$z = x_1 - 2x_2 + 4x_3 + x_1^2 + 2x_2^2 + 3x_3^2 + x_1 x_3$

$x^T \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 2 & 0 \\ \frac{1}{2} & 0 & 3 \end{bmatrix} x + f^T x \qquad f = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

$[x_1\ x_2\ x_3]$ H, f

solver

quad

Energy trading between multiple microgrids for enhancing the grid resilience

Kanyakorn Likhitluecha

May 7, 2021

problem → sub prob

fun main → sub1 → sub2 → sub4

1 Research Objective

The research objective is to develop a framework to keep supplying the electric energy to maximize the social welfare even in case of disasters by trading the electric energy among multiple microgrids.

2 Research plan

- May 2021 : Making the program by considering the imputation in the core of the line between microgrids and solving the problem by gurobi optimizer

3 Research results

- Considering about the interconnection lines between microgrids (1.4851JPN calculated by gurobi)
- The imputation to the interconnection lines between the microgrids : the investment to the lines to increase the capacity by using this imputation, more efficient microgrid operation because If the profit of the grand coalition increases as the capacity of the line increases, the imputation to the line will increase.

KKT ⇒ form dual

$\lambda = f(x)$

Nomenclature

Suffix

i: The number of the players

Constant

c_i: The price coefficient C [JPY/p.u.]
b_i: The resource capacity [p.u.]
$Sline$: Line limit capacity [p.u.]
$Bline$: Line susceptance [p.u.]

Variable

y_i: The shadow price [JPY/p.u.]
θ_i: Phase angle of node i[Rad]
$yline1$: The shadow price of the line at node 1[JPY/p.u.]
$yline2$: The shadow price of the line at node 2[JPY/p.u.]

4 Formulation

The profit maximization problem of each microgrid can be formulated as a bi-level mathematical programming problem with the leader as each microgrid and the follower as a transaction manager as follows.

$$\underset{badd_{i=1}^{100}}{\text{Maximize}} \quad \sum_{i=1}^{100}(b_i + badd_i)y_i + yline1\left(\frac{Sline}{Bline}\right) + yline2\left(\frac{Sline}{Bline}\right) \tag{4.1}$$

$$\text{subject to} \quad badd_i \geq 0 \quad \forall_i \in N \tag{4.2}$$

$$\underset{x_{i=1}^{100},\theta}{\text{Maximize}} \quad \sum_{i=1}^{100} c_i x_i \tag{4.3}$$

$$\text{subject to} \quad 0 \leq x_i \leq b_i + badd_i \quad \forall_i \in N \tag{4.4}$$

$$\sum_{i=1}^{25} x_{2i-1} - \sum_{i=26}^{50} x_{2i-1} = Bline(\theta_1 - \theta_2) \tag{4.5}$$

$$\sum_{i=1}^{25} x_{2i} - \sum_{i=26}^{50} x_{2i} = Bline(\theta_2 - \theta_1) \tag{4.6}$$

$$\left(\frac{-Sline}{Bline}\right) \leq \theta_2 \leq \left(\frac{Sline}{Bline}\right) \tag{4.7}$$

$$\theta_1 = 0 \tag{4.8}$$

The bi-level programming problem can be converted into equivalent constraints by applying KKT (Karush-Kuhn-Tucker) conditions to its lower level problem (4.3 - 4.8). The two-level programming problem can then be converted to a mixed integer linear programming problem.

(1)KKT condition

The KKT (Karush-Kuhn-Tucker) condition is the optimality condition that the optimal solution of various optimization problems should satisfy.The KKT condition is a necessary condition, and is a first-order condition, that is, using first-order derivative vectors and matrices It is a condition to be formulated.[**?**]

Consider the following general mathematical programming problem.

$$\text{Minimize} \quad f(x) \tag{4.9}$$

$$\text{subject to} \quad g(x) \leq 0 \tag{4.10}$$

$$h(x) = 0 \tag{4.11}$$

The Lagrange function corresponding to this problem is

$$L(x, \lambda, \mu) = f(x) + \lambda g(x) + \mu h(x) \tag{4.12}$$

The KKT condition in question is

$$\nabla_x f(x) + \lambda \nabla_x g(x) = 0 \tag{4.13}$$

$$h(x) = 0 \tag{4.14}$$

$$g(x) \leq 0 \tag{4.15}$$

$$\mu g(x) = 0 \tag{4.16}$$

$$\mu \geq 0 \tag{4.17}$$

Where λ, μ is the Lagrange multiplier vector for equations and inequalities, and ∇ is the gradient for x. The constraint expression (4.15)-(4.17) is a complementarity condition,

$$0 \leq \mu \perp g(x) \geq 0 \tag{4.18}$$

Here, $\perp$ indicates complementarity, that is, $\mu g(x) = 0$.

(2)Lagrange function of microgrid i

The Lagrange function of the expression (4.19) is

$0 \leq x \leq b + badd$

$\to$ $x \geq 0$, $x \leq b + badd$ $\}$ 2ineq

$$\begin{aligned} L_i = &\sum_{i=1}^{100} c_i x_i + \mu^{Bline1}\left(\sum_{i=1}^{25} x_{2i-1} - \sum_{i=26}^{50} x_{2i-1} - Bline(\theta_1 - \theta_2)\right. \\ &+ \mu^{Bline2}\left(\sum_{i=1}^{25} x_{2i} - \sum_{i=26}^{50} x_{2i} - Bline(\theta_2 - \theta_1)\right) \\ &+ \rho^{min}_{b_i+badd}\sum_{i=1}^{100}(-x_i) + \rho^{max}_{b_i+badd}\sum_{i=1}^{100}(x_i - (b_i + badd_i)) \\ &+ \phi^{min}_{\theta_2}\left(\left(\frac{-Sline}{Bline}\right) - \theta_2\right) + \phi^{max}_{\theta_2}\left(\theta_2 - \left(\frac{Sline}{Bline}\right)\right) \end{aligned} \tag{4.19}$$

Where μ_i^{Bline1} , μ_i^{Bline2} , $+\rho_{b_i+badd}^{min}$, $\rho_{b_i+badd}^{max}$, $\phi_{\theta_2}^{min}$ and $\phi_{\theta_2}^{max}$ are Lagrange multipier

แทนไปได้

(3)KKT condition of microgrid i

From the Lagrange function(4.19), the KKT condition is as follows

$$\frac{\partial L}{\partial \theta_2} = \mu_i^{Bline1}(Bline) - \mu_i^{Bline2}(Bline) - \phi_{\theta_2}^{min} + \phi_{\theta_2}^{max} = 0, \quad \forall_i \in N \tag{4.20}$$

$$\frac{\partial L}{\partial x_i} = c_i + \mu_i^{Bline1} - \rho_{b_i+badd}^{min} + \rho_{b_i+badd}^{max} = 0, \quad \forall_i \in N, i = 1, 3, 5, ..., 47, 49 \tag{4.21}$$

$$\frac{\partial L}{\partial x_i} = c_i + \mu_i^{Bline2} - \rho_{b_i+badd}^{min} + \rho_{b_i+badd}^{max} = 0, \quad \forall_i \in N, i = 2, 4, 6, ..., 48, 50 \tag{4.22}$$

$$\frac{\partial L}{\partial x_i} = c_i - \mu_i^{Bline1} - \rho_{b_i+badd}^{min} + \rho_{b_i+badd}^{max} = 0, \quad \forall_i \in N, i = 51, 53, 55, ..., 97, 99 \tag{4.23}$$

$$\frac{\partial L}{\partial x_i} = c_i - \mu_i^{Bline2} - \rho_{b_i+badd}^{min} + \rho_{b_i+badd}^{max} = 0, \quad \forall_i \in N, i = 52, 54, 56, ..., 98, 100 \tag{4.24}$$

$$0 \leq x_i \perp \rho_{b_i+badd}^{min} \geq 0, \quad \forall_i \in N \tag{4.25}$$

$$0 \leq (b_i + badd_i) - x_i \perp \rho_{b_i+badd}^{max} \geq 0, \quad \forall_i \in N \tag{4.26}$$

$$0 \leq \theta_2 + \left(\frac{Sline}{Bline}\right) \perp \phi_{\theta_2}^{min} \geq 0, \quad \forall_i \in N \tag{4.27}$$

$$0 \leq \left(\frac{Sline}{Bline}\right) - \theta_2 \perp \phi_{\theta_2}^{max} \geq 0, \quad \forall_i \in N \tag{4.28}$$

$$\sum_{i=1}^{25} x_{2i-1} - \sum_{i=26}^{50} x_{2i-1} - Bline(\theta_1 - \theta_2) = 0 \tag{4.29}$$

$$\sum_{i=1}^{25} x_{2i} - \sum_{i=26}^{50} x_{2i} - Bline(\theta_2 - \theta_1) = 0 \tag{4.30}$$

The equation (4.25 - 4.28) are complementarity constraints which are non - linear equation . By replace them with linear constraint , we need to use Big - M method to solve the problem (4.31 - 4.32)

$$0 \leq \mu \perp g(x) \geq 0 \iff \begin{cases} 0 \leq \mu \leq Mu & (4.31) \\ 0 \leq g(x) \leq M(1-u) & (4.32) \end{cases}$$

The complementarity conditions (4.25 - 4.28) are replaced with the following linear constraints(4.33 - 4.40)

$$0 \leq \rho_{b_i+badd}^{min} \leq M u_{b_i+badd}^{min}, \quad \forall_i \in N \tag{4.33}$$

$$0 \leq x_i \leq M(1 - u_{b_i+badd}^{min}), \quad \forall_i \in N \tag{4.34}$$

$$0 \leq \rho_{b_i+badd}^{max} \leq M u_{b_i+badd}^{max}, \quad \forall_i \in N \tag{4.35}$$

$$0 \leq (b_i + badd) - x_i \leq M(1 - u_{b_i+badd}^{max}), \quad \forall_i \in N \tag{4.36}$$

$$0 \leq \phi_{\theta_2}^{min} \leq M u_{\theta_2}^{min}, \quad \forall_i \in N \tag{4.37}$$

$$0 \leq \theta_2 + \left(\frac{Sline}{Bline}\right) \leq M(1 - u_{\theta_2}^{min}), \quad \forall_i \in N \tag{4.38}$$

$$0 \leq \phi_{\theta_2}^{max} \leq M u_{\theta_2}^{max}, \quad \forall_i \in N \tag{4.39}$$

$$0 \leq \left(\frac{Sline}{Bline}\right) - \theta_2 \leq M(1 - u_{\theta_2}^{max}), \quad \forall_i \in N \tag{4.40}$$

5 Future tasks

Making the program by using gurobi optimizer

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answer

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$$\text{Larg}\,(b\_add, \gamma, \gamma\_line, \lambda) = \sum_{i=1}^{100} (b_i + b\_add_i)\,\gamma_i + \gamma_{line\,1}\left(\frac{S_{line}}{B_{line}}\right)$$
(copper)

$$+ \gamma_{line\,2}\left(\frac{S_{line}}{B_{line}}\right)$$

$$+ \sum_{i=1,3,\dots,49} \gamma_i \left(C_i + \mu_i^{Bline\,1} - \rho^{min}_{b_i + badd} + \rho^{max}_{b_i + badd} \right)$$

$$+ \sum_{i=2,4,\dots,50} \gamma_i \left(C_i + \mu_i^{Bline\,2} - \rho^{min}_{b_i + badd} + \rho_{b_i + badd} \right)$$

$$+ \sum_{i=51,\dots,99} \gamma_i \left(C_i - \mu_i^{Bline\,1} - \rho^{min} + \rho^{max} \right)$$

$$+ \sum_{i=52,\dots,100} \gamma_i \left(C_i - \mu^{Bline\,2} - \rho^{min} + \rho^{max} \right)$$

อาจจะไม่ใช่

แก้ $\mu(b\_add)$

$\rho^{min}(b\_add)$

$\rho^{max}(b\_add)$

$$\min_{x_{i=1}^{100},\,\theta} \; -c^T x$$

subject to

$$\boxed{\begin{aligned} Ax &\le a \\ Bx &= b \end{aligned}}$$

$$\Downarrow$$

$$\begin{aligned} L_{arg}(x, \lambda, v) &= -c^T x + \lambda^T (Ax - a) + v^T (Bx - b) \\ &= -c^T x + \lambda^T A x - \lambda^T a + v^T B x - v^T b \\ &= -\lambda^T a - v^T b + (-c^T + \lambda^T A + v^T B) x \\ &= -\lambda^T a - v^T b + (-c + A^T \lambda + B^T v)^T x \end{aligned}$$

$$\Downarrow$$

$$\max_{\lambda} \; -\lambda^T a - v^T b \quad \Rightarrow \quad \min_{\lambda, v} \; a^T \lambda + b^T v$$

subject $-c + A^T \lambda + B^T v = 0$

$\lambda, v \Rightarrow$ คือ y_i ว y_{line} $\Rightarrow$ ที่ดูตำแหน่ง บกพร่อง

19/May/21

$$\min_{x,\theta} \; c^T x$$

sub to

$$x \geq 0 \quad ; (\lambda_1)$$

$$x \leq b + b_{add} \quad ; (Y_1) \Rightarrow \text{vector}$$

$$k_1^T x - k_2^T x = -Bline\,\theta_2 \quad ; (Y_2) \Rightarrow \text{Scalar}$$

$$k_3^T x - k_4^T x = Bline\,\theta_2 \quad ; (Y_3) \Rightarrow \text{Scalar}$$

$$-\frac{Sline}{Bline} \leq \theta_2 \leq \frac{Sline}{Bline} \quad ; (Y_{line_1}, Y_{line_2}) \Rightarrow \text{Scalar}$$

$$k_1 = [\,\mathbb{1}_{1\times 25} \; 0_{1\times 25} \; 0_{1\times 25} \; 0_{1\times 25}\,]^T$$

$$k_2 = [\,0_{1\times 25} \; \mathbb{1}_{1\times 25} \; 0_{1\times 25} \; 0_{1\times 25}\,]^T$$

$$k_3 = [\,0 \; 0 \; \mathbb{1} \; 0\,]^T$$

$$k_4 = [\,0 \; 0 \; 0 \; \mathbb{1}_{1\times 25}\,]^T$$

$$L = c^T x - \lambda_1^T x + Y_1^T(x - b - b_{add}) + Y_2(k_1^T x - k_2^T x + Bline\,\theta_2) + Y_3(k_3^T x - k_4^T x - Bline\,\theta_2) + Y_{line_1}\left(-\theta_2 - \frac{Sline}{Bline}\right) + Y_{line_2}\left(\theta_2 - \frac{Sline}{Bline}\right)$$

$$= (c^T - \lambda_1^T + Y_1^T + Y_2^T k_1^T - Y_2^T k_2^T + Y_3^T k_3^T - Y_3 k_4^T)\,x + (Y_2^T Bline - Y_3^T Bline - Y_{line_1} + Y_{line_2})\,\theta + Y_1^T(-b - b_{add}) - Y_{line_1}\frac{Sline}{Bline} - Y_{line_2}\frac{Sline}{Bline}$$

$$\Downarrow$$

maximize

$$-1 \times \max \; Y_1^T(-b - b_{add}) - Y_{line_1}\frac{Sline}{Bline} - Y_{line_2}\frac{Sline}{Bline}$$

$$\Downarrow$$

$$\min \; Y_1^T(b + b_{add}) + Y_{line_1}\frac{Sline}{Bline} + Y_{line_2}\frac{Sline}{Bline}$$

Subj to $\lambda_1 \ge 0$, $y_1 \ge 0$

$y_{line_1} \ge 0$, $y_{line_2} \ge 0$

$$C + \lambda_1 + y_1 - (k_1 - k_2)y_2 + (k_3 - k_4)y_3 = 0$$

$$0 \le y_2^T B_{line} - y_3^T B_{line} - y_{line_1} + y_{line_2} \perp \theta \ge 0$$

อาจารย์อยากให้ แต่ตรงนี้ไม่ได้ตัด λ
(เพราะไม่มีใน obj หลัก)

→ ตัดตัด ออก λ ออก เนื่องจาก $\lambda \ge 0$

$$C + y - (k_1 - k_2)y_2 + (k_3 - k_4)y \le 0$$

$$\min f(x)$$

sub $g(x) \le 0 \Rightarrow$ dual

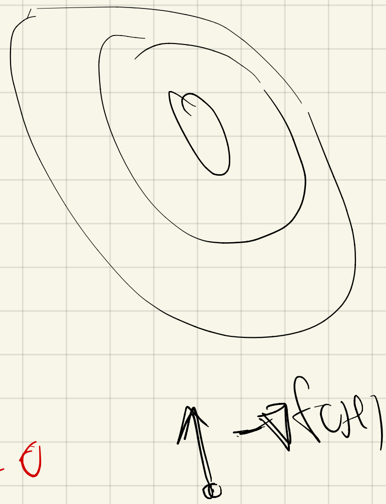
$h(x) = 0$ $\quad \lambda \ge 0$

dual in-eq dual

obj:

$$\underline{L(x,\lambda,\nu)} = f(x) + \lambda^T g(x) + \nu^T \underline{h(x)} \Rightarrow \text{obj}$$

$$\nabla_x L = \nabla f(x) + \lambda^T \nabla g(x) + \nu^T \nabla h(x) = 0$$



PRIMAL	maximize	minimize	DUAL
constraints	$\leq b_i$	≥ 0	*variables*
	$\geq b_i$	≤ 0	
	$= b_i$	*unconstrained*	
variables	≥ 0	$\geq c_j$	*constraints*
	≤ 0	$\leq c_j$	
	unconstrained	$= c_j$	

Note, using the rules in the above table, the dual of

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & A^1 x \leq b^1 \\ & A^2 x \geq b^2 \\ & A^3 x = b^3 \end{array}$$

- Primal form(lower level) x_i, θ_2 ¥top(\top)

$$\begin{array}{llll}
\underset{x_{i=1}^{100},\theta}{\text{Minimize}} & -c^{T} x & & (4.9) \\
\text{subject to} & \cancel{x \geq 0} \quad -x_i \leq 0, \ \forall i \in N & ; \ \cancel{(\lambda_1)} \ \lambda_i^- & (4.10) \\
\lambda_i & \cancel{x} \leq b_i + b_{add_i} \quad , \ \forall i \in N & ; \ \cancel{(y_1)} \ \lambda_i^+ & (4.11) \\
& k_1^T x - k_2^T x = -Bline\theta_2 & ; \ \cancel{(y_2)} \ \mu_1 & (4.12) \\
& k_3^T x - k_4^T x = Bline\theta_2 & ; \ \cancel{(y_3)} \ \mu_2 & (4.13) \\
& \begin{pmatrix} -Sline \\ Bline \end{pmatrix} \leq \theta_2 \leq \begin{pmatrix} Sline \\ Bline \end{pmatrix} & ; \ (y_{line_1}, y_{line_2}) & (4.14)
\end{array}$$

where

$$C = (C_1, C_2, \cdots, C_{100})^T$$

$$\left.\begin{aligned}
k_1 &= [\mathbb{1}_{1\times 25} \quad 0_{1\times 25} \quad 0_{1\times 25} \quad 0_{1\times 25}]^T \\
k_2 &= [0_{1\times 25} \quad \mathbb{1}_{1\times 25} \quad 0_{1\times 25} \quad 0_{1\times 25}]^T \\
k_3 &= [0_{1\times 25} \quad 0_{1\times 25} \quad \mathbb{1}_{1\times 25} \quad 0_{1\times 25}]^T \\
k_4 &= [0_{1\times 25} \quad 0_{1\times 25} \quad 0_{1\times 25} \quad \mathbb{1}_{1\times 25}]^T
\end{aligned}\right\}$$ use ¥top instead of T (\top)

- ~~Using Lagrange multiplier to convert to dual form~~

23:51 Tue 1 Jun

Done Edit

strained variables in the dual is but one aspect that can be observed by looking at the above dual. Other features are summarized in the table below, which describes how to take the dual of a general linear program.

PRIMAL	maximize	minimize	DUAL
constraints	$\leq b_i$	≥ 0	*variables*
	$\geq b_i$	≤ 0	
	$= b_i$	*unconstrained*	
variables	≥ 0	$\geq c_j$	*constraints*
	≤ 0	$\leq c_j$	
	unconstrained	$= c_j$	

Note, using the rules in the above table, the dual of

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & A^1 x \leq b^1 \\ & A^2 x \geq b^2 \\ & A^3 x = b^3 \end{array}$$

becomes

$$\begin{array}{ll} \text{maximize} & (b^1)^T y^1 + (b^2)^T y^2 + (b^3)^T y^3 \\ \text{subject to} & (A^1)^T y^1 + (A^2)^T y^2 + (A^3)^T y^3 = c \\ & y^1 \leq 0 \\ & y^2 \geq 0 \end{array}$$

Since the dual of the dual is the primal, reorganizing the above table yields an alternative procedure for converting primals that involve minimization to their duals.

More

$$\begin{array}{llll}
\underset{x_i,\theta}{\text{Min}} & -c^T x & & \\
\text{subject to} \quad [I \ 0] & x \geq 0 & ; & \lambda^- \\
[I \ 0] & x_i \leq b_i + b_{add_i} & ; & \lambda^+ \\
\end{array}$$

$[(k_1-k_2)^T \ Bline]$

$$k_1^T x - k_2^T x + Bline\,\theta_2 = 0 \Leftarrow k_1^T x - k_2^T x = -Bline \cdot \theta_2 \quad ; \ \mu_1$$

$$k_3^T x - k_4^T x - Bline\,\theta_2 = 0 \Leftarrow k_3^T x - k_4^T x = Bline \cdot \theta_2 \quad ; \ \mu_2$$

$[(k_1-k_2)^T \ Bline]$

$$[0 \ 1] \quad \theta_2 \geq -\frac{Sline}{Bline} \quad ; \ Y_{line_1}$$

$$[0 \ 1] \quad \theta_2 \leq \frac{Sline}{Bline} \quad ; \ Y_{line_2}$$

$$\widehat{\max} \quad 0^T \lambda^- + \lambda^{+T}(b + b_{add}) + 0\mu_1 + 0\mu_2 + Y_{line_1} \cdot \left(-\frac{Sline}{Bline}\right) + Y_{line_2} \cdot \left(\frac{Sline}{Bline}\right)$$

$\lambda^- \leq 0$ $\qquad Y_{line_1} \leq 0$

$\lambda^+ \geq 0$ $\qquad Y_{line_2} \geq 0$

$\mu_1 \Rightarrow$ unconstrain

$\mu_2 \Rightarrow$ unconstrain

$$\begin{bmatrix} I \\ 0 \end{bmatrix} \lambda^- + \begin{bmatrix} I \\ 0 \end{bmatrix} \lambda^+ + \begin{bmatrix} k_1 - k_2 \\ Bline \end{bmatrix} \mu_1 + \begin{bmatrix} k_1 - k_2 \\ Bline \end{bmatrix} \mu_2 = \begin{bmatrix} -c \\ 0 \end{bmatrix}$$

LP

max $c^T x$ → ใช้ solver python gurobi

$A_1 x \le b_1$
$A_2 x = b_2$
$A_3 x = b_3$
$x \geq 0$

$y_1 = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ y_{1}^{line} \\ y_{2}^{line} \end{bmatrix}$

min $c^T x$ dual max $b^{1T} y_1 + b^{2T} y_2$ → $\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$

$A_1 x \geq b^1$ → $y_1 \le 0$
$A_2 x = b^2$ → y_2 → unco
$x \geq 0$ → $A_1^T y_1 + A_2^T y_2 \geq c$

con.

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \theta \end{bmatrix}$ ⇒ $A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \\ & & & & 1 \end{bmatrix}$

var

$\begin{bmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \\ & & & \end{bmatrix} y_1 + \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ B_{line} & -B_{line} \end{bmatrix} y_2 \geq c = \begin{bmatrix} -c_1 \\ -c_2 \\ -c_3 \\ -c_4 \\ 0 \end{bmatrix}$

$x_1 - x_2 + B_{line}\theta_2 = 0$ — ①
$x_3 - x_4 - B_{line}\theta_2 = 0$ — ②

$\begin{bmatrix} 1 & -1 & 0 & 0 & B_{line} \\ 0 & 0 & 1 & -1 & -B_{line} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

A_2

$x \le 0$

$-x \ge 0$

x'

$x \Rightarrow$ uncon.

$\oplus \quad \ominus$

$\Rightarrow x = x^+ - x^-$

$x^+, x^- \ge 0$

$0 \quad 1$

$x^+ - x^-$

$$\min c^T x \Rightarrow c^T x^+ - c^T x^-$$

$$A^1 x \le b_1 \Rightarrow A^1 x^+ - A^1 x^- \le b_1$$

$$A^2 x \ge b_2 \Rightarrow A^2 x^+ - A^2 x^- \ge b_2$$

$$A^3 x = b_3 \Rightarrow A^3 x^+ - A^3 x^- \ge b_3$$

x^+, x^-

$$-\lambda^t + \mu_1^+ - \mu_1^- \le -c_1$$

$$\mu_2^+ - \mu_2^- \le$$

$$y^{lin} + y^{line} + B(\mu_1^+ - \mu_1^-) + B^{lin}(\mu_2^+ - \mu_2^-) \ge 0$$

μ_1

$$\max_{b_{add}} \ (b+b_{add})^T y + y_{line_1}\left(\frac{S_{line}}{B_{line}}\right) + y_{line_2}\left(\frac{S_{line}}{B_{line}}\right)$$

sub to $\quad b_{add} \geq 0$

$$\min \ (b+b_{add})^T y + y_{line_1}\left(\frac{S_{line}}{B_{line}}\right) + y_{line_2}\left(\frac{S_{line}}{B_{line}}\right)$$

$$\begin{bmatrix} I_{100} & \begin{matrix} -\mathbb{1}_{25} & 0 \\ 0 & -\mathbb{1} \\ \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{matrix} \end{bmatrix} \begin{bmatrix} y \\ \mu \end{bmatrix} \geq d \qquad \begin{bmatrix} -I_{100} & \begin{matrix} \mathbb{1}_{25\times1} & 0_{25\times1} \\ 0_{25\times1} & \mathbb{1}_{25\times1} \\ -\mathbb{1}_{25\times1} & 0 \\ 0 & -\mathbb{1}_{25\times1} \end{matrix} \end{bmatrix} \begin{bmatrix} y \\ \mu \end{bmatrix} \leq -d$$

Condition

$$[1 \ \ -1 \ \ B_{line} \ \ -B_{line}] \begin{bmatrix} y_{line} \\ \mu \end{bmatrix} = 0, \quad y \geq 0, \ y_{line_1} \geq 0, \ y_{line_2} \geq 0$$

$$L = (b+b_{add})^T y + y_{line_1}\left(\frac{S_{line}}{B_{line}}\right) + y_{line_2}\left(\frac{S_{line}}{B_{line}}\right) + \rho^T\left(-y + \begin{bmatrix} \mathbb{1}_{25} & 0_{25} \\ 0_{25} & \mathbb{1} \\ -\mathbb{1}_{25} & 0 \\ 0 & -\mathbb{1} \end{bmatrix}\mu + d\right)$$

$$+ \omega\left(y_{line_1} - y_{line_2} + [B_{line} \ \ -B_{line}]\mu\right)$$

$$+ \beta_3^T y + \beta_1 \cdot y_{line_1} + \beta_2 \cdot y_{line}$$

$$\frac{dL}{dy} = b + b_{add} - \rho + \beta = 0$$

max: $d^T \rho$

$$y_{line_1} - y_{line_2} + [B_{line} \ \ -B_{lin}]\mu = 0$$

$$0 \leq \rho \perp \left(y - \begin{bmatrix} \mathbb{1}_{25} & 0 \\ 0 & \mathbb{1} \\ -\mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{bmatrix}\mu\right) \geq 0$$

$$0 \leq y \leq M u_2$$

$$0 \leq \beta \leq M(I - u_2)$$

$$0 \leq y_{line} \leq M u_3$$

$$0 \leq \beta_{lin} \leq M(I - u_3)$$

$$0 \leq \rho \leq M u_1$$

$$0 \leq y - \begin{bmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \\ -\mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{bmatrix}\mu \leq M(I - u_1)$$

Energy trading between multiple microgrids for enhancing the grid resilience

Kanyakorn Likhitluecha

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1 Research Objective

The research objective is to develop a framework to keep supplying the electric energy to maximize the social welfare even in case of disasters by trading the electric energy among multiple microgrids.

2 Research plan

- May 2021 : Making the program by considering the imputation in the core of the line between microgrids and solving the problem by gurobi optimizer

3 Research results

- Considering about the interconnection lines between microgrids (1.4851JPN calculated by gurobi)
- The imputation to the interconnection lines between the microgrids : the investment to the lines to increase the capacity by using this imputation, more efficient microgrid operation because If the profit of the grand coalition increases as the capacity of the line increases, the imputation to the line will increase.

Nomenclature

Suffix

i: The number of the players

Constant

c_i: The price coefficient C [JPY/p.u.]
b_i: The resource capacity [p.u.]
S_{line}: Line limit capacity [p.u.]
B_{line}: Line susceptance [p.u.]

Variable

y_i: The shadow price [JPY/p.u.]
θ_i: Phase angle of node i[rad]
y_{line_1}: The shadow price of the line at node 1[JPY/p.u.]
y_{line_2}: The shadow price of the line at node 2[JPY/p.u.]

4 Formulation

The profit maximization problem of each microgrid can be formulated as a bi-level mathematical programming problem with the leader as each microgrid and the follower as a transaction manager as follows.

$$\underset{b_{add}{}_{i=1}^{100}}{\text{Maximize}} \quad \sum_{i=1}^{100}(b_i + b_{add_i})y_i + y_{line_1}\left(\frac{S_{line}}{B_{line}}\right) + y_{line_2}\left(\frac{S_{line}}{B_{line}}\right) \tag{4.1}$$

$$\text{subject to} \quad b_{add_i} \geq 0 \quad \forall_i \in N \tag{4.2}$$

$$\underset{x_i,\theta_2}{\text{Minimize}} \quad -c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4 \tag{4.3}$$

$$\text{subject to} \quad -x_1 \geq -(b_1 + b_{add_1}) \quad ; \quad (\lambda_1^+) \tag{4.4}$$

$$-x_2 \geq -(b_2 + b_{add_2}) \quad ; \quad (\lambda_2^+) \tag{4.5}$$

$$-x_3 \geq -(b_3 + b_{add_3}) \quad ; \quad (\lambda_3^+) \tag{4.6}$$

$$-x_4 \geq -(b_4 + b_{add_4}) \quad ; \quad (\lambda_4^+) \tag{4.7}$$

$$\theta_2 \geq \left(\frac{-S_{line}}{B_{line}}\right) \quad ; \quad y_{line_1} \tag{4.8}$$

$$\theta_2 \leq \left(\frac{S_{line}}{B_{line}}\right) \quad ; \quad y_{line_2} \tag{4.9}$$

$$x_1 - x_2 + B_{line}\theta_2 = 0 \quad ; \quad (\mu_1) \tag{4.10}$$

$$x_3 - x_4 - B_{line}\theta_2 = 0 \quad ; \quad (\mu_2) \tag{4.11}$$

$$x_1 \geq 0 \quad ; \quad (\beta_1) \tag{4.12}$$

$$x_2 \geq 0 \quad ; \quad (\beta_2) \tag{4.13}$$

$$x_3 \geq 0 \quad ; \quad (\beta_3) \tag{4.14}$$

$$x_4 \geq 0 \quad ; \quad (\beta_4) \tag{4.15}$$

The bi-level programming problem can be converted into equivalent constraints by applying KKT (Karush-Kuhn-Tucker) conditions to its lower level problem (4.3 - 4.15). The two-level programming problem can then be converted to a mixed integer linear programming problem.

(1)KKT condition

The KKT (Karush-Kuhn-Tucker) condition is the optimality condition that the optimal solution of various optimization problems should satisfy.The KKT condition is a necessary condition, and is a first-order condition, that is, using first-order derivative vectors and matrices It is a condition to be formulated.

Consider the following general mathematical programming problem.

$$\text{Minimize} \quad f(x) \tag{4.16}$$

$$\text{subject to} \quad g(x) \leq 0 \tag{4.17}$$

$$h(x) = 0 \tag{4.18}$$

The Lagrange function corresponding to this problem is

$$L(x, \lambda, \mu) = f(x) + \lambda g(x) + \mu h(x) \tag{4.19}$$

The KKT condition in question is

$$\nabla_x f(x) + \lambda \nabla_x g(x) = 0 \tag{4.20}$$

$$h(x) = 0 \tag{4.21}$$

$$g(x) \leq 0 \tag{4.22}$$

$$\mu g(x) = 0 \tag{4.23}$$

$$\mu \geq 0 \tag{4.24}$$

Where λ, μ is the Lagrange multiplier vector for equations and inequalities, and ∇ is the gradient for x. The constraint expression (4.22)-(4.24) is a complementarity condition,

$$0 \leq \mu \perp g(x) \geq 0 \tag{4.25}$$

Here, $\perp$ indicates complementarity, that is, $\mu g(x) = 0$.

(2)Lagrange function of microgrid i

The Lagrange function of the expression (4.26) is

$$\begin{aligned} L_i =&(-c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4) + \mu_1(x_1 - x_2 + B_{line}\theta_2) \\ &+ \mu_2(x_3 - x_4 - B_{line}\theta_2) + \lambda_1^+(-(b_1 + b_{add_1}) + x_1) + \lambda_2^+(-(b_2 + b_{add_2}) + x_2) \\ &+ \lambda_3^+(-(b_3 + b_{add_3}) + x_3) + \lambda_4^+(-(b_4 + b_{add_4}) + x_4) + y_{line_1}(-\left(\frac{S_{line}}{B_{line}}\right) - \theta_2) \\ &+ y_{line_2}(\theta_2 - \left(\frac{S_{line}}{B_{line}}\right)) + \beta_1(-x_1) + \beta_2(-x_2) + \beta_3(-x_3) + \beta_4(-x_4) \end{aligned} \tag{4.26}$$

Where μ_1 , μ_2 , λ_1^+ , λ_2^+ , λ_3^+ , λ_4^+, y_{line_1} , y_{line_2}, β_1 , β_2, $\beta_3$and , β_4 are Lagrange multipier

(3)KKT condition of microgrid i

From the Lagrange function(4.26), the KKT condition is as follows

$$\frac{\partial L}{\partial \theta_2} = \mu_1(B_{line}) - \mu_2(B_{line}) - y_{line_1} + y_{line_2} = 0 \tag{4.27}$$

$$\frac{\partial L}{\partial x_1} = -c_1 + \mu_1 + \lambda_1^+ - \beta_1 = 0 \tag{4.28}$$

$$\frac{\partial L}{\partial x_2} = -c_2 - \mu_1 + \lambda_2^+ - \beta_2 = 0 \tag{4.29}$$

$$\frac{\partial L}{\partial x_3} = -c_3 + \mu_2 + \lambda_3^+ - \beta_3 = 0 \tag{4.30}$$

$$\frac{\partial L}{\partial x_4} = -c_4 - \mu_2 + \lambda_4^+ - \beta_4 = 0 \tag{4.31}$$

$$0 \le -x_1 + (b_1 + b_{add_1}) \perp \lambda_1^+ \ge 0 \tag{4.32}$$

$$0 \le -x_2 + (b_2 + b_{add_2}) \perp \lambda_2^+ \ge 0 \tag{4.33}$$

$$0 \le -x_3 + (b_3 + b_{add_3}) \perp \lambda_3^+ \ge 0 \tag{4.34}$$

$$0 \le -x_4 + (b_4 + b_{add_4}) \perp \lambda_4^+ \ge 0 \tag{4.35}$$

$$0 \le \theta_2 + \left(\frac{Sline}{Bline}\right) \perp y_{line_1} \ge 0 \tag{4.36}$$

2.

$$0 \le \left(\frac{Sline}{Bline}\right) - \theta_2 \perp y_{line_2} \ge 0 \tag{4.37}$$

$$0 \le x_1 \perp \beta_1 \ge 0 \tag{4.38}$$

$$0 \le x_2 \perp \beta_2 \ge 0 \tag{4.39}$$

$$0 \le x_3 \perp \beta_3 \ge 0 \tag{4.40}$$

$$0 \le x_4 \perp \beta_4 \ge 0 \tag{4.41}$$

$$x_1 - x_2 + B_{line}\theta_2 = 0 \tag{4.42}$$

$$x_3 - x_4 - B_{line}\theta_2 = 0 \tag{4.43}$$

The equation (4.32 - 4.41) are complementarity constraints which are non - linear equation . By replace them with linear constraint , we need to use Big - M method to solve the problem (4.44 - 4.45)

$$0 \le \mu \perp g(x) \ge 0 \iff \begin{cases} 0 \le \mu \le Mu & (4.44) \\ 0 \le g(x) \le M(1-u) & (4.45) \end{cases}$$

The complementarity conditions (4.32 - 4.41) are replaced with the following linear constraints(4.46 - 4.65)

$$0 \leq \lambda_1^+ \leq M u_{\lambda_1^+} \tag{4.46}$$

$$0 \leq -x_1 + (b_1 + b_{add_1}) \leq M(1 - u_{\lambda_1^+}) \tag{4.47}$$

$$0 \leq \lambda_2^+ \leq M u_{\lambda_2^+} \tag{4.48}$$

$$0 \leq -x_2 + (b_2 + b_{add_2}) \leq M(1 - u_{\lambda_2^+}) \tag{4.49}$$

$$0 \leq \lambda_3^+ \leq M u_{\lambda_3^+} \tag{4.50}$$

$$0 \leq -x_3 + (b_3 + b_{add_3}) \leq M(1 - u_{\lambda_3^+}) \tag{4.51}$$

$$0 \leq \lambda_4^+ \leq M u_{\lambda_4^+} \tag{4.52}$$

$$0 \leq -x_4 + (b_4 + b_{add_4}) \leq M(1 - u_{\lambda_4^+}) \tag{4.53}$$

$$0 \leq y_{line_1} \leq M u_{y_{line_1}} \tag{4.54}$$

$$0 \leq \theta_2 + \left(\frac{Sline}{Bline}\right) \leq M(1 - u_{y_{line_1}}) \tag{4.55}$$

$$0 \leq y_{line_2} \leq M u_{y_{line_2}} \tag{4.56}$$

$$0 \leq \left(\frac{Sline}{Bline}\right) - \theta_2 \leq M(1 - u_{y_{line_2}}) \tag{4.57}$$

$$0 \leq \beta_1 \leq M u_{\beta_1} \tag{4.58}$$

x_{1-100}

$$0 \leq x_1 \leq M(1 - u_{\beta_1}) \tag{4.59}$$

$$0 \leq \beta_2 \leq M u_{\beta_2} \tag{4.60}$$

$$0 \leq x_2 \leq M(1 - u_{\beta_2}) \tag{4.61}$$

$$0 \leq \beta_2 3 \leq M u_{\beta_3} \tag{4.62}$$

$$0 \leq x_3 \leq M(1 - u_{\beta_3}) \tag{4.63}$$

$$0 \leq \beta_4 \leq M u_{\beta_4} \tag{4.64}$$

$$0 \leq x_4 \leq M(1 - u_{\beta_4}) \tag{4.65}$$

If both the primal and dual problems are feasible, and the KKT condition is satisfied, the strong duality theorem shows ;

$$\begin{aligned} -c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4 = & -\lambda_1^+(b_1 + b_{add_1}) - \lambda_2^+(b_2 + b_{add_2}) - \lambda_3^+(b_3 + b_{add_3}) - \lambda_4^+(b_4 + b_{add_4}) \\ & - y_{line_1}\left(\frac{S_{line}}{B_{line}}\right) - y_{line_2}\left(\frac{S_{line}}{B_{line}}\right) \end{aligned} \tag{4.66}$$

Therefore , the two-level mathematical programming problem is reduced to the following mixed integer linear

① $[0_{1\times100} \mid 0_{1\times100} \mid 0 \mid B_{line} \mid -B_{line} \mid 0_{1-100} \mid 0_{1-100} \mid -1 \mid 1 \mid 1 \mid 0_{202} \mid$

zeros(1, len(x))

$-C$

programming problem as follows ;

no-Big-M

$$
\begin{aligned}
\text{Maximize} \quad & -c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4 && (4.67)\\
\text{subject to} \quad & b_{add_i} \geq 0 \quad \forall_i \in N && (4.68)\\
& \mu_1(B_{line}) - \mu_2(B_{line}) - y_{line_1} + y_{line_2} = 0 && (4.69)\\
& -c_1 + \mu_1 + \lambda_1^+ - \beta_1 = 0 && (4.70)\\
& -c_2 - \mu_1 + \lambda_2^+ - \beta_2 = 0 && (4.71)\\
& -c_3 + \mu_2 + \lambda_3^+ - \beta_3 = 0 && (4.72)\\
& -c_4 - \mu_2 + \lambda_4^+ - \beta_4 = 0 && (4.73)\\
& x_1 - x_2 + B_{line}\theta_2 = 0 && (4.74)\\
& x_3 - x_4 - B_{line}\theta_2 = 0 && (4.75)\\
& 0 \leq \lambda_1^+ \leq Mu_{\lambda_1^+} && (4.76)\\
& 0 \leq -x_1 + (b_1 + b_{add_1}) \leq M(1 - u_{\lambda_1^+}) && (4.77)\\
& 0 \leq \lambda_2^+ \leq Mu_{\lambda_2^+} && (4.78)\\
& 0 \leq -x_2 + (b_2 + b_{add_2}) \leq M(1 - u_{\lambda_2^+}) && (4.79)\\
& 0 \leq \lambda_3^+ \leq Mu_{\lambda_3^+} && (4.80)\\
& 0 \leq -x_3 + (b_3 + b_{add_3}) \leq M(1 - u_{\lambda_3^+}) && (4.81)\\
& 0 \leq \lambda_4^+ \leq Mu_{\lambda_4^+} && (4.82)\\
& 0 \leq -x_4 + (b_4 + b_{add_4}) \leq M(1 - u_{\lambda_4^+}) && (4.83)\\
& 0 \leq y_{line_1} \leq Mu_{y_{line_1}} && (4.84)\\
& 0 \leq \theta_2 + \left(\frac{Sline}{Bline}\right) \leq M(1 - u_{y_{line_1}}) && (4.85)\\
& 0 \leq y_{line_2} \leq Mu_{y_{line_2}} && (4.86)\\
& 0 \leq \left(\frac{Sline}{Bline}\right) - \theta_2 \leq M(1 - u_{y_{line_2}}) && (4.87)\\
& 0 \leq \beta_1 \leq Mu_{\beta_1} && (4.88)\\
& 0 \leq x_1 \leq M(1 - u_{\beta_1}) && (4.89)\\
& 0 \leq \beta_2 \leq Mu_{\beta_2} && (4.90)\\
& 0 \leq x_2 \leq M(1 - u_{\beta_2}) && (4.91)\\
& 0 \leq \beta_2 3 \leq Mu_{\beta_3} && (4.92)\\
& 0 \leq x_3 \leq M(1 - u_{\beta_3}) && (4.93)\\
& 0 \leq \beta_4 \leq Mu_{\beta_4} && (4.94)\\
& 0 \leq x_4 \leq M(1 - u_{\beta_4}) && (4.95)
\end{aligned}
$$

$\theta_2 + \frac{S}{B} \leq M - Mu$

$\theta_2 + Mu \leq M - \frac{S}{B}$

$-\theta_2 + \frac{S}{B} \leq M - Mu$

$-\theta_2 + Mu \leq M - \frac{S}{B}$

- Now , I am in the process of making the program by using gurobi

$x \leq M - Mu_\beta$

$x + Mu_\beta \leq M$

5 Future tasks

Continue making the program by using gurobi optimization

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$$\text{Vara} = \begin{bmatrix} x_{1-100} \\ b_{add,1-100} \\ \theta_2 \\ \mu_1 \\ \mu_2 \\ \beta_{1-100} \\ \lambda_{1-100} \\ y_{line_1} \\ y_{line_2} \\ u_\lambda \\ u_\beta \\ u_{yline} \end{bmatrix}$$

$$\begin{bmatrix} -I_2 & 0 \\ -I_2 & M\cdot I_2 \end{bmatrix}\begin{bmatrix} y \\ u_\lambda \end{bmatrix} \le \begin{bmatrix} 0_2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0_2 \\ -1 & M\cdot I_2 \end{bmatrix}\begin{bmatrix} \theta \\ u_y \end{bmatrix} = \begin{bmatrix} S(B) \\ S(B) \\ M-S(B) \\ M-S(B) \end{bmatrix}$$

$$\begin{array}{cccccccccccc} x & b_{add} & \theta_2 & \mu_1 & \mu_2 & \beta_{1-100} & \lambda_{100} & y_{line_1} & y_{line_1} & u_\lambda & u_\beta & u_{yline} \end{array}$$

$$\begin{bmatrix} 0_{1\times100} & 0_{1\times100} & 0 & -B_{line} & B_{line} & 0_{1\times100} & 0_{1\times100} & 1 & -1 & 0_{1\times100} & 0_{1\times100} & 0_{1\times2} \end{bmatrix} = [0]$$

$$\begin{bmatrix} 0_{100\times100} & 0_{100\times100} & 0_{100\times1} & \begin{matrix} 1_{25\times1} & 0_{25\times1} \\ -1_{25\times1} & 0_{25\times1} \\ 0_{25\times1} & 1_{25\times1} \\ 0_{25\times1} & -1_{25\times1} \end{matrix} & -I_{100} & I_{100} & 0_{100\times1} & 0_{100\times1} & 0_{100\times100} & 0_{100\times100} & 0_{100\times2} \end{bmatrix} = [0]$$

$$\begin{bmatrix} \begin{matrix} 1_{1\times25} & -1_{1\times25} & 0_{1\times25} & 0_{1\times25} \\ 0_{1\times25} & 0_{1\times25} & 1_{1\times25} & -1_{1\times25} \end{matrix} & 0_{2\times100} & \begin{matrix} B_{line} \\ -B_{line} \end{matrix} & 0_{2\times1} & 0_{2\times1} & 0_{2\times100} & 0_{2\times100} & 0 & 0 & 0_{2\times100} & 0_{2\times100} & 0_{2\times2} \end{bmatrix} = [0_2]$$

$0_{100\times100}$ | $-I_{100}$ | $0_{100\times1}$ | $0_{100\times1}$ | $0_{100\times1}$ | $0_{100\times100}$ | $0_{100\times100}$ | $0_{100\times1}$ | $0_{100\times1}$ | $0_{100\times100}$ | $0_{100\times100}$ | $0_{100\times2}$ | $\leq$ | 0_{100}

$0_{100\times100}$ | $0_{100\times100}$ | $0_{100\times1}$ | $0_{100\times1}$ | $0_{100\times1}$ | $0_{100\times100}$ | $-I_{100}$ | $0_{100\times1}$ | $0_{100\times1}$ | $0_{100\times10}$ | 0

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad I_{100}$

$-MI \leq \begin{bmatrix} 0_{10} \\ 0_{100} \end{bmatrix}$

x | bddd | u

$I_{100} \quad -I_{100} \quad 0 \leq \begin{bmatrix} 0 \\ M\cdot b \end{bmatrix}$

$-I_{100} \quad I_{100} \quad M\cdot I \leq$

β_2 | u_B

$-I_{100} \quad 0 \leq 0$

$I_{100} \quad -MI \leq 0$

x

$-I_{100} \quad 0 \quad 0$

$I_{100} \quad 0 \quad M\cdot I \leq \begin{bmatrix} 0 \\ M\cdot_{100} \end{bmatrix}$

0 | y_{line} | u_y

$0 \quad -I_2 \quad 0 \leq 0$

$0 \quad I_2 \quad -MI_2 \leq 0$

$0 \quad 0 \quad s/b$

$0 \quad s/b$

$- \quad 0 \quad M\cdot I \quad M-s/b$

$- \quad M-s/b$