**3D Geometry (concepts)**

1. Direction Ratio of line joining points (x1, y1, z1) and (x2, y2, z2) is x2 – x1, y2 – y1, z2 – z1
2. If a, b, c are DR’s of a direction, then its Direction Cosines are , ,
3. If α, β, γ are angles formed by a direction with x, y, z axes, then direction cosines are l =cos α, m = cos β, n = cos γ
4. Condition for direction ratios to be direction cosine l2 + m2 + n2 = 1
5. If DR’s of 2 directions are a1, b1, c1 and a2, b2, c2 then
6. Condition for parallel = =
7. Condition for perpendicular a1a2 + b1b2 + c1c2 = 0
8. Condition is same if 2 lines are parallel or perpendicular and 2 planes are parallel or perpendicular
9. In case of line and plane
10. If line is parallel to plane a1a2 + b1b2 + c1c2 = 0
11. If line is perpendicular to plane = =
12. Angle between 2 directions cos θ =
13. Formula is same for angle between 2 line or between 2 planes
14. Angle between line and plane sin θ =
15. Equation of line
16. Point on line (x1, y1, z1)
17. DR of line a, b, c

Cartesian form = =

Vector form = + ʎ where = position vector of point and = direction vector of line

Cases for DR

1. Line parallel to given line (same DR)
2. Line perpendicular to given plane (same DR)
3. Line perpendicular to 2 given lines (solve aa1 + bb1 + cc1 = 0 and aa2 + bb2 + cc2 = 0)
4. Line parallel to 2 given planes (solve aa1 + bb1 + cc1 = 0 and aa2 + bb2 + cc2 = 0)
5. Shortest distance between skew lines =
6. Distance between parallel lines =
7. Equation of plane
8. Point in plane (x1, y1, z1)
9. DR of normal to plane a, b, c

Cartesian form a(x – x1) + b(y – y1) + c(z – z1) = 0

Vector form ( - )· = 0

Where is position vector of point and is direction vector of normal to plane

Cases for DR of normal to plane

1. Plane parallel to given plane (same DR)
2. Plane perpendicular to given line (same DR)
3. Plane perpendicular to 2 given planes (solve aa1 + bb1 + cc1 = 0 and aa2 + bb2 + cc2 = 0)
4. Plane parallel to 2 given lines (solve aa1 + bb1 + cc1 = 0 and aa2 + bb2 + cc2 = 0)

Plane through intersection of 2 planes P1 + ʎP2 = 0

Cases for ʎ

1. Plane through given point – put point in equation of plane
2. Plane perpendicular to given plane a1a2 + b1b2 + c1c2 = 0
3. Plane parallel to given line a1a2 + b1b2 + c1c2 = 0

Intercept form + + = 1

Normal form Cartesian form lx + my + nz = p

Vector form · = p where l, m, n are DC’s of normal and p is perpendicular distance of plane from origin

Foot of perpendicular from origin (lp, mp, np)

1. Perpendicular distance of point (x1, y1, z1) from plane Ax + By + Cz + D = 0

(x1, y1, z1)

P =

Ax + By + Cz + D = 0

1. Distance between parallel planes Take any point on plane 1 and find its distance from plane 2.
2. Distance between parallel line and plane – Take a point on line and find its distance from plane.
3. General point of line

Cartesian form = = = ʎ

Vector form = x1  + y1  + z1 + ʎ(a + b + c )

Uses of General Point

1. Point of intersection of line and plane – Find general point of line and put it in equation of plane and find the value of ʎ. Put value of ʎ in general point to get point of intersection.
2. Foot of perpendicular of point on plane ax + by + cz +d = 0

A (x1, y1, z1)

Write equation of AP using DR of plane a, b, c

( as AP is perpendicular to plane, same DR).

Find point of intersection of line AP and plane

P

( that is point P) using above method.

ax + by + cz + d = 0

1. Image of point in plane

A (x1, y1, z1)

P

First find foot of perpendicular P using the

above method. Suppose M (h, k, l) as image

of A and apply midpoint formula ( ,,

) and equate with P, as P is midpoint of AM.

# M (h, k, l)

*= =*

1. Foot of perpendicular P from point A on line

A (x1, y1, z1)

Find general point of line and take it as co-

ordinate of P. Find DR of AP. Apply condition

of perpendicular a1a2 + b1b2 + c1c2 = 0 (as P is

foot of perpendicular) and find the value of ʎ.

P

*= =*

Put value of ʎ in general point to get foot of

perpendicular.

1. Image of point in line can be calculated in the same way as for plane.
2. To prove that lines intersect and also find point of intersection

Find general point of lines. Put x1 = x2 and

Y1 = y2 to get 2 equations in ʎ1 and ʎ2. Solve

The equations and get value of ʎ1 and ʎ2. Put

value of ʎ1 and ʎ2 in z1 and z2. If z1 = z2, lines

intersect. To get point of intersection put ʎ1 in P1 or ʎ2 in P2.

1. To prove that lines intersect – Find shortest distance between the lines. If shortest distance = 0, lines intersect.
2. Coplanar
3. For 2 lines = = and = =

= 0

Equation of plane through these lines = 0

1. 4 points (x1, y1, z1), (x2, y2, z2), (x3, y3, z3) and (x4, y4, z4)

= 0

Equation of line through these points –

= 0