

# Symbolic Gossip

Joris Galema  
Madeleine Gignoux  
Djanira Gomes  
Wouter Smit

Tuesday 28<sup>th</sup> May, 2024

## Abstract

TODO ABSTRACT

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Background</b>	<b>2</b>
<b>3</b>	<b>Gossip Scene Investigation</b>	<b>4</b>
<b>4</b>	<b>Transparent Transformer</b>	<b>6</b>
<b>5</b>	<b>Simple Transformer</b>	<b>7</b>
<b>6</b>	<b>Testing</b>	<b>9</b>
6.1	Gossip Scene Investigation . . . . .	9
6.2	Simple Transformer . . . . .	9
<b>7</b>	<b>Conclusion</b>	<b>10</b>
	<b>Bibliography</b>	<b>10</b>

# 1 Introduction

*The Gossip Problem* or *Gossip* is the problem of sharing information in a network. Many variants of Gossip exist, each with their own computational challenges. Most notably, a distinction is made between the *Transparent* Gossip Problem - the situation where all agents know which agents exchange information at any update - and the *Synchronous* Gossip Problem, where agents know when an update occurs but not which agents exchange information during that update.

For modelling Gossip, an explicit model checker for Gossip called *GoMoChe* exists [Gat23]. Explicit model checkers are generally less efficient than symbolic ones, which aim to cut down on computation time. GoMoChe too is therefore computationally limited to small examples. On the other hand, a symbolic model checker for dynamic epistemic logic (DEL) called SMCDEL exists, which is much more general than *GoMoChe*. SMCDEL is implemented for both  $K$  and  $S5$  and contains symbolic representations for various logic problems, including Gossip [Gat18]. However, in terms of Gossip, SMCDEL only covers an encoding of the Synchronous Gossip Problem (in standard  $S5$  DEL), and the implementation of its update function causes the model to blow up in terms of complexity.

A solution to this exponential blowup was proposed in the unpublished master's thesis by [Rei23], in the shape of a *Simple Knowledge Transformer* that should replace the *Classic Knowledge Transformer* from SMCDEL. An existing implementation by (TODO HAITIAN) extends SMCDEL to incorporate updates with Simple Transformers, but an instance of this transformer tailored to the Gossip problem wasn't included.

This project expands on SMCDEL's functionality. Section 2 contains a description of the Classic Knowledge Transformer in SMCDEL, and specifically how it is used to model updates to the state in [Gat18]. Next, Section 3 contains a number of functions that provide an interpretation of the current state, which makes the Synchronous Gossip Problem already provided in SMCDEL more user-friendly. Next we create a variant of the Classic Transformer for the transparent variant of the Gossip Problem in Section 4. To conclude our work, Section 5 describes our implementation of the Simple Transformer, which cuts down on the complexity of computing the Synchronous Gossip Problem, with the tradeoff of losing higher-order knowledge. Finally, the code of Section 3, 4, and 5 is tested in Subsections 6.1, X (see fixme), and 6.2 respectively.

## 2 Background

For the language and syntax of Gossip, please refer to [Gat18] (Section 6.6). We discuss how the Gossip Problem is approached in SMCDEL using [Gat18] (in particular, Section 6.6.5 on Symbolic Gossip). For an in-depth explanation, please refer to the aforementioned source.

The Gossip Problem models the flow of information called secrets. At the initial state of the problem, no information has been shared and each agent knows only their own secret. The goal is for the agents to exchange all secrets, which happens through *updates* on the model, which is called a *Knowledge Structure*. The Knowledge Structure and the actual state are described by the *vocabulary* ( $V$ ), *state law* ( $\theta$ ), and *observations* ( $O_i$  for each agent  $i$ ). The

vocabulary  $V$  expresses all existing atomic propositions of the form  $S_{ij}$ , where  $S_{ij}$  denotes agent  $i$  knowing agent  $j$ 's secret. Next, the state law  $\theta$  describes the possible worlds in the current model. Following the conceptual assumption that all agents are aware of the model they reside in,  $\theta$  is common knowledge among the agents. Initially,  $\theta$  states that nobody knows anyone else's secret. Finally, the observations  $O_i$  describe which propositional variables agent  $i$  observes; following [Gat18], the observations are initially empty for all agents. Throughout the run of the model, propositions are added to the observables, which encode which calls each agent can observe.

For the sake of simplicity, the notions of knowing one's own secret are completely removed. Equation 1 (from [Gat18]) shows the tuple describing the initial Knowledge Structure.

$$F_{\text{init}} = (V = \{S_{ij} \mid i, j \text{ Agents}, i \neq j\}, \theta = \bigwedge_{i \neq j} \neg S_{ij}, O_i = \emptyset) \quad (1)$$

In order to transform the model after a call happens, we use a Knowledge Transformer. The crux of this paper involves changing the Knowledge Transformer for the Synchronous Gossip Problem provided in SMCDEL to fit our needs.

The Knowledge Transformer explains how the state should change after an update, in this case an arbitrary call. The vocabulary is extended with propositional variables  $q_{ij}$ , which express that agent  $i$  called agent  $j$ . Recalling that we are dealing with the Synchronous Gossip Problem, where agents only know a call occurred, but not which two agents called, we encode this into two laws  $\theta^+$  and  $\theta_-$ , where  $\theta^+$  (also: *preconditions* for a call) expresses that exactly one call happens, and  $\theta_-$  (also: *postconditions* of a call) expresses the conditions under which agent  $i$  can learn agent  $j$ 's secret. Finally, each agent  $i$  observes only calls they participate in, which we describe in  $O_i^+$ .

In short, the Knowledge Transformer for The Synchronous Gossip Problem is the quintuple  $\chi_{\text{call}} = (V^+, \theta^+, V_-, \theta_-, O^+)$  (see [Gat18], page 195 for the exact encoding).

The design of the Knowledge Transformer allows it to encode and check higher-order knowledge, but it also poses a problem in the form of exponential blowup. The state law ( $\theta$ ) keeps track of the updates in the model and is itself updated using  $\theta^+$  and  $\theta_-$ . Essentially, the state law after the final update forms a conjunction of the original state law with event laws ( $\theta^+$ ) for each update and **changelaws** ( $\theta_-$ ), such that the validity of a logical formula on a given Knowledge Structure can be evaluated by solely checking if it's implied by the state law.

However, it is possible for an update to create states that previously were excluded by the state law. In order to allow this type of flexibility, each update causes all propositional variables of the form  $S_{ij}$  to be copied and labelled in the state law. For example, suppose Alice learns Bob's secret during update  $n$ . Any occurrence of the corresponding proposition  $S_a b$  in the state law need to be flagged in update  $n + 1$ , just in case Alice would *forget* Bob's secret in some future update. A copy of  $S_a b$  is added and now exists alongside the flagged version (denoted by  $(S_a b)^o$  to indicate that it is an "old" proposition). The possibility of the truth value  $S_a b$  to change back to false is an unrealistic hypothetical in Gossip, as in this situation agents aren't modelled to forget any secrets. However, SMCDEL is implemented for a wide range of logical problems, which prevents it from making such assumptions.

The existing implementation ([Gat18]) includes optimization functions that discard the redundant propositions (by checking which propositional variables are equivalent), but this optimization is only implemented to be run after running the model and is therefore not optimal.

With this background on how to model Gossip symbolically, we write our own transformer for modelling the transparent variant of The Gossip Problem, implement an adapted optimization that runs in between updates, and a simple transformer based on Daniel Reifsteck’s master thesis.

### 3 Gossip Scene Investigation

This section explains functions that we created to make sense of the current state of a given gossip problem, i.e. gossip scene investigation. We begin with the following imports.

```
module Explain where

import SMCDEL.Examples.GossipS5
import SMCDEL.Symbolic.S5
import SMCDEL.Language
import SMCDEL.Other.BDD2Form
import Data.Maybe
```

One of the differences between SMCDEL and [Gat18] is how the transformer updates the vocabulary by copying all of the secret propositions. This means that in any given transformation, there will be a propositional variable representing a secret, as well as a copy of said variable.

We started by writing `prpLibrary` to decode propositions into whether they were secrets, call propositions, or copies of secrets. The function works by taking in the vocabulary, as well as the number of agents, and returns the library. We also write an (unsafe) function `explainPrp`, which takes in a proposition as well as the library, to return its meaning.

```
prpLibrary :: [Prp] -> Int -> [(Prp,String)]
prpLibrary prps n = zip prps (prpLibraryHelper prps)
  where
    prpLibraryHelper :: [Prp] -> [String]
    prpLibraryHelper [] = []
    prpLibraryHelper prps' = let
      a = secretDecoder (take (n*(n-1)) prps') ++ callDecoder 0
        (take (div (n*(n-1)) 2) (drop (n*(n-1)) prps'))
      in
      a ++ copyDecoder (drop (div (3*n*(n-1)) 2) prps') a ">"
    secretDecoder :: [Prp] -> [String]
    secretDecoder [] = []
    secretDecoder ((P p):ps) = ("S_{" ++ show i ++ "}" ++ show j) : secretDecoder ps
      where (i, j) = (p `quot` n, p `rem` n)
    callDecoder :: Int -> [Prp] -> [String]
    callDecoder k calls | k >= div (n*(n-1)) 2 = []
      | null calls = []
      | otherwise = ("q" ++ show i ++ show j) : callDecoder (k + 1)
        calls
    where
      (i, j) = getCNums k 0
      getCNums :: Int -> Int -> (Int,Int)
      getCNums k' r'' | (k'+1) < n = (r'',k'+1)
        | otherwise = getCNums (k'-n+2+r'') (r''+1)
    copyDecoder :: [Prp] -> [String] -> String -> [String]
    copyDecoder [] _ _ = []
    copyDecoder props lib r = map (++r) lib ++ copyDecoder (drop (length lib) props) lib
      (r++">")
```

```
explainPrp :: Prp -> [(Prp,String)] -> String
explainPrp (P x) prpLib = fromJust (lookup (P x) prpLib)
```

We follow this up with `gsi`, our gossip scene investigation, which takes in a state, the number of agents, and uses `explainPrp` to make sense of the vocabulary and observations. Further work must be done to make sense of the state law.

```
-- Gossip Scene Investigation: GSI. ...like the tv show but with less crime and more gossip
gsi :: KnowScene -> Int -> IO ()
gsi (KnS voc stl obs, s) n = do
  putStrLn "Vocabulary: "
  mapM_ (putStrLn . (++) " -- " . \p -> explainPrp p lib) voc
  putStrLn "State Law: "
  -- Translate BDDS back to formula.
  -- Can be made a bit nicer, maybe make it latex or smth.
  -- Still needs the propositions in the formula translated/explained
  print (formOf stl)
  putStrLn "Observables: "
  mapM_ (putStrLn . (++) " -- " . (\ x -> fst x ++ ": " ++ show (map ('explainPrp' lib)
    (snd x))) ) obs
  putStrLn "Actual state: "
  if null s then putStrLn " -- Nothing is true"
  else
    mapM_ (putStrLn . (++) " -- " . \p -> explainPrp p lib) s
  where
    lib = prpLibrary voc n
```

We can then run the following

```
s0 = gossipInit 3
gsi s0 3
s1 = doCall s0 (0,1)
gsi s1 3
```

which outputs the following

In the future, we hope to also show the law as its BDD using the tool `graphviz`.

```
-- Here a function that takes a BDD or a form and makes a BDD picture.
```

Taking a higher-level view of Gossip, we can see how from an initial state, there are branches depending on which calls are made, leading to a tree. We write now some code to store these states in a tree. Since an infinite amount of calls can be made, we limit the size of the tree using a `depth` parameter.

```
data Tree a = T a [Tree a]
  deriving(Show)

-- explainScene :: KnowScene -> Int -> KnowScene
-- explainScene (KnS voc sLaw obs, s) n = KnS ()

-- explainGossip :: Int -> Int -> Tree KnowScene
-- explainGossip n depth = T (gossipInit n) []

gossipTree :: Int -> Int -> Tree KnowScene
gossipTree n depth = T (gossipInit n) (gossipBranches (gossipInit n) n depth)

gossipBranches :: KnowScene -> Int -> Int -> [Tree KnowScene]
gossipBranches _ 0 = []
```

```
gossipBranches ks n' depth' = [ T (doCall ks (i,j)) (gossipBranches (doCall ks (i,j)) n' (
  depth'-1)) | i <- gossipers n', j <- gossipers n', i < j ]
```

## 4 Transparent Transformer

This section describes how we wrote an implementation of the classic Knowledge Transformer for the Transparent Gossip Problem. This transformer is tailored to the actual call that happens, which makes sure that whenever a call happens, all agents know this and also know which agents participate.

We begin by importing the `GossipS5` and `Symbolic.S5` modules from `SMCDEL`, which define the synchronous classic transformer, and the `Language` module.

```
module Transparent where

import SMCDEL.Examples.GossipS5
import SMCDEL.Language
import SMCDEL.Symbolic.S5
import Data.List
```

We chose to adapt the existing function `callTrf` from `GossipS5`, which is the call transformer for the Synchronous Gossip Problem. Instead of `Int -> KnowTransformer`, the function is now `Int -> Int -> Int -> KnowTransformer`, so that agents  $a$  and  $b$  are arguments for the transformer for call  $ab$ .

As in Section 2, we redefine how to update the vocabulary, law, and observations of each agent.

First, the vocabulary  $V^+$ , called `thisCallHappens`, is now simply the call between agents  $a$  and  $b$ ; as opposed to the synchronous case, we do not need to add any other new call variables, as all agents know exactly which call happens.

We define a helper function `isInCallForm`, which describes the conditions for agent  $k$  to be in a call, and is now not a disjunction of possible calls as in the synchronous case, but requires  $k$  to be either  $a$  or  $b$ . `thisCallHappens` is only defined for the agents performing the actual call.

The `eventlaw`  $\theta^+$  (which originally stated that only one call happens at a time) is simplified to describe that only one call between  $a$  and  $b$  happens. Moreover, `changelaws`  $\theta^-$  are identical to those of the synchronous variant. The `eventobs`  $O_k^+$  are also simplified to the call between  $a$  and  $b$ , as every agent observes the call.

```
callTrfTransparent :: Int -> Int -> Int -> KnowTransformer
callTrfTransparent n a b = KnTrf eventprops eventlaw changelaws eventobs where
  -- agent k is in the call ab if a calls k (so k==b) or k calls b (so k==a)
  isInCallForm k | k == a = Top
                  | k == b = Top
                  | otherwise = Bot

  thisCallHappens = thisCallProp (a,b)
  eventprops = [thisCallHappens]

  -- call ab takes place and no other calls happen
  eventlaw = Conj [PrpF thisCallHappens,
                   Conj [Neg (PrpF $ thisCallProp (i,j)) | i <- gossipers n
                                                             , j <- gossipers n
```

```

, not ((i == a && j == b) || (i == b
      && j == a))
, i < j ]]

changelaws =
  [(hasSof n i j, boolBddOf $
    Disj [ has n i j
          , Conj (map isInCallForm [i,j])
          , Conj [ isInCallForm i
                  , Disj [ Conj [ isInCallForm k, has n k j ]
                        | k <- gossipers n \\ [j], a < k && k < b ]])
    ])
  | i <- gossipers n, j <- gossipers n, i /= j ]

eventobs = [(show k, [thisCallHappens]) | k <- gossipers n]

```

Since the transparent transformer has the same type as the synchronous variant, we inherited its update function. The following functions were adapted from the original implementation to perform the transparent update:

```

callTransparent :: Int -> (Int,Int) -> Event
callTransparent n (a,b) = (callTrfTransparent n a b, [thisCallProp (a,b)])

doCallTransparent :: KnowScene -> (Int,Int) -> KnowScene
doCallTransparent start (a,b) = start 'update' callTransparent (length $ agentsOf start) (a,b)

afterTransparent :: Int -> [(Int,Int)] -> KnowScene
afterTransparent n = foldl doCallTransparent (gossipInit n)

isSuccessTransparent :: Int -> [(Int,Int)] -> Bool
isSuccessTransparent n cs = evalViaBdd (afterTransparent n cs) (allExperts n)

```

## 5 Simple Transformer

This module describes an implementation of the simple transformer as defined by Daniel Reifsteck in his master's thesis [Rei23] (Note: this thesis is not publically available). The simple transformer aims to avoid the exponential blowup of variables that occurs in the classic transformer by copying propositions at each update and storing the "history" of events in the state law. The simple transformer does not change the initial state law throughout the computation. Instead, it directly applies factual change to the actual state.

```

module SimpleTransformer where

import HaitianS5
import SMCDEL.Examples.GossipS5
import SMCDEL.Language
import Data.List ((\\))

```

The model is initialized by the `simpleGossipInit` function, which is based on the `gossipInit` function in the `GossipS5` file. The initial vocabulary contains all propositions of the form "*i* knows the secret of agent *j*", for all agents *i, j*.

Whereas the original state law described the situation in which agents only know their own secrets, this definition is too restrictive for the simple implementation: it prevents the learning of secrets, since the actual state should obey the state law throughout the computation. Thus, in order not to exclude any possible later states, we chose the law to be simply  $\top$ .

The observables for agent  $i$  - which equal the empty set in the classic implementation - now include the proposition " $S_{i,j}$  for all agents  $j$ ". Conceptually, these are the propositions that  $i$  can observe the truth value of these propositions at any point in the model: factual change does not influence the ability of  $i$  to observe them. This is only true for propositions involving  $i$ 's own knowledge. For example, even if Alice can "observe" that Bob does not know Charles' secret in the initial model, she cannot know this fact with certainty after a first call has occurred.

Analogous to the classic implementation, the state `actual` is initially empty, as it describes all true propositions of the form " $i$  knows the secret of agent  $j$ ". While agents do know their own secrets, these are not encoded by propositions and therefore not mentioned in the state.

```
simpleGossipInit :: Int -> KnowScene
simpleGossipInit n = (KnS vocab law obs, actual) where
  vocab = [ hasSof n i j | i <- gossipers n, j <- gossipers n, i /= j ]
  law   = boolBddOf Top
  obs   = [ (show i, allSecretsOf n i) | i <- gossipers n ]
  actual = [ ]
```

The simple transformer is a general call transformer for any calls. This allows the transformer to be synchronous (rather than transparent): agents know that a call has taken place, but not necessarily which call.

The event vocabulary  $V^+$  contains all fresh variables needed to describe the transformation, just like in the classical transformer. Contrary to the classical case,  $V^+$  not to  $V$ , which avoids a quick growth of the vocabulary with each call.

The state law  $\Theta_-$  (`changelaws`) is similarly defined as in the classic transformer, allowing the update to compute the factual change  $V_-$  and modify the state.

The transformation observables in this transformer are empty, as we will show that the specific update function will only need the observables in the original knowledge structure.

The function `simpleGossipTransformer` is the simple analogue of the classic transformer `callTrf` and the transparent variant `callTrfTransparent` from 4.

```
simpleGossipTransformer :: Int -> SimpleTransformerWithFactual
simpleGossipTransformer n = SimTrfWithF eventprops changelaws changeobs where
  -- helper functions to construct the required formulae
  thisCallHappens (i,j) = PrpF $ thisCallProp (i,j)
  isInCallForm k = Disj $ [ thisCallHappens (i,k) | i <- gossipers n \ [k], i < k ]
                        ++ [ thisCallHappens (k,j) | j <- gossipers n \ [k], k < j ]
  allCalls = [ (i,j) | i <- gossipers n, j <- gossipers n, i < j ]

  -- V+ event props stay the same as classic transformer
  eventprops = map thisCallProp allCalls

  -- Theta- change law stays same as classic transformer
  changelaws =
    [(hasSof n i j, boolBddOf $
      Disj [ has n i j -- i already knew j, or
            , Conj (map isInCallForm [i,j]) -- i and j are both in the call or
            , Conj [ isInCallForm i -- i is in the call and there is some k in
                  , Disj [ Conj [ isInCallForm k, has n k j ] -- the call who knew j
                        | k <- gossipers n \ [j] ] ]
      ])
    | i <- gossipers n, j <- gossipers n, i /= j ]

  -- Change obs are empty as they are not used
  changeobs = [ (show i, ([],[])) | i <- gossipers n ]
```



The following functions are analogues of those in `GossipS5.hs` and instead use the simple transformer.

```
-- a single call event with a simple transformer
simpleCall :: Int -> (Int,Int) -> StwfEvent
simpleCall n (a,b) = (simpleGossipTransformer n, [thisCallProp (a,b)])

-- execute a simple call event
doSimpleCall :: KnowScene -> (Int,Int) -> KnowScene
doSimpleCall start (a,b) = start 'update' simpleCall (length $ agentsOf start) (a,b)

-- execute repeated calls using the simple transformer
afterSimple :: Int -> [(Int, Int)] -> KnowScene
afterSimple n = foldl doSimpleCall (simpleGossipInit n)

-- Some helper functions
allSecretsOf :: Int -> Int -> [Prp]
allSecretsOf n x = [ hasSof n x j | j <- gossipers n, j /= x ]
```

## 6 Testing

### 6.1 Gossip Scene Investigation

```
module ExplainTestsSpec where

import Explain
import SMCDEL.Examples.GossipS5
import SMCDEL.Language
import Test.QuickCheck

import Test.Hspec
```

Tests:

- secret propositions are translated correctly
- the vocabulary has correct length
- after a call the state is updated correct

```
spec :: Spec
spec = do
  describe "secret translation:" $ do
    it "init" $ do
      prpLibrary ([ hasSof 2 i j | i <- gossipers 2, j <- gossipers 2, i /= j ]) 2 '
        shouldBe' [(P 1, "S_{0}1"), (P 2, "S_{1}0")]
      --prpLibrary ([ hasSof 3 i j | i <- gossipers 3, j <- gossipers 3, i /= j ]) 3 '
        shouldBe' [(P 1, "S_{0}1"), (P 2, "S_{1}0")]

    --it "after call" $ do
```

### 6.2 Simple Transformer

```

module SimpleTransformerSpec where

import SimpleTransformer

import Test.Hspec hiding ( after )
import SMCDEL.Examples.GossipS5
import SMCDEL.Language
import HaitianS5

```

We test the implementation of the Simple Transformer with the following tests.

```

spec :: Spec
spec = do
  describe "SimpleTransformer" $ do
    it "after same result as individual calls" $ do
      afterSimple 3 [(0,1),(1,2)] 'shouldBe' doSimpleCall (doSimpleCall (
        simpleGossipInit 3) (0,1)) (1,2)
    it "secret was received after 1 call" $ do
      eval (afterSimple 3 [(0,1),(1,2)]) (K "0" $ has 3 0 1) 'shouldBe' True
    it "a knows that b now knows their secret" $ do
      eval (afterSimple 3 [(0,1),(1,2)]) (K "0" $ has 3 1 0) 'shouldBe' True
    it "secret learnt in first was exchanged in second call" $ do
      eval (afterSimple 3 [(0,1),(1,2)]) (K "2" $ has 3 2 0) 'shouldBe' True
    it "SmpTrf: second call shares secrets of other agents" $ do
      eval (afterSimple 3 [(0,1),(1,2)]) (K "2" $ has 3 0 1) 'shouldBe' True
    it "SmpTrf: 3 agents 1 call should infer knowledge but fails" $ do
      eval (afterSimple 3 [(0,1)]) (K "2" $ has 3 0 1) 'shouldBe' False
    it "SmpTrf: higher-order knowledge True" $ do
      eval (afterSimple 3 [(0,1),(1,2)]) (K "2" $ has 3 2 1) 'shouldBe' True

```

Note in particular the test `SmpTrf: higher-order knowledge fails`, which returns false. However, the tested formula  $K_2S_01$  should be true after calls 01;12: after the second call, agent 2 should be able to infer that the prior call was between agents 0 and 1 and conclude that their secrets were exchanged.

## 7 Conclusion

ADD CONCLUSION

## References

- [Gat18] Malvin Gattinger. *New Directions in Model Checking Dynamic Epistemic Logic*. PhD thesis, University of Amsterdam, 2018.
- [Gat23] Malvin Gattinger. Gomoche-gossip model checking. *Branch async from*, 1, 2023.
- [Rei23] Daniel Reifsteck. Comparing state representations for del planning. Master’s thesis, University of Freiburg, April 2023.