Muon Flux Map Simulation

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1 Introduction

In the IceCube, muons detected are mostly atmospheric muons and, with a much smaller number, muons from atmospheric neutrinos. The atmospheric muons are generated from cosmic ray protons, which decay into pions which further decay into atmospheric muons and atmospheric neutrinos. The atmospheric neutrinos can then decay into muons as well. Since those high energy charged muons can travel faster than speed of light in ice, they generate cherenkov radiation and be detected by photomultipliers in IceCube. According to which photomultipliers are activated, the path of a single muon can be reconstructed so its incoming direction is known.

The data of muon flux is then collected. Their incoming azimuth angle and zenith angle are used to allocate them into different blocks in flux map, and the number of muons within certain range of azimuth angle and zenith angle is the flux in this region. Then, a row normalization is performed by dividing flux of each region by average flux of all regions with the same $cos\theta$ (i.e. all regions in this row). The result of this relative flux is shown in Figure 1[5].

As shown in Figure 1, a clear asymmetry can be observed from $cos\theta = 0.4$ to $cos\theta = -0.8$, and the maximum percentage of asymmetry is 30 percent according to the color bar. Since the flux of cosmic ray should be uniform in all directions, this azimuth asymmetry is unexpected, and one possible explanation is the deflection effect of geomagnetic field. However, the reconstruction process of muon path involves a lot of uncertainties, so the direction of incoming muons may not be accurate. As the result, Figure 1 may not reflect the actual flux asymmetry.

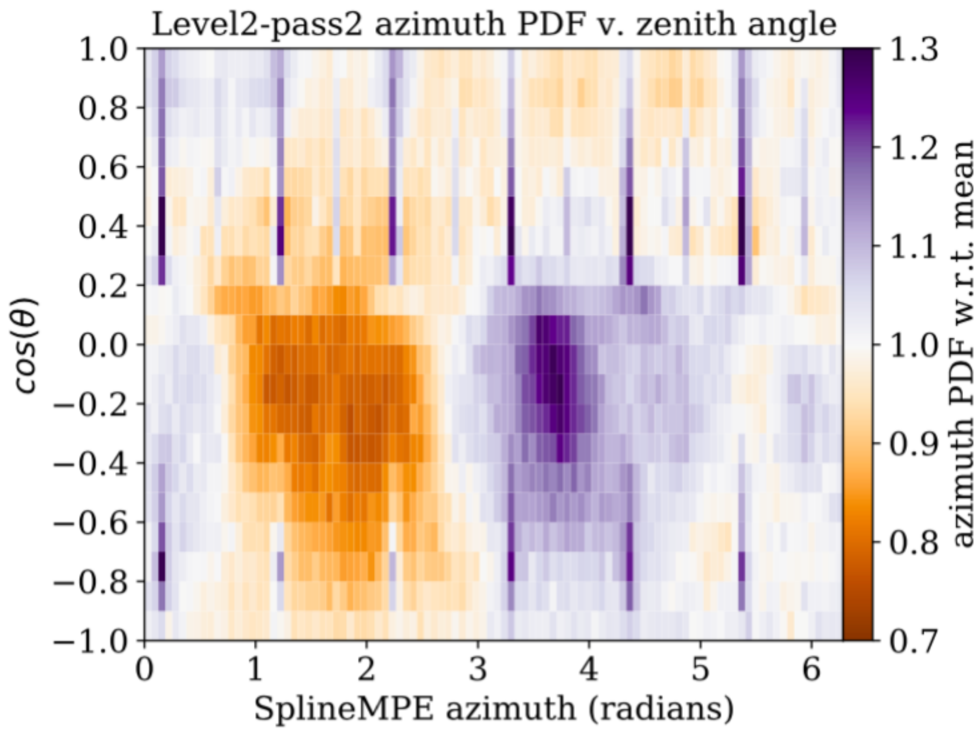


Figure 1: muon flux asymmetry

In order to know what the muon flux map is supposed to look like, a simulation of muon flux distribution is written and run. Its purpose is to generate the expected muon flux combining atmospheric muon, muons from atmospheric neutrinos and effect of earth magnetic field. The row normalized flux map will also be generated and compared with the asymmetric map observed in Figure 1. The construction of this program and the final results are described in following parts.

2 Program Construction

1. overall structure

In this program, a backward simulation is performed to get the muon flux. In other words, instead of simulating a muon coming from atmosphere, losing energy while traveling inside earth, and received by detector like the real scenario, this program simulates muons that come from the detector, gain energy while traveling inside the earth, and stop when reaching surface of the earth. By flipping the direction of geomagnetic field and changing energy loss to energy gain, this backward simulation should generate same trajectory like forward simulation.

In order to get the flux map like Figure 1, the sky of detector is divided uniformly into many small regions, each corresponds to a certain range of $cos\theta$ and phi. In each region one muon was ejected with initial direction of velocity pointing to center of this region and same given initial energy (Since it is a backward simulation, the initial energy here is the energy detected by IceCube when the muon reaches the detector). After the backward trajectory simulation stops when this muon reaches the surface of the earth, the muon's final position, final direction of velocity and final energy will be recorded. The assumption is that if those divided regions are small enough, the final energy, position, and velocity direction of this single muon coming from the center of one region is able to represent the final energy, position, and direction of velocity of all muons that can come from this region. Then, according to equation[1]:

$$\Phi \propto cos^2\theta * E^{-3.7} \ \mathbf{(1)}$$

where θ here is the angle between final velocity direction and normal direction, the muon flux at this region can be determined. Then for every region a flux can be obtained. they are combined to give a all sky muon flux map.

2. trajectory simulation

According to the description above, the key part is that given the initial energy, position, and direction of velocity of a muon, this program should calculate its final energy, position and direction of velocity under the deflection effect of geomagnetic field. In this program a coordinate system with center of earth as origin, south pole as positive z axis and longitude = 0 as positive x axis is established, and the location of IceCube is set to be [0, 0, 6369000], which is 2km beneath the south pole. Starting from this initial position, this muon gets its position, direction of velocity, and energy updated iteratively in a loop with small time interval as following.

Suppose the current position of this muon is (x, y, z) and its current direction of velocity $\hat{v}$ is (v_x, v_y, v_z), then after time interval δt, its new position is:

$$v = \sqrt{c^2 - (\frac{mc^3}{E})^2}$$
$$position_{new} = (x, y, z) + (v_x, v_y, v_z) * v * \delta t$$

Then, $\hat{v}_{new}$, the direction of velocity after δt, is calculated. According to the software from National Geospatial-Intelligence Agency[2], with given position [x, y, z] the geomagnetic field $\vec{B}$ at this position will be generated. Then the current velocity $\vec{v} = (v_x, v_y, v_z) * v$ can be decomposed into a component $\vec{v}_{parallel}$ that is parallel to $\vec{B}$ and another component $\vec{v}_{perpendicular}$ that is perpendicular to $\vec{B}$. Only the perpendicular component $\vec{v}_{perpendicular}$ will be rotated by magnetic field according to:

$$q \mid \vec{v} \times \vec{B} \mid = \frac{|\vec{v}|^2}{R} * \cos^2\theta * \frac{m}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}}$$

By replacing $\vec{v}$ by total energy E and unit direction vector $\hat{\boldsymbol{v}}$, we then get:

$$R = \frac{\sqrt{E^2 - (mc^2)^2} * \cos^2\theta}{q|\hat{\boldsymbol{v}} \times \vec{B}|c} \quad \textbf{(2)}$$

So the angle that $\vec{v}_{perpendicular}$ rotates around $\vec{B}$ is:

$$\theta = \omega * \delta t = \frac{|\vec{v}_{perpendicular}|}{R} * \delta t$$

As the result, due to effect of magnetic field $\vec{v}_{perpendicular}$ gets rotated by angel θ with rotation axis $\hat{\boldsymbol{B}} = (x, y, z)$, and according to the rotation matrix M, the new perpendicular component $\vec{v}_{newperp}$ is:

$$\vec{v}_{newperp} = M * \vec{v}_{perp}$$
$$M = \begin{bmatrix} cos\theta + (1 - cos\theta)x^2 & (1 - cos\theta)xy - zsin\theta & (1 - cos\theta)xz + ysin\theta \\ (1 - cos\theta)yx + zsin\theta & cos\theta + (1 - cos\theta)y^2 & (1 - cos\theta)yz - xsin\theta \\ (1 - cos\theta)zx - ysin\theta & (1 - cos\theta)zy + xsin\theta & cos\theta + (1 - cos\theta)z^2 \end{bmatrix}$$

So the new velocity direction is

$$\hat{v}_{new} = \frac{\vec{v}_{newperp} + \vec{v}_{parallel}}{|\vec{v}_{newperp} + \vec{v}_{parallel}|}$$

and the magnitude is determined by the remaining energy E_{new} after δt.

Next, since in this time period the muon travels distance $x = v * \delta t$, with current energy and energy loss per unit distance the energy of muon after δt also gets updated. According to the result from Muon Monte Carlo Simulation[3], the energy loss per unit distance in ice is:

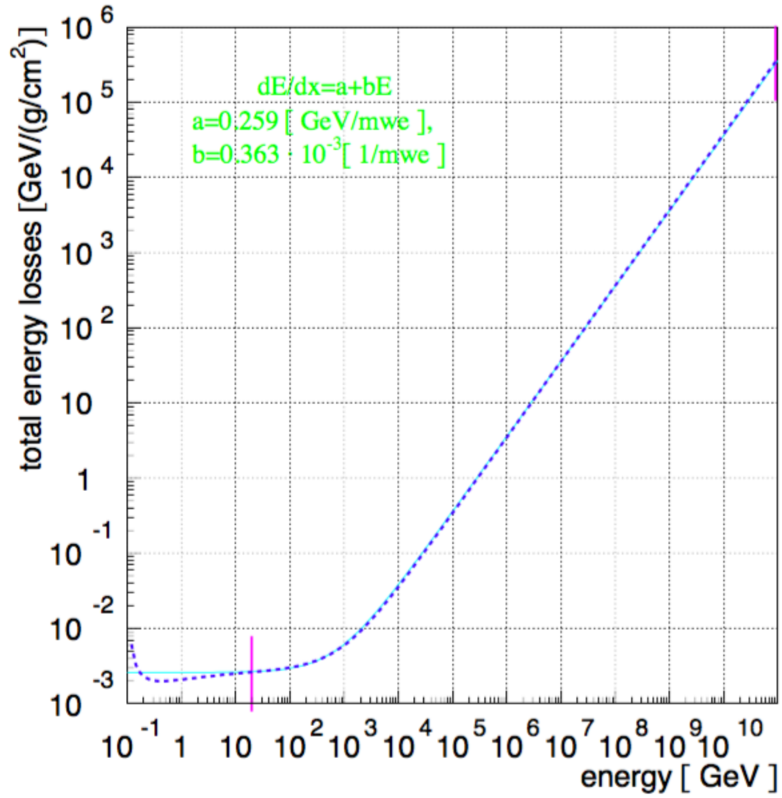


Figure 2: energy loss per unit distance[3]

$$\frac{dE}{dx} = (a + bE) * \rho$$
$$a = 0.259 \text{ GeV/mwe}$$
$$b = 0.363 * 10^{-3} \text{ 1/mwe}$$
$$\rho = 0.9167 \ g/cm^3$$

and we can integral this differential equation to get the relation between distance traveled and energy required:

$$E = \frac{Ce^{b\rho x} - a * \rho}{b * \rho} \ \textbf{(3)}$$

where C is an integration constant determined by initial energy of this muon. So by recording the current total distance traveled the current energy is known.

Then the time period δt used in next iteration is also determined. In this algorithm a constant f is predetermined as how many pieces this algorithm divides a circle into, which represents the accuracy and complexity of this algorithm. So in order to make this muon, with velocity component $| \vec{v}_{perpendicular} |$ in circular motion, travel 1/f of this circle, the time period based on f should be:

$$\delta t' = \frac{2\pi R}{|\vec{v}_{perpendicular}|} * \frac{1}{f}$$

As the result, the energy, position, and direction of velocity of this muon as well as the new time period δt are known and this iterative algorithm comes to its next iteration. By repetitively checking when $| \vec{position} | >= 6371000m$ as stopping condition, The final position, energy, and direction of velocity can be obtained.

Also, in order to improve the accuracy of this program, instead of this Euler method, an improved Euler method is used so that after calculating the new position (x', y', z'), new velocity direction $(v'_x, v'_y.v'_z)$, and new energy E', the program will not come to next iteration but will combine $(v'_x, v'_y.v'_z)$ with $(v_x, v_y.v_z)$ and E with E' to get the average. Then the averaged direction of velocity and energy will be used to get the the energy, position, and direction of velocity of this muon as well as the new time period δt in next iteration.

3. integration over all energy levels

Now the muon flux map for a given initial energy has been made, and the next step is to get the muon flux map for all energy levels that IceCube can detect. This program will run the previous procedure for muons with initial energy 1, $10^{0.1}$, $10^{0.2}$, $10^{0.3}$, $10^{0.4}$ ..... until 10^3 GeV (0.1 gap in log scale), then a numerical integration is used to combine their flux maps to get the flux map of all energy levels.

This method works as follow: Since for each run the theta and phi intervals are divided similarly, now for each theta and phi there are one flux corresponding to an energy. Supposing the relation between energy and flux is $\Phi(E)$, the total flux from all the energy levels is a numerical integration of $\Phi(E)$ with given energy-flux data. Also since the energy is in log scale with gap 0.1, instead of integrating $\Phi(E)$, the integration:

$$\int \Phi(E)dE = \int \Phi(logE)\frac{dE}{dlogE}dlogE$$

is performed numerically by calculating the area under $\Phi(logE)$ curve. Then the flux of all energy levels for this theta and phi region is obtained and this algorithm is performed on the whole map.

After that, the last step is to consider the effect of muon charge. Since positive muons and negative muons get deflected in opposite direction in geomagnetic field, the previous procedure needs to be preformed on both positive and negative muons. Then according to the flux ratio of positive and negative muons[4], the positive muon flux get multiplied by a certain factor according to the muons' energy, and then positive muon flux is added to the negative muon flux to generate the muon flux map for all muon energy levels and charges.

3 Result

1. total flux map

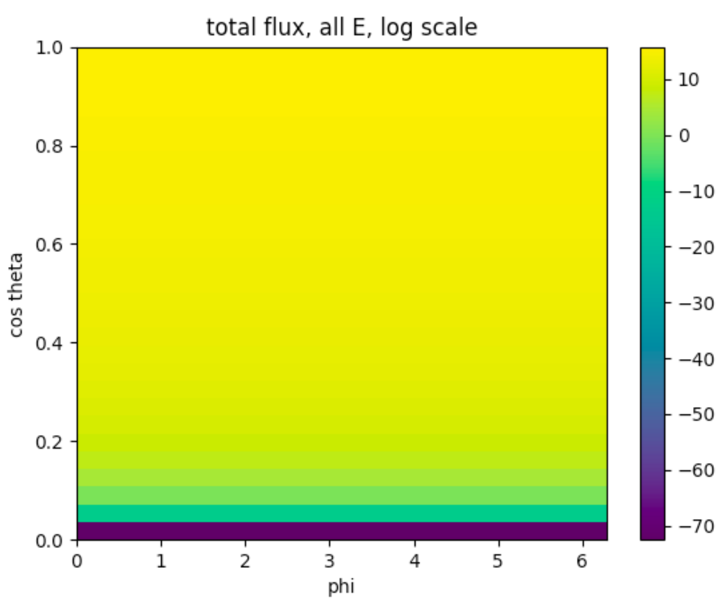


Figure 3: simulated total muon flux

The simulated total muon flux is shown in Figure 3. As mentioned before it is the total flux of all energy levels generated from numerical integration of muons with initial energies of $10^{0.5}$, 10, $10^{1.5}$, 10^2, $10^{2.5}$, and 10^3 GeV. As this plot shows, the muon flux, without row normalization, changes significantly when theta is greater than 84 degrees. This can be explained by the geometric structure in this simulation according to Figure 4.

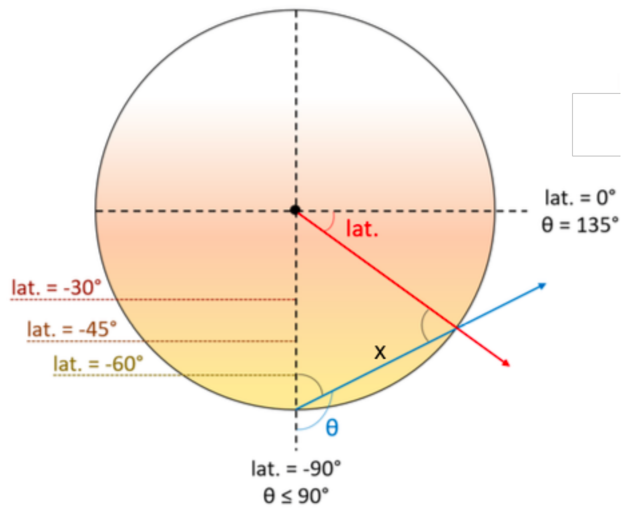


Figure 4: geometric structure of this simulation

As shown in Figure 4, the relation between ideal distance (ideal means without considering deflection effect of earth magnetic field) traveled and θ is:

$$x = \sqrt{R^2 - r^2 * sin^2(\pi - \theta)} + r * cos(\pi - \theta) \quad \mathbf{(4)}$$

$$\mathrm{R} = 6371\mathrm{km},\ \mathrm{r} = 6369\mathrm{km}$$

As shown in Figure 5, when θ is greater than 80 degrees the ideal distance traveled by muon increases significantly. Since the relation between energy and distance traveled is listed in equation(3), change in θ will cause an exponential growth in energy. Then since the flux is related to the muon's final energy and direction of velocity as shown in equation(1), as energy goes large the flux decreases. So it explains the large drop in total flux when $\theta > 84^o$

Then equation(3) can also serves as a check for this simulation. Since relation between energy and radius of muon is shown in equation(2), according to Sam's argument[5], at 1 GeV, a muon in a magnetic field of B = 60 μT has a radius of about 56 km (using an estimate of B in Antarctica from

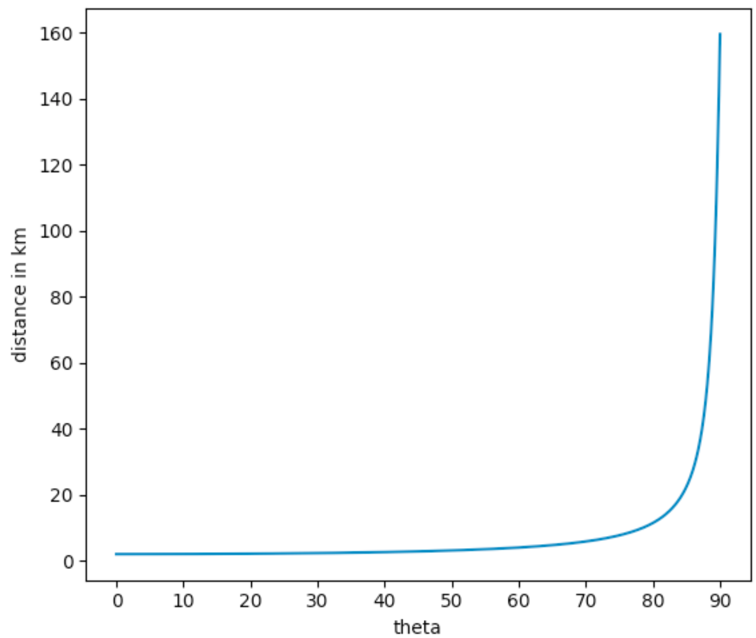
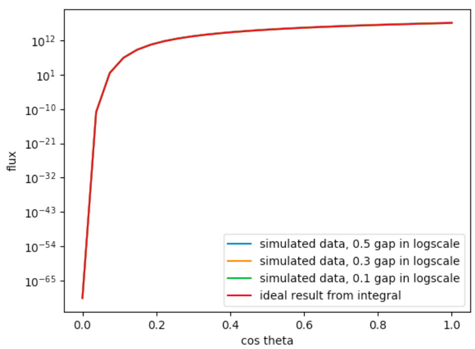


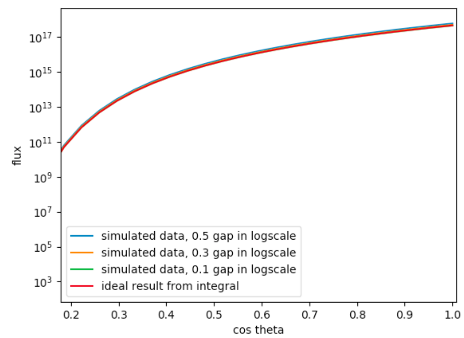
Figure 5: distance that muons travel without magnetic field effect vs theta (in degree)

some geomagnetic maps); at 100 GeV, R is on the order of Earth's radius. So the deflection effect of distance traveled by muon is tiny comparing to the geometric structure effect seen in Figure 5. As the result, the muon flux predicted by ideal distance should match with flux result simulated with geomagnetic field. Figure 6 shows this comparison.

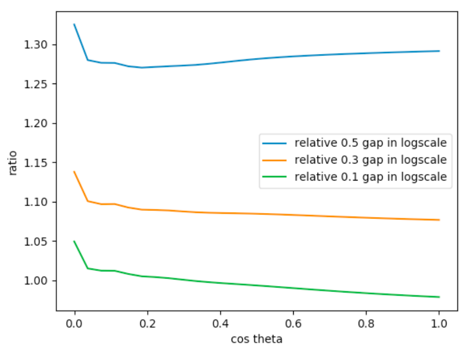
In Figure 6, the blue, yellow and green lines are the summation of muon flux for all phi, obtained from stimulated data but with different energy gaps. (for example 0.5 gap means the energy levels used to do numerical integration are $10^0, 10^{0.5}, 10^1, ...$) So it represents the relation between total flux and cos theta. The red line is the also the summation of muon flux for all phi but is obtained by using the equation (1), (3), and (4) without considering effect of geomagnetic field. In 6(a) the significant drop in flux at $\theta > 84^o$ appears, due to the large change in distance traveled. 6(b) is the zoom in graph of $cos(\theta) > 0.2$ and 6(c) is the ratio between result from different energy gaps and no magnetic field case. In no magnetic field case the energy range 1 to 1000GeV is divided into 1000 intervals, so the assumption is that it should be closest to the actual flux (without considering effect of magnetic field since



(a) different integration intervals

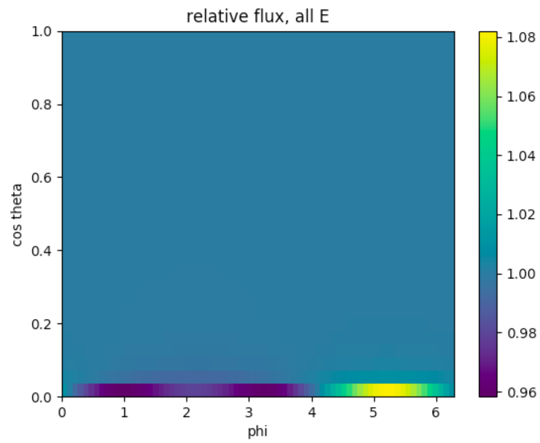


(b) zoom in smaller theta of (a)

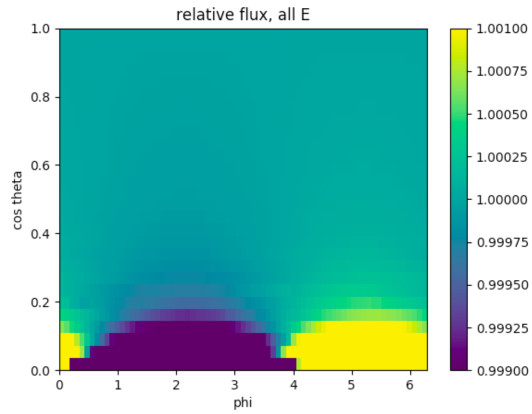


(c) ratio to the no B field case

Figure 6: flux in theta direction



(a) natural scale



(b) fixed scale of +-0.1%

Figure 7: relative flux in different scales

it is tiny). As 6(c) shows, as energy gaps get smaller the ratio gets closer to 1, especially for small theta. For large theta the effect of geomagnetic field increases so the ratio is larger than 1.

2. azimuth asymmetry

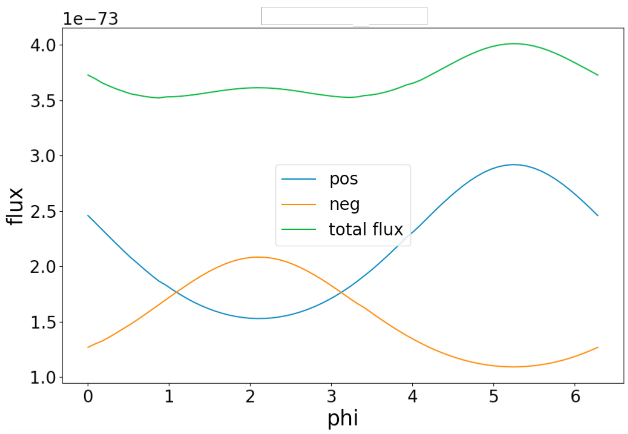
The Figure 7(a) is the row normalized muon asymmetry in default scale, and 7(b) is in 0.1% scale. This asymmetry is generated by the upward deflection effect of geomagnetic field. As shown in Figure 6, change in theta causes a change in muon flux, and as theta goes close to 90^o, the slope of Figure 6 gets steeper so a small difference in theta can cause relatively large effect on muon flux. For the same theta direction of initial velocity, different initial phi gives them different directions of geomagnetic field. As shown in figure 7, at phi from 0.5 to 4 the muon gets deflected downward, which increases theta and decreases flux according to Figure 6. Oppositely, at phi from 4 to 2π the muon gets deflected upward, which decreases theta and increases muon flux. However, instead of around 30% difference observed in Figure 1 from actual data, the maximum muon asymmetry is less than 10%.

Another problem about this result is that this 10% difference shows at $\theta = 90^o$, which in Figure 6 corresponds to super tiny muon flux that could be dominated by atmospheric neutrino effect. The asymmetry could go beyond $\theta = 90^o$, and, since when muons travels a longer distance it also gets deflected more, a larger asymmetry is supposed to appear. However for $\theta = 90^o$, according to equation (4) the distance is 159km, which, for a muon with remaining energy 10GeV, requires 10^{22}TeV to penetrate earth and reaches IceCube according to equation (3). Few muons with such high energy exist so going beyond $\theta = 90^o$ is meaningless. As the result, the maximum asymmetry caused by atmospheric muon should be less than 10%.

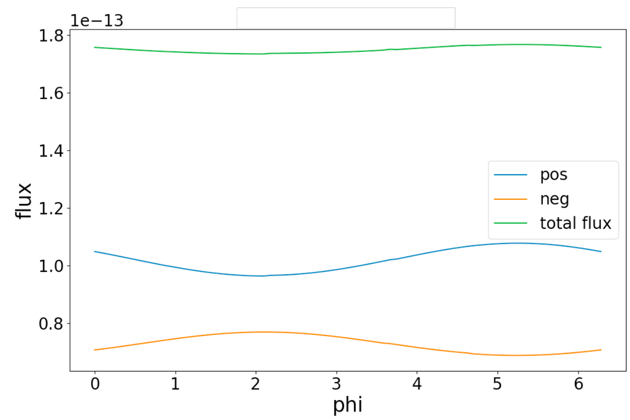
Another way to visualize the azimuth asymmetry is by 1D plots. The Figure 8 and 9 are the 1D version of last three rows in Figure 7, which corresponds to $\theta = 90^o$, $\theta \approx 87^o$, and $\theta \approx 85^o$. The blue line represents the positive muon flux of all energy levels, and the yellow line represents the negative muon flux of all energy. According to the ratio between positive and negative atmospheric muons[4], the positive muon flux is more than the negative one. The green line is the total flux, which is the sum of blue and yellow line.

As shown in Figure 7(a), the last row which corresponds to $\theta = 90^o$ has one maximum and 2 minimum, so does the green line show in Figure 8(a). Also, switching from 8(a) to 7(b) the magnitude of flux changes from 10^{-73} to 10^{-13}, as shown in Figure 6. The extent of asymmetry also decreases as θ gets smaller. At $\theta < 85^o$ the asymmetry is less than, as shown in Figure 9(b), 0.5%.

As the result,if only considering the effect of atmospheric muon, the observed 30% asymmetry in Figure 1[5] could be just the result of misreconstruction. However, atmospheric neutrino could also contribute to the muon flux. As discussed before few muons can travel from $\theta > 90^o$ due to the amount of energy loss; however, on Figure 1 the main asymmetry comes from below the horizon. This phenomenon, if not caused by misreconstruction, could be the effect of atmospheric neutrinos which decay into muons under the detector, and they can be the reason for this asymmetry.

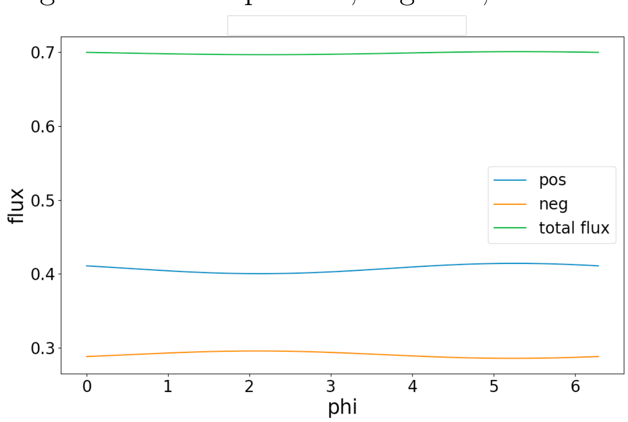


(a) $\theta = 90^o$

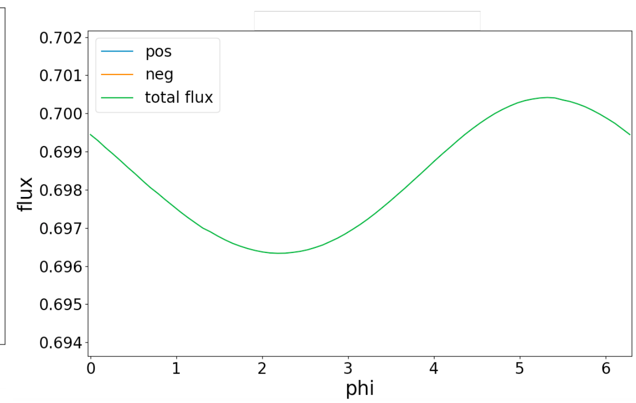


(b) $\theta \approx 87^o$

Figure 8: flux of positive, negative, and total muon at $\theta = 90^o$ and $\theta \approx 87^o$



(a) positive muon flux. negative muon flux and total flux



(b) zoom in total flux at $\theta \approx 85^o$

Figure 9: flux of positive, negative, and total muon at $\theta \approx 85^o$

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[5] Muon Filter Azimuthal Asymmetry, Samuel Joseph Fahey