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What is Strassen’s matrix multiplication ?

Strassen's algorithm is an efficient algorithm for matrix multiplication, named after Volker Strassen who developed it in 1969.

It's based on the divide-and-conquer approach and is particularly efficient for large matrices.

The traditional matrix multiplication algorithm has a time complexity of O(n^3) for multiplying two n × n matrices. Strassen's algorithm reduces this time complexity to approximately O(n^2.81), making it faster for large matrices.

Strassen's algorithm reduces the number of multiplications required from 8 in the standard algorithm to 7, by exploiting symmetries in the multiplication process and using fewer intermediate computations.

The key idea behind Strassen's method is to reduce the number of multiplications needed by using a divide-and-conquer approach. By breaking down the problem into smaller subproblems and combining the results efficiently, it can speed up the matrix multiplication process, especially for large matrices.

.Overall, Strassen's matrix multiplication is a clever algorithm that exploits mathematical properties to multiply matrices more efficiently, saving time and computational resources.

<u>Algorithm for strassen's matrix multiplication :</u>

STEP 1: Start with two square matrices A and B, each of size n×n.

STEP 2: Divide each matrix A and B into four equal-sized submatrices, each of size $n/2 \times n/2$. These submatrices are denoted as follows:

$$A = \begin{pmatrix} A12 & A21 \\ A12 & A22 \end{pmatrix}$$

$$B = \begin{pmatrix} B11 & B21 \\ B12 & B22 \end{pmatrix}$$

STEP 3: Perform the following multiplications recursively to obtain seven products:

P = (A11 + A22) (B11 + B22)

Q= B11 (A21 + A22) R = A11 (B12 – B22)

S = A22 (B21 – B11)

T = B22 (A11 + A12)

U = (A21 – A11) (B11 + B12)

V = (B21 + B22) (A12 – A22)

STEP 4: Compute the following submatrices.

C11= P + S – T + V

C12== R + T

C21= Q + S

C22= P + R – Q + U

STEP 5: The resulting matrix C is obtained by concatenating C11, C12, C21, and C22 appropriately.

$$\begin{pmatrix} C11 & C12 \\ C21 & C22 \end{pmatrix}$$

STEP 6: Return the resulting matrix C.

<u>Problem Statement</u>:- Write Strassen's algorithm to multiply two 2X2 matrices. Apply Strassen's algorithm to multiply following matrices.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

Solution: -

$$\begin{pmatrix} A11 & A12 \\ A21 & A22 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} B11 & B12 \\ B21 & B22 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

Now, we have to find out $\begin{pmatrix} C11 & C12 \\ C21 & C22 \end{pmatrix}$

Formula

P = (A11 + A22) (B11 + B22) => (1 + 1) (2 + 2) => **8**

Q = B11 (A21 + A22) => 2 (1 + 1) => **4**

R = A11 (B12 – B22) => 1 (2 – 2) => **0**

S = A22 (B21 – B11) => 1 (2 – 2) => **0**

T = B22 (A11 + A12) => 2 (1 + 1) => **4**

U = (A21 – A11) (B11 + B12) => (1 – 1) (2 + 2) => **0**

V = (B21 + B22) (A12 – A22) => (2 + 2) (1 – 1) => **0**

C11 = P + S – T + V => 8 + 0 – 4 + 0 => **4**

C12 = R + T => 0 + 4 => **4**

C21 = Q + S => 4 + 0 => **4**

C22 = P + R – Q + U => 8 + 0 – 4 + 0 => **4**

Final Ans :-

$$\begin{pmatrix} C11 & C12 \\ C21 & C22 \end{pmatrix} = \begin{pmatrix} \mathbf{4} & \mathbf{4} \\ \mathbf{4} & \mathbf{4} \end{pmatrix}$$