भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

EE5606/AI5120 Convex Optimization, Spring 2026
Indian Institute of Technology, Hyderabad
Homework 1: Convex Terminologies
Total Marks: **20 points**
Assigned on 28.01.2026, Due date **11:59pm on 02.02.2026**

Instructions

- It is strongly recommended that you work on your homework on an individual basis. If you have any questions or concerns, feel free to talk to the instructor or the TAs.
- Use numpy for basic functions like `log, sqrt, power`. Do not use other built-in functions.
- Please turn in your work with the following notation for the file name: *your-roll-number-hw1.pdf* and *your-roll-number-hw1.ipynb* for the Python notebook.

Question 1 (1 point)
Without invoking the spectral theorem, prove that all 2 x 2 symmetric matrices are always diagonalizable, and the eigenvalues are always real.

Question 2 (5 points)
Given an arbitrary and non empty set S $\subseteq \mathbb{R}^n$, define the set of vectors in S and $\forall\ \underline{y} \in S$

$$C = \underline{x} : \underline{x}^T \underline{y} \geq 0.$$

Identify and justify whether C is a **subspace**, **affine set**, **convex set** or a **cone**. Further does the above answer rely on a specific structure of the set S ?

Question 3 (4 points)
Given a matrix $\mathbf{A}$ of dimensions n x n and a vector $\underline{y} \in \mathbb{R}^n$. Is the function $f_1(\underline{x}) = ||\underline{y} - \mathbf{A}\underline{x}||_2$ convex ? Answer the same for the function $f_2(\underline{x}) = ||\underline{y} - \mathbf{A}\underline{x}||_2^2$.

Question 4 (5 points)
Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function with a well-defined gradient at all points. For any point $\underline{x} \in \mathbb{R}^n$ and any direction $\underline{u} \in \mathbb{R}^n$ with $||\underline{u}||_2 = 1$, define the directional derivative of f at $\underline{x}$ in the direction $\underline{u}$ by

$$f'_{\underline{u}}(\underline{x}) = \lim_{\varepsilon \to 0} \frac{f(\underline{x} + \varepsilon \underline{u}) - f(\underline{x})}{\varepsilon}.$$

1. Using the differentiability of f, derive an expression for the directional derivative $f'_{\underline{u}}(\underline{x})$ in terms of the gradient $\nabla f(\underline{x})$.
2. Show that the directional derivative can be written as an inner product between $\nabla f(\underline{x})$ and $\underline{u}$.

3. Find the direction $\underline{u}$ (with $\|\underline{u}\|_2 = 1$) for which the directional derivative is maximized.
4. Find the direction $\underline{u}$ (with $\|\underline{u}\|_2 = 1$) for which the directional derivative is minimized.
5. Interpret your results in terms of the directions of steepest ascent and steepest descent of the function f at the point $\underline{x}$.

Question 5 (5 points)

Write a Python program (using only basic NumPy operations such as vector addition, subtraction, multiplication, division, summation, conditional statements, and slice operations) to perform Gram–Schmidt orthonormalization. Let $\mathbf{A} \in \mathbb{R}^{m \times n}$ be an input matrix, for any arbitrary m and n. The i-th input vector is given by the i-th row of $\mathbf{A}$, denoted by $\mathbf{A}[i,:]$.The output of the program should be a matrix $\mathbf{Q} \in \mathbb{R}^{r \times n}$, where r is the dimension of the span of the input vectors, and the rows of $\mathbf{Q}$ form an orthonormal basis for the row space of $\mathbf{A}$.
