1 Formulation

- Primal form(lower level)

$$\begin{aligned}
\underset{x_{i=1}^{100},\theta}{\text{Minimize}} \quad & -c^T x && && (1.1)\\
\text{subject to} \quad & x \geq 0 && ; \quad (\lambda_1) && (1.2)\\
& x \leq b_i + b_{add_i} && ; \quad (y_1) && (1.3)\\
& k_1^T x - k_2^T x = -Bline\theta_2 && ; \quad (y_2) && (1.4)\\
& k_3^T x - k_4^T x = Bline\theta_2 && ; \quad (y_3) && (1.5)\\
& \left(\frac{-Sline}{Bline}\right) \leq \theta_2 \leq \left(\frac{Sline}{Bline}\right) && ; \quad (y_{line_1}, y_{line_2}) && (1.6)
\end{aligned}$$

$when \quad k_1 = \begin{bmatrix} \mathbb{1}_{1\times 25} & 0_{1\times 25} & 0_{1\times 25} & 0_{1\times 25} \end{bmatrix}^T$
$k_2 = \begin{bmatrix} 0_{1\times 25} & \mathbb{1}_{1\times 25} & 0_{1\times 25} & 0_{1\times 25} \end{bmatrix}^T$
$k_3 = \begin{bmatrix} 0_{1\times 25} & 0_{1\times 25} & \mathbb{1}_{1\times 25} & 0_{1\times 25} \end{bmatrix}^T$
$k_4 = \begin{bmatrix} 0_{1\times 25} & 0_{1\times 25} & 0_{1\times 25} & \mathbb{1}_{1\times 25} \end{bmatrix}^T$
Using Lagrange multiplyer to convert to dual form

$$\begin{aligned}
\mathcal{L}(x, \lambda_1, y_1, y_2, y_3, y_{line_1}, y_{line_2}) &= -c^T x + \lambda_1^T x + y_1^T (x - b_i - b_{add_i}) \\
&\quad + y_2^T (k_1^T x - k_2^T x + Bline\theta_2) + y_3^T (k_3^T x - k_4^T x - Bline\theta_2) \\
&\quad + y_{line_1}^T (-\theta_2 - \left(\frac{Sline}{Bline}\right)) + y_{line_2}^T (\theta_2 - \left(\frac{Sline}{Bline}\right)) \\
&= (c^T - \lambda_1^T + y_1^T + y_2^T k_1^T - y_2^T k_2^T + y_3^T k_3^T - y_3^T k_4^T) x \\
&\quad + (y_2^T Bline - y_3^T Bline - y_{line_1} + y_{line_2})\theta_2 \\
&\quad + y_1^T (-b_i - badd_i) - y_{line_1} \left(\frac{Sline}{Bline}\right) - y_{line_2} \left(\frac{Sline}{Bline}\right)
\end{aligned} \tag{1.7}$$

Dual form

$$\text{Maximize} \quad y_1^T (-b_i - badd_i) - y_{line_1} \left(\frac{Sline}{Bline}\right) - y_{line_2} \left(\frac{Sline}{Bline}\right) \tag{1.8}$$

or

$$\begin{aligned}
\text{Minimize} \quad & y_1^T (b_i + badd_i) + y_{line_1} \left(\frac{Sline}{Bline}\right) + y_{line_2} \left(\frac{Sline}{Bline}\right) && (1.9)\\
\text{subject to} \quad & \lambda_1 \geq 0 && (1.10)\\
& y_1 \geq 0 && (1.11)\\
& y_{line_1} \geq 0 && (1.12)\\
& y_{line_2} \geq 0 && (1.13)\\
& c + \lambda_1 + y_1 - (k_1 - k_2) y_2 + (k_3 - k_4) y_3 = 0 && (1.14)\\
& 0 \leq y_2^T Bline - y_3^T Bline - y_{line_1} + yline_2 \perp \theta_2 \geq 0 && (1.15)
\end{aligned}$$