Group Coursework Submission Form

Specialist Masters Programme

Please list all names of group members: (Surname, first name) 1. Patel, Prachin 2. Rusakov, Lachezar 3. Shen, Zhao 4. Wan, Jinda	**GROUP NUMBER:** 8
MSc in: Financial Mathematics, Mathematical Trading and Finance, Quantitative Finance	
Module Code: SMM270	
Module Title: Foundations of Econometrics	
Lecturer: Dr. Giovanni Urga	**Submission Date:** **01/12/2023**

Declaration:

Marker's Comments (if not being marked on-line):

Deduction for Late Submission of assignment:

For Students:
Once marked please refer to Moodle for your final coursework grade, including your Peer Assessment grade.

Group 8: Prachin Patel, Lachezar Rusakov, Zhao Shen, Jinda Wan

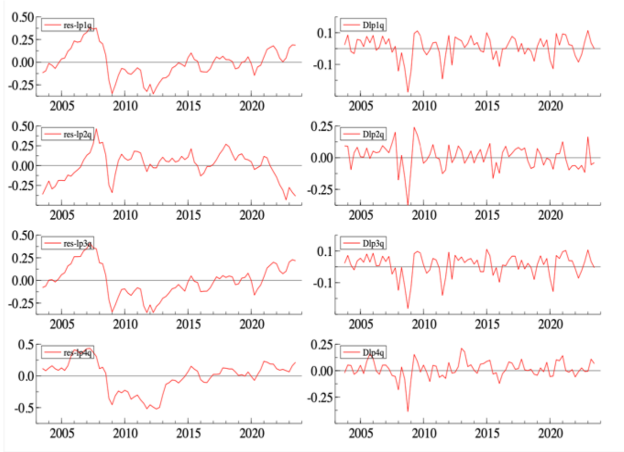
Econometrics Coursework

Question 1:

Conducted data processing by transforming and computing quarterly average log returns for the stock indices on Oxmetrics.

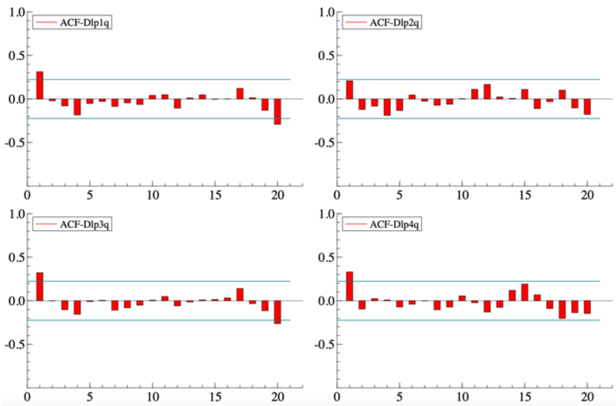
Question 2:

Figure 1 – Residuals & Differenced Series



A)

All four of the series, lp1q, lp2q, lp3q and lp4q are difference stationary since they are mean reverting to 0 as per the first difference plots. However, in the plots of the detrended series i.e. their residuals they are not mean reverting. This suggests the deterministic trend is not prevalent whereas the series exhibits stochastic trend thus they are not trend stationary.



Even looking at the autocorrelation function plots we observe that the 1st lag of the first-difference is significant while the other lags or not. This suggests that differencing the series has made it more stationary.

B)

The Augmented Dickey Fuller (ADF) test is used to determine whether the time series has a unit root. The null hypothesis of the test is that the unit root is present, therefore the time series is non-stationary whereas in the alternative hypothesis the time series is stationary. The test can only be accurate if the error term is not autocorrelated. Upon doing the ADF test with the constant and trend, one observes that the trend factor is not significant hence when testing further we use critical values with the constant only without taking into account the structural breaks in the data. The test requires running the regression as given in equation 1:

$$\Delta lpnq_t = \alpha + \beta t + \gamma lpnq_{t-1} + \delta_1 \Delta lpnq_{t-1} + \cdots + \delta_{p-1} \Delta lpnq_{t-p+1} + \varepsilon_t \ \forall\, n \in \{1,\ 2,\ 3,\ 4\} \qquad \textit{Equation 1}$$

lp1q, lp2q, lp3q and lp4q are non-stationary as they fail to reject the null hypothesis of the ADF test at both 1% and 5% level of significance. However, all four of them are difference stationary since the returns series i.e. the first differences reject the null hypothesis of the ADF as the critical value is smaller than the 1 % level of significance. Moreover, Dlp1q, Dlp2q, Dlp3q and Dlp4q have order of integration 0 i.e. have their Akaike Information Criteria (AIC) lowest at the D-lag 0. Therefore lp1q, lp2q, lp3q are lp4q are difference stationary with the order of integration I(1). As per Holden and Perman (1994), one should reconsider the standard ADF regression when there are structural breaks present in the data. This has come to light since majority of the time series for returns are first difference stationary, but when structural breaks are accounted for, they tend to be trend stationary according to the test. This is because it allows the drift parameter, α, to change due to the integration of dummy variables in the modified ADF regression as given in equation 2:

$$lpnq_t = \alpha + \theta DU_t + \beta t + \gamma DT_t + dD(TB)_t + \rho lpnq_{t-1} + \sum_{i=1}^{k} c_i \Delta lpnq_{t-i} + \varepsilon_t \ \forall\, n \in \{1,\ 2,\ 3,\ 4\}\ \textit{where } DU_t, DT_t \textit{ and } D(TB)_t \textit{ are dummy variables} \qquad \textit{Equation 2}$$

In the null hypothesis we impose the restrictions that $\gamma = \beta = 0$ and $\rho=1$ whereas in the alternative hypothesis $\rho<1$ and β, γ, θ are non-zero resulting the series to be trend stationary. The team found the structural break in the data at the 3rd quarter in 2009 through trial and error. This aligns with the timeline of the 2007-09 Global Financial Crisis where all the major stock indices faced a sharp decline in returns. Thus, the errors follow a trend until the structural breaking point after which they are mean reverting as seen in Figure 1. Even running the ADF test with the data post the structural break one observes that all 4 of the series are still difference stationary with the order of integration 1.

Question 3:

A)

The misspecification tests are used to check whether the model used is a good representation of the relationship in the data.

The misspecification tests implemented are:

1. Error autocorrelation test: The test by Godfrey where the null hypothesis is that there is no serial correlation in the errors, and the alternative hypothesis is that serial correlation is present in the errors.
2. Autoregressive Conditional Heteroscedasticity: The test by Engle where the null hypothesis is that there is no autoregressive conditional heteroscedasticity (ARCH effects) this means that the conditional variance of the error term is constant. The alternative hypothesis is that the data has ARCH effects, which means that the conditional variance varies over time.
3. Normality test: The test by Doornik and Hansen where the null hypothesis is that the errors are normally distributed. The alternative hypothesis is that the errors are not normally distributed. A high test statistic indicates non-normality.
4. Heteroscedasticity test: The test by White where the null hypothesis is that the errors are homoscedastic i.e. constant variance over time. The alternative hypothesis is that the errors are heteroscedastic i.e. time varying variance.
5. RESET test: The test by Ramsey where the null hypothesis is that the functional form of the model is the correct specification. The alternative hypothesis is that the model is misspecified.

The ADL(1,1) model is often considered as the gold standard model since it provides a lot of flexibility, captures the dynamic relationships and is parsimonious. The model makes the following assumptions:

1. No autocorrelation of errors
2. No autoregressive conditional heteroscedastic effects
3. Errors are normally distributed
4. Errors are homoscedastic
5. Model is correctly specified

Figure 2 – ADL(1,1) Model

```
EQ(10) Modelling lp1q by OLS
      The dataset is: /Users/macbook/Downloads/ECOTRIX COURSEWORK LATEST.oxdata
      The estimation sample is: 2003(4) - 2023(3)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
lp1q_1               0.902115    0.05651     16.0  0.0000   0.7703
Constant             0.399444     0.2949     1.35  0.1796   0.0236
lpq4                 0.588060    0.07152     8.22  0.0000   0.4708
lpq4_1              -0.547780    0.08033    -6.82  0.0000   0.3796

sigma               0.0509611  RSS               0.197374378
R^2                  0.919232  F(3,76) =       288.3 [0.000]**
Adj.R^2              0.916044  log-likelihood       126.672
no. of observations        80  no. of parameters          4
mean(lp1q)            8.07651  se(lp1q)            0.175878

AR 1-5 test:      F(5,71)   =   2.8050 [0.0228]*
ARCH 1-4 test:    F(4,72)   =   1.4833 [0.2162]
Normality test:   Chi^2(2)  =   2.7768 [0.2495]
Hetero test:      F(6,73)   =   1.3739 [0.2366]
RESET23 test:     F(2,74)   =  0.45837 [0.6341]
```

The team first regressed the Autoregressive Distributed Lag model, ADL(1,1), between lp1q and lp4q to get the misspecification tests:

$$lp1q_t = \beta_0 + \alpha_1 lp1q_{t-1} + \beta_1 lp4q_t + \beta_2 lp4q_{t-1} + \varepsilon_t \qquad \textit{Equation 3}$$

The misspecification tests for ADL model as seen in Figure 2 suggests that all the assumptions hold except for the errors being autocorrelated. Since we fail to reject the null hypothesis of the ARCH 1-4 test, Normality test, Hetero test and RESET test as the p-values are much larger than 5%. Therefore, the residuals are not independently and identically distributed, so one needs to obtain a model which align with all the assumptions.

The team then imposed restrictions on the ADL(1,1) to derive and estimate the 9 nested models to obtain a better specified and parsimonious model as provided by Hendry, Pagan and Sagan (1984).

B)

After testing the restrictions using F test, it boils down to 3 nested models that collapses from the ADL model which are Differenced Data model, Static model with AR(1) errors and Homogeneous Equilibrium Correction model. Inspecting the misspecification tests for the models one notices Differenced Data model and Homogeneous Equilibrium model have serially correlated errors and are misspecified. On the other hand the Static model with AR(1) error is the closest to the ADL(1,1) model in terms of specification since it only has non-normal errors and has the highest adjusted R^2. Static model with AR(1) errors is given by equation 4:

$$lp1q_t = \alpha + \beta lp4q_t + \varepsilon_t \; where \; \varepsilon_t = \gamma\varepsilon_{t-1} + u_t \qquad \textit{Equation 4}$$

It is simpler to correct for the non-normality of errors than for the serially correlated errors thus to remove the non-normality of errors from Equation 4, the solution is to introduce dummy variables which resulted the model and misspecification test as given in Figure 3. Comparing the adjusted R^2 one observes that the Static model with AR(1) errors and dummy variables provides more predictability than the ADL model, 0.945 > 0.916. As per the graphic analysis from Figure 4 and 5 one sees the change in the errors such that they are not autocorrelated anymore and they are now even more closely normally distributed in the density plot. The fitted line now also captures the extremities and the outliers of the data.

Figure 3 - Static Model with AR(1) Errors & Dummy Variables

```
EQ( 2) Modelling lp1q by OLS
       The dataset is: /Users/macbook/Downloads/ECOTRIX COURSEWORK LATEST.oxdata
       The estimation sample is: 2003(4) - 2023(3)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              4.68750     0.1285     36.5  0.0000   0.9494
lp4q                 0.351323    0.01325     26.5  0.0000   0.9083
res_tp_1             0.869988    0.04514     19.3  0.0000   0.8395
dumm2008            -0.139123    0.04266    -3.26  0.0017   0.1303
dumm2009            -0.138540    0.04273    -3.24  0.0018   0.1289
dumm2011            -0.185686    0.04240    -4.38  0.0000   0.2127
dumm2012            -0.123405    0.04247    -2.91  0.0049   0.1063
dumm2020            -0.119165    0.04187    -2.85  0.0058   0.1024
dumm2023             0.110721    0.04228     2.62  0.0108   0.0881

sigma               0.0413747  RSS               0.121542200
R^2                  0.950264  F(8,71) =      169.6 [0.000]**
Adj.R^2              0.944659  log-likelihood       146.066
no. of observations        80  no. of parameters          9
mean(lp1q)            8.07651  se(lp1q)            0.175878

AR 1-5 test:      F(5,66)   =   1.3393 [0.2588]
ARCH 1-4 test:    F(4,72)   =   1.1691 [0.3317]
Normality test:   Chi^2(2)  =   1.7672 [0.4133]
Hetero test:      F(4,69)   =  0.67761 [0.6098]
RESET23 test:     F(2,69)   =   1.0990 [0.3390]
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Figure 4 – ADL(1,1) Graphic

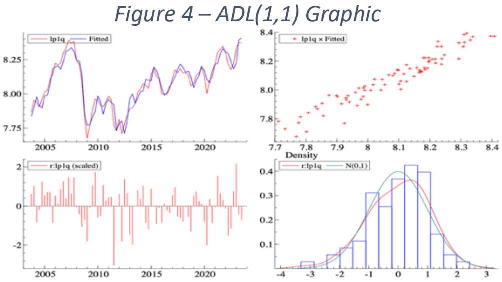
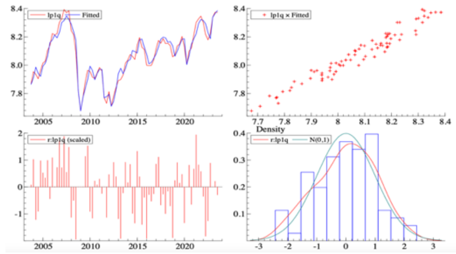


Figure 5 – Static model with AR(1) and Dummy Variables



Question 4:

A)

For testing the cointegration between any two series, they require to be in the same order of integration otherwise one cannot go further with the testing. In the case for lp3q and lp4q, from question 2 it is obtained that they are both first-difference stationary i.e. the order of intergration 1, I(1). Performing the single error correction mechanism requires estimating the ADL(1,1) model as seen from question 3 on the first-differenced data:

$$Dlp3q = \alpha + \beta_0 Dlp4q + \beta_1 lp3q_{t-1} + \beta_2 lp4q_{t-1} + u_t \qquad \text{Equation 5}$$

The next step is to apply the ADF test without the constant and trend on the residuals gathered from running the above regression where the hypothesis testing is as follows:

$$H_0\colon \hat{u}_t \sim I(1) \ vs.\ H_1\colon \hat{u}_t \sim I(0)$$

The outcomes of unit root tests for the residuals from the regression rejects the null hypothesis across all considered lags at the 5% significance level. Furthermore, up to 3 lags, the null hypothesis is rejected at the 1% significance level. This rejection indicates that the unit root is not present. Consequently, cointegration exists between lp3q and lp4q, hence their correlation is not spurious.

B)

Cointegration and spurious regressions are both the concepts in the context of time series analysis, they describe the relationship between variables and time.
The concept of cointegration was introduced by Clive Granger in the early 1980s, the cointegration is defined as if two or more non-stationary time series variables have a long-run equilibrium relationship, then they are said to be cointegrated. The key points of cointegration are the non-stationary time series which described a time series that has a changing mean or variance over time, and the integration order which implies that a linear combination of the individual variables is stationary.
The spurious regression has been recognized as a potential issue in econometrics for many years, however, it was deeply investigated by David Hendry in 1974. The spurious regression occurs when two unrelated, non-stationary time series appear to be correlated but they are not actually correlated, the cause of the spurious regression is the presence of a common, non-stationary factors that influence both variables, as a result, they seem to be correlated.
Both spurious regression and cointegration regression are complex in the economic field, and they are related to some realistic issues and will be discussed below.

In early studies, researchers think that the stock return is predictable, based on the lagged instrumental variables, in the current conditional asset pricing literature, however, the research from Ferson find that the stock returns are not highly autocorrelated, but there is a spurious regression bias in predictive regressions for stock returns *(Ferson*a, 2003)*, furthermore, the data mining and the spurious regression are interacted with each other, and the effects reinforce each other since the high R-squares in predictive regressions will uncover the spurious, persistent regressors *(Ferson*a, 2003).* Ferson highlights the issue of spurious regression bias in financial economics, and the simulation conducted in his study suggest that many regressions in the literature may be spurious, he also examines the properties of stock returns regressions with persistent lagged regressors and discusses the implications of spurious regressions and data mining biases.
In another Ferson's article, he discusses how the spurious regression bias can mislead analysts on stock returns and provide solutions to addressing the problems *(Ferson*b, 2003).* More specifically, he mentions that the variable like dividend yields and yield spreads can appear to be better predictors of returns if the expectations of stock's return are dependent through time, he is warning that searching for predictor variables using historical data increases the likelihood of finding variables with spurious regressions bias *(Ferson*b, 2003),* in addition*,* he suggested that many results in past researches on stock return predictability can be attributed to a combination of data mining and spurious regression *(Ferson*b, 2003).* To overcome these problems associated with the spurious regression in stock returns, Ferson provides three potential solutions: 1. Increase the number of lags in the Newey-West standard errors, which can lead the standard errors capture more persistence in the regression error, but using too many lags will result in inefficient and inaccurate results. 2. Include a lagged value of the dependent variable as an additional right-hand side variable in the regression, which the autocorrelation can be 'soaked up', leaving a clean residual. However, this solution only provides a partial solution and may not fully eliminate the spurious regression problem. 3. Using a practical solution called 'Stochastic detrending'. This approach involves transforming the lagged variable by subtracting the trailing moving average of its own past values. This method is recommended as a simple way to address the spurious regression problem in stock returns *(Ferson*b, 2003).* In conclusion, this article highlights the challenges associated with predicting stock returns and provides insights into how to solve these problems.
Clive Granger is a pioneer on developing the cointegration, he introduced the concept of cointegration which allowed for the merging of economic theory with dynamic econometric models and his work on cointegration has been widely used by major policy institutions for determining long-run outcomes *(Hendry, 2004).* Specifically, Granger developed the Granger representation theorem which states that a vector of stationary ***I(0)*** variables can be expressed as a linear combination of lagged differences of the variables if and only if a linear combination of the levels variables is ***I(0).*** The theorem provides a way to establish cointegration and derive an equilibrium correction model form *(Hendry, 2004)*. Granger also investigated non-linear cointegration, which allows for asymmetric effects in the drift towards equilibrium, this is particularly relevant in macroeconomics, where variables like planned inventories and orders can have asymmetric impacts *(Hendry, 2004).* In conclusion, Granger's work highlights the importance of cointegration in econometric analysis and its role in understanding the long-run relationships between variables *(Hendry, 2004)*
Through discussing some studies about cointegration and spurious regression, we can see that both cointegration and spurious regression are important in realistic economic problems, and they have been already studied for a very long time. Ferson in his study warning that some regression in the literature may be spurious such as the stock return, he also demonstrated the relations between the data mining and spurious regression, and he provided some practical solutions to these problems. Granger developed the concept of cointegration, and he conducted some meaningful studies on cointegration regression, which had a significant impact on the field of economics and has shaped the research agenda of many economists.

C)

$$Dlp3q = Dlp4q + \varepsilon_t \qquad \textit{Equation 6}$$

We implemented an OLS model, as outlined in Equation 6. As we have proved that there is cointegration, from the statistical significance of the coefficient we can conclude that there is long-run relationship between lp3q and lp4q. We obtained the residuals from the regression. To assess the stationarity of these residuals, we conducted an Augmented Dickey-Fuller (ADF) test. At the 1% significance level, the critical value for the ADF test is -2.59, while the corresponding t-statistic is -2.747. We reject the null hypothesis of non-stationarity and conclude that the residuals are stationary. Therefore, we can go ahead with the second step:

$$Dlp3q = Dlp4q + res_{t-1} \qquad \textit{Equation 7}$$

Running the second stage of the 2-step Engel-Granger, as outlined in equation 7, will show us the short-term relationship between the variables. The residuals coefficient is statistically significant at the 1% level, with value of -0.181895. This implies that approximately 18% of the disequilibrium in the previous period is rectified in the current period. In instances of deviation from long-run equilibrium, it is estimated that the system takes 5.5 quarters to return to equilibrium.

Question 5:

A)

Granger causality null hypothesis: no granger causality or H_0: $\beta_1 + \beta_2 + \cdots + \beta_n = 0$

Granger causality alternative hypothesis: H_1: $\beta_1 + \beta_2 + \cdots + \beta_n \neq 0$

$$dlp1q = \alpha + \sum_{i=1}^{4} \beta_{1,i} dlp1q_{t-i} + \sum_{i=1}^{4} \beta_{2,i} dlp4q_{t-i} + \varepsilon_t \quad \textit{Equation 8}$$

We took the first difference of the lp1q and lp4q due to their stationarity, as proven in question 2. Running the model with 4 lags, as shown in equation 8, the residuals are not normally distributed. To clear this misspecification test as one of the assumptions is that the errors are white noise, we excluded the residuals that exceed 1.75 standard errors. The result is a well specified model. The joint F-test for all for lags of lp4q does not provide sufficient evidence to reject the null hypothesis that the past values of the variables in lp4q do not Granger cause lp1q. The p-value associated with the joint F-test is 0.4147 which is well above 0.05 (5% significance level), indicating that there is not enough statistical evidence to conclude that the past values of the variables in lp4q contribute additional information in predicting the future values of lp1q beyond what is captured by the past values of lp1q alone.

$$dlp2q = \alpha + \sum_{i=1}^{4} \beta_{1,i} dlp2q_{t-i} + \sum_{i=1}^{4} \beta_{2,i} dlp4q_{t-i} + \varepsilon_t \quad \textit{Equation 9}$$

Taking the first difference of lp2q and lp4q to test whether lp4q granger causes lp2q we construct a model with four lags, as shown in equation 9. The model is failing ARCH 1-4, Hetero, and RESET23 tests. Excluding the outliers in the residuals which are exceeding 1.75 standard errors we construct a model with seven dummy variables. The model is well specified with the small exception of the ARCH 1-4 test, which rejects the null at 5% level of significance. The joint F-test for all for lags of lp4q does not provide sufficient evidence to reject the null hypothesis that the past values of the variables in lp4q do not Granger cause lp2q. The p-value associated with the joint F-test is 0.3260 which is well above 0.05 (5% significance level), indicating that there is not enough statistical evidence to conclude that the past values of the variables in lp4q contribute additional information in predicting the future values of lp1q beyond what is captured by the past values of lp1q alone.

$$dlp3q = \alpha + \sum_{i=1}^{4} \beta_{1,i} dlp3q_{t-i} + \sum_{i=1}^{4} \beta_{2,i} dlp4q_{t-i} + \varepsilon_t \quad \textit{Equation 10}$$

Taking the first differenced lp3q and lp4q to test whether lp4q granger causes lp3q we construct a model with four lags as shown in equation 10. The model is experiencing non-normality in the residuals. Excluding the outliers in the residuals greater than 1.75 standard error we construct a model with six dummy variables. This new model is well specified. The joint F-test for all for lags of lp4q does not provide sufficient evidence to reject the null hypothesis that the past values of the variables in lp4q do not Granger cause lp3q. The p-value associated with the joint F-test is 0.1984 which is well above 0.05 (5% significance level), indicating that there is not enough statistical evidence to conclude that the past values of the variables in lp4q contribute additional information in predicting the future values of lp1q beyond what is captured by the past values of lp3q alone.

B)

In the field of econometrics, Clive W. Granger's concept of Granger-causality, as elaborated by David F. Hendry, represents a significant shift in understanding causal relationships between economic variables. Granger's seminal work, first published in 1969, emphasised the significance of stochastic variables and the concept of time in defining causality (Granger, 1969). This approach was markedly different from the traditional causal interpretation of simultaneous equation systems, which frequently focused on 'instantaneous' causality with little regard for stochastic variables (Hendry, 2004).

Granger's causality definition was founded on the concept of prediction variance in stationary series. He proposed that if the joint distribution of a subset of observable variables changes when other variables' histories are excluded, then the latter is said to cause the former (Granger, 1969). This operational approach proposed that by including another set of economic variables, one group of economic variables could be predicted more accurately. Sims (1972) developed statistical methods to test the hypothesis of non-causality between economic variables based on the concept of predictability as a basis for causality.

In empirical economic research and policy analysis, Hendry and others have recognised the broad implications of Granger-causality. It is included in Engle, Hendry, and Richard's (1983) definition of strong exogeneity and has important implications for forecasting and policy decisions in econometric models. The relationship between Granger-causality and cointegration is important: if two variables are cointegrated, one must Granger-cause the other (Granger, 1986).

Hendry and Mizon (1999) expanded on the importance of Granger-causality in econometrics, demonstrating its importance in areas such as marginalising, conditioning, distributions of estimators and tests, simulation inference, cointegration, encompassing, forecasting, policy analysis, dynamic simulation, and impulse-response analysis. This broad impact emphasises the unifying aspect of Granger's concept in econometric modelling.
In summary, Granger-causality, as developed by Granger and expanded upon by Hendry, is critical in modern econometric theory and model construction. Its importance stems not only from its ability to represent "true causes," but also from its unifying framework for various aspects of econometric modelling, which improves our understanding of economic relationships and their predictability over time.

Walter N. Thurman and Mark E. Fisher used Granger-causality analysis in their 1988 paper, "Chickens, Eggs, and Causality," to address the age-old question: which came first, the chicken or the egg? This paper is a classic example of how econometric tools, specifically Granger-causality, can be used to analyse temporal relationships in data, even in unusual settings.
Clive W. Granger developed the statistical concept of Granger-causality, which examines whether one time series can predict another. It avoids traditional cause-and-effect relationships in favour of predictability (Granger, 1969). Thurman and Fisher applied this concept to time series data on egg production and chicken populations in the United States from 1930 to 1983 in their study. Their method involved building models to see if previous values of one variable (chickens or eggs) significant predictors of the current value were of the other. The following is the structure of the model for determining whether chickens Granger-cause eggs:

$$Eggs_t = \alpha + \sum \beta_i Eggs_{t-i} + \sum \gamma_i Chickens_{t-i} + \varepsilon_t \qquad \textit{Equation 11}$$

$Eggs_t$ represents the number of eggs at time t, $Eggs_{t-i}$ represents lagged egg values, and $Chikcens_{t-i}$ represents lagged chicken values. The coefficients β_i and γ_i reflect how past egg and chicken values affect current egg production. A statistically significant γ_i indicates that chickens Granger-cause eggs. They also experimented with reverse causality:

$$Chikens_t = \alpha + \sum \delta_i Chickens_{t-I} + \sum \varphi_i Eggs_{t-I} + \varepsilon_t \qquad \textit{Equation 12}$$

Table 1. Granger Causality Tests

Part 1: Did the Chicken Come First?

The following equation was estimated by OLS:

$$Eggs_t = \mu = \sum_{i=1}^{L} \alpha_i Eggs_{t-i} = \sum_{i=1}^{L} \beta_i Chickens_{t-i} + \epsilon_t;$$

$H_0 : \beta_1 = \ldots \beta_L = 0$ (chickens do not Granger cause eggs).

L = no. of lags	F-statistic	P-value	R^2 of the regression
1	.04	.85	.96
2	1.71	.19	.97
3	1.10	.36	.97
4	.79	.54	.97

Part 2: Did the Egg Come First?

The following equation was estimated by OLS:

$$Chickens_t = \mu + \sum_{i=1}^{L} \alpha_i Chickens_{t-i} + \sum_{i=1}^{L} \beta_i Eggs_{t-i};$$

$H_0 : \beta_1 = \ldots = \beta_L = 0$ (eggs do not Granger cause chichens).

L = no. of lags	F-statistic	P-value	R^2 of the regression
1	1.23	.27	.73
2	10.36	.0002	.81
3	5.85	.0019	.81
4	4.71	.0032	.82

Data source: U.S. Department of Agriculture, 1983 and others.
Note: The data are annual, 1930–83.

Figure 6

$Chikens_t$ represents the chicken population at time t, with $Chickens_{t-I}$ and $Eggs_{t-I}$ representing lagged chicken and egg values, respectively. Significant coefficients φ_i would imply Granger-cause eggs.
As shown in Figure 6, the results indicated unidirectional causality from eggs to chickens. The F-statistics and corresponding p-values for 1 to 4 lags (0.04, 1.71, 1.10, 0.79 with p-values of 0.85, 0.19, 0.36, 0.54) for the first model (did the chicken come first?) indicated that the null hypothesis (chickens do not Granger cause eggs) could not be rejected.
For the second model (did the egg come first?), the F-statistics and p-values for 1 to 4 lags (1.23, 10.36, 5.85, 4.71 with p-values of 0.27, 0.0002, 0.0019, 0.0032) clearly rejected the null hypothesis (eggs do not Granger cause chickens), especially for 2 to 4 lags.
Thurman and Fisher deduced from these findings that, within the framework of Granger-causality, the egg came first. They did, however, acknowledge the study's limitations due to the annual sampling period and emphasised that their findings do not provide insights into actual cause-effect relationships, but rather predictability and temporal ordering.
Thurman and Fisher (1988) present a humorous take on a classic question while also demonstrating the applicability and limitations of Granger-causality tests in econometrics. The use of such tests to analyse temporal relationships, even in unconventional settings, demonstrates the versatility and potential of econometric methods in research.

Question 6:

A)

The Engle's ARCH LM tests for autoregressive conditional heteroscedastic (ARCH) effects in the time series that is the volatility of a variable is not constant over time. This tests the phenomenon of volatility clustering which is found in financial time series where periods of high (low) volatility is followed by high (low) volatility. The assumptions of the tests are:

1) Linearity between the dependent variable and its lagged values
2) Residuals are independently and identically distributed (i.i.d)
3) Residuals are normally distributed
4) Residuals are homoscedastic
5) Time series data is stationary

The first step of the test requires obtaining the residual estimates by running the ARCH(1) process as given below.

$$Dlpnq_t = \mu_t + \varepsilon_t \forall\, n \in \{1,\ 2,\ 3,\ 4\}\ where\ Dlpnq_t = \sigma_t u_t\ ,\ u_t\ is\ a\ White\ Noise\ Process \qquad \textit{Equation 13}$$

These residuals are then used to do the second step which is to regress the squared residuals on its lags as given below in equation 14:

$$\widehat{\varepsilon_t^2} = \alpha_0 + \alpha_1 \widehat{\varepsilon_{t-1}^2} + \cdots + \alpha_p \widehat{\varepsilon_{t-p}^2} + u_t \qquad \textit{Equation 14}$$

The null hypothesis is that all the coefficients of the lagged residual squares are equal to 0 i.e. no ARCH effect and the alternative hypothesis is any one of the coefficients of the lagged residual squared is not equal to 0 then ARCH effect is present. The test statistic employed is TR^2 which follows the Chi-Square distribution with p (no of lags) degrees of freedom where T is the total number of observations and R^2 is the R^2 of the regression Equation. As per the empirical analysis conducted on the four stocks reveals presence of ARCH effects. The null hypothesis positing the absence of ARCH effects has been rejected for all four stocks as indicated in Figure 7. The p-values for all stocks are below 0.01 (1% significance). This indicates non-constant and evolving pattern of volatility over the observed time period therefore it is more appropriate to model volatility using more sophisticated models like GARCH(p,q).

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ARCH LM TEST:

Series #1/1: Dlp1
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ARCH 1-2 test:   F(2,5214) =  159.99 [0.0000]**
ARCH 1-5 test:   F(5,5208) =  179.20 [0.0000]**
ARCH 1-10 test:  F(10,5198)=  99.043 [0.0000]**

Series #1/1: Dlp2
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ARCH 1-2 test:   F(2,5214) =  629.08 [0.0000]**
ARCH 1-5 test:   F(5,5208) =  285.50 [0.0000]**
ARCH 1-10 test:  F(10,5198)=  166.41 [0.0000]**
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Series #1/1: Dlp3
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ARCH 1-2 test:   F(2,5214) =  190.68 [0.0000]**
ARCH 1-5 test:   F(5,5208) =  188.10 [0.0000]**
ARCH 1-10 test:  F(10,5198)=  104.35 [0.0000]**

Series #1/1: Dlp4
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ARCH 1-2 test:   F(2,5214) =  680.93 [0.0000]**
ARCH 1-5 test:   F(5,5208) =  328.54 [0.0000]**
ARCH 1-10 test:  F(10,5198)=  185.74 [0.0000]**
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Figure 7 – ARCH LM Test Results

B)

Now to estimate the best univariate GARCH representation amongst the alternative models the team utilized the trial-and-error method. The team used daily frequency data for the 4 stock indices since GARCH models provide better forecastability using more number of observations. As its more likely to then capture the 3 main factors of the volatility of returns: mean reverting, innovation factor and persistence factor. To identify the best univariate GARCH representation model for each of the stock indices the team conducted the following iterative procedure:

1) Financial daily time series empirically have conditional mean returns as 0 thus it is estimated using the ARMA(0,0) model assuming the data is normally distributed and without the regressor in conditional mean.
2) To estimate the conditional volatility GARCH(1,1) model is estimated again making the same assumptions that data is normally distributed and without the regressor in conditional variance.
3) Implement a battery of tests on the estimated model from step 1 and step 2 which are:
- Information Criterion
- Normality test
- Box/Pierce test on standardized residuals for the conditional mean estimates
- Box/Pierce test on standardized squared residuals for the conditional variance estimates
- ARCH test
4) Now iterate through steps 1-3 by assuming the data follows a variety of different distributions like student's t distribution, generalized error distribution (GED) and skewed student's t distribution.
5) Then perform the iteration of step 1-4 using different univariate GARCH representation models like EGARCH(1,1), GJRGARCH(1,1), APARCH(1,1), IGARCH(1,1) and RiskMetrics model to better capture the asymmetry feature of the volatility of returns. This effect is due to the risk averse behavior of investors where investors tend to overreact to bad news rather than good news hence the News Impact Function is skewed.
6) Compare the information criteria, the team selected the Schwarz criteria as it penalizes models with more parameters without the expense of the goodness of fit. The lower the Schwarz criteria the better the model is in terms of parsimony and predictability.

Based on the above procedure and criterion the team obtained the following result for the 4 stock indices:

1) Dlp1: GJRGARCH(1,1) model where the data is assumed to follow the skewed student's t distribution
2) Dlp2: EGARCH(1,1) model where the data is assumed to follow the Gaussian distribution
3) Dlp3: GJRGARCH(1,1) model where the data is assumed to follow the skewed student's t distribution
4) Dlp4: EGARCH(1,1) model where the data is assumed to follow the GED distribution

Looking into the model specifications for EGARCH(1,1), Equation 15 and GJRGARCH(1,1), Equation 16 model specification respectively:

$$\ln(h_t) = \omega + \left| \alpha \left(\left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| - \sqrt{2/\pi} \right) \right| + \left| \gamma \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \beta \ln(h_{t-1}) \qquad \textit{Equation 15}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t\text{-}1}^2 + \beta_1 \sigma_{t\text{-}1}^2 + \eta_1 \varepsilon_{t\text{-}1}^2 I_{t\text{-}1} \qquad \textit{Equation 16}$$

The EGARCH and GJRGARCH model extends the traditional GARCH framework by incorporating an additional term that allows for a differential response to positive and negative shocks. The GJR model allows for a more nuanced response to negative shocks (ε_{t-1}<0), introducing the parameter η_1 to capture the asymmetry in volatility. Since η_1 is positive and statistically significant we can conclude that negative shock increase volatility more than positive one. In the EGARCH model the α coefficient measures the impact of negative shocks on volatility compared to positive shocks. One can visualize the differences between the 3 GARCH models is by looking at the asymmetry through their respective News Impact Curve as seen in Figure 8:

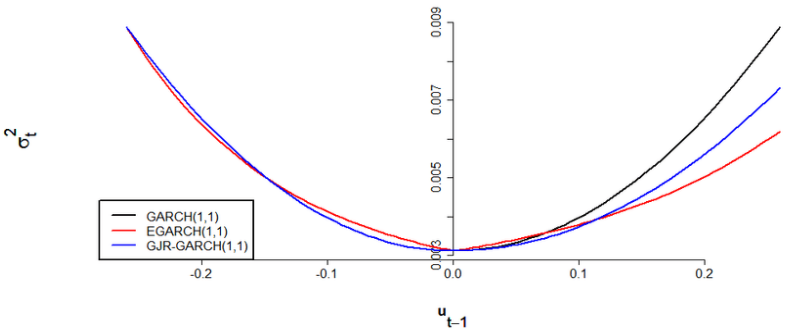

Figure 8 – News Impact Curve for GARCH and its alternatives