MODULE 3

a) D.C. GENERATORS AND D.C. MOTORS

Introduction

- An electrical machine deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form. This process is called **electromechanical energy conversion**.
- An electrical machine which converts mechanical energy into an electrical energy is called an **electric generator**.
- An electrical machine which converts electrical energy into a mechanical energy is called an **electric motor.**
- The D.C. Machines are thus classified as,
    - D.C. Generator
    - D.C. motor

Working Principle of a D.C. Machine as a Generator

- All generators work on the principle of dynamically induced e.m.f.
- The principle is nothing but the faraday`s law of electromagnetic induction. It states that "***Whenever the number of magnetic lines of force i.e. flux linking with the conductor or a coil changes, an electromotive force is set up in that conductor or coil***."
- The magnitude of the induced e.m.f in a conductor is proportional to the rate of change of flux associated with the conductor. This is mathematically given by,

$$\mathbf{e \; \alpha \; \frac{d\varphi}{dt}}$$

- The relative motion can be achieved by rotating conductor with respect to flux or by rotating flux with respect to a conductor.
- So a voltage gets generated in a conductor, as long as there exists a relative motion between the conductor and the flux.
- Such an induced e.m.f which is due to physical movement of the coil or conductor with respect to flux or movement of flux with respect to coil or conductor is called **dynamically induced e.m.f**.
- To have a large voltage at the output, the number of conductors are connected together in a specified manner, to form a winding. This winding is called **armature winding** of a D.C. machine.
- The part on which this winding is kept is called armature of a D.C. machine.
- To have the rotation of conductors, the conductors placed in the armature are rotated with the help of external device. Such an external device is called as **prime movers**.
- The commonly used prime movers are diesel engine, steam engine, steam turbines etc.
- The necessary magnetic flux is produced by current carrying winding which is called field winding.
- The direction of induced e.m.f is obtained by using Fleming`s right hand rule.

- If the angle between the plane of rotation and plane of flux is θ as measured from the axis of the plane of flux then the induced e.m.f is given by

$$E = B\,l\,(v \sin\theta)$$

- Where v sinθ is the component of velocity which is perpendicular to the plane of flux and henceresponsible for the induced e.m.f.

Constructional details of a D.C. Machine

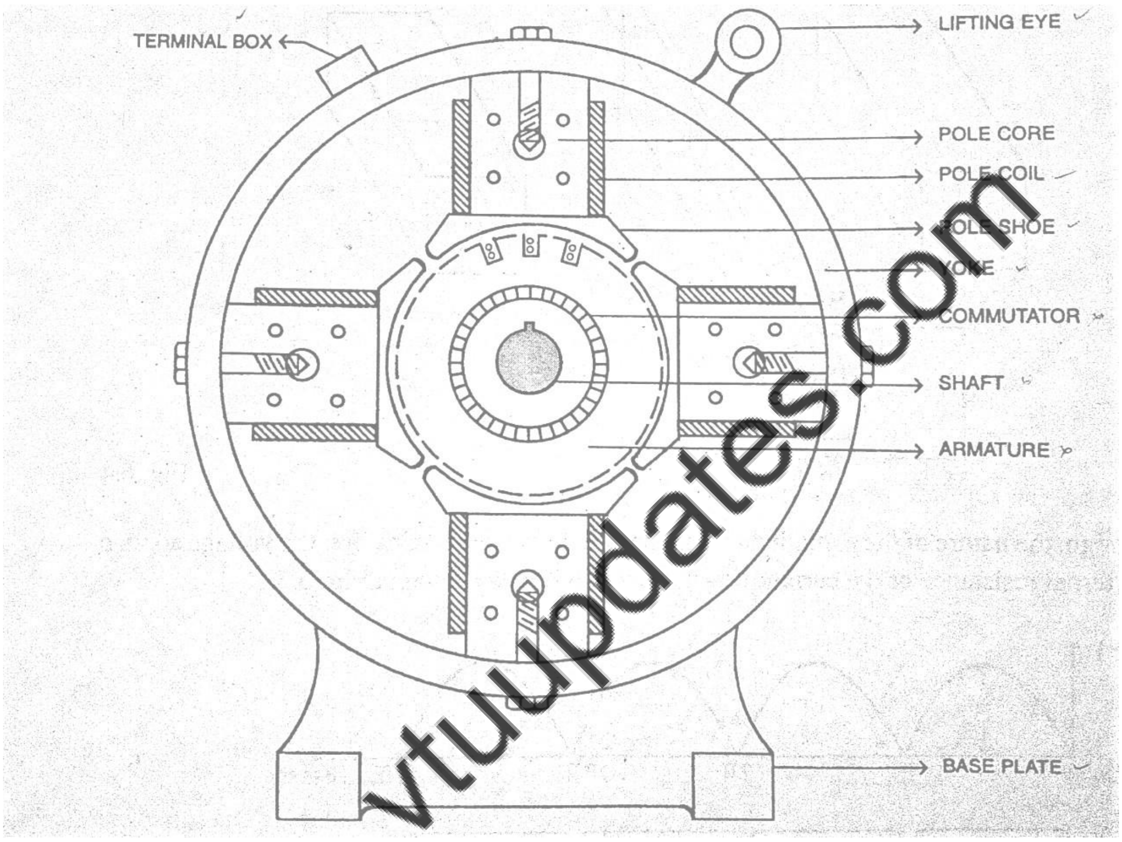


Fig It consists of the following parts

Yoke

A. Functions

a) It serves the purpose of outermost cover of the D.C. machine. So that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases, acidic fumes etc.
b) It provides mechanical support to the poles.
c) It forms the part of the magnetic circuit. It provides the path for low reluctance for magnetic flux. The low reluctance path is important to avoid wastage of power to provide same flux. Large current and hence the power is necessary if the path has high reluctance, to produce the same flux.

B. Choice of material: It is prepared by using cast iron because it is cheapest and provides low reluctance path. For large machines rolled steel, cast steel, silicon steel is used which provides high permeability i.e. low reluctance and gives good mechanical strength.

Poles

Each pole is divided into two parts namely

1.Pole core
2.Pole shoe

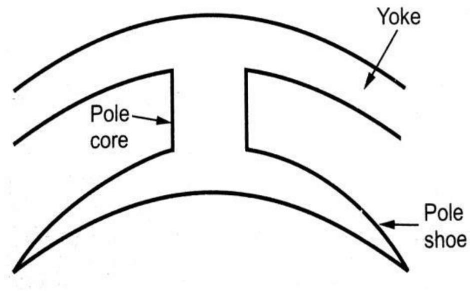


Fig Pole structure

a) **Functions of pole core and pole shoes**
   (1) Pole core basically carries a field winding which is necessary to produce the flux.
   (2) It directs the flux produced through air gap to armature core, to the next pole.
   (3) Pole shoes enlarges the area of armature core to come across the flux, which is necessary to produce larger induced e.m.f. To achieve this, pole shoes has been given a particular shape.

b) **Choice of material:** it is made up of magnetic material like cast iron or cast steel. As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

Field Winding

The field winding is wound on the pole core with a definite direction.

a) **Functions**
   1) The field winding carries current and behaves as an electromagnet, producing necessary flux.
   2) As it helps in producing magnetic field i.e. exciting the pole as an electromagnet it is called field winding or exciting winding.

b) **Choice of material:** It has to carry current hence obviously made up of some conducting material. So aluminum or copper is the choice. But field coils are required to take any type of shape and bend about pole core and copper has good pliability i.e. it can bend easily. So copper is the proper choice.

Armature

The armature is further divided into two parts

1. Armature core

2. Armature winding

1. **Armature core:** Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.
   a. **Functions**
      i. Armature core provides house for armature winding i.e. armature conductors.
      ii. To provide path of low reluctance to the magnetic flux produced by the field winding.

   b. **Choice of material**

      i. As it has to provide low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.
      ii. It is made up of laminated construction to keep eddy current loss as low as possible.

2. **Armature winding:** is nothing but the interconnection of armature conductors, placed in the slots provided on the armature core periphery. When the armature is rotated, in case of generator, magnetic flux gets cut by armature conductors and e.m.f gets induced in them.
   a. **Function**
      i. Generation of e.m.f takes place in the armature winding in case of generators.
      ii. To carry the current supplied in case of D.C. motors.
      iii. To do the useful work in the external circuit.

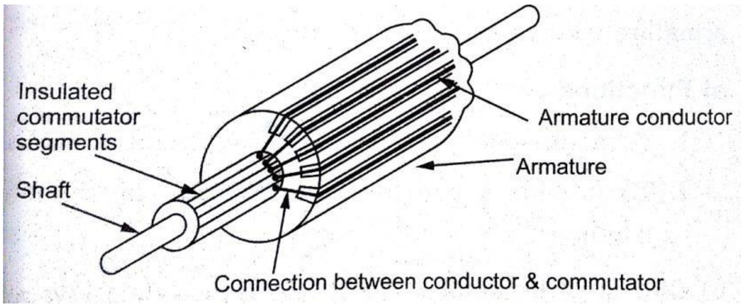
   b. **Choice of material:** As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Commutator

- The basic nature of e.m.f induced in the armature conductor is alternating.
- This needs rectification in case of D.C. generator, which is possible by a device called commutator.

1) **Functions**
   a) To facilitate the collection of current from the armature conductor.
   b) To convert internally developed alternating e.m.f to unidirectional e.m.f.
   c) To produce unidirectional torque in case of motors.

2) **Choice of material:**
   a) As it collects current from the armature, it is also made up of copper segments.
   b) It is cylindrical in shape and is made up of wedge shaped segments of hard drawn high conductivity copper. These segments are insulated from each other by thin layer of mica.
   c) Each commutator segment is connected to the armature conductor by means of copper strip.



Brushes and Brush gear

Brushes are stationary and resting on the surface of the commutator.

1) **Function:** To collect current from commutator and make it available to the stationary external circuit.
2) **Choice of material**: To avoid wear and tear of commutator brushes are normally made up of soft material like carbon.

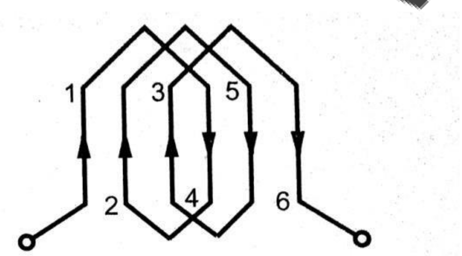
Types of Armature Winding

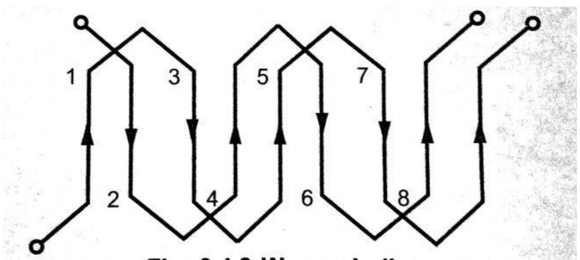
- A number of armature conductors are connected in a specific manner to give armature winding.
- According to the way of connecting the conductors, the armature winding has two types,

  Lap winding

  Wave winding
- In lap type, the connections overlap each other as the winding proceeds.
- Due to this, the number of parallel path in which the conductors are divided is P, where P= number of poles in the machines

  A=P = Number of parallel paths
- Large number of parallel path indicate high current capacity of machine hence lap winding is preferred for high current rating generators.





- In wave winding, the winding travels ahead avoiding the overlap in a progressive fashion.
- Due to this, the armature conductors always get divided into two parallel paths, irrespective of number of poles.

  A=2= number of parallel paths

Comparison between lap and wave type armature winding

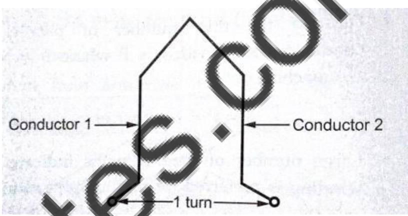
S.no	Lap Winding	Wave Winding
1.	Number of parallel paths (A)= Poles (P)	Number of parallel paths (A) = 2
4.	Number of brush set required is equal to number of poles	Number of brush sets required is always equal to two
3.	Preferable for high current, low voltage capacity generators	Preferable for high voltage, low current capacity generators
4.	Normally used for generators of capacity more than 500A	Normally used for generators of capacity less than 500A

Winding terminology

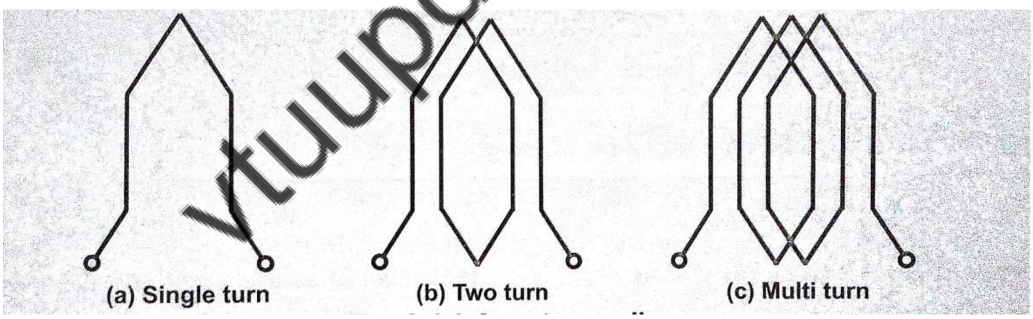
a) **Conductor:** It is the actual armature conductor which is under the influence of the magnetic field, placed in the armature slot.

b) **Turn:** The two conductors placed in different slots when connected together form a turn.

Z = 2 x Number of turns



c) **Coil:** For simplicity of connections, the turns are grouped together to form a coil. If coil contains only one turn it is called single turn coil while coil with more than one turn is called multi turn coil.



d) **Pole pitch:** The distance between the two adjacent poles is called a pole pitch. It is measured in terms of number of slots. Thus total slots along the periphery of the armature divided by total number of poles is called a pole pitch.

E.M.F Equation of D.C.Generator

Let P = Number of poles of the generator

Φ = Flux produced by each pole in weber

N = Speed of armature in r.p.m

Z = Total number of armature conductor

A = Number of parallel path in which the Z number of conductors are divided

So A=P for lap type of winding and A=2 for wave type of winding

E.M.F gets induced in the conductor according to Faraday`s law of electromagnetic induction. Hence the average value of e.m.f induced in each armature conductor is

$$E = \text{rate of cutting of flux} = \frac{d\varphi}{dt}$$

Consider one revolution of conductor. In one revolution, conductor will cut total flux produced by all the poles i.e. φ x P.

While time required to complete one revolution is 60/N sec as speed in N r.p.m

$$\therefore \; E = \frac{\varphi P}{\frac{60}{N}} = \frac{\varphi PN}{60}$$

Now the conductors in one parallel path are always in series. There are total Z conductors with A parallel path, hence Z/A number of conductors are always in series and e.m.f remains same across all the parallel paths

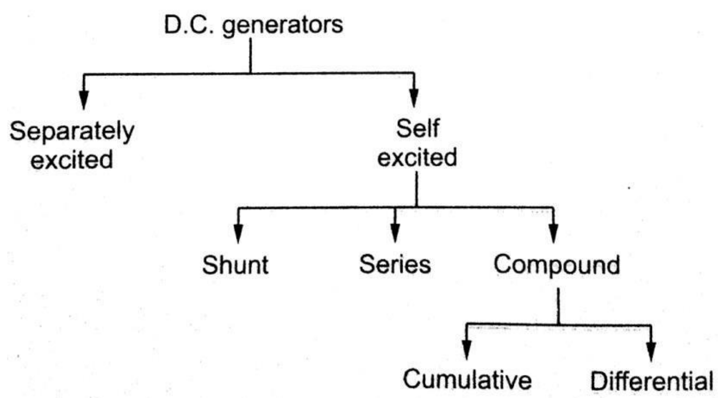
∴ Total e.m.f can be expressed as, $E = \frac{\varphi PN}{60} \times \frac{Z}{A}$

This is nothing but e.m.f equation of a D.C. generator

$$\boxed{\mathbf{E} = \frac{\varphi \mathbf{PNZ}}{\mathbf{60A}} \quad \text{with A= P for lap and A=2 for wave}}$$

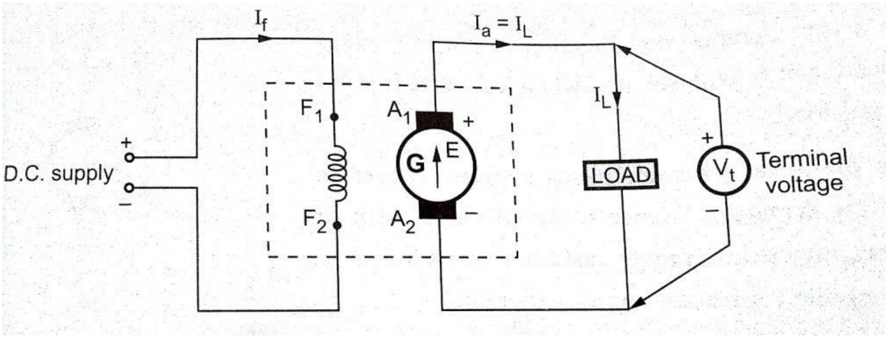
Types of D.C. Generators

- The field winding is also called exciting winding and current carried by the field winding is called an exciting current.
- Thus supplying current to the field winding is called excitation and the way of supplying the exciting current is called method of excitation.
- Depending on the method of excitation used, the D.C. Generators are classified as,



Separately Excited Generator

- When field winding is supplied from external, separate d.c. supply i.e. excitation of field winding is separate then the generator is called **separately excited generator.**
- The schematic representation is shown in fig below



- In the terminology of d.c. machines the various currents are denoted as

I_a = Armature Current I_L = Load Current I_f = field Current

- The field winding is excited separately, so the field current depends on supply voltage and resistance of the field winding.
- For armature side, it is supplying a load, demanding a load current of I_L at a voltage of V_t which is called terminal voltage.

  Now $I_a = I_L$
- The internally induced e.m.f E is supplying the voltage to the load hence terminal voltage V_t is a part of E.
- But E is not equal to V_t while supplying a load. This is because when armature current I_a flows through armature winding, due to armature winding resistance R_a , there is a voltage drop across the armature winding equal to I_aR_a volts. The induced e.m.f has to supply this drop, along with the terminal voltage V_t. To keep I_aR_a drop to minimum the resistance R_a is designed to be very small.
- In addition to this drop, there is some voltage drop at the contacts of the brush called brush contact drop. But this drop is negligible and hence generally neglected.
- When armature carries current. It produces its own flux which distorts the main flux. Due to this, thereis small voltage drop called armature reaction drop. But as small, this drop is also partially neglected.
- So the voltage equation for separately excited generator can be written as,

$$\mathbf{E = V_t + I_aR_a + V_{brush} + \text{armature reaction drop}}$$

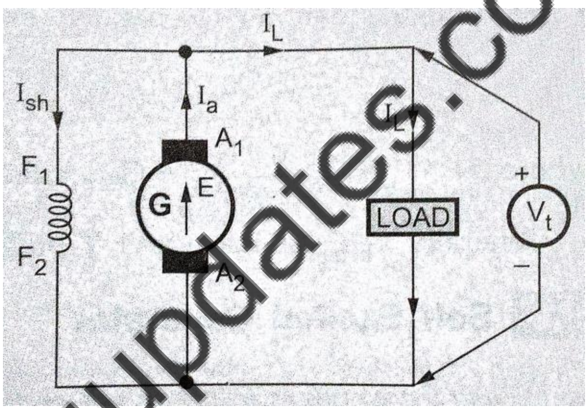
Where $E = \frac{\varphi PNZ}{60A}$ = generated e.m.f

Self Excited Generator

- When the field winding is supplied from the armature of the generator itself then it is said to be self excited generator.
- Based on how field winding is connected to the armature to derive its excitation, this type is further divided into the following three types
- Shunt generator
- Series generator
- Compound generator

Shunt generator

When the field winding is connected in parallel with the armature and the combination across theload then the generator is called shunt generator.



From the fig

$$I_a = I_L + I_{sh}$$

Now the voltage across the load is V_t , which is same across field winding as both are in parallel with each other.

$$I_{sh} = \frac{V_t}{R_{sh}}$$

While induced e.m.f E still requires to supply voltage drop I_aR_a and brush contact drop

$$E = V_t + I_aR_a + V_{brush}$$

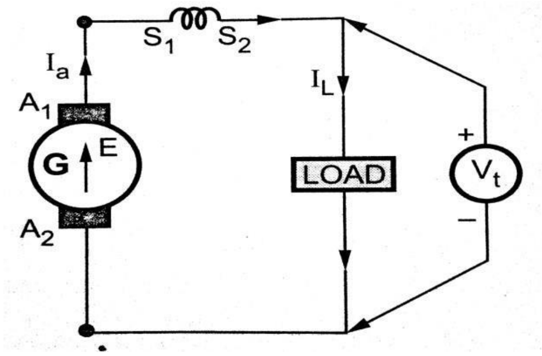
Armature reaction drop is practically neglected.
The power developed by armature is given by the product of induced e.m.f E and armature current I_a

Power developed in armature = EI_a W

While the power available to the load is V_tI_L

Series Generator

When the field winding is connected in series with the armature winding while supplying the load then the generator is called series generator.



As all armature, field and load are in series they carry the same current

$$\therefore\ I_a = I_{se} = I_L$$

Now in addition to drop I_aR_a , induced e.m.f has to supply voltage drop across series field winding too. This is I_aR_{se} **.** so voltage equation can be written as

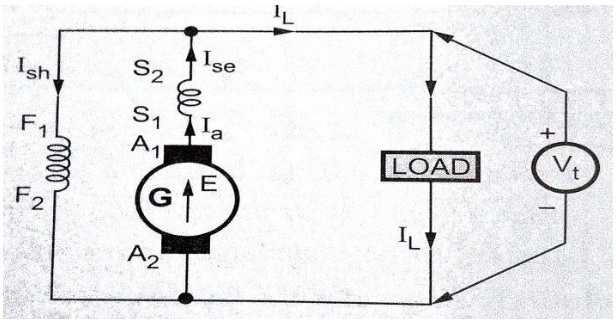
$$E = V_t + I_aR_a + I_aR_{se} + V_{brush}$$

Compound Generator

- In this type, the part of the field winding is connected in parallel with armature and part in series with armature.
- Depending on the connection of shunt and series field winding, compound generators are further classified as
  Long shunt compound generators
  Short shunt compound generators.

Long shunt compound generators

- In this type, shunt field winding is connected across the entire series combination of armature and series field winding.



- From the fig $I_a = I_{se}$

$$I_a = I_L + I_{sh}$$

- Voltage across the shunt field winding is V_t

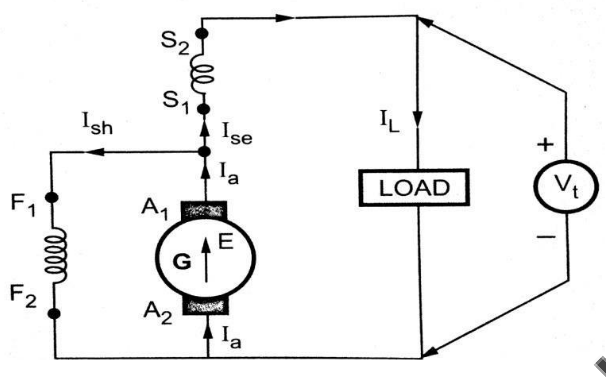
$$I_{sh} = \frac{V_t}{R_{sh}}$$

- Voltage equation is

$$E = V_t + I_a R_a + I_a R_{se} + V_{brush}$$

Short Shunt Compound Generator

- In this type, shunt field winding is connected only across the armature, excluding series field winding.



- From the fig

$$I_a = I_{se} + I_{sh}$$

And

$$I_{se} = I_L$$

∴

$$I_a = I_L + I_{sh}$$

- the drop across shunt field winding is drop across the armature only and not the total V_t
- so the drop across the shunt field winding is E- $I_a R_a$

$$I_{sh} = \frac{E - I_a R_a}{R_{sh}}$$

- now the voltage equation is $E = V_t + I_a R_a + I_{se} R_{se} + V_{brush}$

Now

$$I_{se} = I_L$$

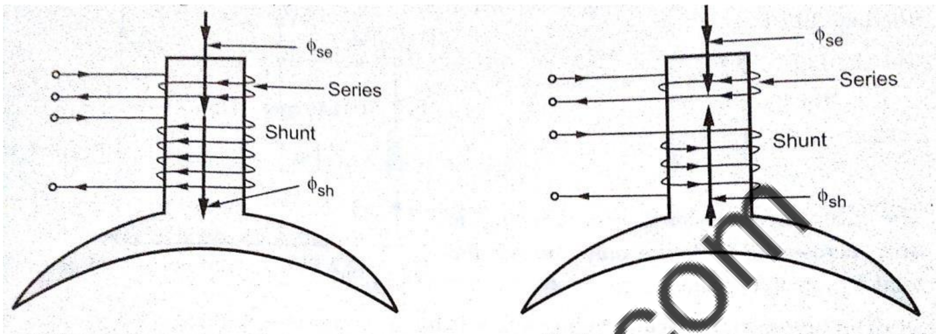
Hence $E = V_t + I_a R_a + I_L R_{se} + V_{brush}$

- Neglecting V_{brush} we can write $E = V_t + I_a R_a + I_L R_{se}$ i.e. $E - I_a R_a = V_t + I_L R_{se}$

$$\therefore \quad I_{sh} = \frac{V_t + I_L R_{se}}{R_{sh}}$$

Cumulative and Differential Compound Generator

- The two windings shunt and series field are wound on the same pole.
- Depending on the direction of winding on the pole, two fluxes produced by shunt and series field may help or oppose each other. This fact decides whether generator is cumulative or differential compound.
- If the two fluxes help each other the generator is called cumulative compound generator.
- If the two windings are wound in such a direction that the fluxes produced by them oppose each other then the generator is called differential compound generator.



(a) Cumulative compound generator (b) Differential compound generator

Application of various type of D.C.Generator

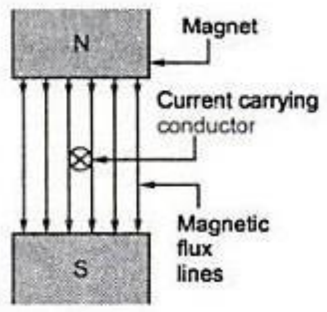
1. **Separately Excited Generator**
   - As a separate supply is required to excite field, the use is restricted to some special applications like electro-plating, electro-refining of materials.
2. **Shunt Generators**
   - Commonly used in battery charging and ordinary lighting purpose.
3. **Series Generator**
   - Commonly used as boosters on d.c. feeders, as a constant current generator for welding generators and arc lamps.
4. **Cumulatively compound generators**
   - These are used for domestic lighting purpose and to transmit energy over long distance.
5. **Differential Compound generator**
   - These are used for special application like electric arc welding.

Principle of operation of a D.C. machine as a motor

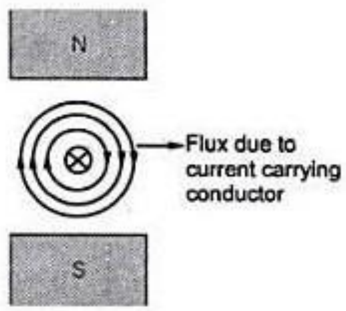
- The principle of operation of a D.C. motor can be stated in a single statement as ‘when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force’.

- In a practical d.c.motor, field winding produces a required magnetic field while armature conductors play a role of a current carrying conductor and hence armature conductors experiencea force.

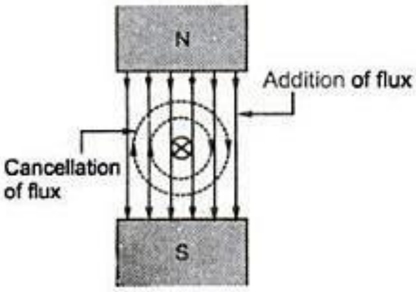
Consider a single conductor placed in a magnetic field as shown in fig



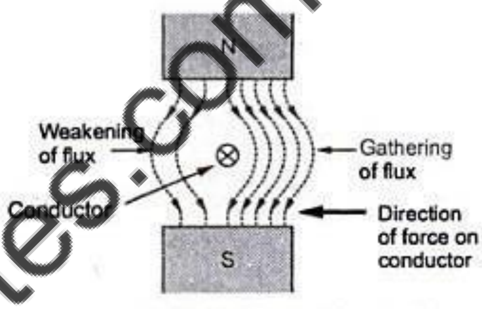
(a) Flux due to magnet



(b) Flux due to current carrying conductor



(a) Presence of the two fluxes



(b) Resultant flux pattern

- Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it carries a current away from an observer as shown in fig.
- Any current carrying conductor produces its own magnetic field around it; hence this conductor also produces its own flux. The direction of this flux can be determined by right hand thumb rule. For direction of current considered the direction of flux around a clockwise.
- Now there are two fluxes present,
  - The flux produced by permanent magnet called main flux
  - The flux produced by the current carrying conductor
- From this it is clear that on one side of the conductor, both the fluxes are in the same direction. In this case on the right hand side of the conductor there is gathering of the flux lines as two fluxes help each other.
- As against this on the left hand side of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this density of flux in this area gets weekend. So on the right, there exists high flux density area while on the left there exist low density area.
- This flux distribution around the conductor acts like a stretched rubberband under tension. This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area.

- In a practical d.c.motor, the permanent magnet is replaced by a field winding which produces the required flux called main flux and all armature conductors, mounted on the periphery of the armature drum, gets subjected to the mechanical force.
- Due to this, overall armature experiences a twisting force called torque and armature of the motor starts rotating.
- The magnitude of force experienced by conductor in a motor is given by

$$\mathbf{F = B\, \mathit{l}\, I \ Newtons}$$

B= flux density due to the flux produced by field winding

l= active length of the conductor

I= magnitude of current passing through the conductor

- The direction of such force i.e. direction of rotation of motor can be determined by Flemings left hand rule.

Back E.M.F in a D.C.Motor

- It is seen in the generating action, that when a conductor cuts the lines of flux, e.m.f gets induced in the conductor. In a d.c.motor, after a motoring action, armature starts rotating and the armature conductors cut the main flux. So there is generating action existing in a motor after motoring action.

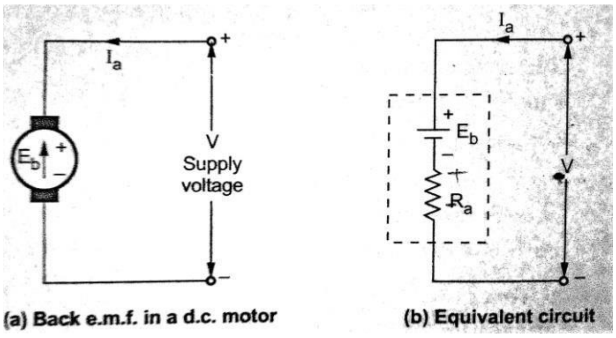
- There is an induced e.m.f in the rotating armature conductors according to Faraday`s law of electromagnetic induction. This induced e.m.f in the armature always acts in the opposite direction to the supply voltage. This is according to Lenz`s law which states that the direction of the induced e.m.f is always so as to oppose the cause producing it.

- In a D.C. motor, electrical input i.e. supply voltage is the main cause for the armature current and motoring action and hence this induced e.m.f opposes the supply voltage. This e.m.f tries to set up a current through the armature which is in the opposite direction to that, which supply is forcing through the conductor.

- As this e.m.f always opposes the supply voltage, it is called back e.m.f and denoted as E_b. Though it is denoted as E_b, basically it gets generated by the generating action. So its magnitude can be determined by the e.m.f equation

$$\mathbf{E_b} = \frac{\varphi \mathbf{PNZ}}{\mathbf{60A}} \text{ volts}$$

This e.m.f is shown schematically in the fig (a). So if V is supply voltage in volts and $\mathbf{R_a}$ is the value of the armature resistance, the equivalent electric circuit can be shown as in fig (b)



Voltage Equation of D.C. Motor

- From the equivalent circuit, the voltage equation for a D.C.Motor can be obtained as

$$\mathbf{V = E_b + I_aR_a + \text{brush drop}}$$

- The brush drop is practically neglected
- Hence the armature current $\mathbf{I_a}$ can be expressed as

$$\mathbf{I_a = \frac{V - E_b}{R_a}}$$

Significance of Back E.M.F

- Due to the presence of back e.m.f the d.c.motor becomes a regulating machine. i.e. motor adjusts itself to draw the armature current just enough to satisfy the load demand.
- The basic principle of this fact is that the back e.m.f. is proportional to speed, $\mathbf{E_b \ \alpha \ N}$.
- When the load is suddenly put on the motor, motor tries to slow down. So speed of the motor reduces due to which back e.m.f also decreases. So the net voltage across the armature $\mathbf{V - E_b}$ increases and motor draws more armature current.
- Due to the increased armature current, force experienced by the conductor and hence the torque on the armature increases. The increase in the torque is just sufficient to satisfy increased load demand.
- When the load on the motor is decreased, the speed of the motor tries to increase. Hence back e.m.f increases. This causes $\mathbf{V - E_b}$ to reduce which eventually reduces the current drawn by the armature. The motor speeds stops increasing when the armature current is just enough to produce the less torque required by the new load.
- So back e.m.f regulates the flow of armature current and it automatically alters the armature current to meet the load requirement.

Power equation of a D.C.Motor

- The voltage equation of a D.C.Motor is given by

$$\mathbf{V = E_b + I_aR_a}$$

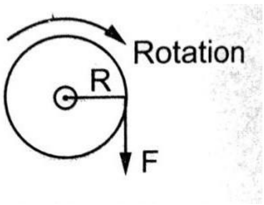
- Multiplying both sides of the above equation by $\mathbf{I_a}$ we get

$$\mathbf{VI_a = E_bI_a + I_aR_a}$$

- This equation is called power equation of d.c.motor

Torque Equation of a d.c. Motor

- The turning or twisting force about an axis is called torque.
- Consider a wheel of radius R meters acted upon by a circumferential force F newtons



- The wheel is rotating at a speed of N rpm then its angular speed is

$$\boldsymbol{\omega} = \frac{\mathbf{2\,\pi\,N}}{\mathbf{60}} \text{ rad/sec}$$

so work done in one revolution is

W= F x distance travelled in one revolution = F x 2 π R

$$P = \text{Power developed} = \frac{\text{work done}}{\text{time}} = \frac{F \times 2\pi R}{60/N} = (F \times R) \times \frac{2\pi N}{60}$$

$\therefore$ P = T x ω watts

Let T_a be the gross torque developed by the armature of the motor. It is also called armature torque.
The gross mechanical power developed in the armature is $\mathbf{E_b I_a}$.
So if the speed of the motor is N rpm, then

Power in armature = Armature torque x ω

$$E_b I_a = T_a \times \frac{2\pi N}{60}$$

$$\text{But } E_b = \frac{\varphi PNZ}{60A}$$

$$\therefore \quad \frac{\varphi PNZ}{60A} \times I_a = T_a \times \frac{2\pi N}{60}$$

$$\therefore \quad \mathbf{T_a} = \frac{\mathbf{1}}{\mathbf{2\pi}}\, \varphi\, \mathbf{I_a} \ \mathbf{x}\ \frac{\mathbf{PZ}}{\mathbf{A}} = \mathbf{0.159}\ \boldsymbol{\varphi} \mathbf{I_a} \ .\ \frac{\mathbf{PZ}}{\mathbf{A}}$$

Type of Torque in the Motor

- The mechanical power developed in the armature is transmitted to the load through the shaft of the motor.

- It is impossible to transmit the entire power developed by the armature to the load. This is because while transmitting the power through the shaft, there is a power loss due to the friction, windage and the iron loss.

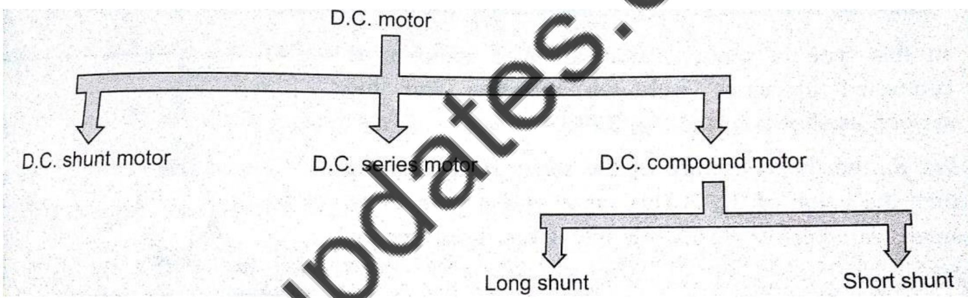
- The torque required to overcome these losses is called lost torque, denoted as T_f. These losses arealso called stray losses.

- The torque which is available at the shaft for doing the useful work is known as load torque or shaft torque denoted as T_{sh}

  $T_f = T_a + T_{sh}$
- The shaft torque magnitude is always less than the armature torque.
- The speed of the motor remains same say N rpm. Then the product of shaft torque T_{sh} and angular speed ω rad/sec is called power available at the shaft. i.e. net output of the motor. A maximum power a motor can deliver to the load safely is called output rating of a motor. Generally it is expressed as H.P.

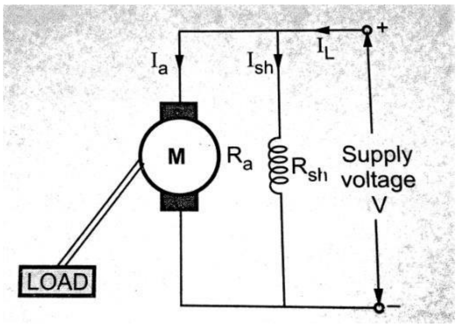
  Net output of motor = Pout = $T_{sh} \times \omega$

Types of D.C.Motors



D.C. Shunt Motor

- In this type, the field winding is connected across the armature winding and the combination is connected across the supply.



- Let R_{sh} be the resistance of shunt field winding and R_a be the resistance of armature winding.
- The value of R_a is very small while R_{sh} is quite large. Hence shunt field winding has more number of turns.

Voltage and current relationship

- The voltage across the armature and field winding is same equal to supply voltage V.

- The total current drawn from the supply is denoted as line current I_L.

$$I_L = I_a + I_{sh} \quad \text{and} \quad I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_b + I_a R_a + V_{brush}$$

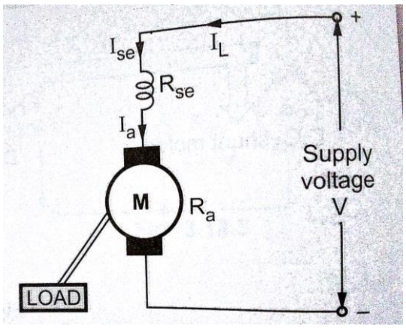
- Now flux produced by the field winding is proportional to the current passing through it. i.e. I_{sh}

$$\Phi \;\alpha\; I_{sh}$$

- As long as supply voltage is constant, the flux produced is constant. Hence D.C. shunt motor is called constant flux motor.

D.C.Series Motor

- In this type of motor, the series field winding is connected in series with the armature and the supply.
- Let R_{se} be the resistance of the series field winding and the value of R_{se} is very small.



Voltage and Current Relationship

- Let I_L be the total current drawn from the supply.

So, $$I_L = I_{se} = I_a$$

And $$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a(R_a + R_{se}) + V_{brush}$$

- Supply voltage has to overcome the drop across series field winding in addition to E_b and drop across armature winding.
- In series motor, entire armature current is passing through the series field winding. So flux produced is proportional to armature current.

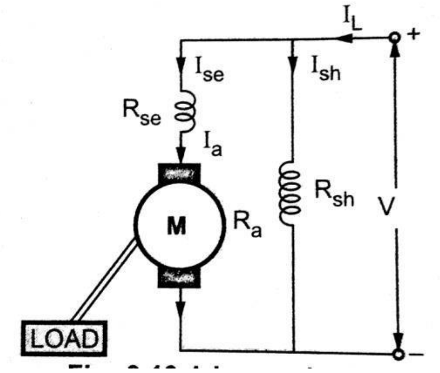
$$\Phi \;\alpha\; I_{se} \alpha\; I_a \quad \text{for series motor}$$

D.C. Compound Motor

- The compound motor consists of part of the field winding connected in series and part of the field winding connected in parallel with armature. It is further classified as short shunt compound and long shunt compound.

1. Long Shunt Compound Motor

- In this type, the shunt field winding is connected across the combination of armature and the series field winding.



Voltage current relationship

- Let R_{se} be the resistance of series field winding and R_{sh} be the resistance of shunt field winding.
- The total current drawn from the supply is I_L.

$$\mathbf{I_L = I_{se} + I_{sh}} \quad \textbf{but} \quad \mathbf{I_a = I_{se}}$$

i.e. $$\mathbf{I_L = I_a + I_{sh}}$$

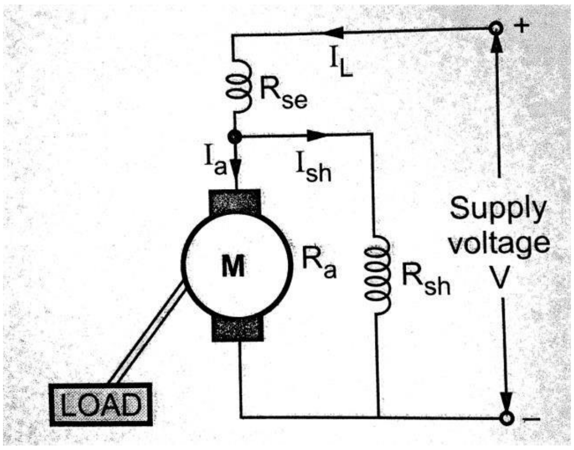
and $$\mathbf{I_{sh} = \frac{V}{R_{sh}}}$$

and $$\mathbf{V = E_b + I_aR_a + I_{se}R_{se} + V_{brush}}$$

$$\mathbf{V = E_b + I_a(R_a + R_{se}) + V_{brush}}$$

4. Short shunt compound Motor

- In this type, the shunt field is connected purely in parallel with armature and the series field is connected in series with this combination.



- The entire line current is passing through the series field winding

$$I_L = I_{se} \quad \text{and} \quad I_L = I_a + I_{sh}$$

- Now the drop across the shunt field winding is to be calculated from the voltage equation

$$V = E_b + I_aR_a + I_{se}R_{se} + V_{brush}$$

But $I_L = I_{se}$

$\therefore$ $V = E_b + I_aR_a + I_LR_{se} + V_{brush}$

- Drop across shunt field winding = $V - I_LR_{se} = E_b + I_aR_a + V_{brush}$

$$\therefore I_{sh} = \frac{V - I_LR_{se}}{R_{sh}} = \frac{E_b + I_aR_a + V_{brush}}{R_{sh}}$$

Torque and Speed Equation

From torque Equation

$$T \; \alpha \; \varphi I_a$$

Now φ is the flux produced by the field winding and is proportional to the current passing through the field winding

$$\Phi \; \alpha \; I_{field}$$

For a d.c.shunt motor, I_{sh} is constant as long as supply voltage is constant. Hence φ flux is also constant.

$$\therefore \quad T \; \alpha \; I_a$$

For d.c. series motor, I_{se} is same as I_a. hence flux φ is proportional to armature current I_a

$$\therefore \; T \; \alpha \; I_a^2$$

Similarly as $E_b = \dfrac{\varphi PNZ}{60A}$ we can write the speed equation as

$$E_b \; \alpha \; \varphi N \qquad \text{i.e} \qquad N \; \alpha \; \frac{E_b}{\varphi}$$

$\therefore$ $E_b = V - I_aR_a$ neglecting brush drop

For shunt motor as flux φ is constant

$$N \; \alpha \; V - I_aR_a$$

While for series motor, flux φ is proportional to I_a

$$N \; \alpha \; \frac{V - I_aR_a - I_aR_{se}}{I_a}$$

CHARACTERISTICS OF DC MOTORS:

There are three important characteristics.

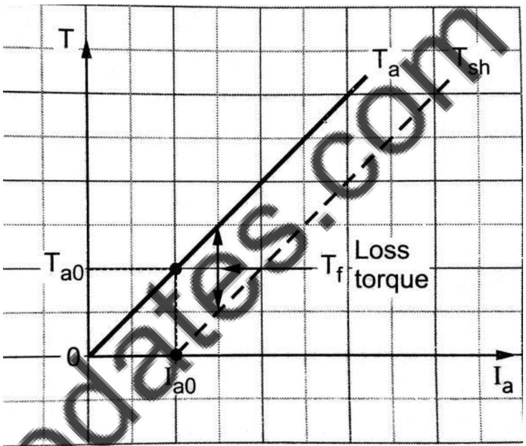
1. Armature torque vs. Armature current ; T_a vs I_a (Electrical characteristics)
2. Speed vs. armature current characteristic
3. Speed vs. torque N vs. T_a (Mechanical characteristics)

CHARACTERISTICS OF SHUNT MOTORS

1. Armature torque vs. Armature current

- For a constant values of R_{sh} and supply voltage V, I_{sh} is also constant and hence flux is also constant.

$$\therefore \ T_a \ \alpha \ I_a$$

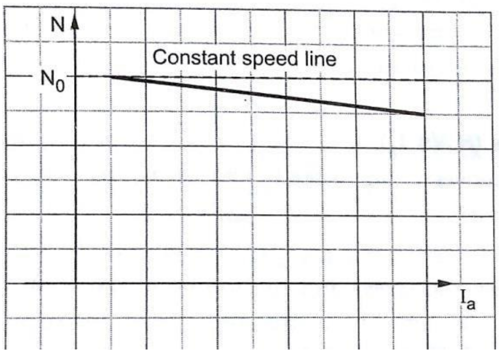


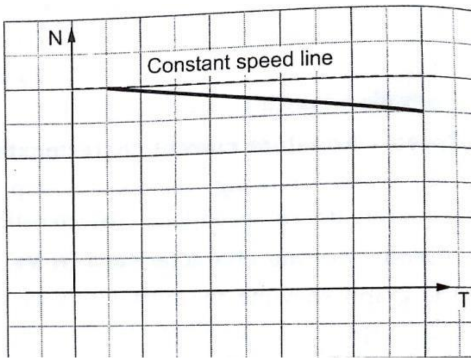
- The equation represents a straight line passing through the origin.
- Torque increases linearly with armature current. So as load increases, armature current increases, increasing the torque developed linearly.

2. Speed vs. Armature current

- From the speed equation we get

$$N \ \alpha \ V - I_aR_a$$





- So as load increases, the armature current increases and hence drop I_aR_a also increases.
- But as R_a is very small, for change in I_a from no load to full load, drop I_aR_a is very small and hence drop in speed is also not significant from no load to full load.

3. Speed vs. Armature torque

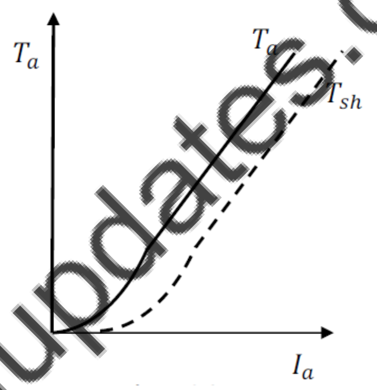
- These characteristic can be derived from the above two characteristic.
- This graph is similar to speed- armature current characteristic as torque is proportional to armature current.

CHARACTERISTIC OF SERIES MOTOR

4. Armature torque vs. Armature current

- For the series motor the series field winding is carrying the entire armature current, hence

$$\therefore\ T \ \alpha\ I_a^2$$



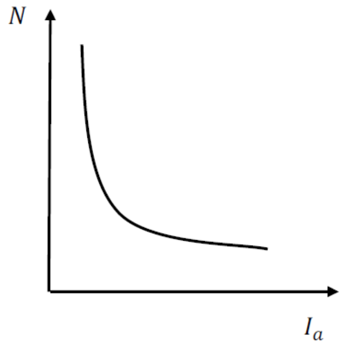
- Thus torque in case of series motor is proportional to the square of the armature current. This relation is parabolic in nature.
- As load increases armature current increases and torque produced increases proportional to the square of the armature current up to certain limit.
- As the entire armature current flows through series field, there is a property of electromagnet called saturation, may occur.
- Saturation means though the current through the winding increases, the flux produced remains constant. Hence after saturation the characteristic takes the shape of straight line as flux becomes constant.

5. Speed vs. Armature current

- From the speed equation we get

$$N\ \alpha\ \frac{V - I_aR_a - I_aR_{se}}{I_a}$$

- The values of $\mathbf{R_a}$ **and** $\mathbf{R_{se}}$ are so small that the effect of change in $\mathbf{I_a}$ on speed overrides the effect of change in $\mathbf{V - I_aR_a - I_aR_{se}}$ on the speed.
- Hence in speed equation $\mathbf{E_b \approx V}$ and can be assumed constant.



- So speed equation reduces to

$$\mathbf{N\ \alpha\ \frac{1}{I_a}}$$

- So speed armature current characteristic is rectangular hyperbola as shown in fig.

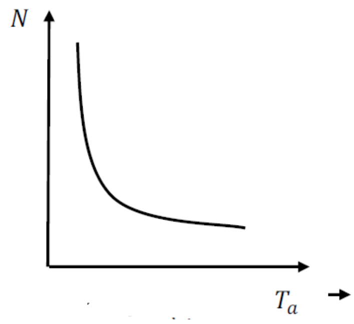
6. Speed vs. Armature torque

- In case of series motor

$$\mathbf{T\ \alpha\ I_a^2 \quad and \quad N\ \alpha\ \frac{1}{I_a}}$$

- Hence we can write

$$\mathbf{N\ \alpha\ \frac{1}{\sqrt{T}}}$$



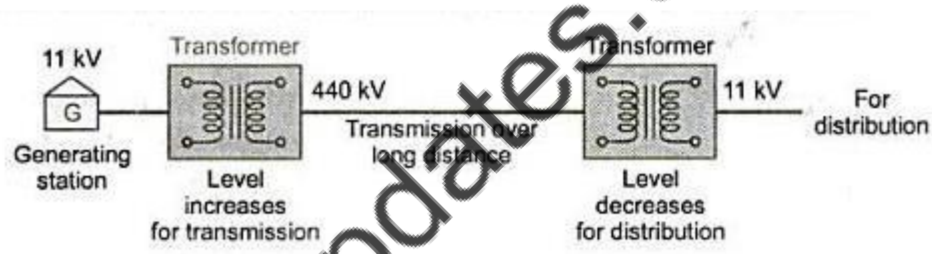
- Thus as torque increases when load increases, the speed decreases.
- On no load torque is very less and hence speed increases to dangerously high value.

Module 3 b: Single Phase Transformers

INTRODUCTION

The main advantage of alternating currents over direct currents is that, the alternating currents can be easily transferable from low voltage to high or high voltage to low. Alternating voltages can be raised or lowered as per requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called transformer. The transformer works on the principle of mutual induction. It transfers an electric energy from one circuit to other when there is no electrical connection between the two circuits. Thus we can define transformer as below.

The transformer is a static piece of apparatus by means of which an electrical power is transformed from one alternating current circuit to another with the desired change in voltage and current, without any change in the frequency.



Use of transformers in transmission system

Principle of operation

A transformer works on the principle of electromagnetic induction and mutual inductance between the two coils. The general arrangement of the transformer is shown in fig 3.2.1

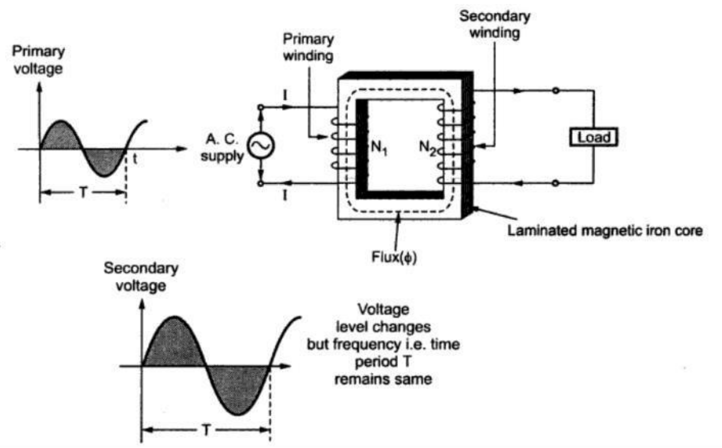


Fig Basic Transformer

A steel core consists of laminated sheets about 0.4-0.7mm thick insulated from each other. The core is laminated to reduce eddy current loss. The vertical parts of the core are known as limbs, while the top and bottom parts are called yokes.

There are two separate electrical windings, which are linked through a common magnetic part. These windings are isolated from each other electrically.

The coil into which electrical energy is fed is called primary winding (P), while the other coil from which electrical energy is drawn is called the secondary winding (S). The primary winding has N_1 number of turns while the secondary winding has N_2 number of turns.

When the primary winding is connected to an alternating voltage V_1 , an alternating current flows through the primary winding P and this current produces an alternating flux φ in the steel core, the mean path of this flux being indicated by the dotted lines. If the entire flux produced by P passes through S, the EMF induced in each turn is the same for P and S.

The above mentioned alternating flux φ produces self induced EMF E_1 in the primary winding P, while due to mutual induction i.e., due to flux produced by primary linking the secondary, it produces mutually induced EMF E_2 in the secondary winding S.

Then EMFs are $E_1 = -N_1 \frac{d\varphi}{dt}$ and $E_2 = -N_2 \frac{d\varphi}{dt}$

Therefore $\frac{E_1}{E_2} = \frac{N1}{N_2} = K$

K is known as the ***voltage transformation ratio.***

The frequency of the two EMFs is the same. The voltage transformation ratio may be alternatively obtained as follows.

The EMF per turn is the same for P and S. hence,

$$\frac{\text{total EMF induced in S}}{\text{total EMF induced in P}} = \frac{N_2 * \text{EMF per turn}}{N_1 * \text{EMF per turn}} = \frac{N_2}{N_1} = K$$

When the secondary is on open circuit, its terminal voltage is the same as the induced EMF. The primary current is then small, so that the applied voltage V_1 is practically equal and opposite to the EMF induced in P. Hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = K \qquad (1)$$

As the full load efficiency of a transformer is almost 100%

$V_1 I_1$* Primary power factor = $V_2 I_2$* secondary power factor

As both the primary and secondary power factor are almost equal on full load

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = K \qquad (2)$$

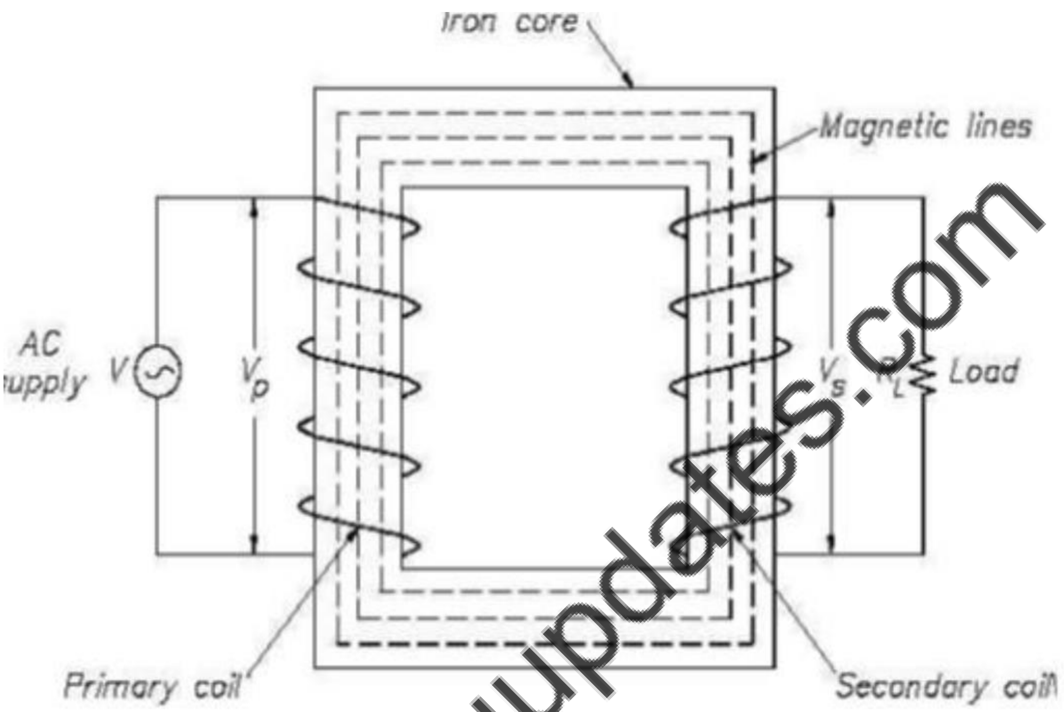
From eqns (1) and (2)

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Construction of transformers

- There are two basic parts of a transformer:
  1) Magnetic core 2) winding
- The core of the transformer is either rectangular or square in size.
- The core is divided into i) Yoke ii)Limb
- Core is made up of silicon steel which has high permeability and low hysteresis co-efficient.
- The vertical portion on which the winding is wound is called Limb.
- The top and bottom horizontal portion is called Yoke.
- The core forms the magnetic circuit
- There are 2 windings i) Primary winding ii) Secondary winding which forms the Electric circuit. made up of conducting material like copper.
- The winding which is connected to the supply is called primary winding and having 'N1' number of turns.

- The winding which is connected to a load is secondary winding and having 'N2' number of turns.
- Lamination of the core minimises eddy current loss.
- These laminations are insulated from each other by a thin coating of suitable varnish.
- The thickness of the lamination ranges from 0.35mm for a frequency of 25Hz to 0.5mm for a frequency of 50Hz.
- The lamination strips are assembled, where the joints are staggered to avoid narrow gaps all through the cross section of the core



Types of Transformer

The two main types of transformers are

1. core type
2. shell type

3.4.1 Core type transformer

- It has a single magnetic circuit.
- The core is rectangular having two limbs.
- The winding encircles the core.
- The coils used are of cylindrical type.
- The coils are wound in helical layer with different layers insulated from each other by paper or mica.

- Both the coils are placed on both the limbs.
- The low voltage coil is placed inside, near the core while the high voltage coil surrounds the low voltage coil.
- Core is made up of large number of thin laminations.
- As the windings are uniformly distributed over the two limbs the natural cooling is more effective.
- The coils can be easily removed by removing the lamination of the top yoke , for maintenance.
- Fig (a) shows the schematic representation of the core type transformer while(b) shows the view of actual construction of the core type transformer.

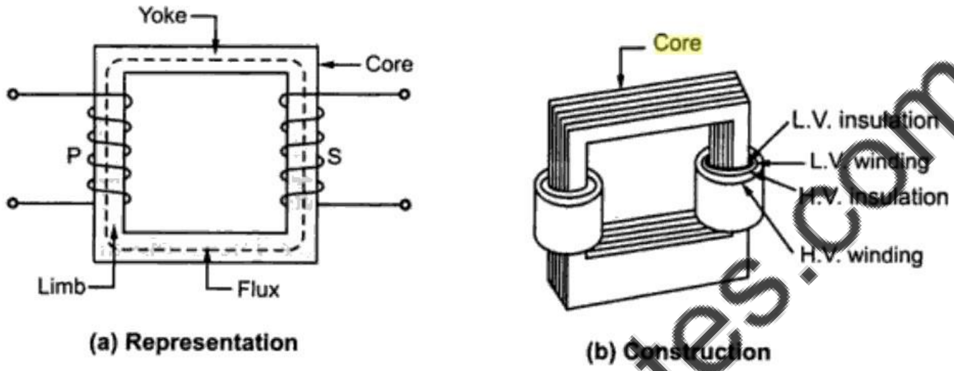


Fig Core Type of Transformer

Shell type transformer

- It has a double magnetic circuit.
- The core has three limbs.
- Both the windings are placed on the central limb.
- The core encircles most part of the windings.
- The coils used a re generally multilayer disc type or sandwich coils.
- Each high voltage coil is in between low voltage coils and low voltage coils are nearest to top and bottom of the yokes.
- The core is laminated.
- While arranging the lamination of the core, the care is taken that all the joints at alternate layers are staggered.
- This is done to avoid narrow air gap at the joints, right through the cross section of the core. Such joints are called overlapped or imbricated joints.
- Generally for very high voltage transformers, the shell type construction is preferred.
- As the winding is surrounded by the core, the natural cooling does not exist.

- Fig (a) shows the schematic representation of the shell type transformer while (b) shows the view of actual construction of the shell type transformer

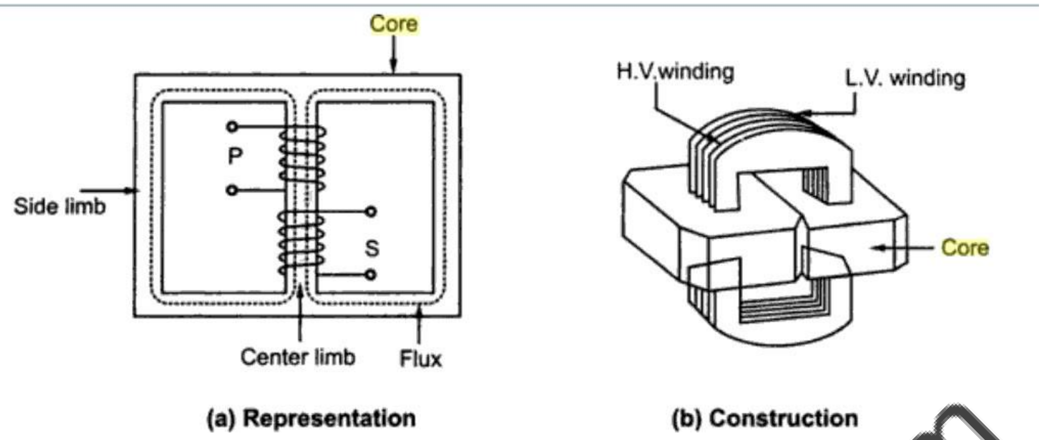


Fig 3.4.2 Shell Type Transformer

EMF Equation

Let us consider a transformer having:

N_1 = primary turns

N_2 = secondary turns

φ_m = maximum value of the flux in the core linking both the windings

= B_mA

Where B_m= maximum flux density in the core (Wb/ m^2)

A = area of cross section of the core (m^2)

f = frequency of AC input in hertz (Hz)

The flux in the core will wary sinusoidally as shown in fig 3.5, so that it increases from zero to maximum value φ_m in one quarter of the cycle i.e in $\frac{1}{4f}$ seconds

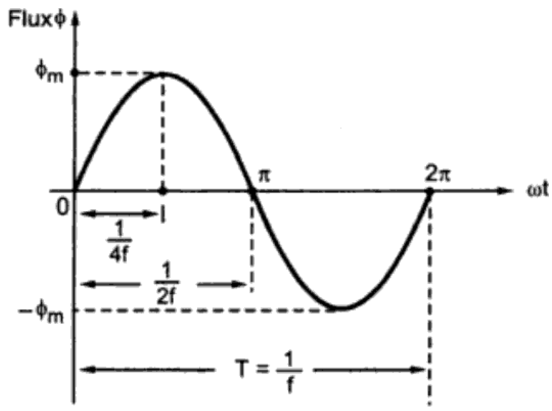


Fig 3.5 Sinusoidal Flux

Therefore, average rate of change of flux = $\frac{\varphi m}{\frac{1}{4f}}$ = 4 f φ_m

We know that rate of change of flux per turn means induced EMF in volts

Therefore average EMF/ turn = 4 f φ_m

Since the flux is varying sinusoidally, the RMS value of the induced EMF is obtained by multiplying the average value by the form factor.

Therefore RMS value of the EMF induced/ turn = 1.11 * 4 f φ_m

= 4.44 f φ_m volts

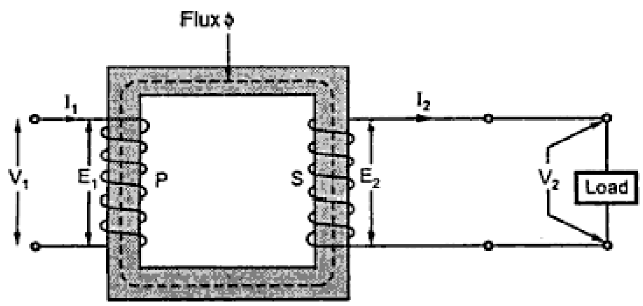
Therefore RMS value of induced EMF in the entire primary winding = (induced EMF/turn) * no. of primary turns

$E_1 = 4.44$ f $\varphi_m N_1$ = 4.44 f B_m A N_1------------------------ (1)

In the similar manner, the RMS value of induced EMF in the entire secondary winding is

$E_2 = 4.44$ f $\varphi_m N_2$ = 4.44 f B_m A N_2----------------------- (2)

Ratios of a Transformer



Current Ratio

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1 is same as the product of secondary voltage V_2 and the secondary current I_2 .

V_1I_1 = input KVA and V_2I_2 = output KVA

For an ideal transformer

$$V_1I_1 = V_2I_2$$

$$\frac{V_2}{V_1} = \frac{I1}{I_2} = K$$

Volt ampere rating

When electrical power is transferred from primary winding to secondary, there are few power losses in between. These power losses appear in the form of heat which increases the temperature of the device. Now this temperature must be maintained below certain limiting value as it is always harmful from insulation point of view. As current is the main cause in producing heat, the output maximum rating is generally specified as the product of output voltage and output current i.e. V_2I_2. This always indicates that when transformer is operated under this specified rating, its temperature rise will not be excessive. The copper losses depend on current and iron losses depend on voltage. These losses are independent of the load power factor cosφ_2. Hence though the output power depends on cos φ_2 , the transformer losses are functions of V and I and the rating of the transformer is specified as the product of voltage and current called VA rating. This rating is generally expressed in kVA (kilovolt amperes rating).

Now ,

$$\frac{V_2}{V_1} = \frac{I1}{I_2} = K$$

$$V_1I_1 = V_2I_2$$

$$\text{KVA rating of transformer} = \frac{V1I1}{1000} = \frac{V_2I_2}{1000}$$

If V_1 and V_2 are the terminal voltages of primary and secondary then from specified kVA rating we can decide full load currents of primary and secondary, I_1 and I_2. This is the safe maximum current limit which may carry, keeping temperature rise below its limiting value.

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \quad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Losses in Transformer

There are two types of power losses occur in a transformer

1) Iron loss 2) Copper loss

1) **Iron Loss (Pi)**: This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.
a) Eddy current loss b) Hysteresis loss

The Iron losses are called as the constant losses.

a) Eddy current loss (We) :

This power loss is due to the alternating flux linking the core, which will induced an emf , due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss.

$$\boxed{\text{Eddy current loss} = K_e B_m^2 f^2 t^2 \text{ watts/unit volume}}$$

where K_e = eddy current constant

t = thickness of the core

 Eddy current loss is proportional to the square of the supply frequency.
 Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents.

b) Hysteresis loss (W_h): This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

$$\boxed{\text{Hysteresis loss} = K_h B_m^{1.67} f v \text{ watts}}$$

K_h = hysteresis constant depends on material

B_m = maximum flux density

f = frequency.

v = volume of the core

Hysteresis loss can be minimized by using the core material having high permeability.

Total Iron loss Pi = We + Wh

The flux in the core is almost constant as supply voltage V_1 at rated frequency f is always constant. Hence the flux density B_m in the core and hence both hysteresis and eddy current losses are constants at all the loads. Hence the core or iron losses are also called constant losses. The iron losses are denoted as W_i.

The iron losses are minimized by using high grade core material like silicon steel having very low hysteresis loop and by manufacturing the core in the form of laminations.

2) Copper loss or I²R losses (Pcu) :

The copper losses are due to the power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

$$\begin{aligned}\text{Total Cu loss} &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 (R_1 + R_2') = I_2^2 (R_2 + R_1') \\ &= I_1^2 R_{1e} = I_2^2 R_{2e}\end{aligned}$$

The copper losses are denoted by W_{cu}. If the current through the winding is full load current, we get copper losses at full load. If the load on the transformer is half then we get copper losses at half loads which are less than full load copper losses. Thus copper losses are called variable losses. For transformer VA rating is V_1I_1 or V_2I_2 . As V_1 is constant we can say that copper losses are proportional to the square of the KVA rating and square of the current.

So, $W_{cu}\, \alpha\, I^2\, \alpha\, (KVA)^2$

Thus for a transformer

Total loss = iron losses + copper losses = $W_i + W_{cu}$

Thus if current is full load then copper losses are full load losses denoted by $W_{cu}\,(F.L)$.
If current is fraction of full load where n is the fraction then new copper losses are $n^2 W_{cu}\,(F.L)$.

Efficiency of Transformer

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied

Therefore Power output = Power input – total losses

Therefore Power input = Power output + total losses

= Power output + $W_i + W_{cu}$

The efficiency of any device is defined as the ratio of power output to power input. So for a transformer the efficiency can be expressed as

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$$\eta = \frac{\text{Power output}}{\text{Power output} + W_i + W_{cu}}$$

Now power output = $V_2 I_2 cos\varphi$

Where $cos\varphi$ = load power factor

The transformer supplies full load of current I_2 and with terminal voltage V_2

W_{cu} = copper loss on full load = $I_2^2 R_{2e}$

Therefore, $\eta = \dfrac{V_2 I_2 cos\varphi}{V_2 I_2 cos\varphi + W_i + I_2^2 R_{2e}}$

but $V_2 I_2$ = VA rating of a transformer

Therefore $\eta = \dfrac{(VA\ \ rating) cos\varphi}{(VA\ rating\ ) cos\varphi + W_i + I_2^2 R_{2e}}$

$$\boldsymbol{\eta = \frac{(VA\ rating) cos\varphi}{(VA\ rating\ ) cos\varphi + W_i + I_2^2 R_{2e}} * 100}$$ ……………………Full load efficiency

$$\boldsymbol{\eta = \frac{(VA\ rating) cos\varphi}{(VA\ rating\ ) cos\varphi + W_i + W_{cu}(F.L)} * 100}$$ …………………… full load efficiency

But if the transformer is subjected to fractional load then using the appropriate values of the quantities, the efficiency can be obtained.

When load changes, the load current changes by same proportion

Therefore new $I_2 = n\ I_2$ (F.L)

Similarly as copper losses are proportional to the square of the current then,

New $(W_{cu}) = n^2 W_{cu}(F.L)$

In general for fractional load the efficiency is given by

$$\boldsymbol{\eta = \frac{(VA\ rating) cos\varphi}{(VA\ rating\ ) cos\varphi + W_i + n^2 W_{cu}(F.L)} * 100}$$

Condition for Maximum Efficiency

When a transformer works on a constant Input voltage and frequency then efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further the efficiency starts decreasing.

Let us determine,

1. Condition for maximum efficiency.

2. Load current at which η_{max} occurs.

The efficiency is functions of loads i.e. load current I_2 assuming $\cos\varphi_2$ constant. The secondary terminal voltage V_2 is also assumed constant So for maximum efficiency,

$$\frac{d\eta}{dI_2} = 0$$

Now $$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}}$$

$$\therefore \quad \frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}}\right] = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}) \frac{d}{dI_2}(V_2 I_2 \cos\phi_2)$$
$$- (V_2 I_2 \cos\phi_2)\cdot \frac{d}{dI_2}(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}) = 0$$

$$(V_2 I_2 \cos\varphi_2 + W_i + I_2^2 R_{2e})(V_2\cos\varphi_2) - (V_2 I_2 \cos\varphi_2)(V_2\cos\varphi_2 + 2I_2R_{2e}) = 0$$

Cancelling $V_2\cos\varphi_2$ from both the terms we get

$$V_2 I_2 \cos\varphi_2 + W_i + I_2^2 R_{2e} - V_2 I_2 \cos\varphi_2 - 2I_2^2 R_{2e} = 0$$

$$W_i - I_2^2 R_{2e} = 0$$

$$\mathbf{W_i = I_2^2 R_{2e} = W_{cu}}$$

So condition to achieve maximum efficiency is that

Copper loss = Iron Loss i.e. $W_i = W_{cu}$

Load corresponding to maximum efficiency

If X is the load under maximum condition, W_i becomes cu loss for X kVA. We know that Cu loss is directly proportional to $(kVA)^2$, so

W_{cu} α (full load kVA)^2

Or W_i α $(X)^2$

Therefore , $$\left(\frac{X}{full\ load\ kVA}\right)^2 = \frac{W_i}{W_{cu}}$$

$$X = \text{Full load kVA} * \sqrt{\frac{W_i}{W_{cu}}}$$

$$\mathbf{X = Full\ load\ kVA} * \sqrt{\frac{\textit{Iron loss}}{\textit{full load copper loss}}}$$