Testing Bunch Properties of IOTA Proton Beam with pyORBIT

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1 Bunch Setup

In all the following discussion, the bunch is initialized according to the IOTA proton ring setup in Figure 1.

Parameter	*Value*
Beam kinetic energy, E	2.5 MeV
Revolution period, C_0, T_0	39.97 m, 1.83 μs
RF voltage, frequency, revolution harmonic	400 V, 2.18 MHz, 4
Synchrotron tune, Q_s	7×10^{-3}
Number of particles, beam current, N_e, I_e	9×10^{10}, 8 mA
Equilibrium beam emittance, ε_x, ε_y	0.3, 0.3 μm
Beam energy spread, bunch length, σ_E, σ_z	1.5×10^{-3}, 1.7 m
Space charge tune shift (unbunched, bunched)	-0.5, -1.2

Figure 1: IOTA Proton Beam

In pyORBIT, 10^4 macro particles are initialized, and each one represents 10^5 real particles. This set up represents totally 10^9 particles, which is the low end intensity of IOTA since due to current computing power, higher intensity may significantly increase the time complexity. These macro particles are initialized in a truncated gaussian distribution on x, x', y, y', and z. The emittance on both x and y plane are set to be $\epsilon = 0.3\mu m$ as shown in figure 1, so the rms size on transverse plane is determined as $\sqrt{\epsilon\gamma}$ and $\sqrt{\epsilon\beta}$. On longitudinal part, the rms size on z is set to be 1.7m according to figure 1, with 0 in initial momentum dispersion (although due to rf cavity the momentum dispersion is still expected to show up). Also, all of these gaussian distributions are truncated at 3σ.

For the IOTA sequence used in the following tests, all sextupoles are removed to avoid nonlinear effects other than space charge effect. The space charge effect in pyORBIT is simulated by a 2.5D model which splits every element in lattice into multiple parts where a kicker node is added to kick the bunch like space charge force. Both the number of space charge nodes and the gird of such nodes determines the accuracy and speed of this simulation. In the following discussion, all elements, including dirft space, quadrupoles, and dipoles, are set to have 2 space charge nodes with size 32 x 32 x 16.

2 Testing Longitudinal Tune Shift

Before testing the space charge effect, the longitudinal tune of single particles initialized at different initial z is plotted to check if the bunch is defined properly longitudinally. 120 particles are initialized at ($\sigma_x/2$, 0, $\sigma_y/2$, 0, z_i, 0) where z_i uniformly ranges from 0 to $3\sigma_z$. This bunch is tracked for 5000 turns, and each time when it passes through the initial position, the z coordinate of every particle in this bunch is recorded. For every particle in this bunch, an FFT algorithm is performed on its z value list and the place of peak shows its longitudinal tune. For example, figure 2 shows the FFT result of synchronous particle.

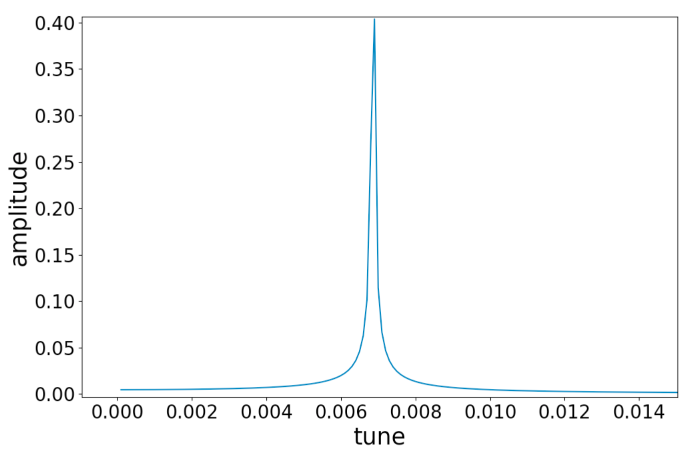

Figure 2: Synchronous Tune

According to figure 1, for synchronous particle initialized at z = 0, the longitudinal tune is $7 * 10^{-3}$ which corresponds to the location of peak at figure 2. Then the relation between initial z coordinate and longitudinal tune determined by all 120 test particles is plotted in figure 3.

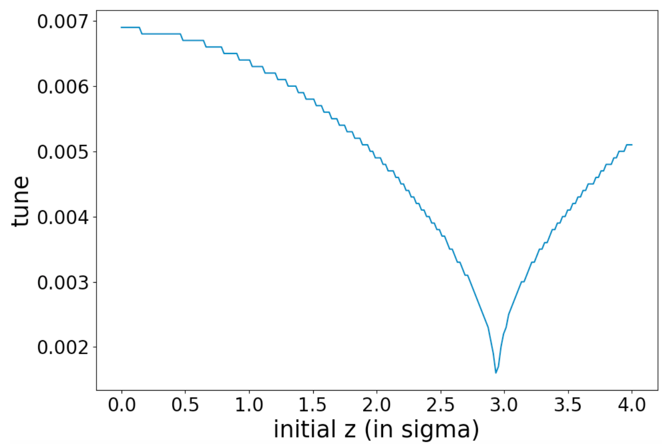

Figure 3: Initial z vs Longitudinal Tune

As shown in figure 3, as initial longitudinal coordinate deviates further from 0, the longitudinal tune decreases from synchronous tune $0.7*10^{-3}$ until the initial z gets larger than $3\sigma_z$. After this point, the longitudinal tune increases symmetrically. This result shows that the longitudinal size of bucket is roughly $\pm 3\sigma_z$, and by using a gaussian with truncation at $3\sigma_z$ it is guaranteed that the bunch is within the same bucket and is stable longitudinally.

3 Testing Bunch Emittance and $\beta_{x,y}$

In this test, a bunch is initialized as described in part 1 and is tracked for 5000 turns. Every time this bunch passes through the initial point, its $< x >$, $< x' >$, $< xx' >$, and $< x^2 >$ are calculated for both horizontal and vertical plane. Then the emittance is calculated by

$$\sigma_x = \sqrt{< x^2 > - < x >^2}$$
$$\sigma'_x = \sqrt{< x'^2 > - < x' >^2}$$
$$\sigma_{xx'} =< xx' > - < x >< x' >$$
$$\epsilon = \sqrt{\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}^2}$$

and the relation between turns and $\epsilon_{x,y}$ is plotted, for no space charge case and with space charge case.

One thing that needs to be clarified is that the $<x>$, $<x'>$, $<xx'>$, and $<x^2>$ are all calculated after removing the dispersion. Since the rf cavity exists in the sequence and initial longitudinal distribution is in random gaussian, the momentum separation will exist which causes dispersion. Such dispersion changes the result of emittance calculation, so, in order to compare the emittance with the expected result of $0.3\mu m$ as shown in figure 1, $<x>$ is calculated as:

$$<x>=\frac{1}{N}\sum_{i=1}^{N} x_i - \frac{\delta p_i}{p_0} * D_x$$

and so do other quantities. Another thing noteworthy is that in MADX the dispersion D_x is defined as $\frac{\partial x}{\partial p_t}$ where $p_t = \beta * \frac{\delta p}{p_0}$. So in order to use the dispersion value from MADX in the equation above, an extra Lorentz factor β needs to be multiplied to D_x, and for D_{px} this factor will be β^2.

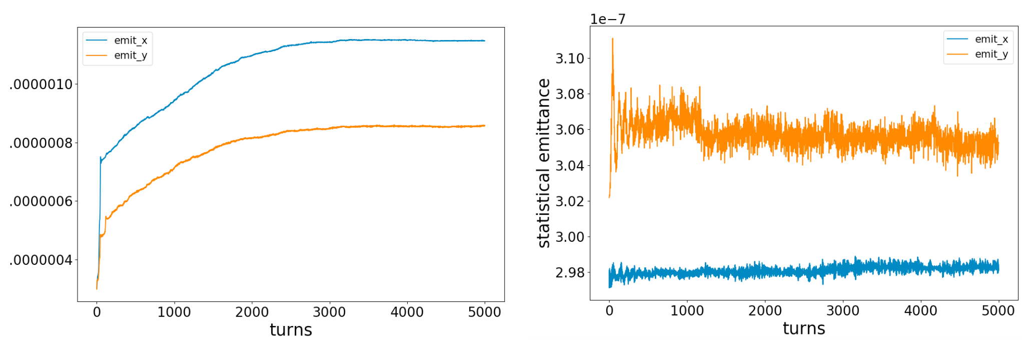

Figure 4: Emittance with (left) and without (right) space charge

Figure 4 shows the calculated emittance in each turn with and without considering space charge effect. According to the right plot, the emittance on both x and y plane is close to $0.3\mu m$, which is close to expectation value shown in figure 1. The left plot shows that with the space charge effect the beam emittance gets much larger due to the column force. The emittance on x is also much larger than emittance on y. The reason could be that since initially all particles' x and y coordinates are assigned in gaussian distribution with rms: $\sigma_{x,y} = \sqrt{\beta_{x,y}\epsilon_{x,y}}$ (as discussed in part 1), where $\beta_{x,y}$ are twiss function at initial position from MADX. Since $\beta_x = 0.68$ and $\beta_y = 3.28$ initially, rms on x is smaller than rms on y, which means that initially particles on x are distributed closer. This causes larger space charge force on x plane which makes the emittance on x larger.

As another cross check of the bunch property, value of the β twiss function at the beginning of lattice is calculated statistically in no space charge case, according to $\beta_{x,y} = \frac{\sigma^2_{x,y}}{\epsilon_{x,y}}$, and it is plotted against turn in figure 5.

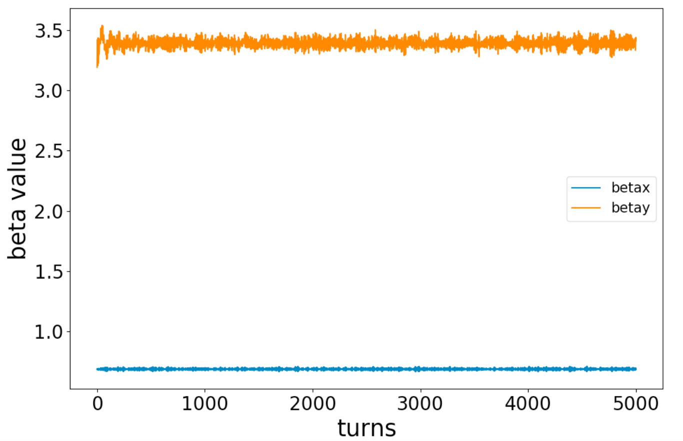

Figure 5: Beta Function w.r.t Turns

According to this plot, the value of β_x is 0.68 and β_y is 3.3. This result is close to the initial $\beta_{x,y}$ values solved by MADX.

4 FFT of Transverse Tune $Q_{x,y}$ without Space Charge

In this part an FFT is performed to get transverse tune $Q_{x,y}$ of bunch described in part 1 without space charge effect. The bunch is initialized and tracked for 7000 turns, and for each turn when it passes through the initial position, the $< x >$, $< x^2 >$, $< y >$, and $< y^2 >$ are recorded after removing dispersion effect. After the bunch has been tracked for the first 5000 turns and reached stable, in order to excite the transverse dipole mode a dipole kicker is applied such that every particle in this bunch is shifted by $\frac{\sigma_{x,y}}{2}$ in x and y axis. Although an actual dipole kicker changes x' and y' instead if x and y, this pseudo-kicker works since its only purpose is to make the bunch deviate from its original stable motion and excite the dipole mode. After gotten kicked, this bunch is tracked for another 2000 turns. Figure 6 shows the $< x >$ and $< y >$ w.r.t turns and it can be shown clearly the effect of dipole kicker at 5000 turns.

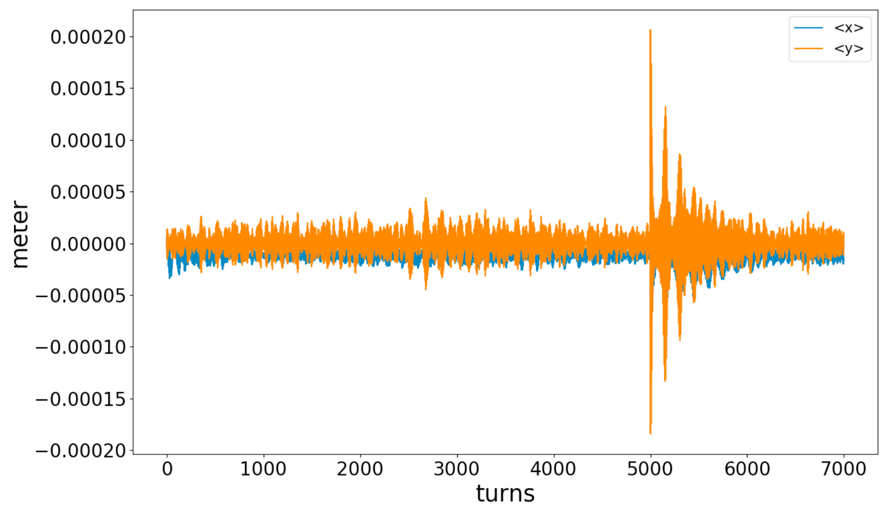

Figure 6: $< x >$ and $< y >$ w.r.t Turns

Then an FFT is performed on $< x >$ and $< y >$ from 5000 to 7000 turns to get the transverse tune, and the result is shown in figure 7.

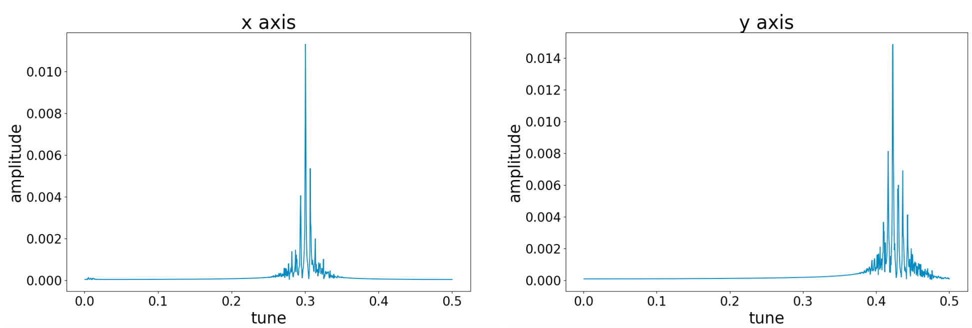

Figure 7: FFT of $< x >$ and $< y >$

According to MADX output, the IOTA sequence has $Q_x = 5.3$ and $Q_y = 5.42$. Since the result of FFT only shows the fractional part in 0 - 0.5 scale, the peak at 0.3 and 0.42 in figure 7 does correspond to the transverse tune. The secondary peaks around the largest peak are separated roughly by 0.007, which is the close to the synchronous tune. This fact implies that these peaks are caused by synchrotron oscillation due to the momentum deviation.

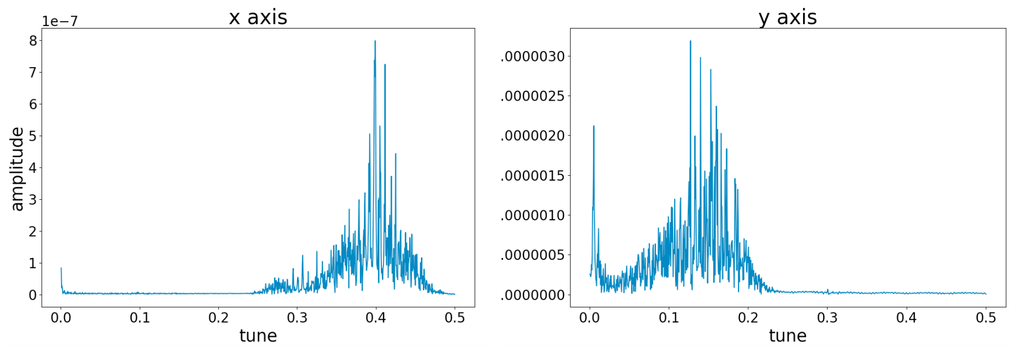

Figure 8: FFT of $< x^2 >$ and $< y^2 >$

Figure 8 shows the FFT result of $< x^2 >$ and $< y^2 >$. Since tune of $< x^2 >$ and $< y^2 >$ are two times the transverse tune, the value is expected to be $0.3 * 2 = 0.6$ and $0.42 * 2 = 0.84$, which corresponds to 0.4 and 0.16 in 0 - 0.5 scale of FFT result. This value also matches the peak shown in figure 8.

5 Testing Space Charge Tune Shift of Single Particle

In this part an FFT analysis similar to that of part 4 is performed on one single particle's x and y (instead of statistical $< x >$ and $< y >$). The bunch is set up in the same way as described in part 1, and a test particle is added at (σ_x, 0, σ_y, 0, 0, 0). This bunch is also tracked for 7000 turns with a dipole kicker applied at 5000-th turn, and for each turn the x and y position of this test particle is recorded. Such test is performed for two times, once with space charge effect and once without. For each test an FFT is performed on x and y records from 5000-th - 7000-th turn, and the result is shown in figure 9. According to this plot, in both x and y plane the space charge effect shifted the transverse tune of this test particle by 0.02.

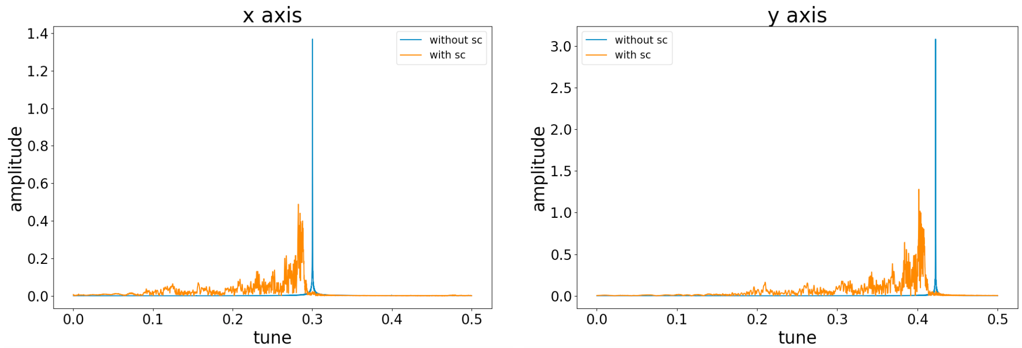

Figure 9: FFT of x and y with and without SC effect

6 Next Step

In the next step, a quadrupole kicker will be added to excite the quadrupolar mode which causes oscillation of bunch envelop. In the existence of space charge effect, the same bunch will be tracked for 5000 turns and the quadrupole kicker will be applied. Then σ_x and σ_y will be recorded. An FFT will be applied then to extract the quadrupolar mode tune.