Report August 31

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1 IOTA Lattice with Octupole Strength t-0.4

In this part, octupoles are added to IOTA to simulate the lowest order of nonlinear potential that nonlinear lens brings. First of all, the chromaticity of new lattice is measured. In MADX, twiss command is run with $\frac{\delta p}{p}$ ranging from -0.002 to 0.002 with step size 0.0004, and the tunes of all particles are plotted against $\frac{\delta p}{p}$. The plot looks like: The slope of linear regression result

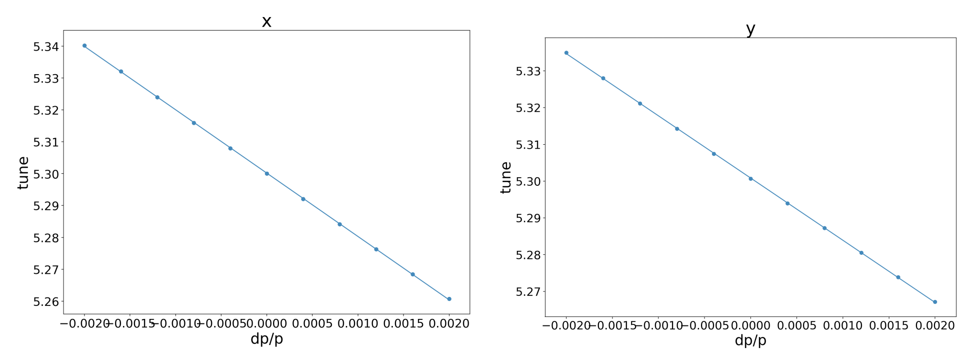


Figure 1: $q_{x,y}$ vs $\frac{dp}{p}$

shows the chromaticity, which is q_1 = -19.876 and q_2 = -16.937.

The next check focuses on the symplecticity of IOTA with octupole. In this part, the Jacobian matrix is calculated according to:

$$J = \begin{bmatrix} \frac{\partial x_f}{\partial x_i} & \frac{\partial x_f}{\partial p_{x_i}} & \frac{\partial x_f}{y_i} & \frac{\partial x_f}{\partial p_{y_i}} \\ \frac{\partial p_{x_f}}{\partial x_i} & \frac{\partial p_{x_f}}{\partial p_{x_i}} & \frac{\partial p_{x_f}}{y_i} & \frac{\partial p_{x_f}}{\partial p_{y_i}} \\ \frac{\partial y_f}{\partial x_i} & \frac{\partial y_f}{\partial p_{x_i}} & \frac{\partial y_f}{y_i} & \frac{\partial y_f}{\partial p_{y_i}} \\ \frac{\partial p_{y_f}}{\partial x_i} & \frac{\partial p_{y_f}}{\partial p_{x_i}} & \frac{\partial p_{y_f}}{y_i} & \frac{\partial p_{y_f}}{\partial p_{y_i}} \end{bmatrix}$$

where the coordinate $(x_i,\ p_{x_i},\ y_i,\ p_{y_i})$ represents the initial position of test particle in 4D phase space and $(x_f,\ p_{x_f},\ y_f,\ p_{y_f})$ is its final position after one turn's tracking. By shooting test particles with small difference in initial position, tracking them for one turn, and recording their final positions, we can calculate the numerical partial derivative and then the Jacobian matrix. The determine of J is calculated and the symplecticity test is performed by checking whether $J^T SJ = S$ holds, where S is the symplectic matrix.

For IOTA lattice with octupoles, this test is performed on (0, 0, 0, 0), $(\sigma_x,\ \sigma_{p_x},\ \sigma_y,\ \sigma_{p_y})$, 2 * $(\sigma_x,\ \sigma_{p_x},\ \sigma_y,\ \sigma_{p_y})$, and 2.6 * $(\sigma_x,\ \sigma_{p_x},\ \sigma_y,\ \sigma_{p_y})$, where σ is calculated based on normalized emittance $0.3\mu m$. The results are shown in the following figures:

```
Jacobin is
[[-3.09039598e-01  7.51884943e-01 -1.63834808e-11  4.54951697e-10]
 [-1.20297267e+00 -3.09035406e-01  8.34839086e-12 -2.31854063e-10]
 [ 2.62834908e-10 -3.37526364e-10 -3.13067564e-01  1.10472985e+00]
 [ 2.07172475e-10 -2.66046017e-10 -8.16478977e-01 -3.13067570e-01]]

det is 1.0000012130068225

J_tran S J is
[[-1.77765769e-18  1.00000121e+00 -1.72029058e-10  3.07791840e-10]
 [-1.00000121e+00  3.73296236e-18  1.93506756e-10  3.65846139e-10]
 [ 1.72029058e-10 -1.93506756e-10 -1.00664819e-17  1.00000000e+00]
 [-3.07791840e-10 -3.65846139e-10 -1.00000000e+00  1.64818562e-17]]
```

(a) 0 σ

```
Jacobin is
[[-0.31658051  0.72710601  0.1303579   0.38652194]
 [-1.15574118 -0.38267385 -0.00449798  0.28198945]
 [ 0.29825978  0.17563816 -0.33322655  1.10189761]
 [ 0.28406971  0.29657616 -0.82754241 -0.14897506]]

det is 1.0000642751953328

J_tran S J is
[[ 2.73144212e-18  1.00005670e+00 -7.90810408e-05 -2.05058295e-06]
 [-1.00005670e+00  3.02959366e-17  9.30740561e-05 -1.42073386e-05]
 [ 7.90810408e-05 -9.30740561e-05  2.55529972e-17  1.00000758e+00]
 [ 2.05058295e-06  1.42073386e-05 -1.00000758e+00  9.08317981e-18]]
```

(b) 1 σ

```
Jacobin is
[[ 0.64550319  1.33954517  0.45335424  2.33848641]
 [ 1.98931635  1.61754274 -0.44640364  3.47791223]
 [ 1.3031692   0.70151733 -0.34349989  1.08007249]
 [ 2.37399338  3.28884908  0.28738879  3.81470288]]

det is 0.9997635719068217

J_tran S J is
[[-1.31926755e-16  9.99879199e-01 -3.72897902e-05  1.32537118e-04]
 [-9.99879199e-01  2.37108745e-17  2.98190491e-05  1.03591642e-04]
 [ 3.72897902e-05 -2.98190491e-05  6.49480524e-18  9.99884351e-01]
 [-1.32537118e-04 -1.03591642e-04 -9.99884351e-01  4.46465938e-16]]
```

(c) 2 σ

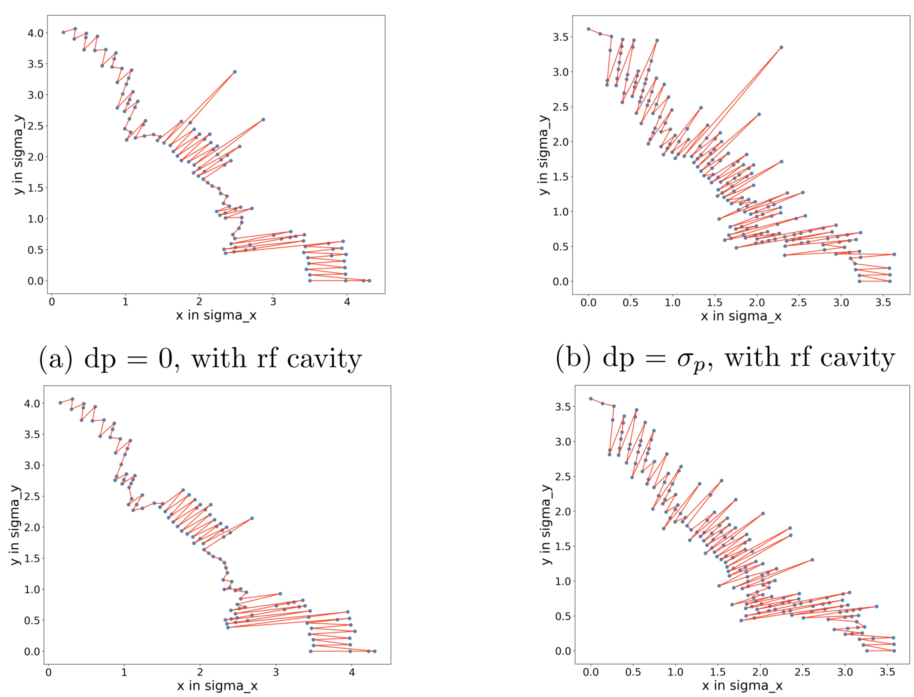
```
Jacobin is
[[-3437.95208464 -3315.74233492 -1014.48277717 -5048.25930237]
 [-3275.96918319 -3159.45303805  -966.62557482 -4810.21395582]
 [ -452.82119627  -436.89726345  -133.11441394  -661.73250098]
 [ -445.25230894  -429.99213143  -131.38582783  -651.7372715 ]]

det is -126.46213568003252

J_tran S J is
[[ 6.25259373e-10 -4.15140915e+01  2.28827166e+01 -1.74279348e+02]
 [ 4.15140915e+01 -2.45113074e-10  3.46061416e+01 -1.05674506e+02]
 [-2.28827166e+01 -3.46061416e+01  3.66699745e-11 -8.39848291e+01]
 [ 1.74279348e+02  1.05674506e+02  8.39848291e+01  1.80921970e-09]]
```

(d) 2.6 σ

Figure 2: Symplecticity test at different places for lattice

As shown in figure 2, when the initial position deviates further from center of orbit, the determinant of Jacobian deviates further away from 1 and $J^T SJ$ differs more from S. When the initial position is 2.6 * $(\sigma_x,\ \sigma_{p_x},\ \sigma_y,\ \sigma_{p_y})$, the one turn Jacobian matrix cannot be called symplectic anymore, and all test particles start even further away are lost in one turn.

Since we find that IOTA lattice with octupole strength t - 0.4 becomes non-symplectic after 2σ, in this part we try to find the dynamic aperture of this lattice to get a basic idea about how we should initialize the bunch to prevent particle loss. With normalized emittance $0.3\mu m$, 10000 particles are initialized at ($lcos\theta$, 0, $lsin\theta$, 0, 0, dp) where l ranges from 0 to $8\sigma_x$ and θ ranges from 0 to $\frac{\pi}{2}$. The energy is set to be either all 0 or σ_p for all test particles, and with rf cavity on and off we are able to see the effect of chromaticity on dynamic aperture. These particles are tracked for 100000 turns, and the initial position in x-y plane of surviving particles are recorded. The outmost particles represents the largest deviation from center where particles can survive in 10000 turns of tracking, so they are linked to be the dynamic aperture.



(a) dp = 0, with rf cavity

(b) dp = σ_p, with rf cavity

(c) dp = 0, no rf cavity

(d) dp = σ_p, no rf cavity

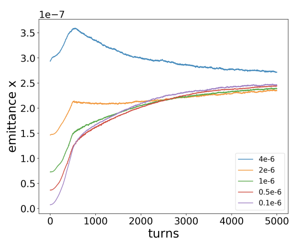
Figure 3: Dynamic aperture for different setups

In this figure, we do see chromaticity effect reduces the size of aperture.

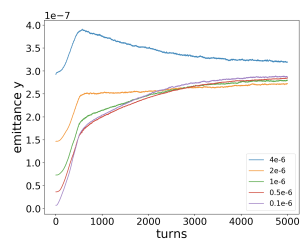
2 IOTA with Octupoles and Space Charge Effect

In this part, both octupoles and space charge effect are considered. However, as shown in the previous section, for the octupole lattice with t - 0.4 both dynamic aperture and symplecticity test show a small stable region. Since space charge will increase the beam size, we expect to see even large particle loss. In order to reduce the beam loss, in the following tests octupoles with reduced strength t - 0.2 are used.

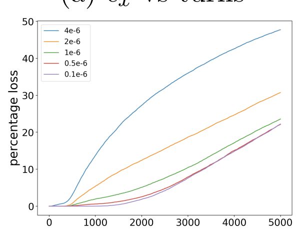
The bunch setup is mostly the same as previous. The transverse plane x-px and y-py is initialized with a gaussian distribution, and for longitudinal plane it is uniform within a contour inside separatrix. The only change is that the transverse emittance has been changed in order to find the smallest particle loss we can get. The parameters of IOTA listed in paper shows unnormalized emittance $\epsilon_x = \epsilon_y = 4.11 * 10^{-6}$, and in the following tests a series of values from $4 * 10^{-6}$ to $0.1 * 10^{-6}$ are used. With total intensity 10^{10}, firstly the test is performed with 10^5 macro particles, then, after we get a basic idea about for which initial emittance the particle loss and emittance converge, 10^6 macro particles are used to make accurate check. (Note: in the following plots a slow initialization is performed in the first 500 turns)



(a) ϵ_x vs turns



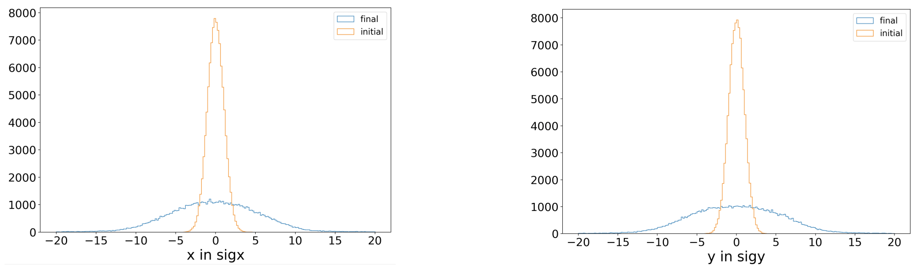
(b) ϵ_x vs turns



(c) percentage loss vs turns

Figure 4: Emittance and particle loss using 10^5 Mps

In these plots using 10^5 macro particles, we see that a converging particle loss plot is reached by initial emittance values less than $1 * 10^{-6}$. If we plot the initial and final distribution on x and y, we see that the space charge effect expands the bunch, however, since the fringe particles are removed due to the nonlinear effect and aperture, the emittance gets stable in the end.

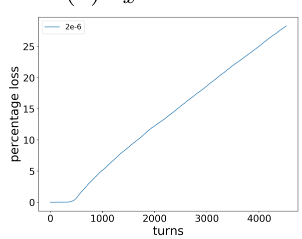


(a) Distribution on x (b) Distribution on y

Figure 5: Initial and final distribution for initial $\epsilon = 0.1e-6$

A more accurate check is done with 10^6 macro particles, as shown in the following plots.

(a) ϵ_x vs turns (b) ϵ_x vs turns



(c) percentage loss vs turns

Figure 6: Emittance and particle loss using 10^6 Mps

We see that after switching to 10^6 macro particles, the basic shape of plot does not change, but emittance growth and particle loss decrease a little bit due to the reduction of statistical error.

3 Next Step

1. get the tune spectrum for IOTA with octupole, without space charge effect

2. get the tune spectrum & incoherent tune shift for IOTA with octupole, without space charge effect

For this part, since we keeps losing particles, how to make the result as accurate as possible? I am thinking about first initialize the bunch with emittance 1e-6, then track the bunch for 3000 turns so that the emittance is basically stable but particle loss is around 10%. Then add test particles, track for 2000 turns and do FFT.

3. Nonlinear lens may provides a more stable beam?