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1 Convergence Test with intensity 10^9

In this part, IOTA lattice is modified such that its tune on x changes from 5.3 to 5.305 and on y from 5.3 to 5.295, in order to avoid the coupling resonance at qx = qy. Then a 2.5MeV proton beam with intensity 10^9 is used to test the stability of bunch inside the IOTA ring.

1. Bunch Initialization

In all the following tests, the bunch is initialized in the same distribution as described next: For total intensity m*n, m is the number of macro particles and n is how many real protons one macro particle represents. These macro particles are added to the bunch one by one, and for each of them the initial 6D coordinate (x, px, y, py, z, dE) is determined as following: Since the energy of this beam is 2.5MeV, rf voltage is 400 Volt, and harmonic number is 4, the stationary longitudinal bucket can be determined. In longitudinal phase space, (z, dE) coordinates of these 100,000 macro particles are uniformly randomly distributed inside an inner contour within the bucket to prevent debunch. In transverse phase space, according to IOTA parameters the normalized beam emittance is 0.3μm, which corresponds to unnormalized emittance $\epsilon = \frac{0.3e^{-6}}{\beta\gamma}$, so the rms length in transverse phase space can be calculated according to $\sigma_x = \sqrt{\beta_x \epsilon}$ and $\sigma_{px} = \sqrt{\gamma_x \epsilon}$. In both x-px and y-py plane, we use a random gaussian distribution with these rms values to determine (x, px) and (y, py) for each particle. Also, after (x, px, y, py, z, dE) has been determined for this particle, its x and px values are modified due to the dispersion effect. According to MADX twiss result, the dispersion of x at initial position is D_x =-2.832 and for px it is $D_{px} = -9.84 * 10^{-9}$ for this lattice. As a result $\frac{1}{\beta^2}\frac{dE_i}{E_0} * D_x$ is added to this particle's x and $\frac{1}{\beta^2}\frac{dE_i}{E_0} * D_{px}$ is added to its px. For a beam with intensity 10^9, the following example models it with 100,000 macro particles and macro size 10,000, and the distribution in phase space is shown in figure 1.

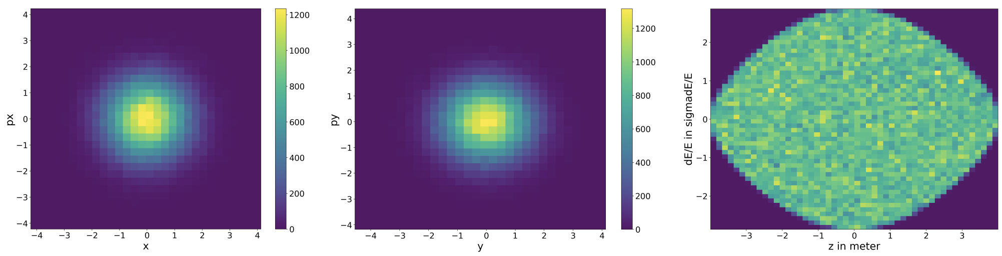

Figure 1: Phase space distribution of macro particles in bunch, axis scaled by σ

Then this bunch is tracked for 3000 turns for test under space charge effect, the final phase space distribution becomes:

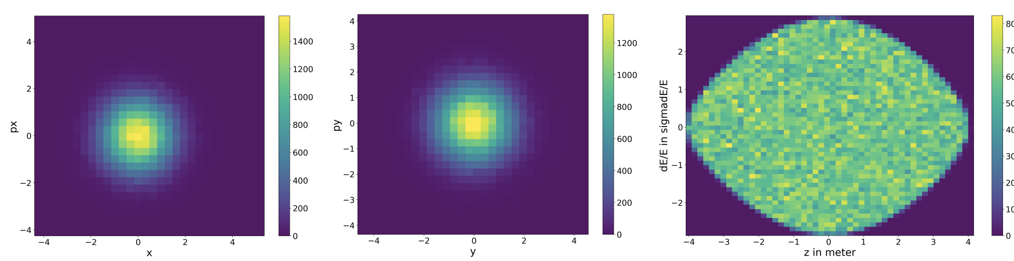

Figure 2: Phase space distribution of macro particles in bunch, axis scaled by σ

We see that the bunch remains mostly unchanged in both transverse and longitudinal phase space. Since the space charge effect is weak in 10^9 intensity, all particles survive after tracking.

2. Convergence Test on Number of Macro Particles

In this test, we examine the minimum number of macro particles that leads to a simulation with acceptable error. The bunch is initialized with 10000, 100000, and 1000000 macro particles in each trail, with macro size corresponding to maximum intensity 10^9. Also, instead of using the full intensity 10^9 in the beginning we perform a slow initialization with the first 200 turns, and after this the bunch is tracked for another 800 turns. The result of emittance vs turns and beta vs turns plots are shown for each number of macro particles we choose:

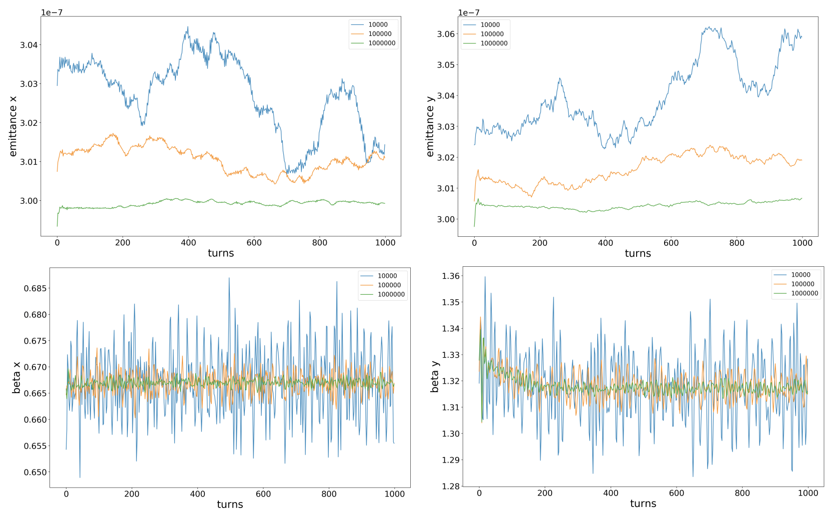

Figure 3: Emittance and β_0 vs turns for different number of macro particles

As shown in these plots, as number of macro particles increases we see a much more stable emittance and a much smaller variation of initial β value. It seems that in order to get small enough statistical uncertainty, using 10^6 macro particles is preferred.

According to MADX twiss function, the value of β initially is $\beta_{x0} = 0.6655$ and $\beta_{y0} = 1.3267$. As shown in figure 3 we see space charge effect shifts value of β, and after the first 200 turns if slow initialization, β_0 converges to a stable value.

3. Convergence Test on Number of SC Kicks

In pyORBIT, for every element there is a variable called nparts, which determines how many subnodes this element is divided into. Since pyORBIT uses kickers to model the effect of space charge and kickers are added in subnodes of each element, the nparts also determines the accuracy in modeling space charge effect. In this test, nparts = 2, 4, 8 are used to track the bunch with 100,000 macro particles with space charge effect in order to determine the effect of SC kickers in modeling. The result is shown in figure 4:

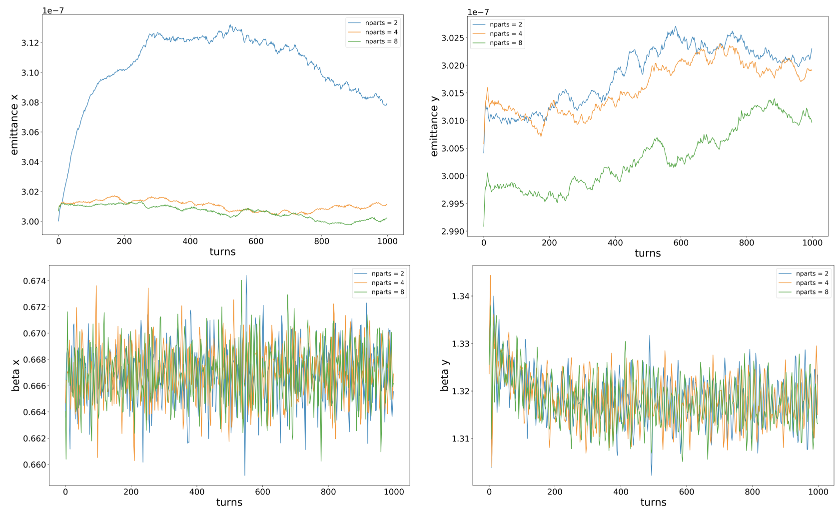

Figure 4: Emittance and β_0 vs turns for different nparts

We see that for y axis, the change in emittance is mostly the same, and value of β also converges for different nparts values. However, on x axis the nparts = 2 creates a large deviation. This is not caused by modeling of space charge effect, instead it is caused by nonlinear effect. Since for each subnode of this element, both linear and nonlinear transform are performed to calculate the particle's motion, nparts also determines the accuracy of modeling. Figure 5 shows the emittance vs turns without considering space charge effect, as we see here nparts = 2 also makes a bigh difference in x.

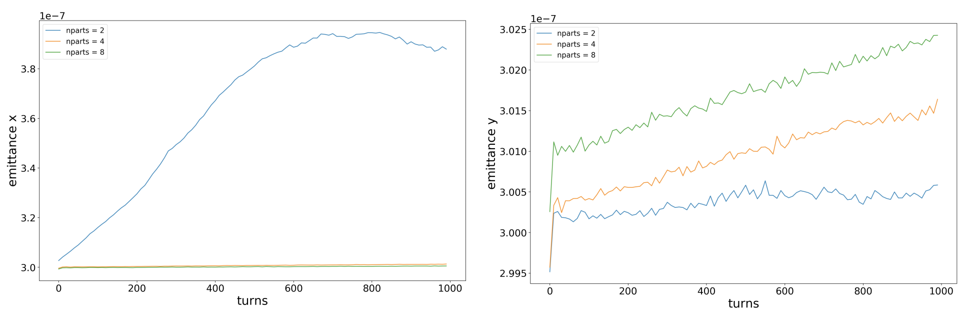

Figure 5: Emittance vs turns for different nparts at 0 SC effect

As the result, nparts should be set to at least 4 in order to correctly modeling the nonlinear effect in dipoles and quadrupoles.

4. Tune Footprint and 0 Amplitude Tune Shift

In this test we measure the tune footprint of IOTA with intensity 10^9 initialized as described in part 1. The bunch is firstly tracked for 3000 turns, with first 1000 turns as slow initialization. Then 400 test particles are assigned in ($lcos\theta$, px, $lsin\theta$, py, 0, 0) where l ranges from 0.1 to $6\sigma_x$, θ ranges from 0 to $\frac{\pi}{2}$, and px and py are non 0 only when $\theta = 0$ or $\frac{\pi}{2}$. The bunch is tracked for 3000 turns, and for each of these particles its x and y coordinates are recorded and used to perform the FFT. The result is made into a scatter plot with resonance lines as shown in figure 6. Since for the modified IOTA lattice, the bare tune is qx = 5.305 and qy = 5.295, we see that the pace charge effect shifts the tune of test particles away from this value.

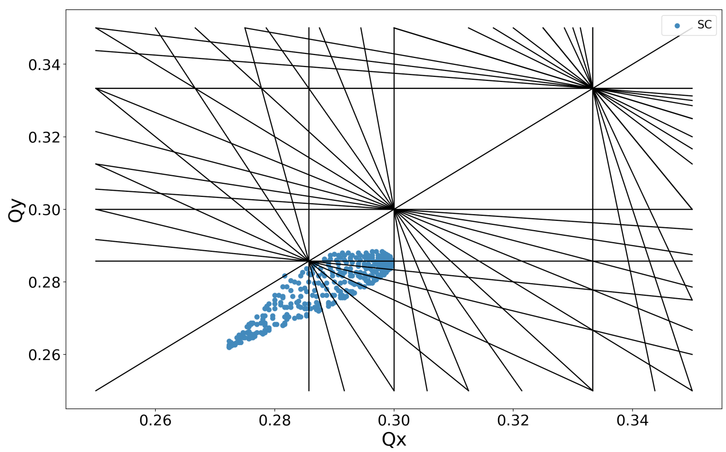

Figure 6: Tune footprint

In order to get the 0 amplitude tune shift, we add one single test particle with (0, $\frac{\sigma_{px}}{10}$, 0, $\frac{\sigma_{py}}{10}$, 0, 0) in the stable bunch we get after 3000 turns. Then this bunch is tracked for 3000 turns and the x and y records of this test [article is used for FFT. This test is performed for 2 times, first with slice on z = 4 and then with slice on z = 1. The former one is a more realistic simulation since longitudinal charge distribution is considered, however, the latter one assumes uniform longitudinal charge distribution so its result can be used to compare with analytical solution. The result is qx = 0.272 and qy = 0.264 for slice z = 4, and qx = 0.282 and qy = 0.273 for slice z =1. This difference makes sense since that according to figure 1, the water bag distribution in z-dE plane corresponds to a linear charge density larger in the middle. For the test particle added at z = 0, if longitudinal charge distribution is considered the space charge effect it experiences will be stronger than treating line charge density as uniform, which causes tune shift with slice z = 4 to be larger.

2 Machine Nonlinearlity

In previous discussion, the IOTA lattice we used does not contain sextupoles, in order to remove most of the nonlinear effect. In this section sextupoles are added back and adjusted so that the lattice has qx = 5.305, qy = 5.295, and dqx = dqy = 1.

1. Dynamic Aperture

First, this test determines the dynamic aperture of the new lattice. 10000 test particles are assigned in ($lcos\theta$, 0, $lsin\theta$, 0, 0, 0) where l ranges from 0 to $15\sigma_x$ and θ ranges from 0 to $\frac{\pi}{2}$. These particles are tracked for 100000 turns, and the initial position in x-y plane of surviving particles are recorded. The outmost particles represents the largest deviation from center where particles can survive in 10000 turns of tracking, so they are linked to be the dynamic aperture.

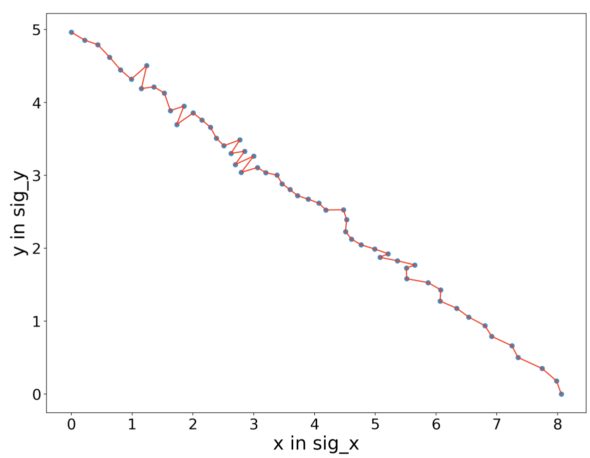

Figure 7: Dynamic aperture scaled by $\sigma_{x,y}$