Digital Filter Design For Low Frequency Assets

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Financial time series is a series of digital data points in successive order taken over a specified interval of time. The data points considered for this project are the closing prices of asset. The financial asset i.e. the stock price follows trends and drives the market in a certain direction. Financial analysts all over the world use financial time series to predict market movement using mathematical models or estimators. Two of the most popular estimators used for estimation of the trends are leading and lagging estimators. The leading estimators have high prediction rate, whereas the lagging estimators have a large latency period, as a result lagging estimators are always behind the market movement. The goal of the project is to improve the design of a lagging estimator i.e. a Moving Average Convergence Divergence (MACD) Filter using filter design techniques. We have used minimum phase filter design methodology to improve the design of lagging estimator by minimizing the Group Delay (grd).

I. INTRODUCTION

In financial markets, time series is used to track specific data points, whether that data point is closing price of an asset or a mean value over a specified time, where the data points are recorded at regular intervals[1]. The intervals at which the digital data points are recorded are decided by the investor or the analyst looking to draw relevant information for further analysis. The raw financial data points collected are random in nature and represent the characteristics of a non-stationary process. In order to draw insights from the data, we need the data points to be stochastic in nature i.e. a weakly stationary process as the parameters such as mean and variance exhibit stability in time, and therefore such a signal can be used for forecasting market behaviour[2].

If data points exhibit a stationary process, then such a signal can't be used for generating trends as statistical parameters do not change with time shift. When the data points collected are a weakly stationary process and represent a low-frequency signal, such a signal is optimal for further processing using tools in digital signal processing. For generating trends, we use a Moving Average Convergence Divergence (MACD) indicator which is a trend following momentum indicator, which calculates the difference between two moving averages. The indicator is popularly known as lagging estimator as it is based on past inputs of the signal which results in a high latency period/lag when compared to the current market movement as it considers the inputs from the past, which ultimately results in low accuracy in trend prediction[3]. In order to reduce the latency period and increase the accuracy of the MCAD indicator, we use filter design techniques in signal processing to reduce the group delay, false crossover points and attenuate the noise. We plan to improve the design of MACD filter and increase the accuracy of estimating trends and forecasting market movements.

II. SYSTEM OVERVIEW

At any time instant *t*, ROA for a given asset with price P_t is defined as follows[2]:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \qquad \dots (1)$$

During financial time series analysis, ROA, i.e. return signal was considered as a reference signal to estimate trend because of favourable statistical properties and resemblance to a weakly stationary process. The digital signal comprises of two parts namely Trends and Noise. The trend represents the relevant information i.e. the changes the signal undergoes with respect to time, whereas the noise represents the false information which can be interpreted as relevant information[4]. The presence of noise may increase the risk involved by increasing transaction cost of the concerned asset.

In a low-frequency trading scenario, the information significant for trend estimation is contained in low frequency range and the noise lies in high frequency range. In signal processing, the circuits designed to perform frequency selective operation are known as filters. One such filter that is used to filter the noise present in a low-frequency signal is called moving-average convergence divergence filter. As we know, the associated lag is a major factor which delimits its performance in trend prediction. This lag in the output signal is mapped with non-linearity in the phase response of a filter. For a low pass filter, group delay gives us the measure of delay of modulation envelope or how much the output signal is delayed in time with respect to input signal. It is calculated by differentiating the phase lag with respect to frequency[5] i.e.

$$grd(\omega) = -\frac{\partial\theta(\omega)}{\partial\omega} \qquad \dots (2)$$

While we aim to design a filter using lagging estimator, the filter is modified to reduce the group delay. The group delay is inversely proportional to the cut-off frequency as seen in equation (2) and is directly proportional to the attenuation level as the input signal is delayed by infinite samples when the filter has an infinite attenuation level.

The group delay of the MA filter is almost equal to half the length of the filter and is defined as follows[6]:

$$grd(\omega) = \frac{L}{2} - \frac{1}{2} \qquad \dots (3)$$

Hence, greater the length of moving average, larger is the delay experienced by the input signal. Therefore, to reduce the group delay of a linear phase filter, we implement the concept of a Minimum Phase system. For a strictly minimum phase filter, the key features are[7]:

- Both Poles and Zeros are inside the unit circle.
- The phase response is nonlinear because of which the group delay takes positive and negative values.
- Energy of the system is concentrated at the start of the impulse response. Hence, the delay of energy is reduced.

In a Minimum Phase System, the magnitude and phase response are dependent on each other, as if we are given either of the two i.e. magnitude or phase response, we can calculate the other using Hilbert relations[8].

III. ALGORITHM

The methodology implemented in optimising the filter design and analysing the performance of the output signal is defined below:

Step 1: Calculation of return signal, i.e. *roa(t)*

Step 2: Set l_1 to an initial value and vary l_2 within a defined range

where, l_1 - length of MA_1 and l_2- length of MA_2; $l_2>l_1$

Step 3: For every step change in $l=l_2-l_1$, calculate cut-off frequency for low-pass filter, i.e. $\omega = 2 * pi/l$

Step 4: Implement MACD and Minimum phase filter design.

Step 5: Compare the frequency response obtained in Step 4 on the following basis:

- Lower the group delay, better is the filter design
- The poles and zeros should lie within the unit circle for a stable and causal model
- Attenuation level should be kept at an acceptable level to maintain a trade-off between group delay and high frequency noise

Step 6: Pass the *roa(t)* signal calculated on Step 1 as an input to filter and compare the output signal, *o(t)* for statistical properties like mean, variance and cross-over points.

The input signal in consideration, i.e. the asset signal represents the real time financial data indicating S&P 500 stock market index which is collected from Yahoo Finance. The ROA signal is calculated by taking the difference of the prices at time instant t and $t-1$. The resultant signal is fed as an input to the estimator under consideration. The filter length, l and the range of pass band which filters the dominant information present at low frequencies low-frequency behaviour of the asset was varied to study the impact of filter design parameters on the performance of estimator and output signal.

IV. SOFTWARE PACKAGE DESCRIPTION

MATLAB (matrix Laboratory) is a proprietary programming language developed by Math Works, MATLAB is a high performance and fourth generation programming language for numerical/technical computing and multi-domain simulations. It allows you to manipulate matrices, plot functions and data, implement various algorithms, create user interfaces and has the capability to interface with programs written in other languages including C, C++, C#, Java, SQL, Hadoop and Python. It's one of the most widely used environments for Digital Signal Processing, Machine Learning, Image Processing, Computer Vision, Communications, Computational Finance, Robotics and much more[9][10].

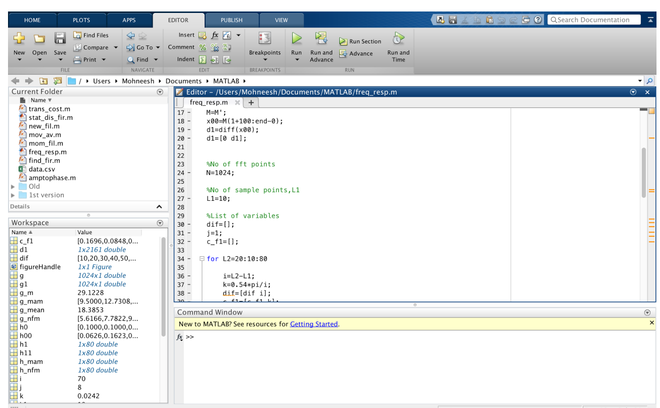

Figure 1: MATLAB User Interface

V. EXPERIMENT RESULTS

The frequency response of MACD and Minimum phase estimators was compared on the basis of group delay. The MACD filter was implemented where l_1 was fixed at 10 and

l_2 was varied from 20 to 80 which resulted in varying the filter length l from 20 to 80. For filter length l= 20, the difference between the samples for the given filter is 10 and it progressively increases by a step size of 10 as the filter length increases. Keeping the range of the pass band as similar to MACD filter and reducing the attenuation level for the same amplitude response, the minimum phase response was estimated using the Hilbert relationships to observe minimum or less group delay than MACD indicator. The frequency response and group delay for filter length, l=20 is shown in Figure 2.

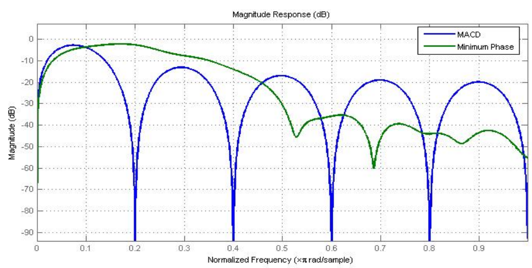

Figure 2: Comparison of Frequency Response of MACD and Minimum Phase Filter

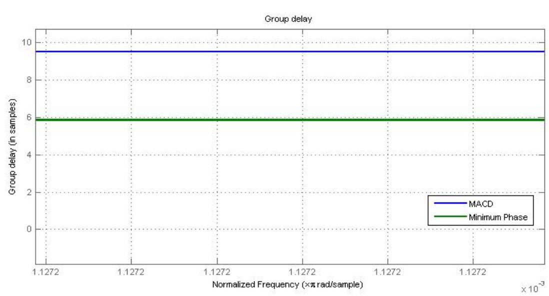

Figure 3: Comparison of Group Delay of MACD and Minimum Phase Filter

As observed from Figure 3, the group delay has been reduced in the proposed filter design. The group delay exhibited by MACD for $l=20$ is around 9.5 samples whereas Minimum Phase exhibits a delay of 6 samples. This reinforces the fact that the new output signal estimated by minimum phase filter design is more accurate for studying the trend because the lag between the input and output signal has been reduced by a margin of 41%. Since these filters represent a nonlinear phase system, the group delay was observed within the pass band of the filter for computational purposes.

The mean group delay was calculated within the pass band range for MACD and MP filter and the dependence graph between group delay and cut-off frequency was plotted which can be seen in Figure 4.

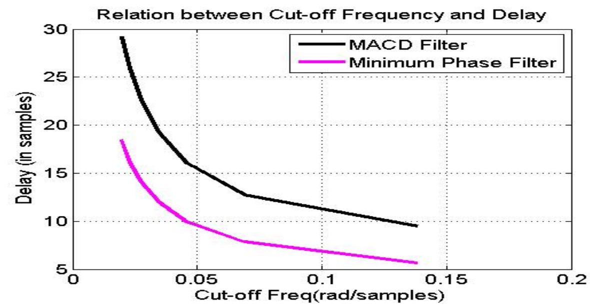

Figure 4: Dependence Graph between Group Delay & Cut-off Frequency

Table 1: Mean Group Delay of MACD & Minimum Phase

Parameters (Group Delay)	**MACD**	**Minimum Phase**
$\boldsymbol{grd}(\omega) for\ L = 30$	12.73	7.78
$\boldsymbol{grd}(\omega) for\ L = 60$	22.5	13.97

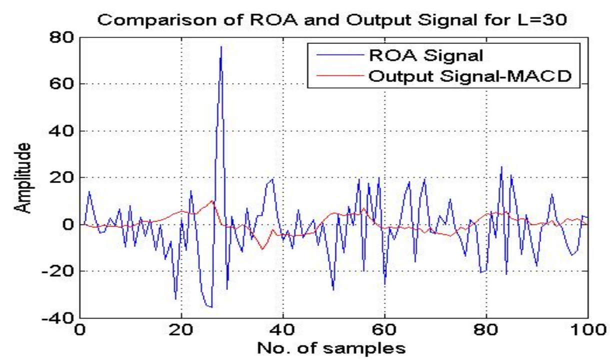

Figure 5: Plot of ROA v/s output signal for MACD

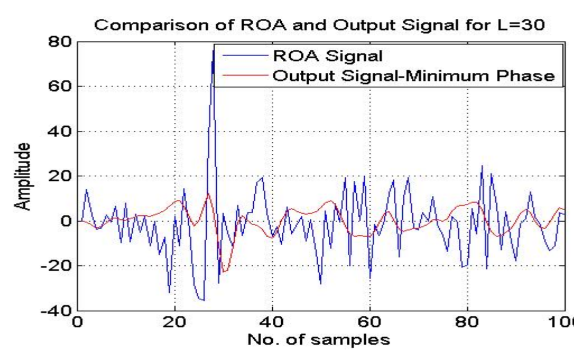

Figure 6: Plot of ROA v/s output signal for Minimum Phase

Table 2: Variance and CP for MACD

	L=30		L=60	
Parameters	**ROA**	**O/P Sig**	**ROA**	**O/P Sig**
σ	286.36	14.10	286.36	18.43
CP	1128	374	1128	340

Table 3: Variance and CP for Minimum Phase Filter

	L=30		L=60	
Parameters	**ROA**	**O/P Sig**	**ROA**	**O/P Sig**
σ	286.36	40.30	286.36	44.50
CP	1128	375	1128	352

After analysing the frequency response, ROA signal was fed as an input to both MACD and Minimum Phase filter design models and the parameters like variance and cross-over points (CP) of output signal were analysed for different filter lengths. The ROA and output signal was plotted for different filter lengths and the results for l=30 can be seen in Figure 5 & 6.

As observed from Table 3, for filter length l=30, the variance of original signal, i.e. ROA signal was reduced by 95.07% for MACD indicator and 85.92% when passed through minimum phase filter. This indicates that output signal is a filtered version of the original ROA signal and can be used to study the trend cycle. As a consequence, the false transaction points involved in buying or selling an asset were also reduced from 1128 to 374 and 375 for MACD and Minimum Phase filter respectively. The values of variance and CP are least for MACD as the attenuation levels are highest for MA which also accounts for the extensive delay. Hence, by implementing minimum phase filter design, we have achieved a trade-off between group delay and attenuation levels and have improved the filter design and quality of the estimated trend.

VI. Conclusion

The frequency response and other parameters of the Moving Average Convergence Divergence (MACD) filter are rectified. The new filter design shows less group delay. The improved Minimum Phase filter displays larger accuracy in predicting trends of a low frequency signal when compared to MACD. As the goal of an investor in financial markets is to maximize profit returns, this is possible when trend analysis happens in real time. It's safe to say, investors can mitigate their risks by using the Minimum Phase Filter design compared to Moving Average Convergence Divergence.

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