MS-E2121 Homework 1

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January 31, 2024

Use of AI: AI was used to decode some errors in some parts of the code for the model. AI was also used in understanding some parts of problem 1.3.

1 Deterministic capacity expansion problem

For a syntax example, this problem can be formulated as follows:

$$\begin{aligned}
\text{min.} \quad & \sum_{i \in I} C_i x_i + \sum_{i \in I} \sum_{t \in T} (H_i k_{it} + M_i p_{it}) + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} F_{ij} e_{ijt} + \sum_{j \in J} \sum_{t \in T} Q_j u_{jt} && (1) \\
\text{s.t.} \quad & p_{it} \leq x_i, \ \forall i \in I, \forall t \in T && (2) \\
& p_{it} + k_{i(t-1)} = \sum_{j \in J} e_{ijt} + k_{it}, \ \forall i \in I, \forall t \in T && (3) \\
& \sum_{i \in I} e_{ijt} = D_{jt} - u_{jt}, \ \forall j \in J, \forall t \in T && (4) \\
& k_{i0} = 0, \ \forall i \in I && (5) \\
& x_i \geq 0, \ \forall i \in I && (6) \\
& p_{it}, k_{it} \geq 0, \ \forall i \in I, \forall t \in T && (7) \\
& e_{ijt} \geq 0, \ \forall i \in I, \forall j \in J, \forall t \in T && (8) \\
& u_{jt} \geq 0, \ \forall j \in J, \forall t \in T && (9)
\end{aligned}$$

1.a First-period storage constraint

The storage constraint for the first period is

$$k_{i0} = 0, \ \forall i \in I$$

1.b Objective value

The optimal objective value for the provided instance is 166.57689375. Which is approximately 166.58.

2 Stochastic capacity expansion problem

2.a Problem formulation

$$
\begin{aligned}
\text{min. } & \sum_{i\in I} C_i x_i + \sum_{s\in S} Ps_s\Big(\sum_{i\in I}\sum_{t\in T}(H_i k_{it} + M_i p_{it}) + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} F_{ij} e_{ijt} + \sum_{j\in J}\sum_{t\in T} Q_j u_{jt}\Big) && (10)\\
\text{s.t. } & p_{it} \le x_i,\ \forall i \in I, \forall t \in T && (11)\\
& p_{it} + k_{i(t-1)} = \sum_{j\in J} e_{ijt} + k_{it},\ \forall i \in I, \forall t \in T && (12)\\
& \sum_{i\in I} e_{ijt} = D_{jt} - u_{jt},\ \forall j \in J, \forall t \in T && (13)\\
& k_{i0} = 0,\ \forall i \in I && (14)\\
& x_i \ge 0,\ \forall i \in I && (15)\\
& p_{it}, k_{it} \ge 0,\ \forall i \in I, \forall t \in T && (16)\\
& e_{ijt} \ge 0,\ \forall i \in I, \forall j \in J, \forall t \in T && (17)\\
& u_{jt} \ge 0,\ \forall j \in J, \forall t \in T && (18)
\end{aligned}
$$

2.b Objective value and running time

The objective value is approximately 166.9947 and the running time of the model is 0.06214878 seconds. [HiGHS runtime is 0.03]

2.c Computational considerations

- In figure 1, the stochastic solutions with fewer scenarios seemed to be more spread out (have more variability), with the deterministic solution intersecting with the points but as the number of scenarios increased the overall behaviour of the stochastic solutions become more stable and consistent and thus, converge. Thus with less scenarios, there is a high level of uncertainty in the results. The increased number of scenarios allows the model to better adapt and respond to various situations. Increasing the number of scenarios has the advantage of allowing the model to better capture underlying patterns or trends, resulting in a more reliable and convergent set of solutions.

- The main challenges in increasing the number of scenarios are that the solution times of the models increase and more memory usage of the models is required. The solution times does not seem linear but the memory requirements seem linear.

3 Standard form polyhedra

3.a Number of feasible solutions

(a) False: If $n = m + 1$, it means the number of variables is greater than the number of constraints. In this case, the polyhedron P might have more than two basic feasible solutions. In a situation where $m = 2$ and $n = 3$, the polyhedron P could be unbounded, and there could be infinitely many basic feasible solutions.

3.b Set of optimal solutions

(b) False: The set of all optimal solutions may not be bounded. In a case where the feasible region is unbounded, and the objective function can be improved indefinitely, the set of optimal solutions is not bounded.

3.c Positive variables at optimum

(c) True: At every optimal solution, the number of positive variables cannot exceed m. This is because the polyhedron P is defined as $Ax = b$ where $x \geq 0$. At an optimal solution, the objective function is optimized, and the value of each variable is non-negative. If more than m variables are positive, it implies that there is redundancy in the representation, and that the solution is not optimal.

3.d Number of optimal solutions

(d) False: Having more than one optimal solution does not necessarily mean there are infinitely many optimal solutions. There could be a finite number of optimal solutions. For example, a case where the objective function is constant over a line segment in the feasible region.

3.e Optimal and basic feasible solutions

(e) True: If there are several optimal solutions, it signifies that the objective function is optimized at more than one point. Per the definition of optimality, each optimal solution must correspond to an extreme point/vertex of the feasible region. These extreme points are basic feasible solutions. If there are several optimal solutions, there must be at least two distinct extreme points, each corresponding to a different basic feasible solution.