Using Gradient Descent to Correct Unitary Errors in Quantum Devices

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March 6, 2025

Abstract

Quantum error correction is essential for mitigating the high error rates inherent in quantum circuits. In this experiment, I focus on addressing unitary errors—rotational perturbations on the Bloch sphere that preserve a quantum state's probability amplitude—within a quantum teleportation circuit. Noise is introduced by applying pseudorandom unitary operators, modeled as various Pauli matrices and their tensor products, at each gate. A gradient descent optimization algorithm is then employed to determine the optimal rotation parameters (θ, φ, and λ) for a unitary gate that reorients the teleported state toward a target configuration. Starting from an initial guess for θ, derived from the fidelity between the target and teleported states, and arbitrarily setting φ and λ to 0, the algorithm refines these parameters based on fidelity measurements from 100 trial runs. Subsequent trials with the optimized unitary gate demonstrate an average fidelity improvement of approximately 6.05 % across the course of the simulation. These results underscore the potential of gradient descent and similar machine-learning techniques to statistically mitigate quantum errors.

I. Introduction

Quantum computing is a field that aims to perform computations that classical computers would be rendered incapable of doing [1]. These circuits rely on using qubits—quantum equivalent of classical bits of information—to perform computations. However, quantum states are easily disturbed by environmental noise that is unable to be eliminated completely. Quantum error correction is a set of techniques to mitigate these errors created within quantum circuits. Measurement errors are reported to occur from 8 to 30% on current quantum devices [2]. Because quantum devices are highly sensitive to these perturbations, quantum error correction is an essential part of conducting experimental research concerning quantum computing. For the focus of my research, I will be examining unitary errors in quantum circuit schemes. Unitary errors are formed by applying a unitary operator to a quantum state. These errors are represented by rotations on the Bloch sphere of a quantum state and preserve the probability amplitude of the wavefunction.

Methods in Artificial Intelligence are currently being explored as an area to correct and interpret information from quantum devices by combining machine-learning techniques [3]. Here, I use a method to statistically mitigate unitary errors on a simulation by means of mathematical optimization through gradient descent, a common technique used in machine learning. Gradient descent works by starting at a given input and then calculating the gradient of points surrounding the starting point. The algorithm will move to a neighboring point in the direction of the gradient until the function is minimized [4]. We will run a simulation on a quantum teleportation circuit that utilizes an EPR-pair to recreate a quantum state, in the process destroying the initial state [Figure 1]. Quantum teleportation circuits are often crucial to building quantum devices, as they enable the transfer of quantum information.

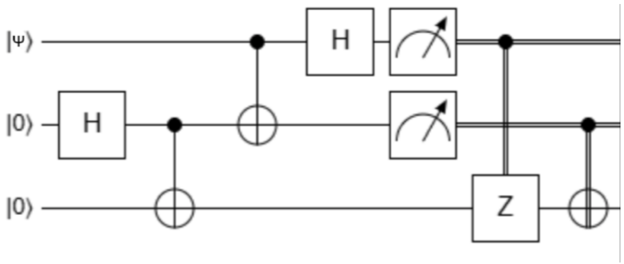

Figure 1: Quantum Teleportation Circuit. An entangled pair is created at the bottom two qubits. The top qubit is teleported to the last qubit.

The main idea is to use a gradient descent algorithm to find optimized parameters of a unitary gate that will mitigate errors. This unitary gate will be applied to the teleported state and reorient the Bloch sphere representation into a statistically more advantageous position by tending towards the direction of greater state fidelity.

II. Experiment

I start with a quantum teleportation circuit shown in Figure 1. I introduced noise to our circuit by applying pseudorandom unitary operators at each gate. These unitary operators are a range of Pauli matrices and their tensor products. I run the simulation for 100 trials, calculating the fidelity between target state $|\phi\rangle$ and teleported state $|\psi\rangle$ for each trial. Ideally, we want $|\langle\phi|\psi\rangle|^2 = 1$ since that means that our target state $|\phi\rangle$ was successfully teleported.

Subsequently, the average fidelity across trials is computed and used as the objective function for the gradient descent algorithm. The gradient descent function will attempt to find optimal parameters of a unitary operator defined by a rotation in θ-, φ-, and λ-directions. This unitary gate will be applied to teleported state's qubit at the end of the circuit and will take on the form:

$$U(\theta,\varphi) = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -e^{i\lambda}\sin\left(\frac{\theta}{2}\right) \\ e^{i\varphi}\sin\left(\frac{\theta}{2}\right) & e^{i(\varphi+\lambda)}\cos\left(\frac{\theta}{2}\right) \end{bmatrix} \quad (1)$$

I use the Batch gradient descent algorithm described by Ruder in reference 4 to search for optimal $\theta, \varphi, \lambda \in [0\,, 2\pi]$ with η being the learning rate [Figure 2]:

$$\Theta_i = \Theta_i - \eta(\nabla f)_i\,, \qquad \Theta = \{\theta, \varphi, \lambda\} \quad (2)$$

In Eq. 2, $f(\theta, \varphi, \lambda)$ represent the function that maps our beginning parameters for θ, φ, λ into a fidelity of target state $|\phi\rangle$ and teleported state $|\psi\rangle$ from the circuit, i.e.:

$$f(\theta, \varphi, \lambda) := \theta, \varphi, \lambda \rightarrow |\langle\phi|\psi\rangle|^2 \qquad (3)$$

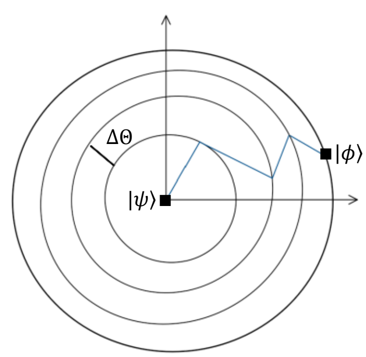

Figure 2: 2D projection of gradient descent algorithm working its way from $|\psi\rangle$ to $|\phi\rangle$ along the blue path by gradient steps of ΔΘ.

The initial guess for θ is given by $\theta = \cos^{-1}(\,|\langle\phi|\psi\rangle|^2)$, as it will rotate $|\psi\rangle$ to the line that $|\phi\rangle$'s endpoint exists on as shown in Figure 3. Guesses for ϕ and λ are arbitrarily set to 0.

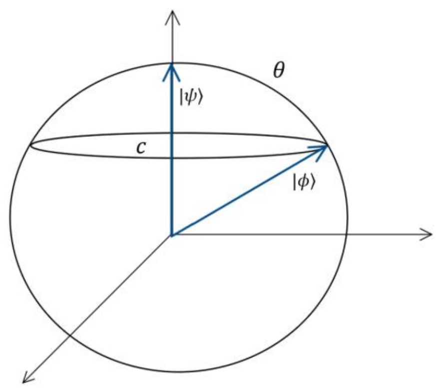

Figure 3: $|\phi\rangle$ endpoint exists on circumference line c. Rotating $|\psi\rangle$ by θ will move $|\psi\rangle$'s endpoint to line c.

After running the algorithm, we receive parameters for θ, φ, and λ that we use to construct unitary operator in Eq. 1. I then ran the simulation for 100 more trials with the corrected parameters.

III. Results

The results are taken from 20 sets of 100 trials of the base circuit and circuit with corrected parameters for the unitary operator. The probability that a unitary error operator appears at a gate is tuned differently for each set corresponding to 20 data points. These results are summarized in Figure 4 below. From the data, we can measure the overall fidelity gained by the corrected circuits across all error ranges by adding up the change in fidelity between the circuits. The calculated fidelity gained by the corrected circuit is about 6.05 % with a standard deviation of ~ 0.135.

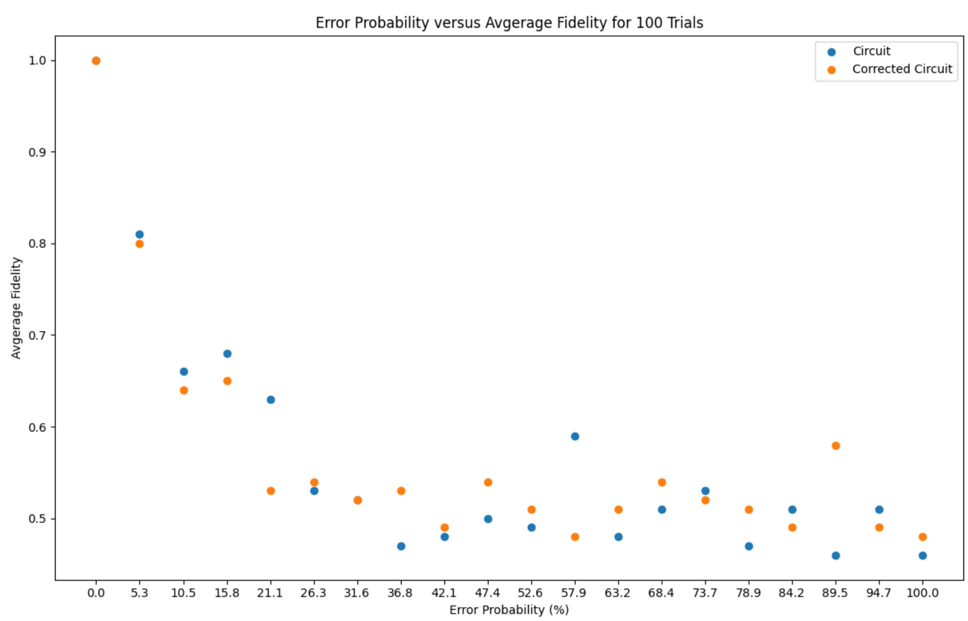

Figure 4: The probability of unitary error in percent is plotted on the x-axis. The average fidelity between the target and teleported state for 100 trials is plotted on the y-axis.

IV. Conclusion

The error curve in the base and corrected circuits are both in agreement with results produced in reference 5. Both results show average fidelity as a graph in the form of exponential decay.

An important limiting factor in this experiment was computational resources, as quantum computing simulation is a resource-intense process on traditional computing devices. My home device is not the most suited to this task, and as such, the algorithm was limited in the amount of training data it had access to. In addition, this limiting factor made it difficult to sample for more points to really compare how well each circuit performed.

Despite these setbacks, the results show a slight improvement in the circuit with corrected parameters as previously stated. Gradient descent may be useful to apply in unison with other error-correcting techniques, but the number of trials required to train the algorithms may not warrant its use. It also seems like the gradient descent algorithm may need to be tuned differently to earlier points on the curve as they seem more difficult for the algorithm to optimize.

However, the results do warrant investigation into other machine-learning techniques which can be used in deep-neural networks to filter noise in a more resource-efficient manner. There should be ways to improve this search algorithm or find another equivalent to transport the fidelity of the system to a statistically more advantageous state. Generally, AI-powered quantum error suffers from the low amount of data it can train on and the constant fluctuations to the noise models in real quantum devices [1]. Due to this fact, AI training seems to be a tool reserved for quantum devices characterized by noise models with slight bias. An important part of future research should revolve around identifying algorithms that are useful in training off a small amount of data and finding a paradigm consistent with that of quantum systems to interpret data.

V. Code Availability

The code used to run this experiment can be found at the following GitHub repository:

https://github.com/PatentLlama/UGDE

VI. References

[1] Wang, Tang, "Artificial Intelligence for Quantum Error Correction: A Comprehensive Review", arXiv, 2024

[2] Tannu, Qureshi, "Mitigating Measurement Errors in Quantum Computers by Exploiting State-Dependent Bias", In Proceedings of the 52nd Annual IEEE/ACM International Symposium on Microarchitecture, 2019.

[3] Mafu, "Advances in artificial intelligence and machine-learning for quantum communication applications", The Institution of Engineering and Technology, p. 204-211 2023.

[4] Ruder, "An overview of gradient descent optimization algorithms", arXiv, 2017.

[5] Oh, Lee, Lee, "Fidelity of Quantum Teleportation through Noisy Channels", The American Physical Society, p. 1, 6, 2018