EE5606/AI5120 Convex Optimization, Spring 2026
Indian Institute of Technology Hyderabad
Ungraded Assignment

Note: Let 'a' denote a **scalar**. Similarly, '$\underline{a}$' denotes a **vector**, '$\mathbf{A}$' denotes a **matrix** of suitable dimension, and '$\mathbb{A}$' represents a **set**.

1) Find whether the given are open, closed, and bounded.

   1. $\mathbb{R}^n$ for any positive integer n,
   2. The unit open ball $\mathbb{B}_n = \{\underline{x} \in \mathbb{R}^n : \|\underline{x}\| < 1\}$,
   3. The unit closed ball $\mathbb{B}_n = \{\underline{x} \in \mathbb{R}^n : \|\underline{x}\| \leq 1\}$,
   4. For a continuous function $f : \mathbb{R} \to \mathbb{R}$, $\mathbb{X} = \{x \in [0, 1] : f(x) \leq 1\}$.

2) Recall the definition of a convex function. For all real numbers $\underline{x_1}, \underline{x_2} \in \mathbb{R}^n$ and $\theta \in [0, 1]$,

$$f(\theta\underline{x_1} + (1-\theta)\underline{x_2}) \leq \theta f(\underline{x_1}) + (1-\theta)f(\underline{x_2}).$$

   Prove that if $f''(\underline{x}) > 0$, then f is convex. Conversely, if f is convex and $f''(\underline{x})$ exists for all $\underline{x}$, show that $f''(\underline{x}) \geq 0$.

3) Justify whether the following functions are convex.

   1. $f(x) = \log(x)$,
   2. $f(x) = x\log(x)$,
   3. Cross-entropy function,
   4. $f(x) = |x|$.

4) Without invoking the spectral theorem, prove that all 2×2 symmetric matrices are diagonalizable and have real eigenvalues.

5) Find the eigenvalues, eigenvectors, and square roots of:

$$\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & -3 \\ -4 & 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 4 \\ 6 & 3 \end{bmatrix}, \quad \begin{bmatrix} 3 & 5 & -2 \\ 9 & 4 & -3 \\ -4 & 8 & 0 \end{bmatrix}$$

   Also find the rank, nullity, column space, and right null space.

6) Using `cvxpy` and `numpy`, solve:

   1. $\min(x_1^2 + x_2^2)$, $\quad (x_1, x_2$ are scalars)
   2. Lasso regression: $\|\mathbf{A}\underline{x} - \underline{b}\|^2 + \lambda\|\underline{x}\|_1$,
   3. Quadratic program: $\min \frac{1}{2}(\underline{x}^T\mathbf{P}\underline{x}) + \underline{q}^T\underline{x}$ subject to $\mathbf{G}\underline{x} \leq \underline{h}$ and $\mathbf{A}\underline{x} = \underline{b}$.