More Fault Tolerant Consensus

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21 Oct 2020

1. TurpinCoan Sync Multi

2. BenOr Async Stop

Outline

TurpinCoan Sync Multi

- General agreement, over arbitrary finite set V, $|V| \geq 2$
- TurpinCoan: two extra rounds $+$ binary Byz

TurpinCoan Init aka Round 0 (Process $\#i$)

- Initial choice: $x \in V$
  - determined by other means, or
  - received from outside
- Proposal: $y \in V \cup \perp = \perp$
- Candidate: $z \in V \cup \perp = \perp$
- Vote: $\hat{v} \in \{0, 1\} = 0$

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TurpinCoan Round 1 (Process $\#i$)

- Send x to all processes
- Let $W \subseteq V \cup \perp$ = multiset of all received messages
- $|W| = N$: sync, $\perp$
- If $\exists v \in V, |W|_v \geq N - F = 2F + 1$, then $y = v$
- Else, keep $y = \perp$
- $y \in V \cup \perp$ is our proposal
- Note: all non-faulty processes select the same $y \in V \cup \perp$
  - $aaa\underline{b} \Rightarrow y = a$, $aaa\underline{c} \Rightarrow y = a$
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TurpinCoan Round 2 (Process $\#i$)

- Send $y \in V \cup \perp$ to all processes
- Let $W \subseteq V \cup \perp$ = multiset of all received messages
- $|W| = N$: sync, $\perp$
- If $\exists v \in V, |W|_v \geq N - F = 2F + 1$, then $z = v, \hat{v} = 1$
  - We vote for candidate $z \in V$
- Else if $\exists v \in V, \arg\max_v |W|_v$, arbitrary tie break, then $z = v$ $(\hat{v} = 0)$
  - We do NOT vote for candidate $z \in V$, but this may be the final decision
- Else i.e. $|W| \cap V = \emptyset$. $(z = \perp, \hat{v} = 0)$
  - No candidate, no vote

TurpinCoan Round 3, ... (Process $\#i$)

- Binary Byz agreement on $\hat{v} \in \{0, 1\}$, for the candidate $z \in V \cup \perp$
- If this Byz decision is 1 and $z \in V$ (i.e. $z \neq \perp$), then final decision z
- Else, final decision $v_0 \in V$

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TurpinCoan Other Agreement Examples

The three loyal processes start with aab, $a, b \in V$, the last process is faulty

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- Still, this is a fundamental problem that needs solutions!
- Way around: stronger model, and weaker termination
- Stronger model: processes use randomisation
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BenOr Init aka Round 0 (Process $\#i$)

- Initial choice: $x \in \{0, 1\}$
  - determined by other means, or
  - received from outside
- Proposal: $y \in \{0, 1, \perp\} = \perp$
- Step: $s \geq 0 = 0$, unbounded
- Each step has two rounds

BenOr Step s, Round 1 (Process $\#i$)

- Send (I, s, x) to all processes
- Let M = multiset of first $N - F = 2F + 1$ received messages $(I, s, *)$
- If all $m \in M$ have same value $v \in \{0, 1\}$, then $y = v$
- Else, $y = \perp$

BenOr Step s, Round 1 (Process $\#i$)

- Send (I, s, x) to all processes
- Let M = multiset of first $N - F = 2F + 1$ received messages $(I, s, *)$
- If all $m \in M$ have same value $v \in \{0, 1\}$, then $y = v$
- Else, $y = \perp$

BenOr Step s, Round 2 (Process $\#i$)

- Send (II, s, y) to all processes
- Let $M =$ multiset of first $N - F = 2F + 1$ received messages $(II, s, *)$
- If all $m \in M$ have same value $v \in \{0, 1\}$, then $x = v$, decide v (if not already), and continue
- If at least $N - 2F = F + 1$ $m \in M$ have same value $v \in \{0, 1\}$, then $x = v$, but do not decide
- Else i.e. $x =$ random $\in \{0, 1\}$.

BenOr Step s, Round 2 (Process $\#i$)

- Send (II, s, y) to all processes
- Let M = multiset of first $N - F = 2F + 1$ received messages $(II, s, *)$
- If all $m \in M$ have same value $v \in \{0, 1\}$, then $x = v$, decide v (if not already), and continue
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