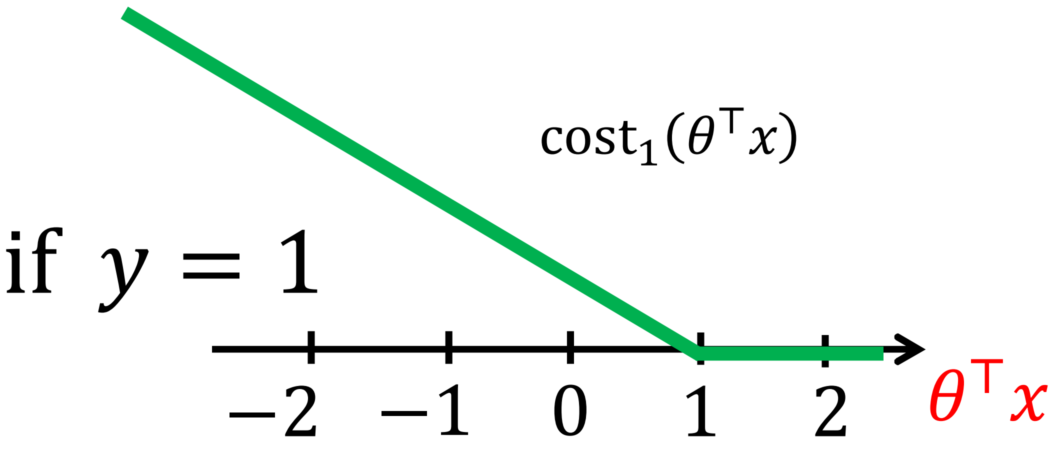
Large Margin Classifier

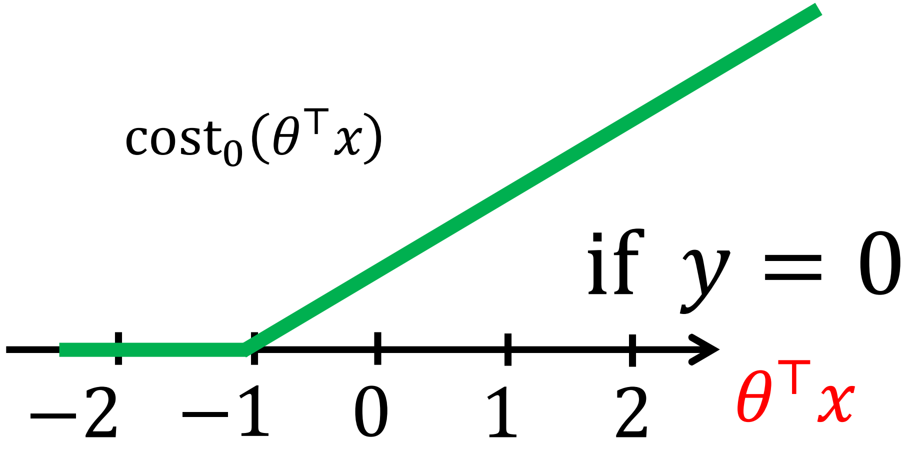
Name : Shashank Patel

Roll no. :220103027

Support vector machine

$$\min_{\theta} C \left[\sum_{i=1}^{m} y^{(i)} \ \mathrm{cost}_1\left(\theta^\top x^{(i)}\right) + (1 - y^{(i)}) \ \mathrm{cost}_0\left(\theta^\top x^{(i)}\right) \right] + \sum_{j=1}^{n} \theta_j^2$$





If "y = 1", we want $\theta^\top x \geq \ 1$ (not just ≥ 0)

If "y = 0", we want $\theta^\top x \leq -1$ (not just < 0)

SVM decision boundary

$$\min_{\theta} C\left[\sum_{i=1}^{m} y^{(i)}\ \text{cost}_1\left(\theta^\top x^{(i)}\right) + (1 - y^{(i)})\ \text{cost}_0\left(\theta^\top x^{(i)}\right)\right] + \frac{1}{2}\sum_{j=1}^{n} \theta_j^2$$

- Let's say we have a very large C...

- Whenever $y^{(i)} = 1$:

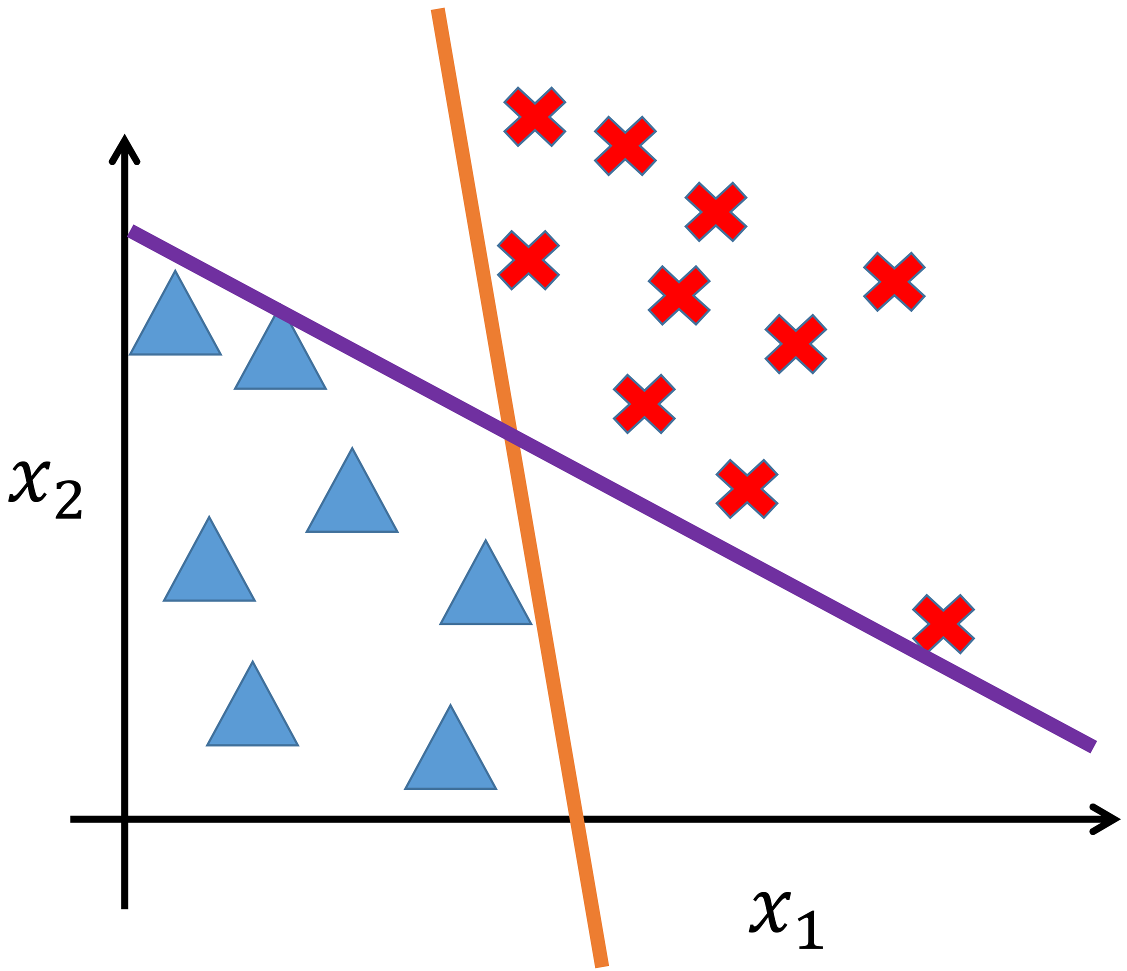
  $\theta^\top x^{(i)} \geq 1$

- Whenever $y^{(i)} = 0$:

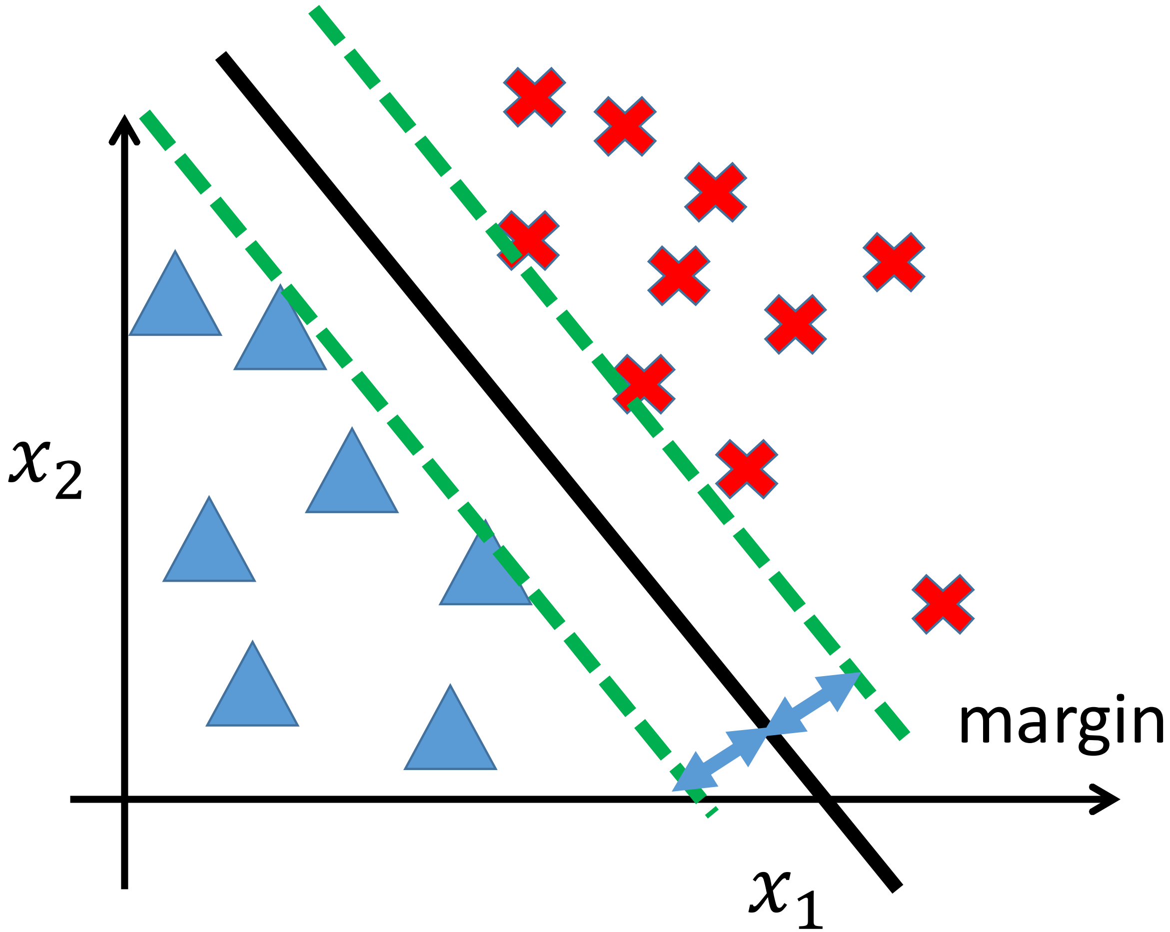
  $\theta^\top x^{(i)} \leq -1$

$$\boxed{\begin{aligned} &\min_{\theta} \frac{1}{2}\textstyle\sum_{j=1}^{n} \theta_j^2 \\ &\text{s.t.} \quad \theta^\top x^{(i)} \geq 1 \quad \text{if} \quad y^{(i)} = 1 \\ &\qquad\;\; \theta^\top x^{(i)} \leq -1 \quad \text{if} \quad y^{(i)} = 0 \end{aligned}}$$

SVM decision boundary: Linearly separable case



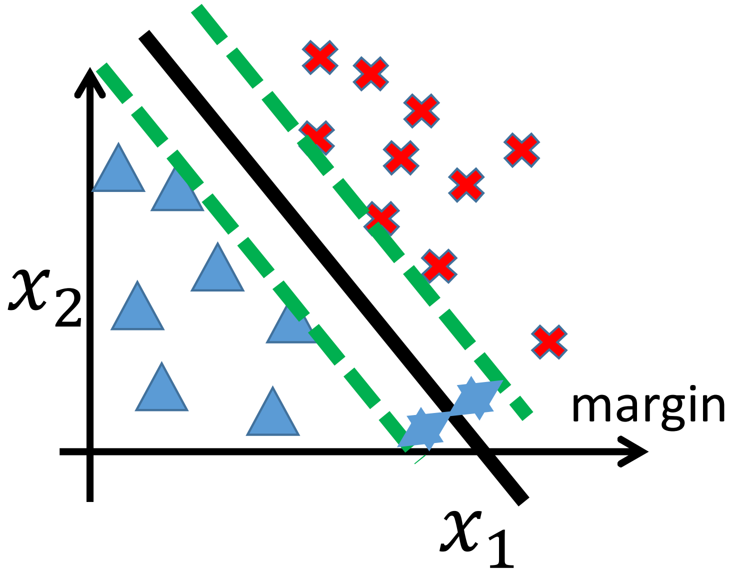
SVM decision boundary: Linearly separable case



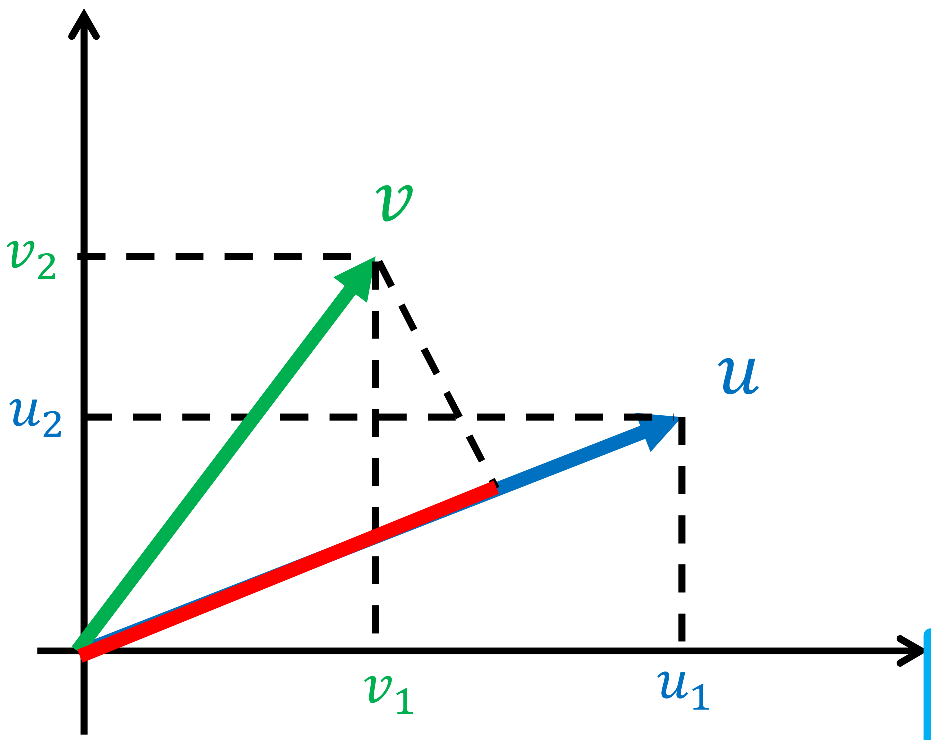
Why large margin classifiers?

$$\min_{\theta} \frac{1}{2}\sum_{j=1}^{n} \theta_j^2$$

$$\text{s.t.} \quad \theta^\top x^{(i)} \geq 1 \quad \text{if} \quad y^{(i)} = 1$$
$$\theta^\top x^{(i)} \leq -1 \quad \text{if} \quad y^{(i)} = 0$$



Vector inner product



$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ $\quad$ $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$\|u\| =$ length of vector u

$= \sqrt{u_1^2 + u_2^2} \in \mathbb{R}$

$p =$ length of projection of v onto u

$$\boxed{\begin{aligned} u^\top v &= p \cdot \|u\| \\ &= \mathrm{u}_1\mathrm{v}_1 + \mathrm{u}_2\mathrm{v}_2 \end{aligned}}$$

SVM decision boundary

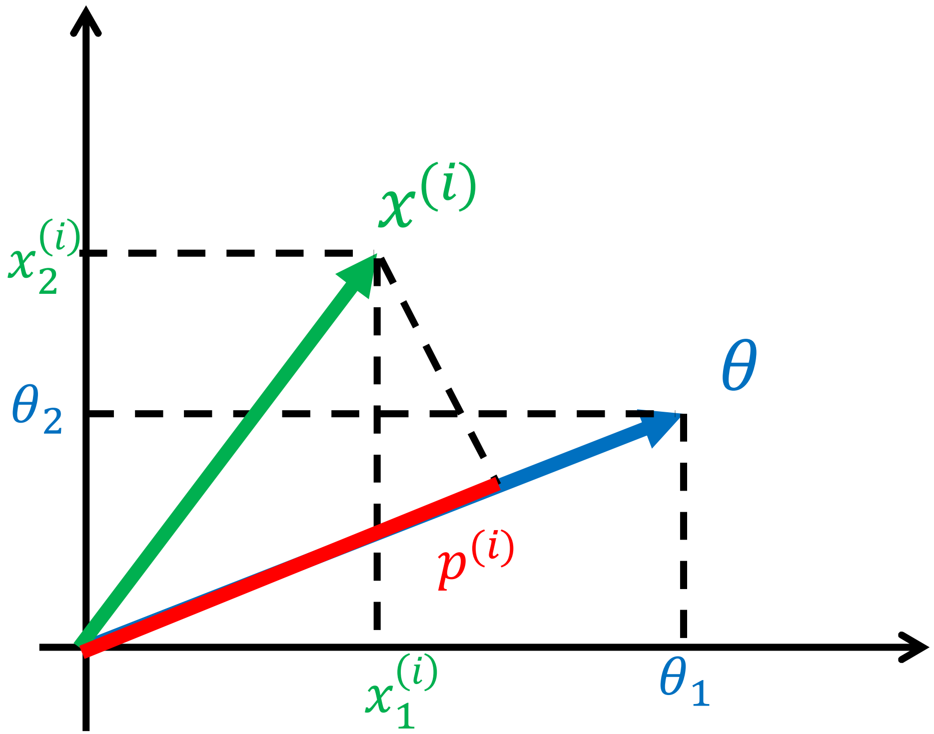
$$\min_{\theta} \frac{1}{2}\sum_{j=1}^{n} \theta_j^2$$

$$\frac{1}{2}\sum_{j=1}^{n} \theta_j^2 = \frac{1}{2}(\theta_1^2 + \theta_2^2) = \frac{1}{2}\left(\sqrt{\theta_1^2 + \theta_2^2}\right)^2 = \frac{1}{2}\|\theta\|^2$$

$$\text{s.t.} \quad \theta^\top x^{(i)} \geq 1 \quad \text{if} \quad \mathrm{y}^{(i)} = 1$$
$$\theta^\top x^{(i)} \leq -1 \quad \text{if} \quad \mathrm{y}^{(i)} = 0$$

Simplication: $\theta_0 = 0,\ n = 2$

What's $\theta^\top x^{(i)}$?



$$\theta^\top x^{(i)} = p^{(i)}\|\theta\|^2$$

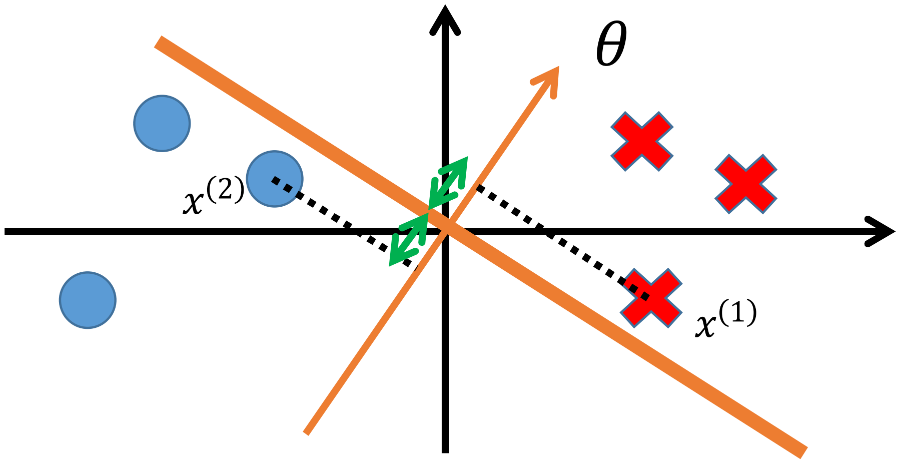
SVM decision boundary

$$\min_{\theta} \frac{1}{2}\|\theta\|^2$$

$$\text{s.t.} \quad p^{(i)}\|\theta\|^2 \geq 1 \quad \text{if} \quad y^{(i)} = 1$$
$$\qquad p^{(i)}\|\theta\|^2 \leq -1 \quad \text{if} \quad y^{(i)} = 0$$

Simplication: $\theta_0 = 0,\ n = 2$

$p^{(1)}, p^{(2)}$ small $\rightarrow \|\theta\|^2$ large



$p^{(1)}, p^{(2)}$ large $\rightarrow \|\theta\|^2$ can be small