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Runze Li

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1 Obtaining Stable Bunch with Space Charge Effect

Before we could test properties of intense proton bunch, this bunch much reach a stable state in lattice under space charge effect. As a result, in this part we first prepare the bunch and track it until a stable state is reached. In the beginning, the bunch is set up according to the proton beam used in IOTA[1]:

Parameter	*Value*
Beam kinetic energy, E	2.5 MeV
Revolution period, C_0, T_0	39.97 m, 1.83 μs
RF voltage, frequency, revolution harmonic	400 V, 2.18 MHz, 4
Synchrotron tune, Q_s	7×10^{-3}
Number of particles, beam current, N_e, I_e	9×10^{10}, 8 mA
Equilibrium beam emittance, ε_x, ε_y	0.3, 0.3 μm
Beam energy spread, bunch length, σ_E, σ_z	1.5×10^{-3}, 1.7 m
Space charge tune shift (unbunched, bunched)	-0.5, -1.2

Figure 1: parameters of the proton beam in IOTA

In this test, the intensity we used is actually 10^9 which is the low end intensity of IOTA. This intensity is modeled by 100,000 macro particles, and each one of them represents 10000 protons. These macro particles are added to the bunch one by one, and for each of them the initial 6D coordinate (x, px, y, py, z, dE) is determined as following: Since the energy of this beam is 2.5MeV, rf voltage is 400 Volt, and harmonic number is 4, the stationary longitudinal bucket can be determined as discussed in previous report. In longitudinal phase space, (z, dE) coordinates of these 100,000 macro particles are uniformly randomly distributed inside the bucket. In transverse phase space, according to figure 1 the normalized beam emittance is 0.3μm, which corresponds to unnormalized emittance $\epsilon = \frac{0.3e^{-6}}{\beta\gamma}$, so the rms length in transverse phase space can be calculated according to $\sigma_x = \sqrt{\beta_x \epsilon}$ and $\sigma_{px} = \sqrt{\gamma_x \epsilon}$. In both x-px and y-py plane, we use a random gaussian distribution with these rms values to determine (x, px) and (y, py) for each particle. Also, after (x, px, y, py, z, dE) has been determined for this particle, its x and px values are modified due to the dispersion effect. According to MADX twiss result, the dispersion of x at initial position is D_x =-2.832 and for px it is $D_{px} = -9.84 * 10^{-9}$ for this lattice. As a result $\frac{1}{\beta^2}\frac{dE_i}{E_0} * D_x$ is added to this particle's x and $\frac{1}{\beta^2}\frac{dE_i}{E_0} * D_{px}$ is added to its px.

After all macro particles are added to the bunch, this bunch needs to be tracked until equilibrium. In pyORBIT the precision of tracking under space charge effect is determined by many terms. First, for every element there is a variable called nparts, which determines how many subnodes this element is divided into. Since pyORBIT uses kickers to model the effect of space charge and kickers are added in subnodes of each element, the nparts also determines the accuracy in modeling space charge effect. Here in this test, the nparts is set to be 4 for all elements including drift space to guarantee computational efficiency as well as accuracy. Also, the grid size of space charge calculator determines the accuracy of space charge modeling. The grid size is defined as (x, y, z) where x, y define the transverse grid and z defines the longitudinal slices. Larger grid size increases the accuracy, however, it also requires more macro particles since the resolution increases. For the 100,000 macro particles we have in this test, the grid size is set to be (64, 64, 4).

When tracking this bunch with high intensity instead of using the full intensity 10^9 in the beginning we perform a slow initialization. In the beginning this bunch is set to have 100,000 macro particles and each of them represents 10 protons. Then during each turn the number of protons each macro particle represents is increased by 10 so the full intensity of 10^9 is reached at 1000 turns. Since the full intensity will cause strong space charge effect that may cause the emittance to blow up suddenly, the slow initialization helps to avoid the sudden increment in the beginning and make a gradually increasing emittance until stable.

After the first 1000 turns for initialization, this bunch is tracked for another 2000 turns. In each turn when this bunch passes through the initial point, its $< x >$, $< x' >$, $< xx' >$, and $< x^2 >$ are calculated for both horizontal and vertical plane. Then the emittance is calculated by

$$\sigma_x = \sqrt{< x^2 > - < x >^2}$$
$$\sigma_x' = \sqrt{< x'^2 > - < x' >^2}$$
$$\sigma_{xx'} =< xx' > - < x >< x' >$$
$$\epsilon = \sqrt{\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}^2}$$

Here all expectation values are calculated after removing the dispersion effect, for example, $< x >$ is calculated according to:

$$< x >= \tfrac{1}{N} \sum_{i=1}^{N} x_i - \frac{\delta p_i}{p_0} * D_x$$

Also, value of beta function in the beginning can be calculated as:

$$\beta_x = \tfrac{\sigma_x^2}{\epsilon_x}$$

The normalized emittance on x-px and y-py plane, as well as value of beta functions, are plotted with respect to turns. The result is shown in figure 2 and 3.

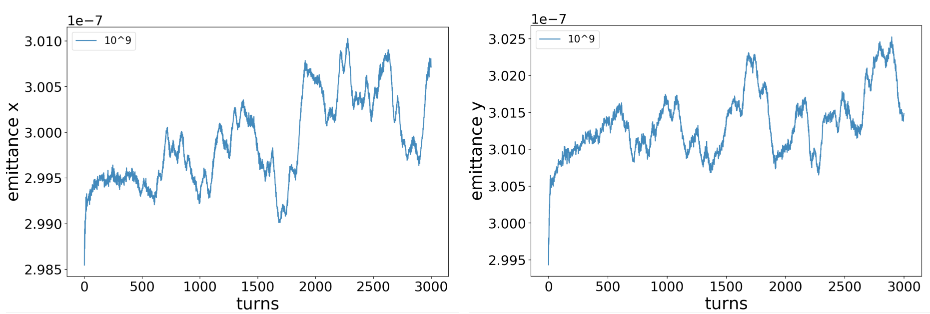

Figure 2: $\epsilon_{x,y}$ vs turns

The initial beta value without space charge, according to MADX twiss output, is 0.687 in x and 1.297 in y. According to figure 3, we see that for y the beta value calculated begins closely to 1.297 and then converges to a smaller value after the first 1000 turns where space charge effect gets stronger with increasing intnsity, and for x it stays around 0.688. The change in beta function is due to the effect of space charge, and since the value gets stable in the end we know that we now have a stable bunch for further analysis.

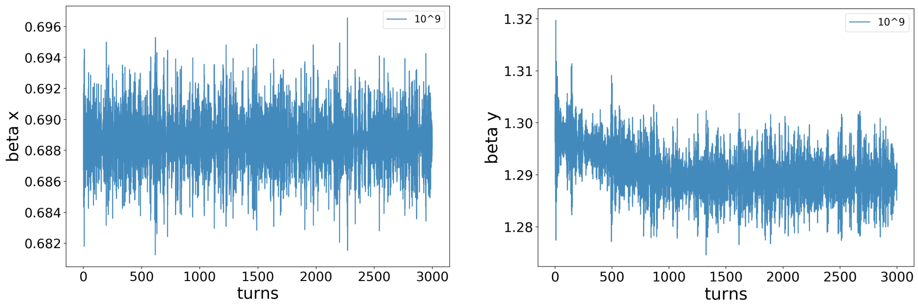

Figure 3: $\beta_{x,y}$ vs turns

2 Tune Spectrum

With the stable bunch under space charge effect, we try to get the tune spectrum. The rms size of current bunch is calculated as σ_x and σ_y, and in a rectangle with size $\pm 6\sigma_x$ and $\pm 6\sigma_y$ 625 test particles are added in the bunch on a uniform 25 x 25 grid with (x_i, p_x, y_i, p_y, 0, 0), where p_x and p_y are only non 0 for test particles with $x_i = 0$ or $y_i = 0$ to excite betatron oscillation. This bunch is then tracked for another 3000 turns, and the coordinates x and y for each of these 625 test particles are recorded. After tracking, for each test particle its coordinate list is used to perform the Fast Fourier Fransform to get its tune. Then the tune spectrum is made as a 2D scatter plot with horizontal and vertical axis the tune in x and y, as shown in figure 4.

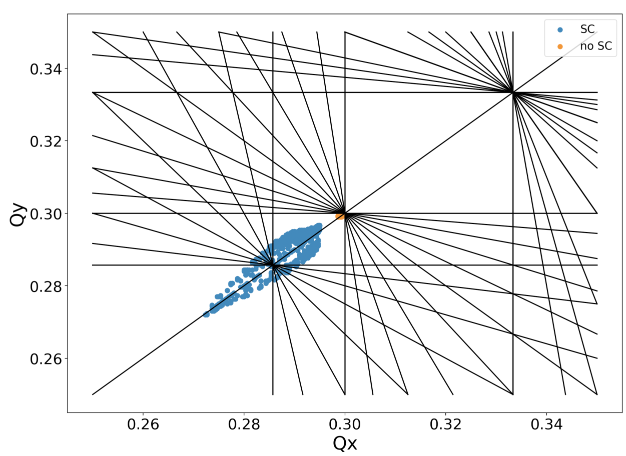

Figure 4: tune spectrum with and without space charge effect

Figure 4 shows the tune spectrum with and without space charge effect. Since the tune on x and y plane for IOTA is both 5.3, we see that in no space charge case those points are around (0.3, 0.3). With the space charge effect the tune of test particles are shifted.

3 Result of Other Intensity

Currently the intensity we use is 10^9 which is the lowest intensity IOTA is designed with. For higher intensity like 10^{10}, we choose to use 100,000 macro particles, each representing 100,000 protons, with the same distribution to try to get stable bunch. However, the emittance and beta function plot for the first 3000 turns like like this:

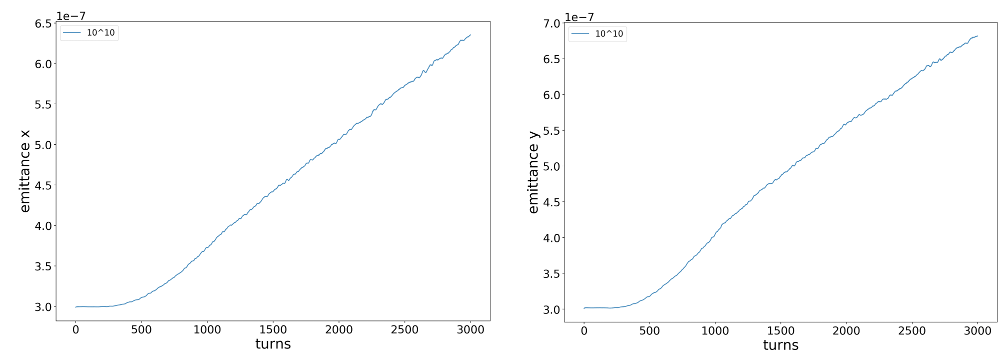

Figure 5: $\epsilon_{x,y}$ vs turns for intensity 10^{10}

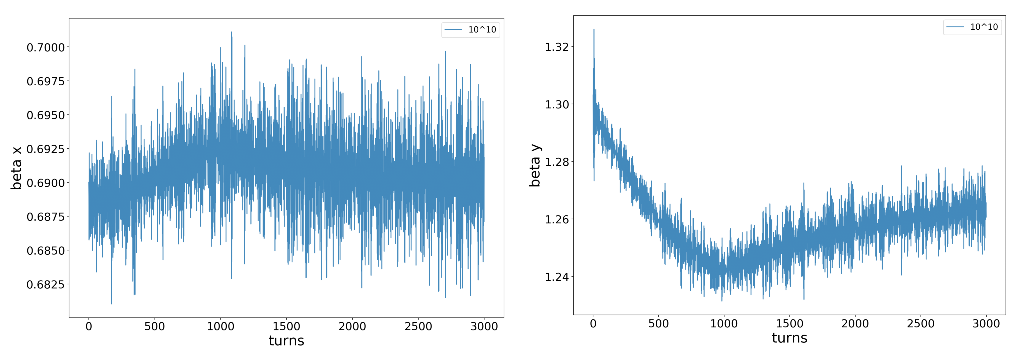

Figure 6: $\beta_{x,y}$ vs turns for intensity 10^{10}

Since the intensity is 10 times larger, we see clearly change in $\beta_{x,y}$ during first 1000 slow initializing turns, and then it slowly converges. However, as shown in emittance plot, the emittance does not show any sign of converging even if we track for 2000 more turns.

References

[1] Sergei Antipov, Daniel Broemmelsiek, David Bruhwiler, and et al. IOTA (Integrable Optics Test Accelerator): Facility and Experimental Beam Physics Program