Energy trading between multiple microgrids for enhancing the grid resilience

Kanyakorn Likhitluecha

June 11, 2021

1 Research Objective

The research objective is to develop a framework to keep supplying the electric energy to maximize the social welfare even in case of disasters by trading the electric energy among multiple microgrids.

2 Research plan

- May 2021 : Making the program by considering the imputation in the core of the line between microgrids and solving the problem by gurobi optimizer

3 Research results

- Considering about the interconnection lines between microgrids (1.4851JPN calculated by gurobi)
- The imputation to the interconnection lines between the microgrids : the investment to the lines to increase the capacity by using this imputation, more efficient microgrid operation because If the profit of the grand coalition increases as the capacity of the line increases, the imputation to the line will increase.

Nomenclature

Suffix

i: The number of the players

Constant

c_i: The price coefficient C [JPY/p.u.]
b_i: The resource capacity [p.u.]
S_{line}: Line limit capacity [p.u.]
B_{line}: Line susceptance [p.u.]

Variable

y_i: The shadow price [JPY/p.u.]
θ_i: Phase angle of node i[rad]
y_{line_1}: The shadow price of the line at node 1[JPY/p.u.]
y_{line_2}: The shadow price of the line at node 2[JPY/p.u.]

4 Formulation

The profit maximization problem of each microgrid can be formulated as a bi-level mathematical programming problem with the leader as each microgrid and the follower as a transaction manager as follows.

$$\underset{b_{add}{}_{i=1}^{100}}{\text{Maximize}} \quad \sum_{i=1}^{100}(b_i + b_{add_i})y_i + y_{line_1}\left(\frac{S_{line}}{B_{line}}\right) + y_{line_2}\left(\frac{S_{line}}{B_{line}}\right) \tag{4.1}$$

$$\text{subject to} \quad b_{add_i} \geq 0 \quad \forall_i \in N \tag{4.2}$$

$$\underset{x_i,\theta_2}{\text{Minimize}} \quad -c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4 \tag{4.3}$$

$$\text{subject to} \quad -x_1 \geq -(b_1 + b_{add_1}) \quad ; \quad (\lambda_1^+) \tag{4.4}$$

$$-x_2 \geq -(b_2 + b_{add_2}) \quad ; \quad (\lambda_2^+) \tag{4.5}$$

$$-x_3 \geq -(b_3 + b_{add_3}) \quad ; \quad (\lambda_3^+) \tag{4.6}$$

$$-x_4 \geq -(b_4 + b_{add_4}) \quad ; \quad (\lambda_4^+) \tag{4.7}$$

$$\theta_2 \geq \left(\frac{-S_{line}}{B_{line}}\right) \quad ; \quad y_{line_1} \tag{4.8}$$

$$\theta_2 \leq \left(\frac{S_{line}}{B_{line}}\right) \quad ; \quad y_{line_2} \tag{4.9}$$

$$x_1 - x_2 + B_{line}\theta_2 = 0 \quad ; \quad (\mu_1) \tag{4.10}$$

$$x_3 - x_4 - B_{line}\theta_2 = 0 \quad ; \quad (\mu_2) \tag{4.11}$$

$$x_1 \geq 0 \quad ; \quad (\beta_1) \tag{4.12}$$

$$x_2 \geq 0 \quad ; \quad (\beta_2) \tag{4.13}$$

$$x_3 \geq 0 \quad ; \quad (\beta_3) \tag{4.14}$$

$$x_4 \geq 0 \quad ; \quad (\beta_4) \tag{4.15}$$

The bi-level programming problem can be converted into equivalent constraints by applying KKT (Karush-Kuhn-Tucker) conditions to its lower level problem (4.3 - 4.15). The two-level programming problem can then be converted to a mixed integer linear programming problem.

(1)KKT condition

The KKT (Karush-Kuhn-Tucker) condition is the optimality condition that the optimal solution of various optimization problems should satisfy.The KKT condition is a necessary condition, and is a first-order condition, that is, using first-order derivative vectors and matrices It is a condition to be formulated.

Consider the following general mathematical programming problem.

$$\text{Minimize} \quad f(x) \tag{4.16}$$

$$\text{subject to} \quad g(x) \leq 0 \tag{4.17}$$

$$h(x) = 0 \tag{4.18}$$

The Lagrange function corresponding to this problem is

$$L(x, \lambda, \mu) = f(x) + \lambda g(x) + \mu h(x) \tag{4.19}$$

The KKT condition in question is

$$\nabla_x f(x) + \lambda \nabla_x g(x) = 0 \tag{4.20}$$

$$h(x) = 0 \tag{4.21}$$

$$g(x) \leq 0 \tag{4.22}$$

$$\mu g(x) = 0 \tag{4.23}$$

$$\mu \geq 0 \tag{4.24}$$

Where λ, μ is the Lagrange multiplier vector for equations and inequalities, and ∇ is the gradient for x. The constraint expression (4.22)-(4.24) is a complementarity condition,

$$0 \leq \mu \perp g(x) \geq 0 \tag{4.25}$$

Here, $\perp$ indicates complementarity, that is, $\mu g(x) = 0$.

(2)Lagrange function of microgrid i

The Lagrange function of the expression (4.26) is

$$
\begin{aligned}
L_i =&(-c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4) + \mu_1(x_1 - x_2 + B_{line}\theta_2) \\
&+ \mu_2(x_3 - x_4 - B_{line}\theta_2) + \lambda_1^+(-(b_1 + b_{add_1}) + x_1) + \lambda_2^+(-(b_2 + b_{add_2}) + x_2) \\
&+ \lambda_3^+(-(b_3 + b_{add_3}) + x_3) + \lambda_4^+(-(b_4 + b_{add_4}) + x_4) + y_{line_1}(-\left(\frac{S_{line}}{B_{line}}\right) - \theta_2) \\
&+ y_{line_2}(\theta_2 - \left(\frac{S_{line}}{B_{line}}\right)) + \beta_1(-x_1) + \beta_2(-x_2) + \beta_3(-x_3) + \beta_4(-x_4) \qquad (4.26)
\end{aligned}
$$

Where μ_1 , μ_2 , λ_1^+ , λ_2^+ , λ_3^+ , λ_4^+, y_{line_1} , y_{line_2}, β_1 , β_2, $\beta_3$and , β_4 are Lagrange multipier

(3)KKT condition of microgrid i

From the Lagrange function(4.26), the KKT condition is as follows

$$
\begin{aligned}
&\frac{\partial L}{\partial \theta_2} = \mu_1(B_{line}) - \mu_2(B_{line}) - y_{line_1} + y_{line_2} = 0 && (4.27)\\
&\frac{\partial L}{\partial x_1} = -c_1 + \mu_1 + \lambda_1^+ - \beta_1 = 0 && (4.28)\\
&\frac{\partial L}{\partial x_2} = -c_2 - \mu_1 + \lambda_2^+ - \beta_2 = 0 && (4.29)\\
&\frac{\partial L}{\partial x_3} = -c_3 + \mu_2 + \lambda_3^+ - \beta_3 = 0 && (4.30)\\
&\frac{\partial L}{\partial x_4} = -c_4 - \mu_2 + \lambda_4^+ - \beta_4 = 0 && (4.31)\\
&0 \le -x_1 + (b_1 + b_{add_1}) \perp \lambda_1^+ \ge 0 && (4.32)\\
&0 \le -x_2 + (b_2 + b_{add_2}) \perp \lambda_2^+ \ge 0 && (4.33)\\
&0 \le -x_3 + (b_3 + b_{add_3}) \perp \lambda_3^+ \ge 0 && (4.34)\\
&0 \le -x_4 + (b_4 + b_{add_4}) \perp \lambda_4^+ \ge 0 && (4.35)\\
&0 \le \theta_2 + \left(\frac{Sline}{Bline}\right) \perp y_{line_1} \ge 0 && (4.36)\\
&0 \le \left(\frac{Sline}{Bline}\right) - \theta_2 \perp y_{line_2} \ge 0 && (4.37)\\
&0 \le x_1 \perp \beta_1 \ge 0 && (4.38)\\
&0 \le x_2 \perp \beta_2 \ge 0 && (4.39)\\
&0 \le x_3 \perp \beta_3 \ge 0 && (4.40)\\
&0 \le x_4 \perp \beta_4 \ge 0 && (4.41)\\
&x_1 - x_2 + B_{line}\theta_2 = 0 && (4.42)\\
&x_3 - x_4 - B_{line}\theta_2 = 0 && (4.43)
\end{aligned}
$$

The equation (4.32 - 4.41) are complementarity constraints which are non - linear equation . By replace them with linear constraint , we need to use Big - M method to solve the problem (4.44 - 4.45)

$$
0 \le \mu \perp g(x) \ge 0 \iff \begin{cases} 0 \le \mu \le Mu & (4.44)\\ 0 \le g(x) \le M(1-u) & (4.45) \end{cases}
$$

The complementarity conditions (4.32 - 4.41) are replaced with the following linear constraints(4.46 - 4.65)

$$0 \leq \lambda_1^+ \leq Mu_{\lambda_1^+} \tag{4.46}$$
$$0 \leq -x_1 + (b_1 + b_{add_1}) \leq M(1 - u_{\lambda_1^+}) \tag{4.47}$$
$$0 \leq \lambda_2^+ \leq Mu_{\lambda_2^+} \tag{4.48}$$
$$0 \leq -x_2 + (b_2 + b_{add_2}) \leq M(1 - u_{\lambda_2^+}) \tag{4.49}$$
$$0 \leq \lambda_3^+ \leq Mu_{\lambda_3^+} \tag{4.50}$$
$$0 \leq -x_3 + (b_3 + b_{add_3}) \leq M(1 - u_{\lambda_3^+}) \tag{4.51}$$
$$0 \leq \lambda_4^+ \leq Mu_{\lambda_4^+} \tag{4.52}$$
$$0 \leq -x_4 + (b_4 + b_{add_4}) \leq M(1 - u_{\lambda_4^+}) \tag{4.53}$$
$$0 \leq y_{line_1} \leq Mu_{y_{line_1}} \tag{4.54}$$
$$0 \leq \theta_2 + \left(\frac{Sline}{Bline}\right) \leq M(1 - u_{y_{line_1}}) \tag{4.55}$$
$$0 \leq y_{line_2} \leq Mu_{y_{line_2}} \tag{4.56}$$
$$0 \leq \left(\frac{Sline}{Bline}\right) - \theta_2 \leq M(1 - u_{y_{line_2}}) \tag{4.57}$$
$$0 \leq \beta_1 \leq Mu_{\beta_1} \tag{4.58}$$
$$0 \leq x_1 \leq M(1 - u_{\beta_1}) \tag{4.59}$$
$$0 \leq \beta_2 \leq Mu_{\beta_2} \tag{4.60}$$
$$0 \leq x_2 \leq M(1 - u_{\beta_2}) \tag{4.61}$$
$$0 \leq \beta_2 3 \leq Mu_{\beta_3} \tag{4.62}$$
$$0 \leq x_3 \leq M(1 - u_{\beta_3}) \tag{4.63}$$
$$0 \leq \beta_4 \leq Mu_{\beta_4} \tag{4.64}$$
$$0 \leq x_4 \leq M(1 - u_{\beta_4}) \tag{4.65}$$

If both the primal and dual problems are feasible, and the KKT condition is satisfied, the strong duality theorem shows ;

$$\begin{aligned} -c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4 = &- \lambda_1^+(b_1 + b_{add_1}) - \lambda_2^+(b_2 + b_{add_2}) - \lambda_3^+(b_3 + b_{add_3}) - \lambda_4^+(b_4 + b_{add_4}) \\ &- y_{line_1}\left(\frac{S_{line}}{B_{line}}\right) - y_{line_2}\left(\frac{S_{line}}{B_{line}}\right) \end{aligned} \tag{4.66}$$

Therefore , the two-level mathematical programming problem is reduced to the following mixed integer linear programming problem as follows ;

$$
\begin{aligned}
\text{Maximize} \quad & -c_1x_1 - c_2x_2 - c_3x_3 - c_4x_4 && (4.67)\\
\text{subject to} \quad & b_{add_i} \geq 0 \quad \forall_i \in N && (4.68)\\
& \mu_1(B_{line}) - \mu_2(B_{line}) - y_{line_1} + y_{line_2} = 0 && (4.69)\\
& -c_1 + \mu_1 + \lambda_1^+ - \beta_1 = 0 && (4.70)\\
& -c_2 - \mu_1 + \lambda_2^+ - \beta_2 = 0 && (4.71)\\
& -c_3 + \mu_2 + \lambda_3^+ - \beta_3 = 0 && (4.72)\\
& -c_4 - \mu_2 + \lambda_4^+ - \beta_4 = 0 && (4.73)\\
& x_1 - x_2 + B_{line}\theta_2 = 0 && (4.74)\\
& x_3 - x_4 - B_{line}\theta_2 = 0 && (4.75)\\
& 0 \leq \lambda_1^+ \leq Mu_{\lambda_1^+} && (4.76)\\
& 0 \leq -x_1 + (b_1 + b_{add_1}) \leq M(1 - u_{\lambda_1^+}) && (4.77)\\
& 0 \leq \lambda_2^+ \leq Mu_{\lambda_2^+} && (4.78)\\
& 0 \leq -x_2 + (b_2 + b_{add_2}) \leq M(1 - u_{\lambda_2^+}) && (4.79)\\
& 0 \leq \lambda_3^+ \leq Mu_{\lambda_3^+} && (4.80)\\
& 0 \leq -x_3 + (b_3 + b_{add_3}) \leq M(1 - u_{\lambda_3^+}) && (4.81)\\
& 0 \leq \lambda_4^+ \leq Mu_{\lambda_4^+} && (4.82)\\
& 0 \leq -x_4 + (b_4 + b_{add_4}) \leq M(1 - u_{\lambda_4^+}) && (4.83)\\
& 0 \leq y_{line_1} \leq Mu_{y_{line_1}} && (4.84)\\
& 0 \leq \theta_2 + \left(\frac{Sline}{Bline}\right) \leq M(1 - u_{y_{line_1}}) && (4.85)\\
& 0 \leq y_{line_2} \leq Mu_{y_{line_2}} && (4.86)\\
& 0 \leq \left(\frac{Sline}{Bline}\right) - \theta_2 \leq M(1 - u_{y_{line_2}}) && (4.87)\\
& 0 \leq \beta_1 \leq Mu_{\beta_1} && (4.88)\\
& 0 \leq x_1 \leq M(1 - u_{\beta_1}) && (4.89)\\
& 0 \leq \beta_2 \leq Mu_{\beta_2} && (4.90)\\
& 0 \leq x_2 \leq M(1 - u_{\beta_2}) && (4.91)\\
& 0 \leq \beta_2 3 \leq Mu_{\beta_3} && (4.92)\\
& 0 \leq x_3 \leq M(1 - u_{\beta_3}) && (4.93)\\
& 0 \leq \beta_4 \leq Mu_{\beta_4} && (4.94)\\
& 0 \leq x_4 \leq M(1 - u_{\beta_4}) && (4.95)
\end{aligned}
$$

- Now , I am in the process of making the program by using gurobi

5 Future tasks

Continue making the program by using gurobi optimization

References

[1] Gabriel,S.,Conejo,A.,Fuller,D.,Hobbs,B.and Ruiz,C.2013. Complementrity Modeling in Energy Markets: KarushKuhn-Tucker Conditions. Springer, New York. 321 pp.

[2] Fortuny-Amat,J.,McCarl.B, “A Representation and Economic Interpretation of a Two-Level Programming Problem ” .

[3] Salvador,P.,Juan.M, “ Solving Linear Bilevel Problems Using Big-Ms :Not all that glitters is gold ” .

[4] Carlos.R,Antonio.C,Yves.S, “ Equilibria in an Oligopolistic Electricity Pool With Stepwise Offer Curves ”” .

[5] J. Fortuny-Amat and B. McCarl, “ A representation and economic interpretation of a two-level programming problem ” , Journal of the Operational Research Society,Vol. 32, No. 9, pp. 783-792, 1981.

[6] G.Owen, “ On the core of linear production games ”, North-Holland Publishing Company, pp. 358-370, 1975.

[7] C.Prete and B. Hobbs,"A cooperative game theretic analysis of incentives for microgrids in regulated electricity markets". pp524-541,2016

[8] B.Roy and K.Mason,Formulation and Analysis of Linear Programs: Chapter4 Duality.Standford University, pp83-110,2005

[9] Y.Jia,P.Wen,Y.Yan and L.Huo,"Joint operation and transaction mode of rural multi microgrid and distribution network",2021