**Proof by Mathematical Induction**

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**Let F(0) = 0  and   F(n+1) = 6F(n)+20   for n >= 0**

**Using induction, prove that  F(n) = 4\*( (6^n) – 1 )**

**Identify the basis step, inductive hypothesis, and inductive step.**

I will start this proof by induction by evaluating the basis case of n=0

Note that for [ F(n) = 4\*( (6^n) – 1 ) ] and n=0:

F(0) = 4\*((6^0) – 1) = 4\*(1-1) = 4\*0 = 0

F(n+1) = F(0+1) = F(1) = 4\*((6^1) - 1) = 4\*(6-1) = 4\*5 = 20

In addition, note that for [F(n+1) = 6\*F(n) + 20], [F(0)=0] and n=0:

F(n+1) = F(1) = 6\*F(0) + 20 = 6\*0 + 20 = 20

By substitution, we have shown that **[ F(n) = 4\*( (6^n) – 1 ) ]** yields 0 when n=0. In addition, both equation **[ F(n+1) = 4\*( ( 6^n+1 ) – 1 ) ]** and equation **[ F(n+1) = 6\*F(n) + 20 ]** yield 20 when n=0. Based on these observations, we have shown that **[ F(n+1) = 6\*F(n) + 20 ]** and **F(0)=0** imply **[ F(n) = 4\*( (6^n) – 1 ) ]** when n=0. This shows that the basis case is true.

Next, consider the arbitrary integer k, with k>=0. In the inductive hypothesis, we assume that equation **[ F(k+1) = 6\*F(k) + 20 ]** and **F(0)=0** implies **[ F(k) = 4\*( (6^k) – 1 ) ]** for this arbitrary value k. In the inductive step, I will show that whenever we assume the inductive hypothesis is true, equation **[F(k+1) = 6\*F(k) + 20]** and **F(0)=0** implies **[F(k+1) = 4\*((6^(k+1)) - 1)]**.

F(k+1) = 6\*F(k) + 20 = 6 \* ( 4\*( (6^k) – 1 ) ) + 20 = 6\*4\*(6^k) – 6\*4 + 20 = 4\*( 6^(k+1) ) – 4

F(k+1) = 4\*(6^(k+1)) – 4 = 4\*((6^(k+1)) – 1)

In the above transformations we have seen that whenever we assume the inductive hypothesis that **[ F(k+1) = 6\*F(k) + 20 ]** and **F(0)=0** implies **[ F(k) = 4\*( (6^k) – 1 ) ]**  for some non-negative value k, it follows that **[ F(k+1) = 6\*F(k) + 20 ]** and **F(0)=0** implies **[ F(k+1) = 4\*( ( 6^(k+1) ) – 1 ) ]**. Because we have demonstrated that the basis case is true and that case k being true implies that case k+1 is true, we have proven via induction that equation **[ F(n+1) = 6\*F(n) + 20 ]** and **F(0)=0** implies **[ F(n) = 4\*( (6^n) – 1 ) ]** for all values of n, with n>=0.