భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

EE5606/AI5120 Convex Optimization, Spring 2026
Indian Institute of Technology, Hyderabad
Homework 2: Convex Optimization
Total Marks: 20 points
Assigned: 05.02.2026,
Due date: 11.02.2026
Time: 11:59 PM

Instructions

- It is strongly recommended that you work on your homework on an individual basis. If you have any questions or concerns, feel free to talk to the instructor or the TAs.
- Please turn in your work with the following notation for the file name: *your-roll-number-hw2.pdf* and *your-roll-number-hw2.ipynb* for the Python notebook.
- You may submit the assignment either handwritten or in LaTeX (preferred).

Note: We will use the notation a for scalars, $\underline{a}$ for vectors, $\mathbf{A}$ for matrices, and A for sets throughout.

Question 1 (1 point)

Let $C \equiv \left\{ \underline{x} \in \mathbb{R}^n \mid \mathbf{A}_0 - \left(\sum_{i=1}^{n} x_i \mathbf{A}_i \right) \succeq 0 \right\}$, where $\mathbf{A}_i$ are all symmetric matrices of size m. Show that C is a convex set.

Question 2 (1 point)

Let P be the set of probability density functions. Show that P is a convex set.

Question 3 (1 point)

Consider the set $K_{\mathbf{M}} = \left\{ \begin{bmatrix} \underline{x} \\ y \end{bmatrix} \mid \| \underline{x} \|_{\mathbf{M}} \leq y, \underline{x} \in \mathbb{R}^n, y \in \mathbb{R} \right\}$, where $\| \underline{x} \|_{\mathbf{M}}$ denotes the $\mathbf{M}-$norm of $\underline{x}$: $\| \underline{x} \|_{\mathbf{M}} \equiv \sqrt{\underline{x}^\top \mathbf{M} \underline{x}}$ if $\mathbf{M} \succ 0$. What is the dual cone of $K_{\mathbf{M}}$ (simplify your expression as much as possible)? **Hint**: Use EVD of $\mathbf{M}$ in your simplifications.

Question 4 (1 point)

Show that the hyperbolic cone $K_{\mathbf{M}} = \left\{ \underline{x} \mid \underline{x}^\top \mathbf{M} \underline{x} \leq (\underline{c}^\top \underline{x})^2, \underline{x} \in \mathbb{R}^n, \underline{c}^\top \underline{x} \geq 0 \right\}$ is a convex set

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Question 5 (2 marks)

Geometric interpretation of positive semidefinite cones

Consider the set of symmetric 2×2 matrices

$$C = \left\{ \mathbf{A} \in \mathbb{S}^2 \;\middle|\; \mathbf{A} \succeq 0 \right\},$$

where $\mathbb{S}^2$ denotes the space of 2×2 symmetric matrices, and $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is positive semidefinite.

(a) Write a complete Python script that plots the set C in the three-dimensional coordinate system (x, y, z), where

$$\mathbf{A} = \begin{bmatrix} x & y \\ y & z \end{bmatrix}.$$

Clearly label the boundary surface and indicate the region corresponding to $\mathbf{A} \succeq 0$.

(b) In 3–5 sentences, explain why the set C is convex from a geometric point of view. In particular, address the following:

- Describe the shape of the boundary surface.
- Explain whether the boundary curves inward or outward (i.e., its curvature).
- Relate this curvature to the convexity of the set C.

Hint: A symmetric 2×2 matrix is positive semidefinite if and only if

$$\det(\mathbf{A}) \geq 0. \quad \text{(derive constraints for } x, y, z\text{)}$$

Question 6 (6 points)

Which of the following sets are convex?

For each part, clearly draw a low-dimensional figure (in $\mathbb{R}$) illustrating the set and provide a brief but precise mathematical justification for your answer.

(a) A *slab*, i.e., a set of the form

$$\{\underline{x} \in \mathbb{R}^n \mid \alpha \leq \underline{a}^T \underline{x} \leq \beta\}.$$

(b) A *rectangle*, i.e., a set of the form

$$\{\underline{x} \in \mathbb{R}^n \mid \alpha_i \leq x_i \leq \beta_i,\ i = 1, \ldots, n\}.$$

A rectangle is sometimes called a *hyperrectangle* when $n > 2$.

(c) A *wedge*, i.e.,

$$\{\underline{x} \in \mathbb{R}^n \mid \underline{a}_1^T \underline{x} \leq b_1,\ \underline{a}_2^T \underline{x} \leq b_2\}.$$

(d) The set of points closer to a given point than a given set, i.e.,

$$\{\underline{x} \mid \|\underline{x} - \underline{x}_0\|_2 \leq \|\underline{x} - \underline{y}\|_2 \text{ for all } \underline{y} \in S\},$$

where $S \subseteq \mathbb{R}^n$.

(e) The set

$$\{\underline{x} \mid \underline{x} + S_2 \subseteq S_1\},$$

where $S_1, S_2 \subseteq \mathbb{R}^n$ with S_1 convex.

(f) The set of points whose distance to $\underline{a}$ does not exceed a fixed fraction θ of the distance to $\underline{b}$, i.e., the set

$$\{\underline{x} \mid \|\underline{x} - \underline{a}\|_2 \leq \theta \|\underline{x} - \underline{b}\|_2\}.$$

You can assume $\underline{a} \neq \underline{b}$ and $0 \leq \theta \leq 1$.

Question 7 (2 points)

Voronoi sets and polyhedral decomposition. Let $\underline{x}_0, \ldots, \underline{x}_K \in \mathbb{R}^n$ be distinct. Consider the set of points that are closer (in Euclidean norm) to $\underline{x}_0$ than to the other $\underline{x}_i$, i.e.,

$$V = \{\underline{x} \in \mathbb{R}^n \mid \|\underline{x} - \underline{x}_0\|_2 \leq \|\underline{x} - \underline{x}_i\|_2,\ i = 1, \ldots, K\}.$$

The set V is called the *Voronoi region* around $\underline{x}_0$ with respect to $\underline{x}_1, \ldots, \underline{x}_K$.

(a) Show that V is a polyhedron. Express V in the form

$$V = \{\underline{x} \mid \mathbf{A}\underline{x} \leq \underline{b}\}.$$

(b) Conversely, given a polyhedron P with nonempty interior, show how to find $\underline{x}_0, \ldots, \underline{x}_K$ such that the polyhedron is the Voronoi region of $\underline{x}_0$ with respect to $\underline{x}_1, \ldots, \underline{x}_K$.

Question 8 (6 points)

Consider the set

$$S = \left\{\underline{x} \in \mathbb{R}^m \;\middle|\; |p(t)| \leq 1 \text{ for all } |t| \leq \pi/3\right\},$$

where the function $p(t)$ is the trigonometric polynomial

$$p(t) = \sum_{k=1}^{m} x_k \cos(kt).$$

It is known that S can be expressed as the intersection of infinitely many slabs:

$$S = \bigcap_{|t| \leq \pi/3} S_t,$$

where each slab is defined by

$$S_t = \left\{\underline{x} \in \mathbb{R}^m \;\middle|\; -1 \leq \big(\cos t,\ \cos 2t,\ \ldots,\ \cos(mt)\big)^\top \underline{x} \leq 1\right\}.$$

(a) For $m = 2$,

(i) explicitly write the expression for $p(t)$, and

(ii) describe (in words or mathematically) the set S in the (x_1, x_2)-plane.

(b) Write a complete Python script that numerically approximates the set S for $m = 2$ and generates **two separate plots**:

(i) **First plot** (trigonometric polynomials vs. t):

- Plot $p^{(1)}(t)$ and $p^{(2)}(t)$ versus t corresponding to two distinct points $\underline{x}^{(1)}, \underline{x}^{(2)} \in S \in \mathbb{R}^2$.
- Also plot the average curve (as a dashed line):

$$p_{\text{avg}}(t) = \frac{1}{2}\big(p^{(1)}(t) + p^{(2)}(t)\big).$$

- Use $t \in [0, \pi]$ (or a slightly larger interval if helpful).
- Clearly indicate the horizontal lines $y = \pm 1$ and the vertical line $t = \pi/3$ (boundary of the constraint interval).

(ii) **Second plot** (feasible region in the (x_1, x_2)-plane):

- Show the boundary of S by plotting many slab boundary lines

$$\cos t \cdot x_1 + \cos 2t \cdot x_2 = \pm 1$$

for different values of $t \in [0, \pi/3]$.

- **Do not shade or fill** the feasible region — display only the boundary lines.
- Use enough lines (e.g., 30–100) so that the shape of S is clearly visible as their intersection region.

(c) Include both plots (or clearly describe them) and comment on the geometric properties of the set S for $m = 2$. In particular, discuss:

- the **smoothness** of the boundary of S (smooth everywhere, or are there corners/flat segments?),
- the **symmetries** of the set S (with respect to axes, origin, etc.),
- and explain **why** the average curve $p_{\text{avg}}(t)$ also satisfies the constraint $|p_{\text{avg}}(t)| \leq 1$ for all $|t| \leq \pi/3$.

Note: You may use `numpy` and `matplotlib` (or equivalent libraries) for plotting. Ensure the code is self-contained and easy to run.