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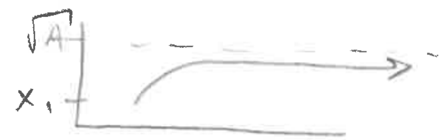
Halley's algorithm: $f(n) = X_{n+1} = X_n\left[\dfrac{15-\left(\frac{1}{A}X_n^2\right)\left(10-3\left(\frac{1}{A}X_n^2\right)\right)}{8}\right]$

Declare $\left(\frac{1}{A}X_n^2\right) = Y_n$

$f(n) = X_n\left[\dfrac{15-10Y_n+3Y_n^2}{8}\right] = X_{n+1}$

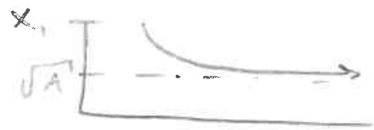
notice trend:

if $X_1 < \sqrt{A}$:



X_n <u>increases</u> until $\sqrt{A}$ reached by some criteria ϵ

if $X_1 > \sqrt{A}$



X_n <u>decreases</u> until $\sqrt{A}$ reached by some criteria ϵ

$\therefore$ for $X_1 > \sqrt{A}$ if X_n is increasing, algorithm is diverging.

notice equation:

$f(n) = X_n\left[\dfrac{15-10Y_n+3Y_n^2}{8}\right]$

This part of equation creates a coefficient on X_n.

if X_n is required to decrease, then <u>coefficient < 1</u>

$\therefore \quad \dfrac{15-10Y_n+3Y_n^2}{8} < 1$

TI-84 or whatever flavor of quadratic solver:

$Y_n = 2.\overline{3} = 7/3$

$Y_n = 7/3 = \frac{1}{A}X_1^2$

$\therefore$ upper bound on guess, (critical guess) $= \sqrt{\dfrac{7A}{3}}$