Introduction to control systems
TI0118/TH0178

Homework I

Exercise 1

30 Points

Consider the input-output model in (1):

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 5y(t) = \frac{du(t)}{dt} + 5u(t) \tag{1}$$

1. Using the Laplace transform, find the free evolution of the system, assuming the initial conditions given in (2):

$$y(t)\Big|_{t=0} = 2, \quad \frac{dy(t)}{dt}\Big|_{t=0} = 1 \tag{2}$$

2. Find the transfer function for the system and, taking the inverse Laplace transform, the impulse response of the system.

3. Find the forced response of the system subject to a unit step input.

4. Plot the response and comment your results.

Exercise 2

30 Points

Given the state-space model in (3):

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{4} \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 13 & 9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{cases} \tag{3}$$

1. Find a the corresponding transfer function.

2. Find an input-output model equivalent to the state-space model (3).

3. Find, using the Laplace transform, the state and output forced evolution as response of the input signal $u(t) = e^{-t}$.

Exercise 3

30 Points

For a system with the transfer function in (4):

$$G(s) = \frac{\alpha s + 3}{s^2 + 4s + 12} \tag{4}$$

1. For $\alpha = 1$, find and plot the unit step and impulse response.
2. For $\alpha = [-4,\ -1,\ 0,\ 1,\ 4]$, plot and compare the unit step response.
3. Discuss how the system varies its response for the different values of α.

Exercise 4

10 Points

Find the inverse Laplace transform by hand calculations of the following:

1. $F_1(s) = \dfrac{s - 10}{(s + 2)(s + 5)}$
2. $F_2(s) = \dfrac{100}{(s + 1)(s^2 + 4s + 13)}$
3. $F_3(s) = \dfrac{s + 18}{s(s + 3)^2}$

Verify your results using the Symbolic toolbox.

Guidelines

You must solve each exercise using hand calculations and compare/simulate your results on a programming language of your choice. The report for the homework must be sent in a single document (preferably in pdf format), which must include:

- The relevant steps, the results and your comments on each exercise (pictures of your hand written text will be disregarded).
- The code developed for solving each exercise. This can be included as part of the single exercise or in a separate section of the report (for instance, as an appendix).
- The graphs required in the exercises.

The report for the homework must be sent through SIGAA by **April 9, 2018**. Note that **delays will be penalized** (<24h: 20% penalty; <48h: 40% penalty; etc.).