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Sia $X \sim P(\lambda)$, sapendo che $P(X=8) = P(X=9)$

1) CALCOLARE λ

$$P(X=8) = P(X=9) \rightarrow \frac{\cancel{e^{-\lambda}} \cdot \lambda^8}{8!} = \frac{\cancel{e^{-\lambda}} \cdot \lambda^9}{9!} \Rightarrow \frac{\lambda^8}{\cancel{8!}} = \frac{\lambda^9}{\cancel{8!} \cdot 9}$$

$$9\lambda^8 - \lambda^9 = 0 \rightarrow \lambda^8(\lambda - 9)$$

$$\lambda = 0 \quad \lambda = 9$$

2) $P(X=10 \mid Z=15)$ $\qquad Y \sim P(\lambda)$ con media 8

$$p_{X|Z=n}(k) = \frac{P(X=k, Z=n)}{P(Z=n)} = \frac{P(X=k)\,P(Y=n-k)}{P(Z=n)}$$

$$= \frac{e^{-\lambda_1} \cdot \frac{\lambda_1^k}{k!} \cdot e^{-\lambda_2} \cdot \frac{\lambda_2^{n-k}}{(n-k)!}}{e^{-(\lambda_1+\lambda_2)} \cdot \frac{(\lambda_1+\lambda_2)^n}{n!}}$$

$$= \binom{n}{k} \cdot \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \cdot \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k}$$

$$= \binom{15}{10} \cdot \left(\frac{9}{17}\right)^{10} \cdot \left(\frac{8}{17}\right)^5 = 0.1198661$$

= dbinom(10, size = 15, prob = 9/17)

3) DETERMINARE IL VALORE ATTESO CONDIZIONATO $E(X \mid Z=15)$

~~E(X|Z)~~ Essendo una binomiale : $p \cdot n$

$$E(X \mid Z=15) = \frac{\lambda_1}{\lambda_2+\lambda_1} \cdot n = \frac{9}{17} \cdot 15 = 7.941176$$