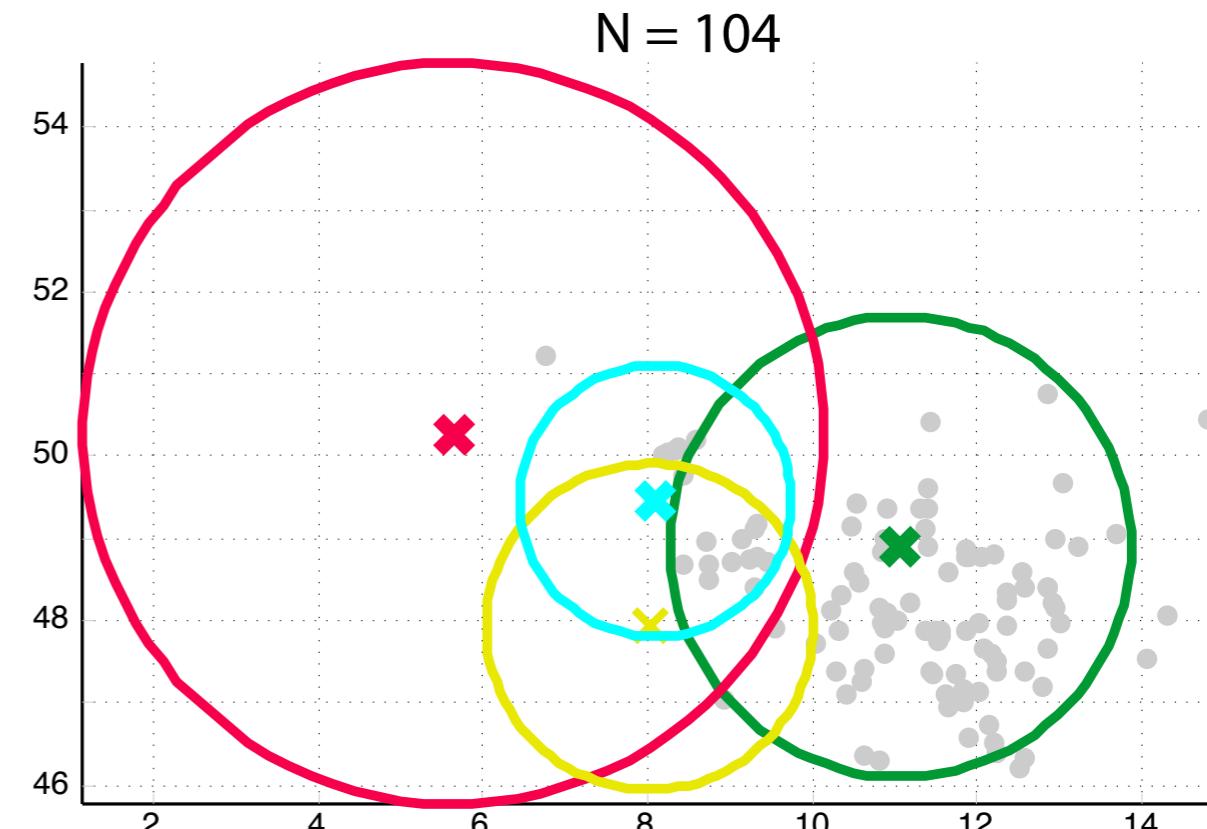
DP-EM: Abortion data

[destatis.de]

Raw data



Privatized MoG using DP-EM

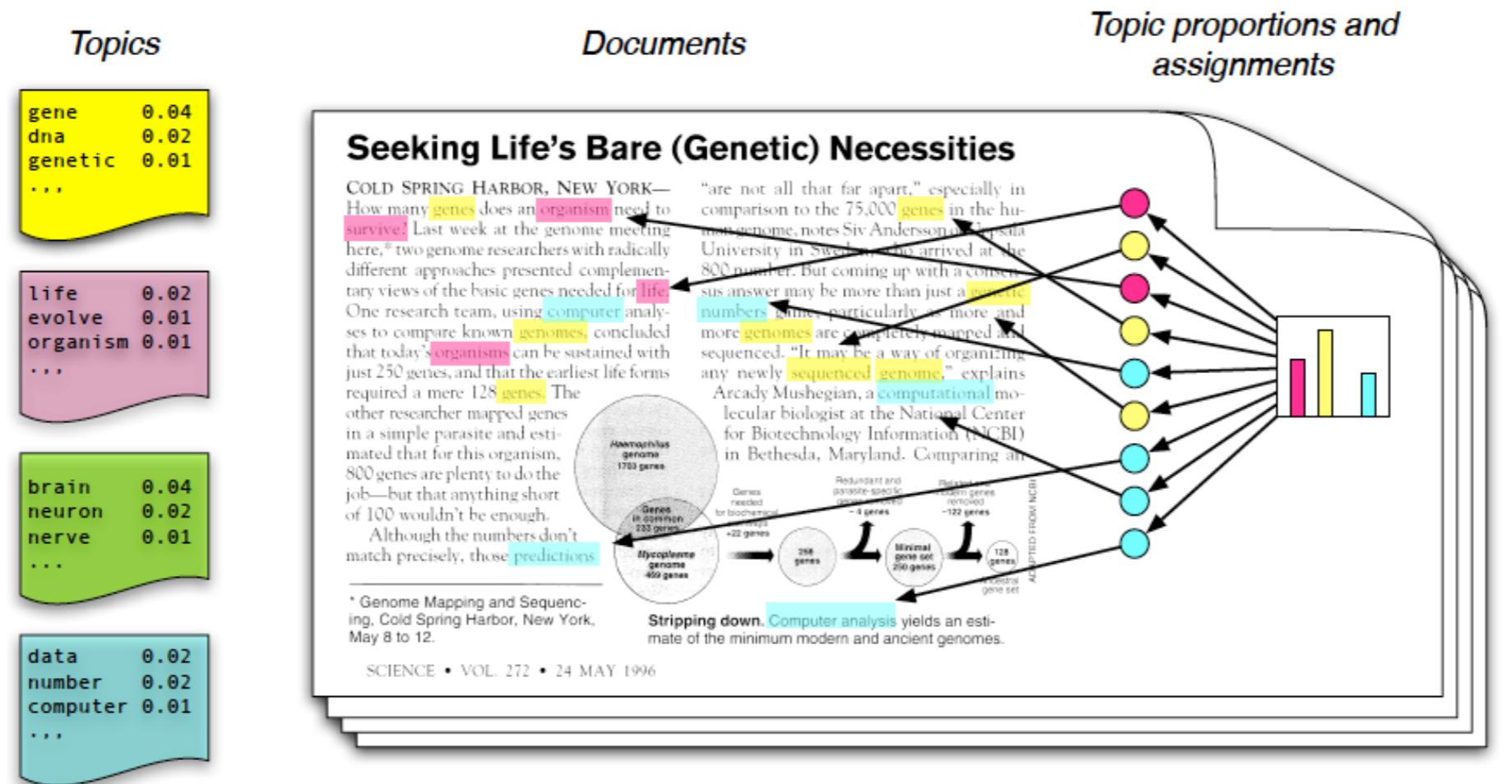


Even for the cluster with a single datapoint (**red cluster**), one can't accurately identify the geographical location of that individual from outputs of DP-EM

Topic modeling

Topic modeling

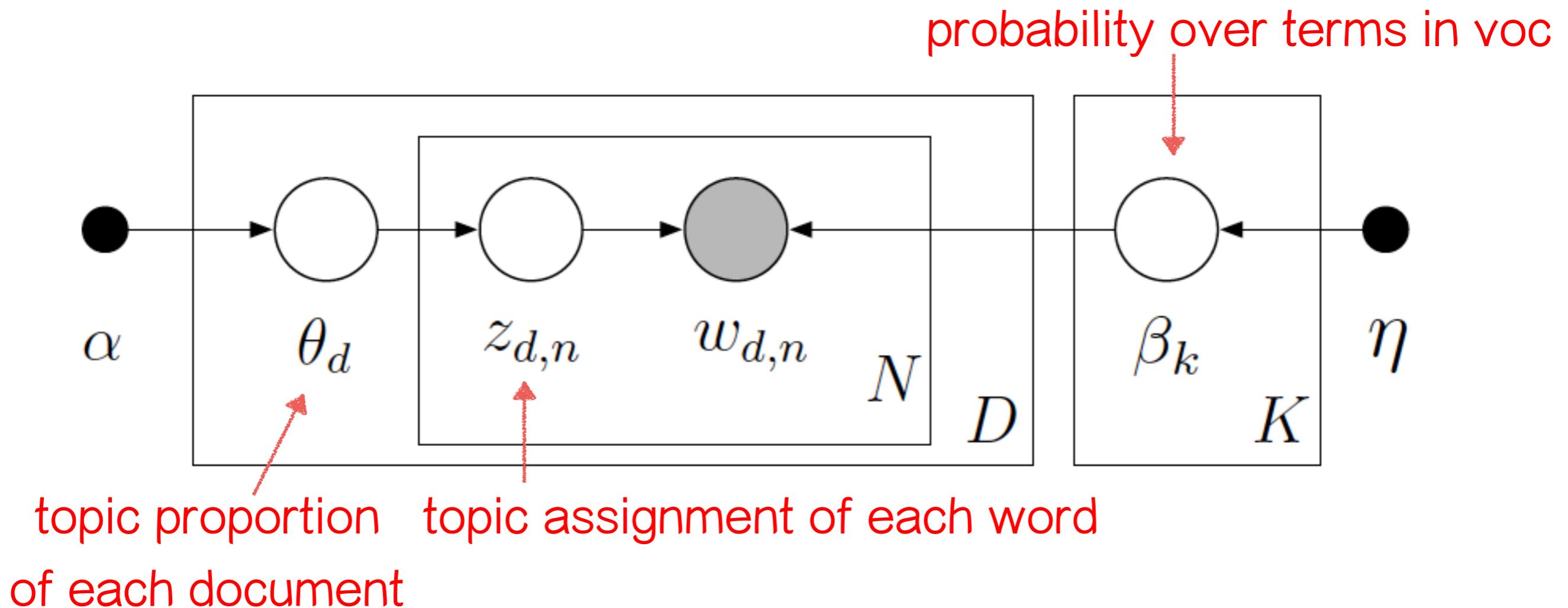
- Given documents, **describe how** documents are organised!



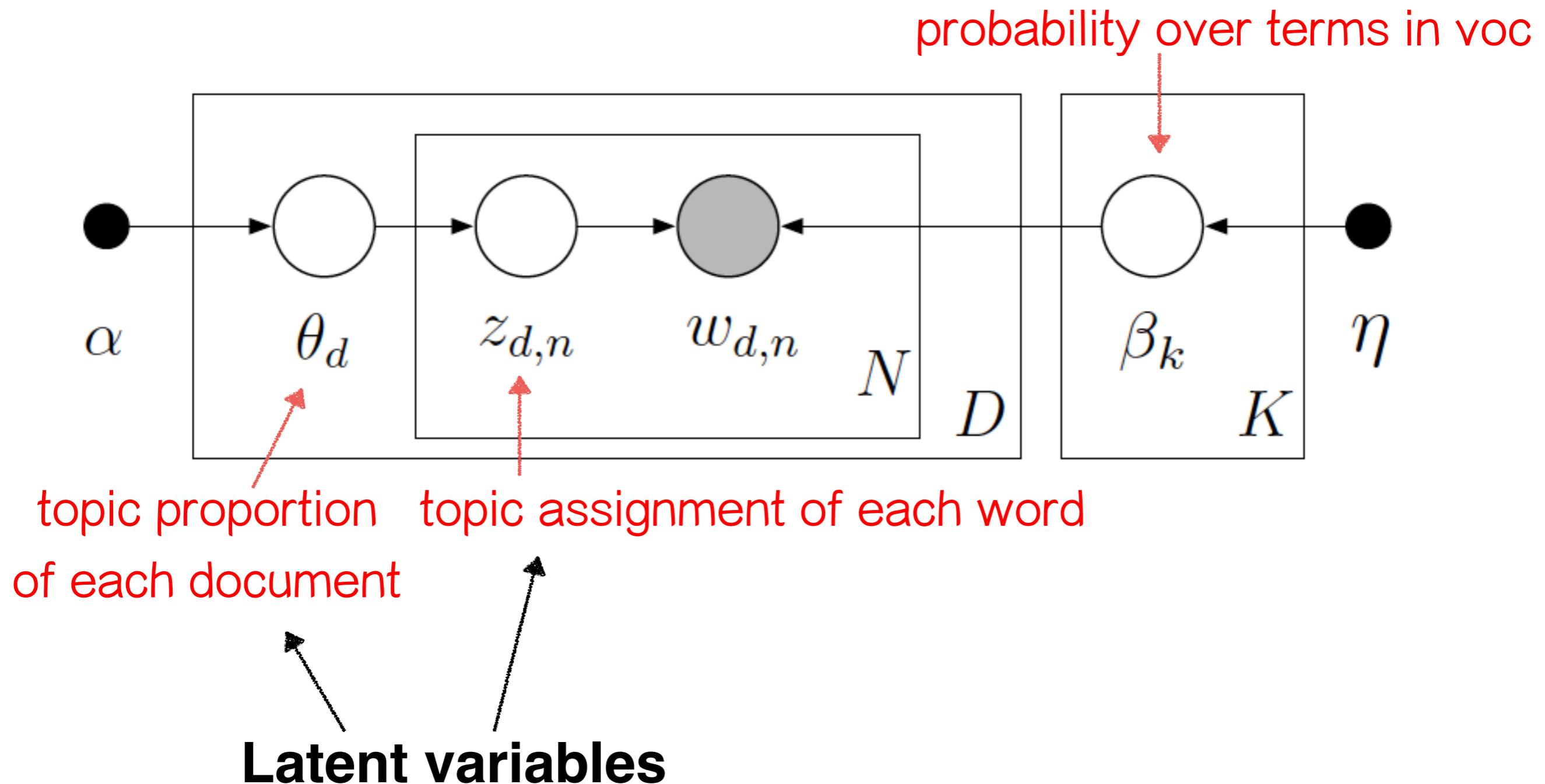
[Figure taken from D. Blei's slides]

Goal : allocate words to topics & assign probability to terms in each topic.

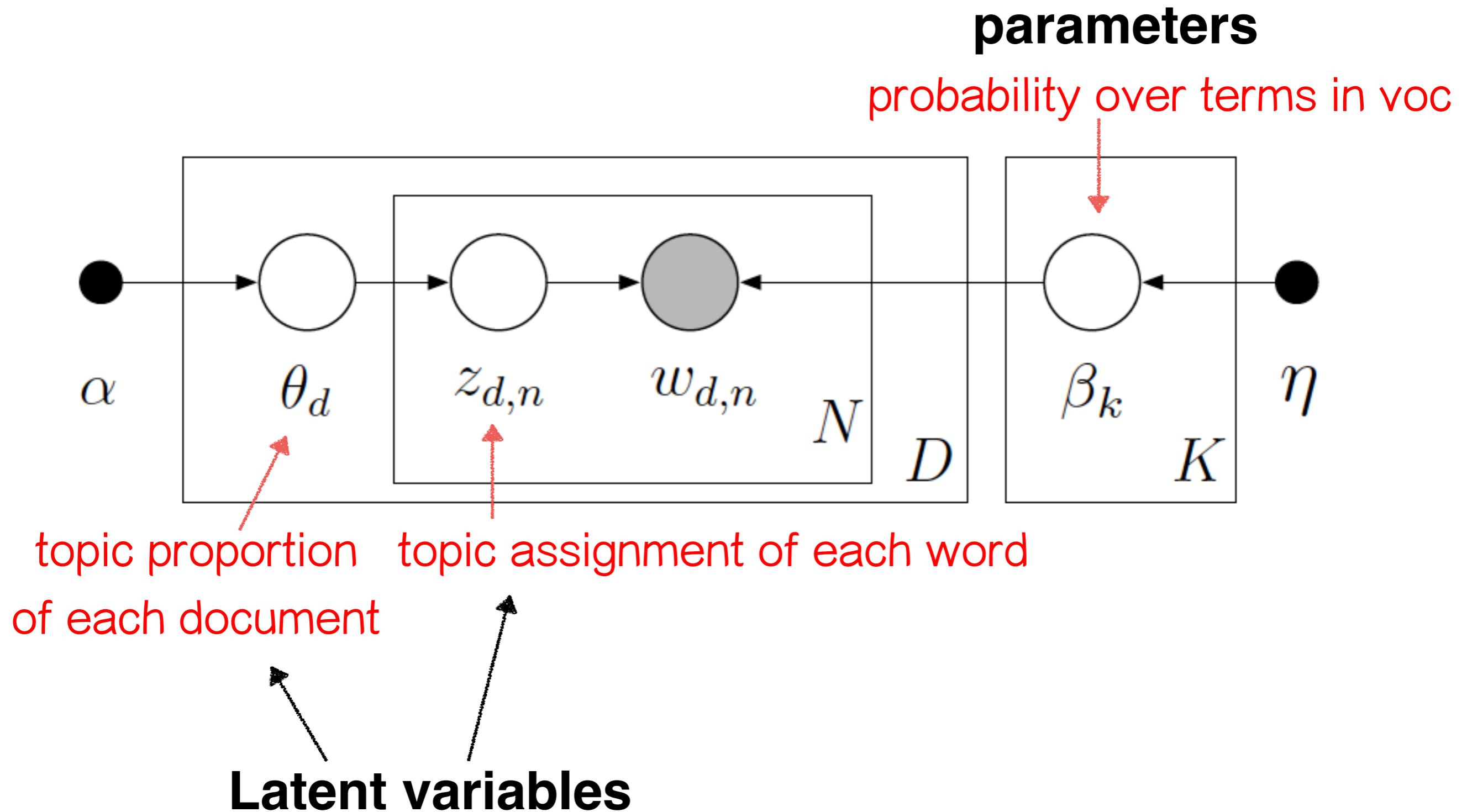
Latent Dirichlet Allocation (LDA)



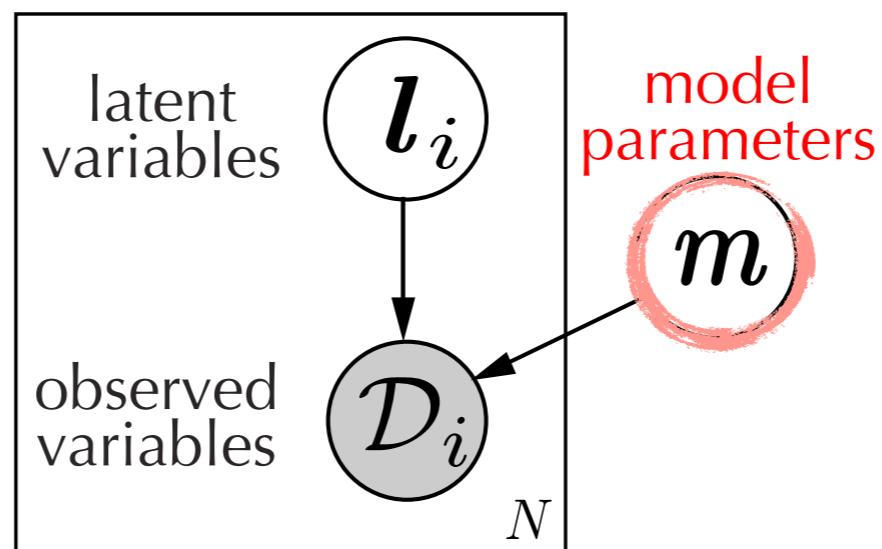
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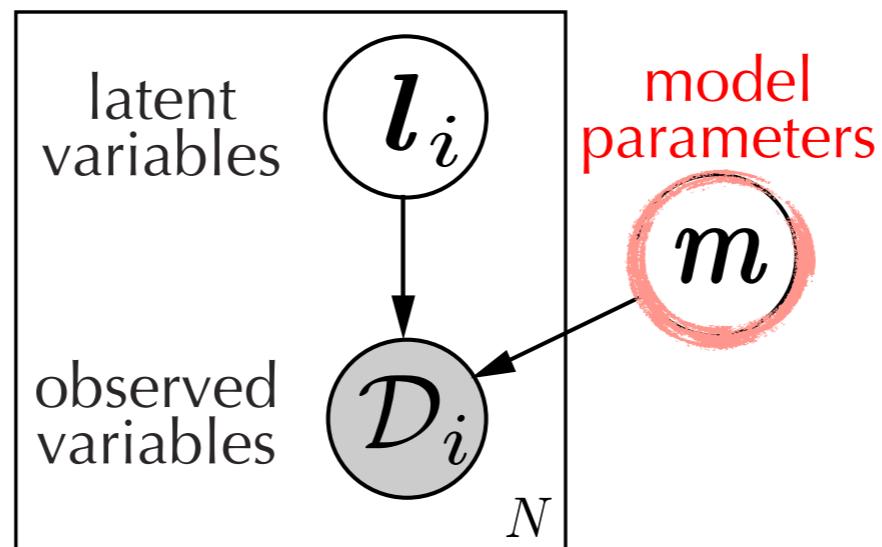
Latent Dirichlet Allocation (LDA)



Variational inference

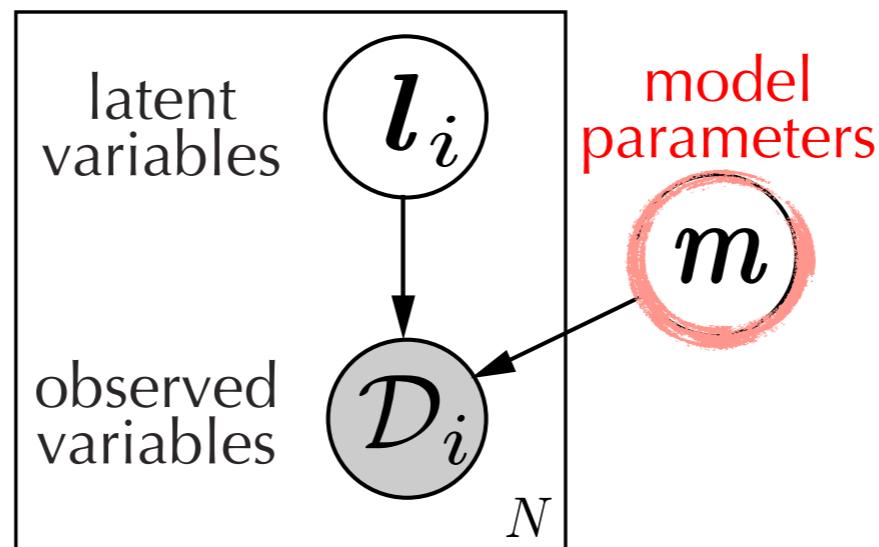


Variational inference



log-marginal-likelihood

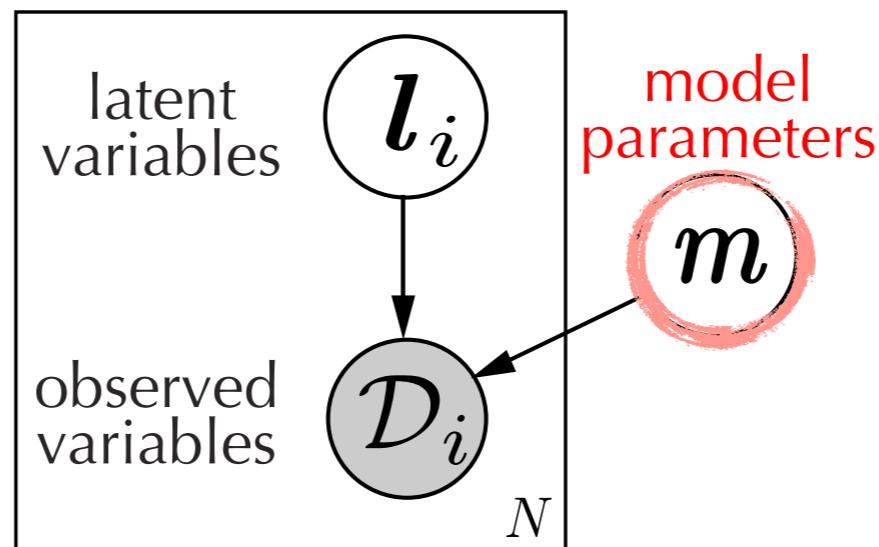
Variational inference



log-marginal-likelihood

$$\log p(\mathcal{D}) \geq \int q(\mathbf{l})q(\mathbf{m}) \log \frac{p(\mathcal{D}, \mathbf{l}|\mathbf{m})p(\mathbf{m})}{q(\mathbf{l})q(\mathbf{m})}$$

Variational inference

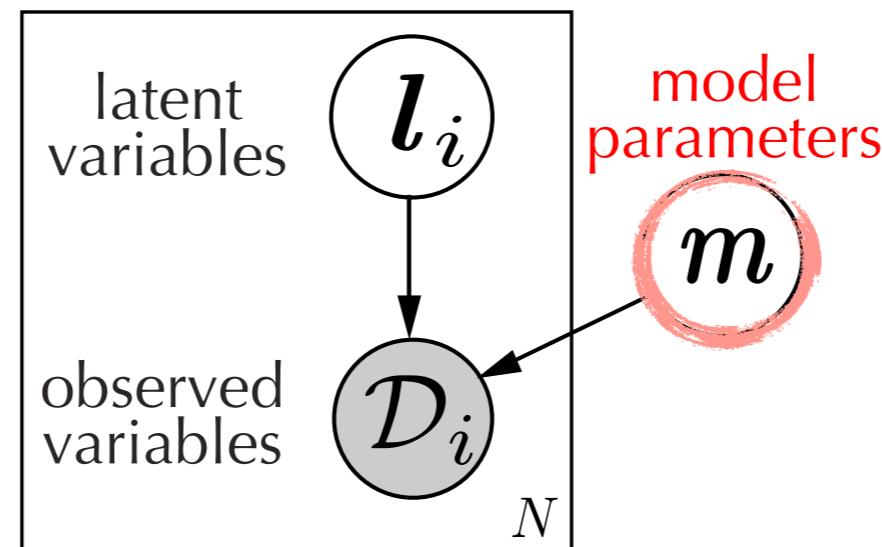


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variational posterior
over latent variables

Variational inference



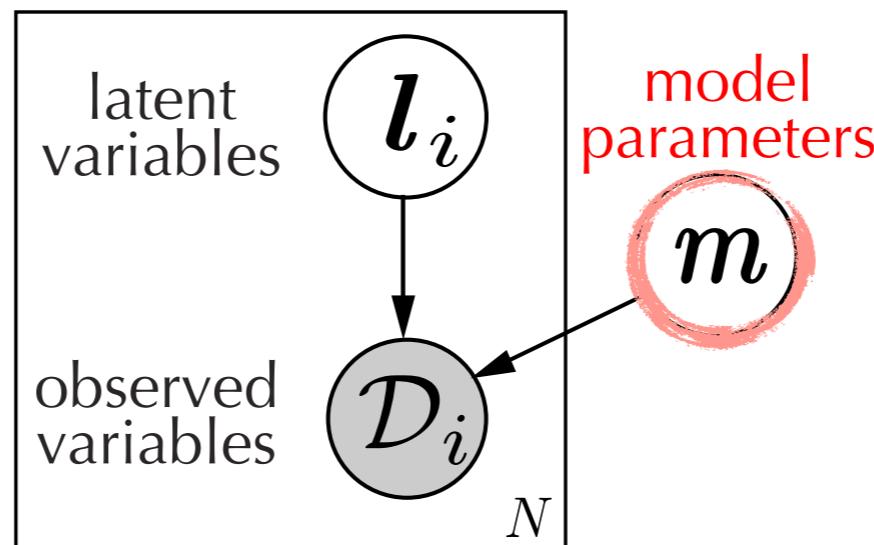
log-marginal-likelihood

$$\log p(\mathcal{D}) \geq \int q(\mathbf{l})q(m) \log \frac{p(\mathcal{D}, \mathbf{l}|m)p(m)}{q(\mathbf{l})q(m)}$$

variational posterior
over latent variables

variational posterior
over model params

Variational inference



Variational lower bound

$$\log p(\mathcal{D}) \geq \mathcal{L} = \int q(\mathbf{l})q(\mathbf{m}) \log \frac{p(\mathcal{D}, \mathbf{l}| \mathbf{m})p(\mathbf{m})}{q(\mathbf{l})q(\mathbf{m})}$$

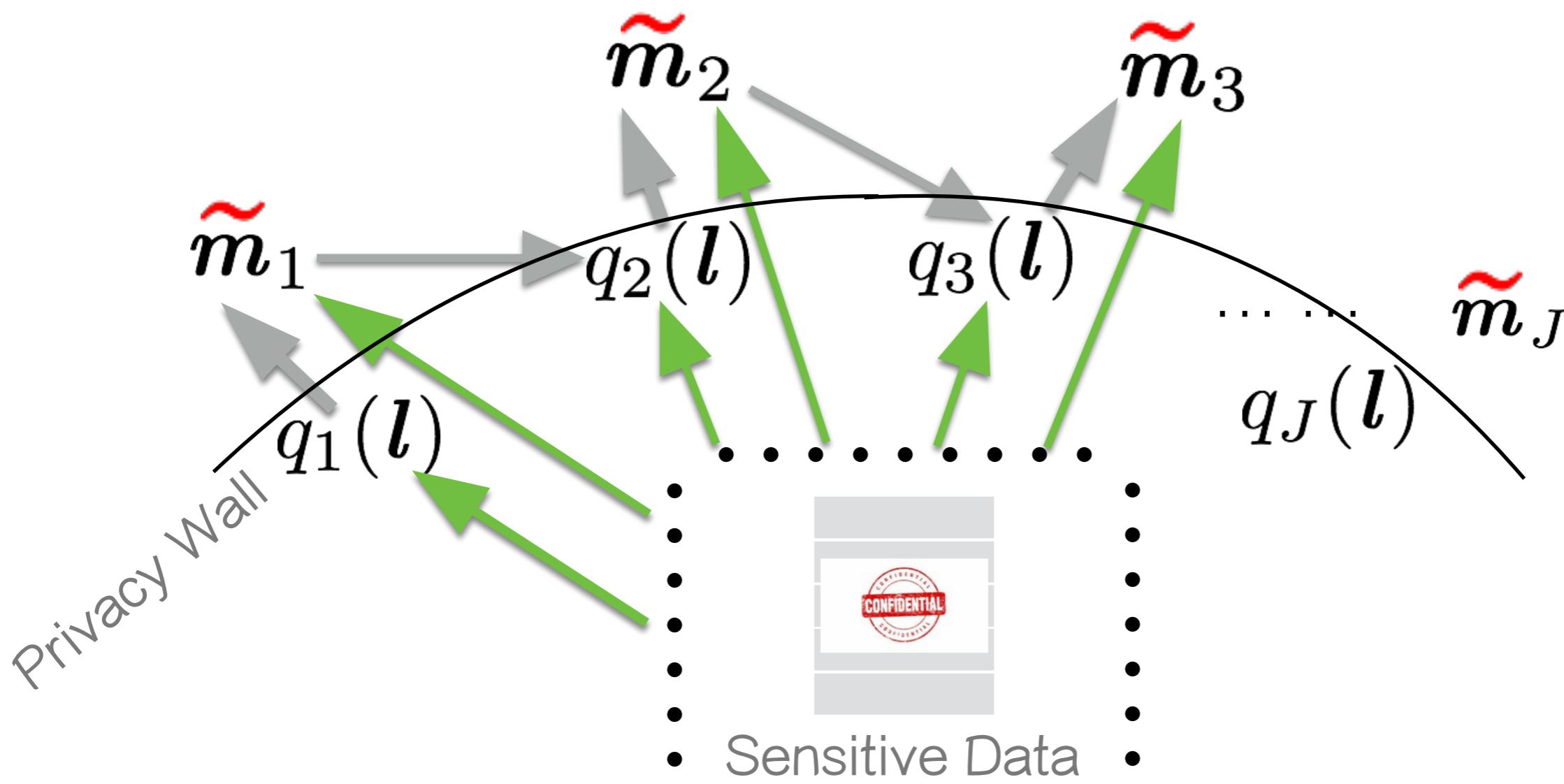
log-marginal-likelihood

variational posterior over latent variables

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DP-VI

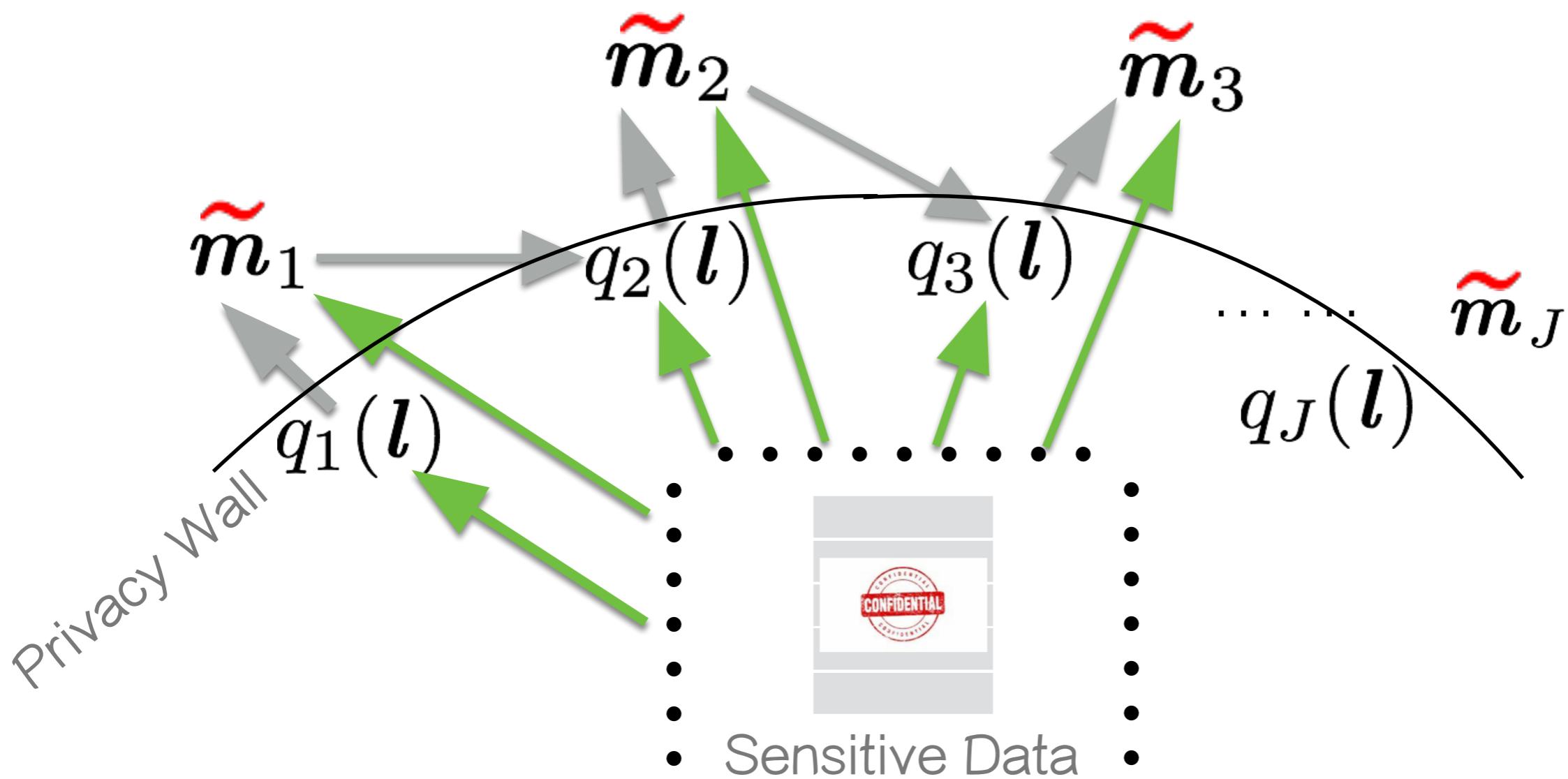
[Park et al, 2019]



DP-VI

[Park et al, 2019]

exponential family: $p(\mathcal{D}_n, \mathbf{l}_n | \mathbf{m}) \propto \exp(\mathbf{n}(\mathbf{m})^\top \mathbf{s}(\mathcal{D}_n, \mathbf{l}_n))$

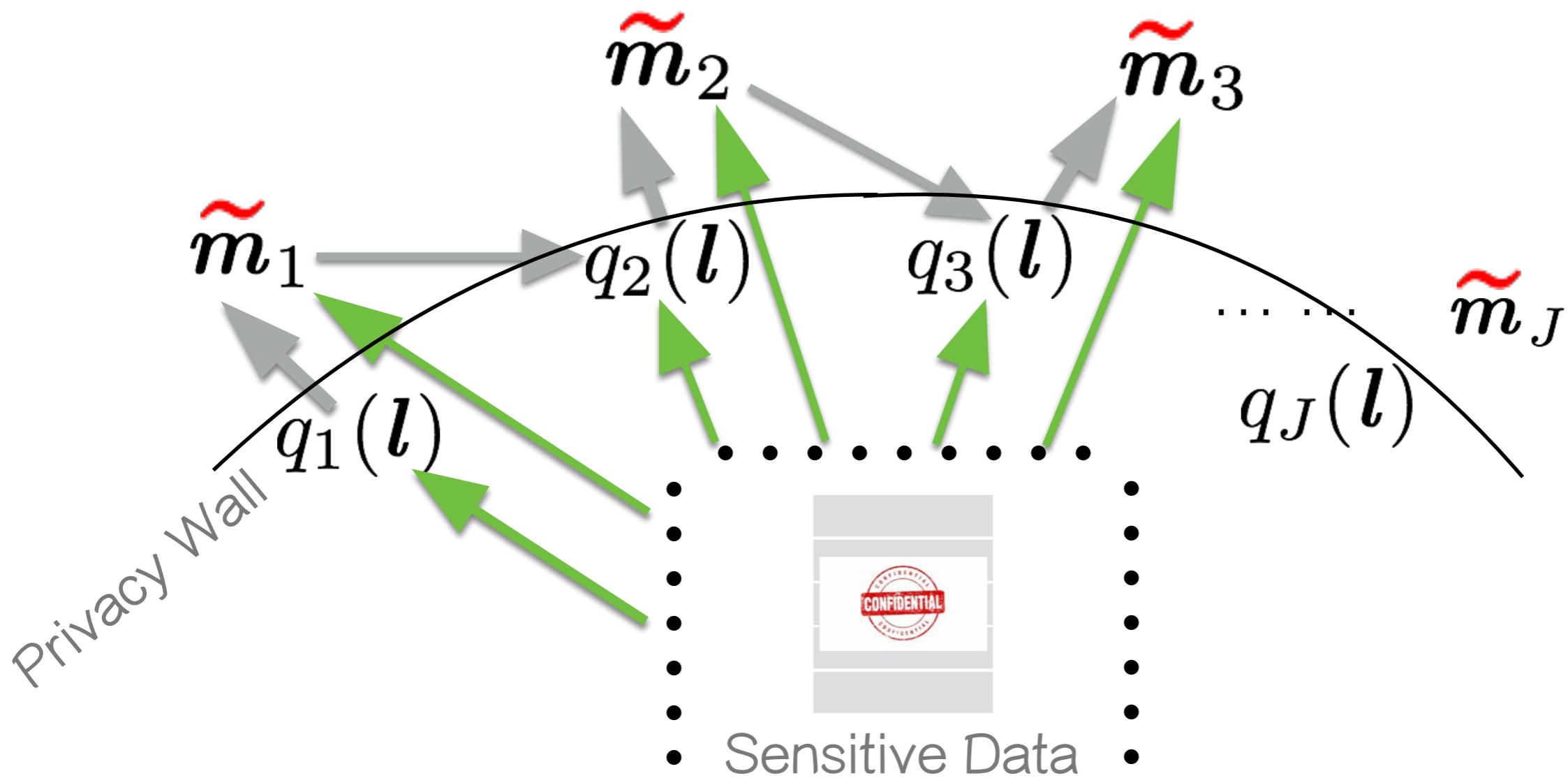


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natural params

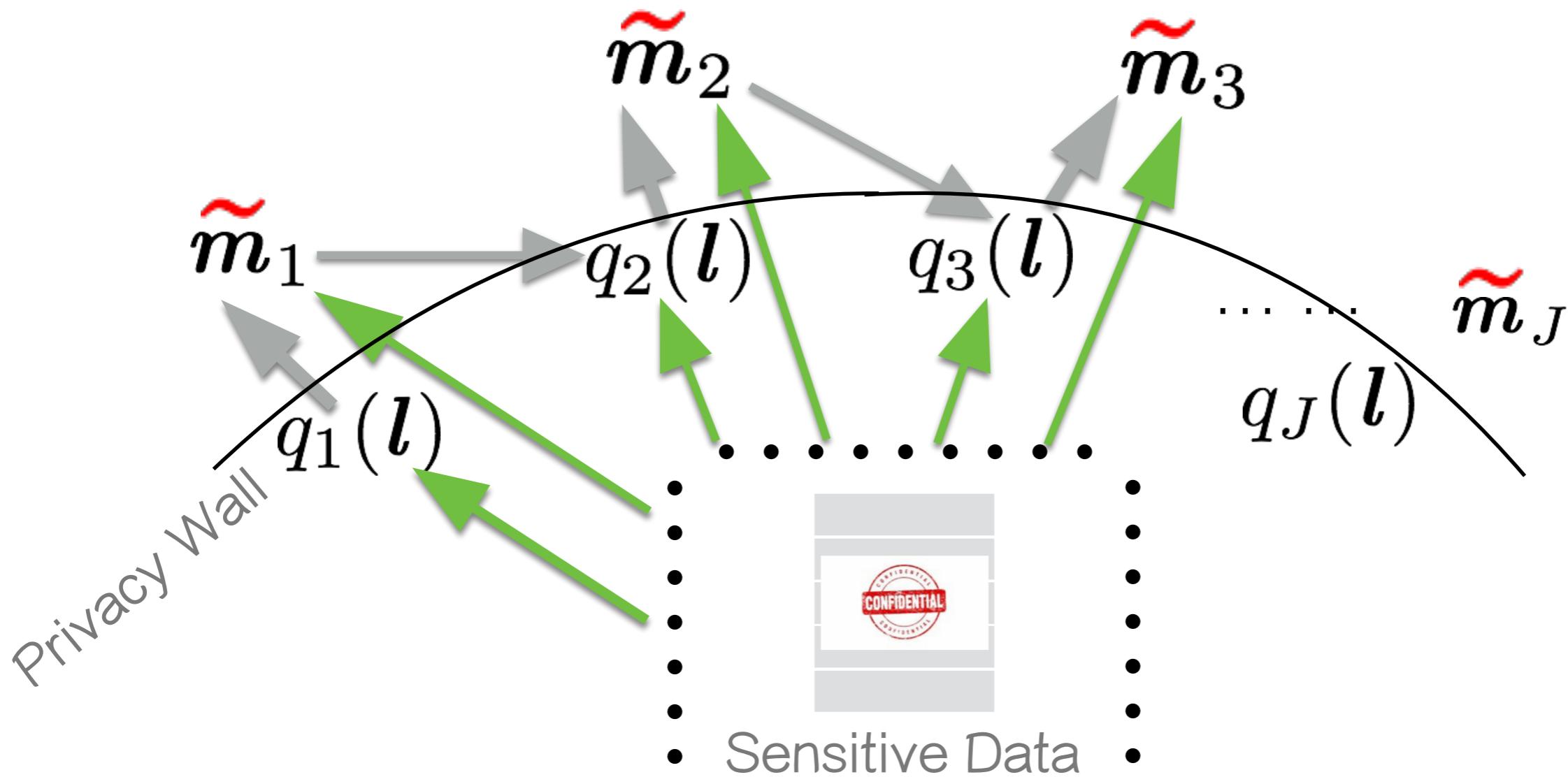


DP-VI

[Park et al, 2019]

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natural params suff stats



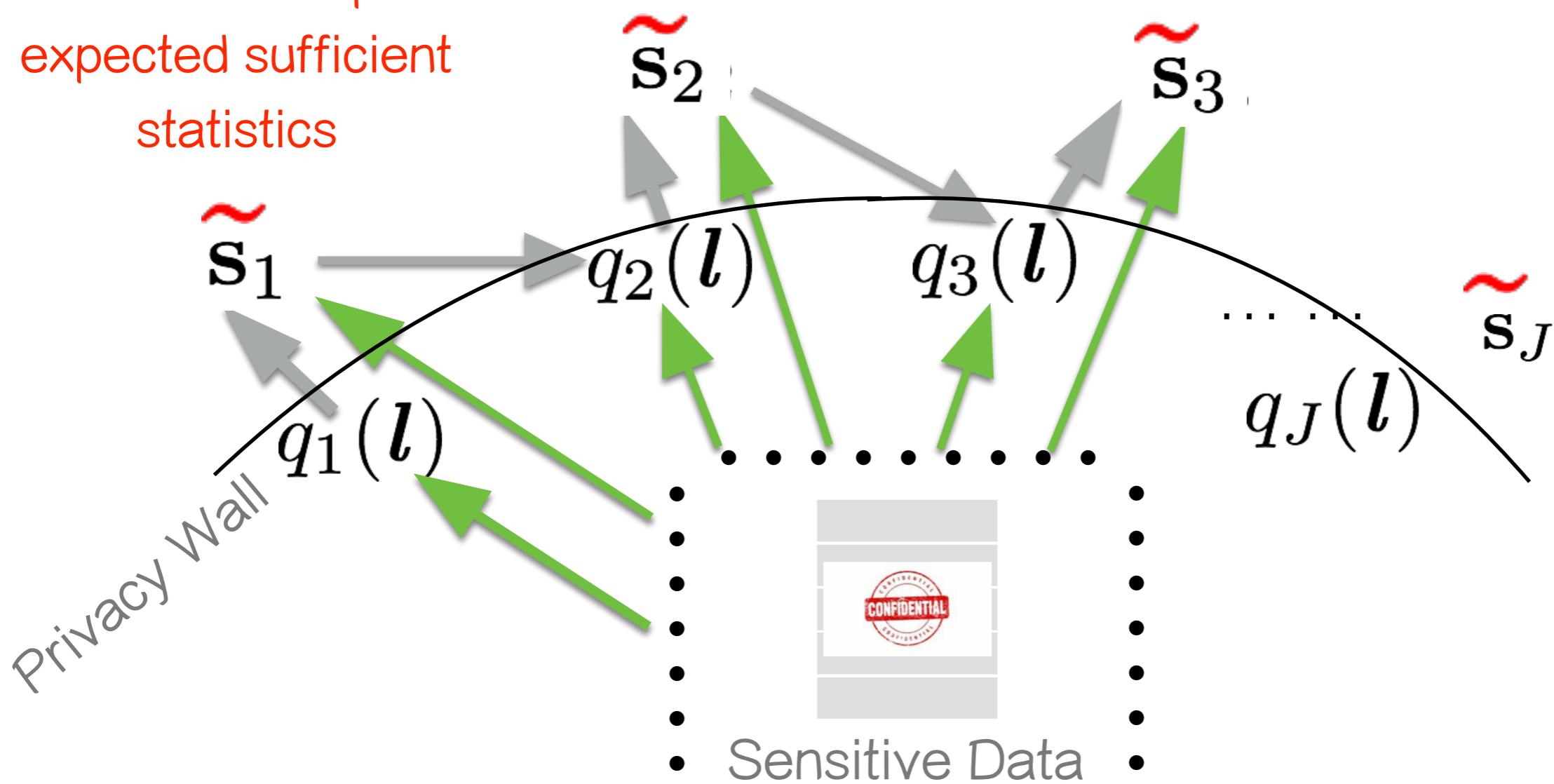
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natural params suff stats

Perturb & output
expected sufficient
statistics



Large document datasets : stochastic learning

Privacy Amplification Effect due to subsampling

Theorem 1 (*Theorem 1 in (Li et al., 2012)*) Any $(\epsilon_{iter}, \delta_{iter})$ -DP mechanism running on a uniformly sampled subset of data with a sampling ratio ν guarantees $(\log(1 + \nu(\exp(\epsilon_{iter}) - 1)), \nu\delta_{iter})$ -differential privacy. [Li et al,12]

Large document datasets : stochastic learning

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sampling rate	total privacy loss
1	1
0,1	0,159
0,01	0,017

Posterior topics by DP-VI

Non-private LDA

topic 3:	
david	0.0667
king	0.0318
god	0.0304
son	0.0197
israel	0.0186
bible	0.0156
hebrew	0.0123
story	0.0102
book	0.0095
adam	0.0092

Private LDA (Moments Accountant)

topic 83:	
david	0.0410
jonathan	0.0199
king	0.0188
samuel	0.0186
israel	0.0112
saul	0.0075
son	0.0068
dan	0.0067
god	0.0053
story	0.0048

Private LDA (Advanced Composition)

topic 73:	
mount	0.0034
display	0.0011
animal	0.0011
equipment	0.0011
cynthia	0.0009
position	0.0008
systems	0.0008
support	0.0008
software	0.0008
heavy	0.0008

DP-VI with MA composition + privacy amplification outputs more coherent topics (**red box**) than DP-VI with AC (**blue box**) + privacy amplification

Deep learning

DP-Stochastic gradient descent

[Abadi et al, 2016]

Given a mini-batch,

$$\mathbf{(A)} \quad \mathbf{g}_t(x_i) \leftarrow \nabla_{\theta_t} \mathcal{L}(\theta_t, x_i)$$

$$\mathbf{(B)} \quad \mathbf{g}_t = \frac{1}{L} \sum_i \mathbf{g}_t(x_i)$$

$$\mathbf{(C)} \quad \theta_{t+1} \leftarrow \theta_t - \eta_t \mathbf{g}_t$$

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Learning rate
↓

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Each gradient has
a limited sensitivity

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Each gradient has
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(B) $\mathbf{g}_t = \frac{1}{L} \sum_i \mathbf{g}_t(x_i) \longrightarrow \tilde{\mathbf{g}}_t \leftarrow \frac{1}{L} \sum_i [\bar{\mathbf{g}}_t(x_i) + \mathcal{N}(0, \sigma^2 C^2 I)]$

Learning rate
↓
(C) $\theta_{t+1} \leftarrow \theta_t - \eta_t \mathbf{g}_t$

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Learning rate
DP parameters

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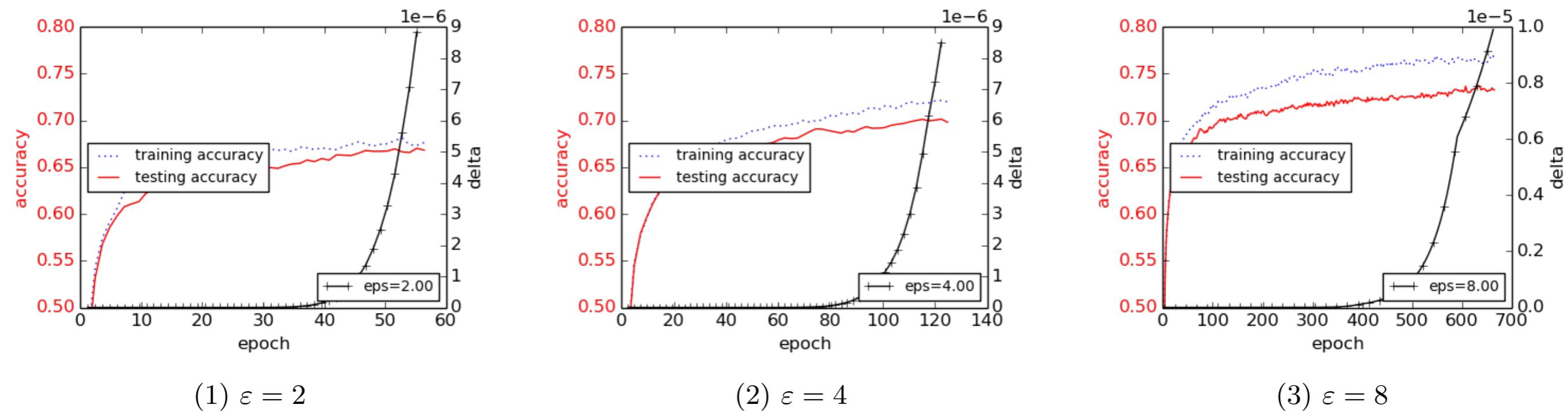
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Learning rate
↓
DP parameters

- Moments accountant composition to compute the total privacy loss incurred during training

DP-SGD on CIFAR10 data

[Abadi et al, 2016]



- DP-SGD with a high level of noise ($\text{epsilon}=2$) provides a less accurate classifier than DP-SGD with a lowe level of noise ($\text{epsilon}=8$)
- The gap between training and test accuracies gets smaller with more additive noise. Noise added for privacy seems helpful for better generalization.

Examples of DP-ML algorithms

- DP-EM for clustering & density estimation
- DP-VI for topic modeling
- DP-SGD for deep learning

DP in industry

DP in machine learning systems

Google's Randomized Aggregatable Privacy-Preserving
Ordinal Response (RAPPOR)

- A DP technique for crowdsourcing statistics, e.g., for collecting data about the Google Chrome users.
- Based on [Randomised response technique!](#)

DP in machine learning systems

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Introduce Randomness

1. flip a coin
2. if tails, respond truthfully
3. if heads, flip another coin
4. if heads, “yes”. Say “no” otherwise.

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Introduce Randomness

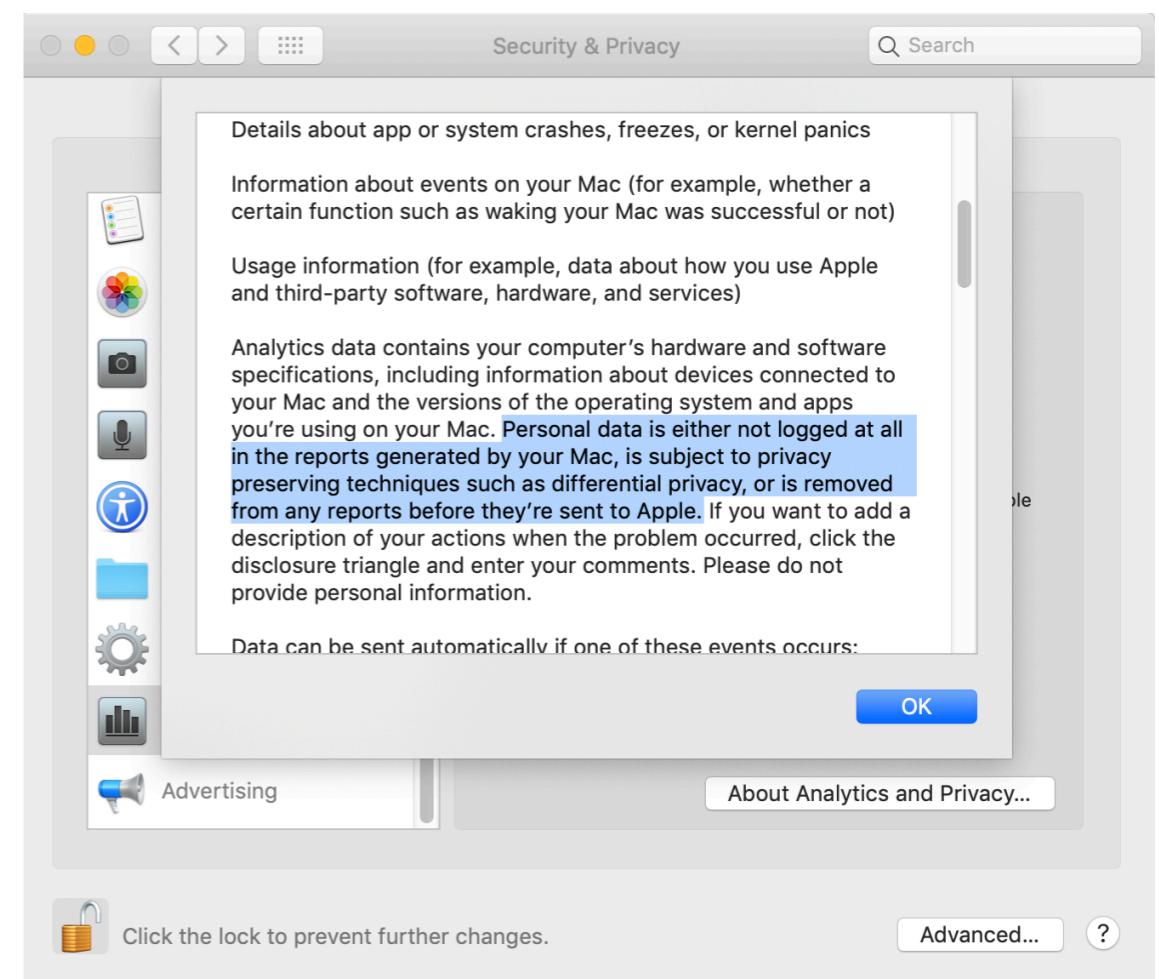
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4. if heads, “yes”. Say “no” otherwise.

True answer 75% of time
accurate population statistics
while preserving the privacy
of the individual answers!

DP in data collection



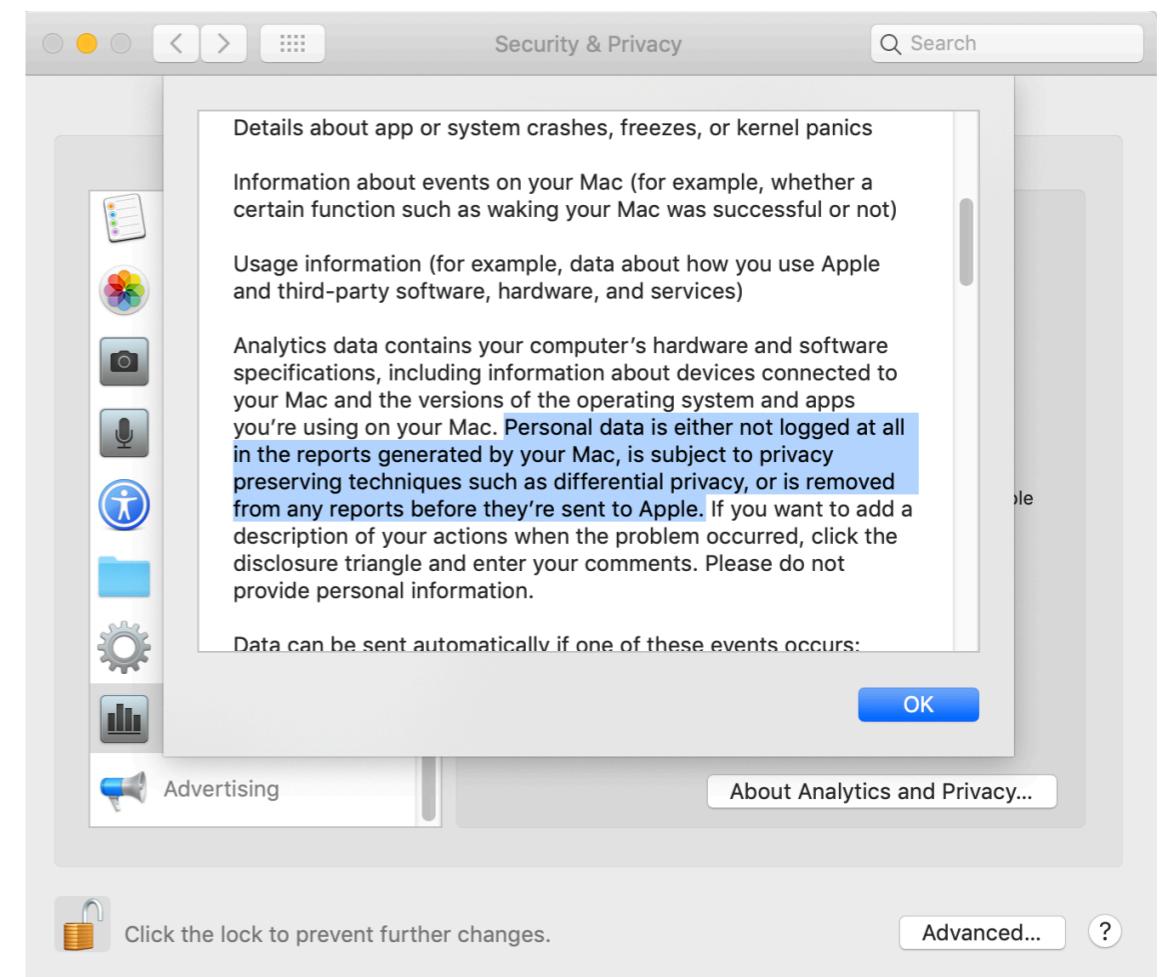
- DP techniques are used to send my own personal data collected by iMAC (analytics).



DP in data collection



- DP techniques are used to send my own personal data collected by iMAC (analytics).



- Specific techniques on a new word and Emoji: [OneBitHistogram](#), [CountMedianSketch](#) -> adding noise to data before sending to server!
- Issue: [non-transparent & questionable “epsilon” selection \[Tang et al. 2017\]](#)

Summary

- Privacy is necessary to consider not only for the sake of protection against threats, but also for **philanthropic** reasons in data sharing.
- **Differential privacy** seems to dominate the field due to the mathematically provable guarantees.
- Although many progresses have been made in privacy-preserving machine learning, there are still **A LOT of work that needs to be done** to apply DP in many machine learning/data sharing applications.