Simulation for Inverted Pendulum

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```
clear;
clc;
```

Constants

```
M   = 4.800;    % Cart mass
m   = 0.356;    % Pendulum mass
L   = 0.560;    % Pole length
bth = 0.035;    % Joint Friction
bx  = 4.900;    % Cart Friction
I   = 0.006;    % Pole inertia moment
g   = 9.806;    % Gravitational acceleration
```

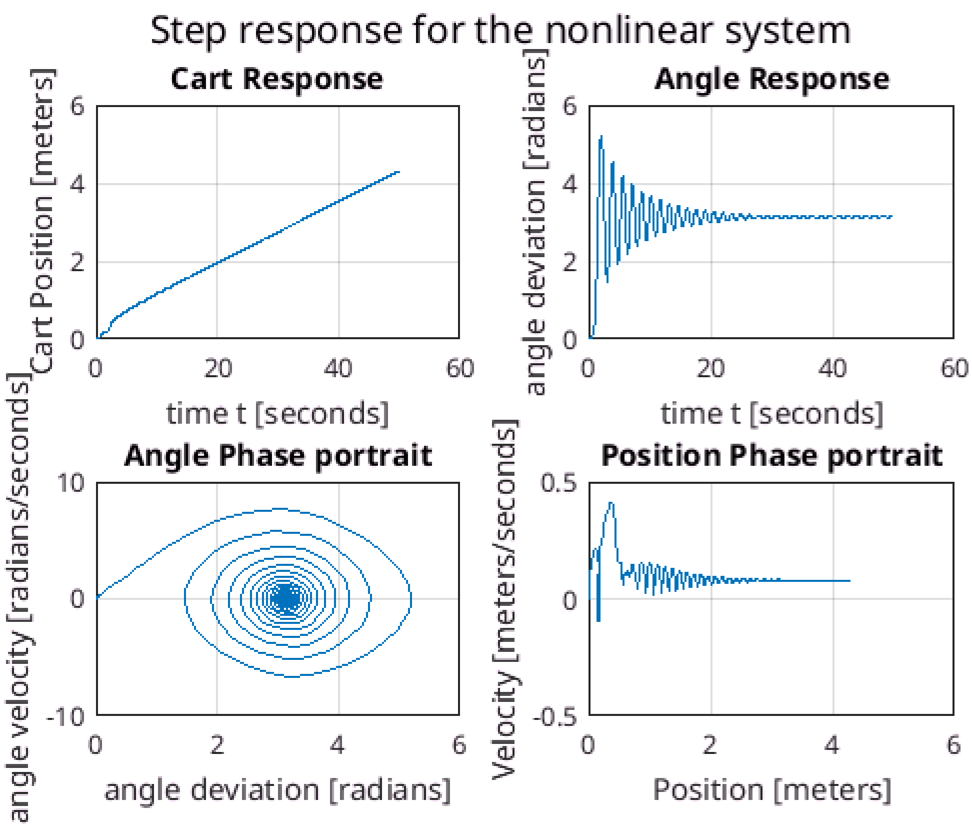
Nonlinear system simulation

```
to = 0; tF = 50; sampling = 1000;

t = linspace(to, tF, sampling);
[~,y] = ode23s(@nonlinear_equations, t, [0 0 0 0]);
```

Step response

```
figure();
% Cart Position
ax1 = subplot(2,2,1);
plot( t, y(:,1) );
grid(ax1, 'on');
title('Cart Response');
ylabel('Cart Position [meters]');
xlabel('time t [seconds]');

% Angle deviation
ax2 = subplot(2,2,2);
plot( t, y(:,3) );
grid(ax2, 'on');
title('Angle Response');
ylabel('angle deviation [radians]');
xlabel('time t [seconds]');

% Phase portrait
ax3 = subplot(2,2,3);
plot( y(:,3), y(:,4) );
grid(ax3, 'on');
title('Angle Phase portrait');
ylabel('angle velocity [radians/seconds]');
xlabel('angle deviation [radians]');
```

```
% Phase portrait
ax4 = subplot(2,2,4);
plot( y(:,1), y(:,2) );
grid(ax4, 'on');
title('Position Phase portrait');
ylabel('Velocity [meters/seconds]');
xlabel('Position [meters]');

suptitle('Step response for the nonlinear system')
```



Linear system simulation

```
[A_unstab, B_unstab] = linear_CartPend(M, m, L, I, bx, bth, 0); % Simulation around pi
```

```
     0
```

```
[A_stable, B_stable] = linear_CartPend(M, m, L, I, bx, bth, 1); % Simulation around pi
C = eye(4);
D = zeros(4,1);

ss_unst = ss(A_unstab, B_unstab, C, D);
ss_stab = ss(A_stable, B_stable, C, D);

ss_unst
```

```
ss_unst =

  A =
             x1        x2        x3        x4
   x1         0         1         0         0
   x2         0    -1.017    0.6876  -0.01231
```

```
  x3         0         0         0         1
  x4         0    -1.723     17.78   -0.3184

 B =
         u1
  x1       0
  x2  0.2075
  x3       0
  x4  0.3517

 C =
      x1  x2  x3  x4
  y1   1   0   0   0
  y2   0   1   0   0
  y3   0   0   1   0
  y4   0   0   0   1

 D =
      u1
  y1   0
  y2   0
  y3   0
  y4   0

Continuous-time state-space model.
```

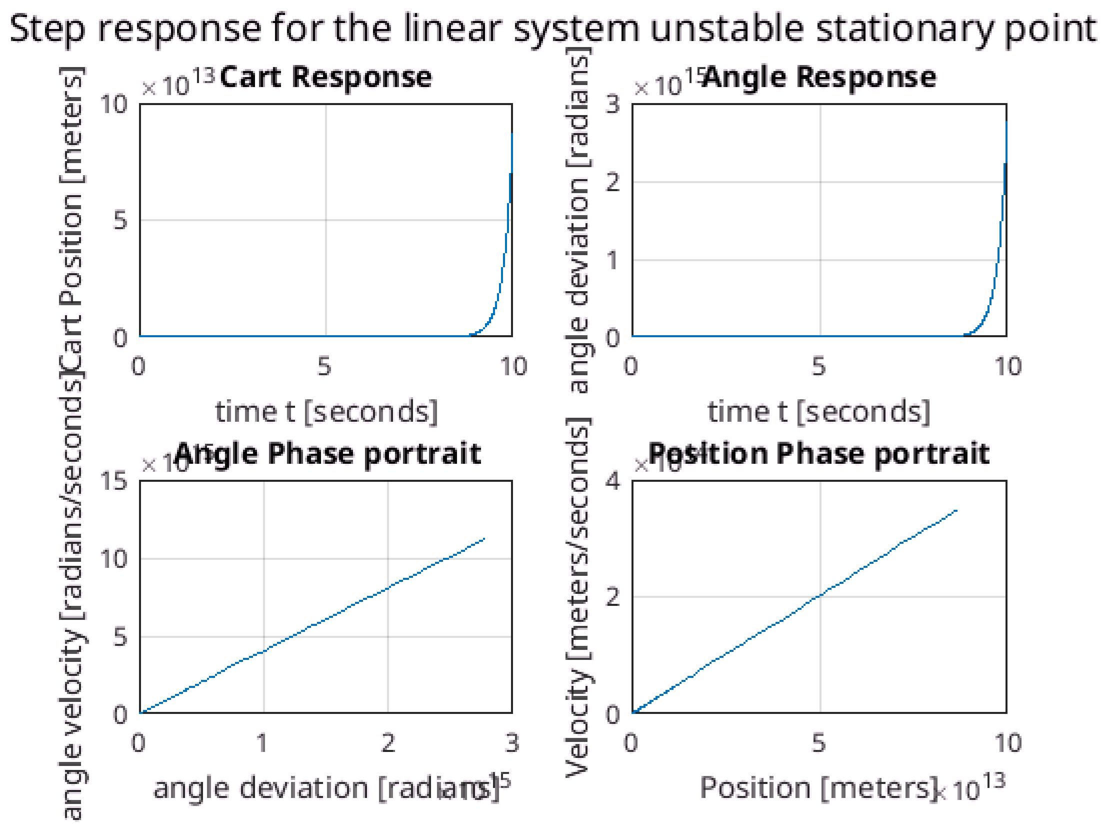
```
ss_stab
```

```
ss_stab =

 A =
           x1       x2       x3       x4
  x1        0        1        0        0
  x2        0   -1.017  -0.6876  0.01231
  x3        0        0        0        1
  x4        0    1.723   -17.78  -0.3184

 B =
           u1
  x1        0
  x2   0.2075
  x3        0
  x4  -0.3517

 C =
      x1  x2  x3  x4
  y1   1   0   0   0
  y2   0   1   0   0
  y3   0   0   1   0
  y4   0   0   0   1

 D =
      u1
  y1   0
  y2   0
  y3   0
  y4   0

Continuous-time state-space model.
```

Unstable Step response

```
t_lin = linspace(0,10,1000);
[y_linear,~] = step(ss_unst,t_lin);
```

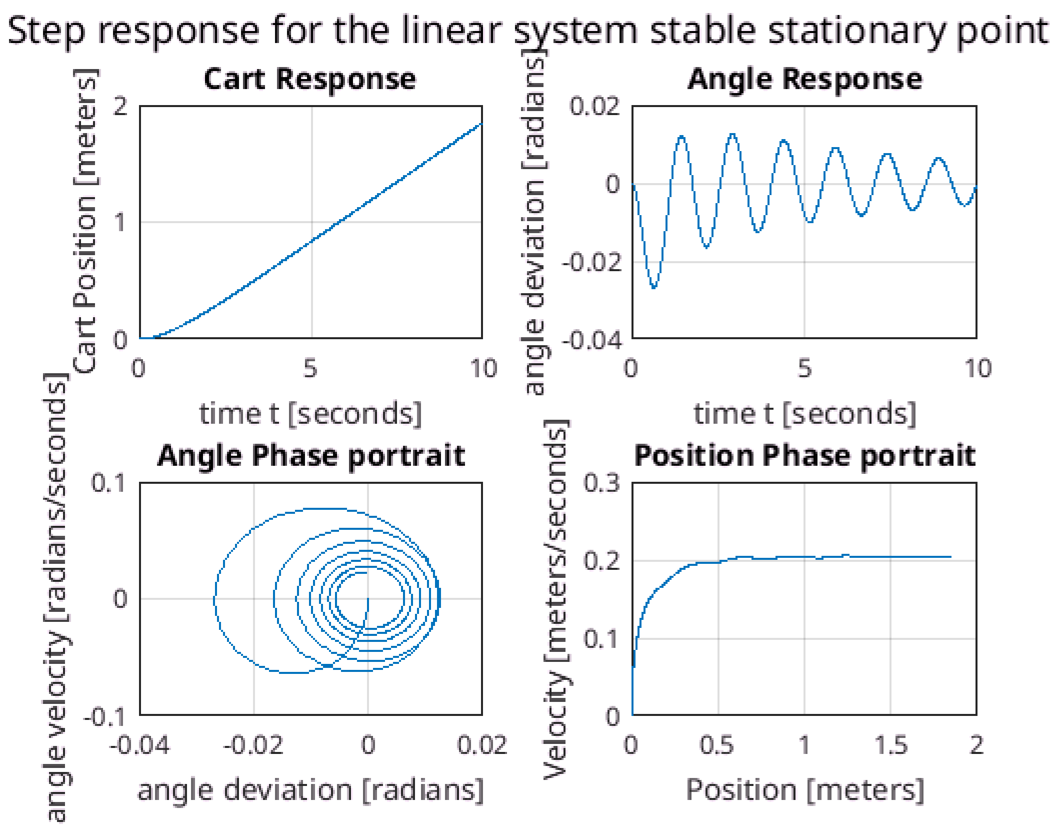
```
figure()

% Cart Position
ax1 = subplot(2,2,1);
plot( t_lin, y_linear(:,1) );
grid(ax1, 'on');
title('Cart Response');
ylabel('Cart Position [meters]');
xlabel('time t [seconds]');

% Angle deviation
ax2 = subplot(2,2,2);
plot( t_lin, y_linear(:,3) );
grid(ax2, 'on');
title('Angle Response');
ylabel('angle deviation [radians]');
xlabel('time t [seconds]');

% Phase portrait
ax3 = subplot(2,2,3);
plot( y_linear(:,3), y_linear(:,4) );
grid(ax3, 'on');
title('Angle Phase portrait');
ylabel('angle velocity [radians/seconds]');
xlabel('angle deviation [radians]');

% Phase portrait
ax4 = subplot(2,2,4);
plot( y_linear(:,1), y_linear(:,2) );
grid(ax4, 'on');
title('Position Phase portrait');
ylabel('Velocity [meters/seconds]');
xlabel('Position [meters]');

suptitle('Step response for the linear system unstable stationary point')
```



Stable Step response

```
t_lin = linspace(0,10,1000);
[y_linear,~] = step(ss_stab,t_lin);
figure()

% Cart Position
ax1 = subplot(2,2,1);
plot( t_lin, y_linear(:,1) );
grid(ax1, 'on');
title('Cart Response');
ylabel('Cart Position [meters]');
xlabel('time t [seconds]');

% Angle deviation
ax2 = subplot(2,2,2);
plot( t_lin, y_linear(:,3) );
grid(ax2, 'on');
title('Angle Response');
ylabel('angle deviation [radians]');
xlabel('time t [seconds]');

% Phase portrait
ax3 = subplot(2,2,3);
plot( y_linear(:,3), y_linear(:,4) );
grid(ax3, 'on');
title('Angle Phase portrait');
ylabel('angle velocity [radians/seconds]');
xlabel('angle deviation [radians]');
```

```
% Phase portrait
ax4 = subplot(2,2,4);
plot( y_linear(:,1), y_linear(:,2) );
grid(ax4, 'on');
title('Position Phase portrait');
ylabel('Velocity [meters/seconds]');
xlabel('Position [meters]');

suptitle('Step response for the linear system stable stationary point')
```



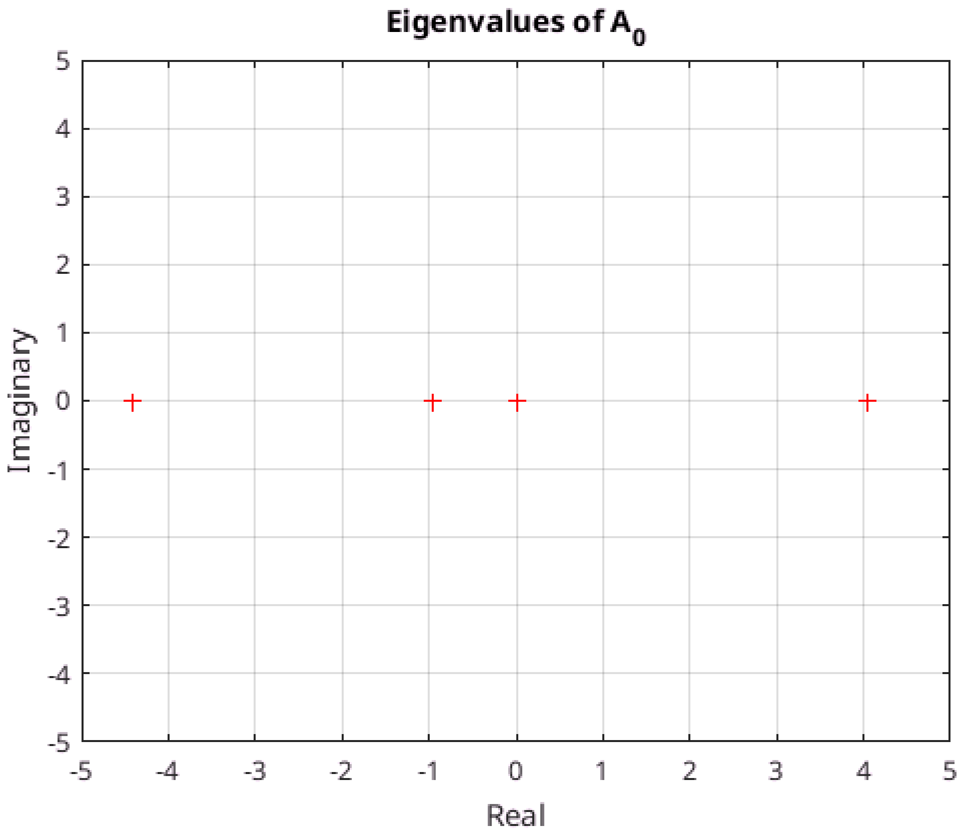
Linearized system eigenvalues

```
[A_unst, B_unst] = linear_CartPend(M, m, L, I, bx, bth, 0); % Simulation around 0
```

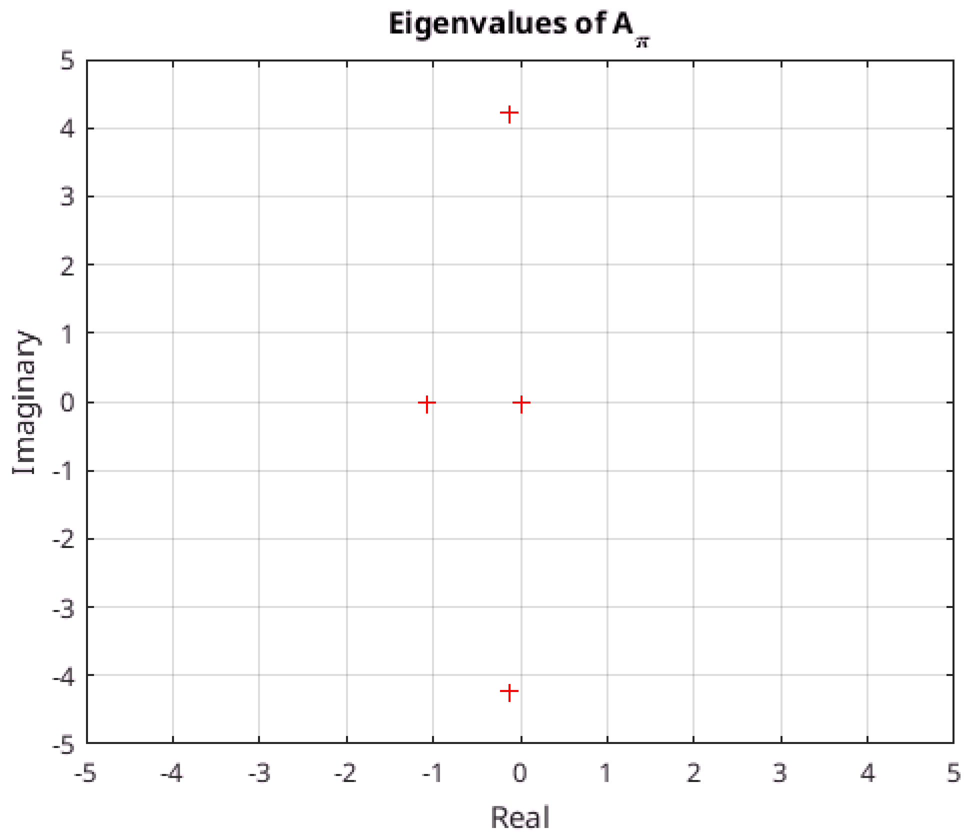
```
0
```

```
[A_stab, B_stab] = linear_CartPend(M, m, L, I, bx, bth, 1); % Simulation around 0

poles_unst = eig(A_unst);
poles_stab = eig(A_stab);

poles_unst = [poles_unst, zeros(4,1)];
%poles_stab = [poles_stab, zeros(4,1)];

figure();
plot(poles_unst(:,1), poles_unst(:,2),'r+');
title('Eigenvalues of A_0');
ylabel('Imaginary')
xlabel('Real')
grid();
```

```
xlim([-5,5]);
ylim([-5,5]);
```




```
figure();
plot(poles_stab,'r+');
title('Eigenvalues of A_\pi');
ylabel('Imaginary')
xlabel('Real')
grid();
xlim([-5,5]);
ylim([-5,5]);
```


Eigenvalues of A_π
Imaginary
Real
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5