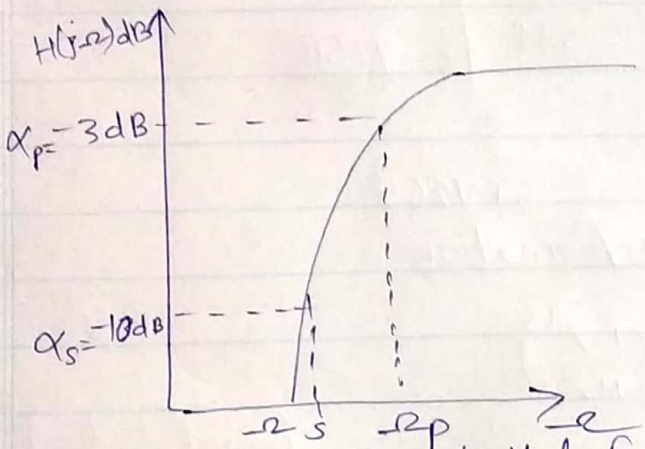
Q. Using the Bilinear transformation, design a highpass filter, Monotonic in passband with cutoff frequency 1KHz and down 10dB at 350Hz. the sampling frequency is 5000Hz.

Given. $\omega_c = \omega_p = 2\times\pi\times1000 = 2000\pi$ rad/sec.

$\omega_s = 2\times\pi\times350 = 700\pi$ rad/sec.

$T = \frac{1}{f_s} = \frac{1}{5000} = 2\times10^{-4}$ sec.



So, prewarping the digital frequencies we have.

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times10^{-4}}\tan\frac{(2000\pi\times2\times10^{-4})}{2} = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{700\pi\times2\times10^{-4}}{2}\right) = 2235 \text{ rad/sec}$$

To check order of filter

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log\frac{7265}{2235}} = 0.932 \simeq 1$$

∴ the order of ~~frequ~~ filter $N = 1$

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~~$\Omega_c = \frac{1}{\text{rad}}$~~

The 1st order Butterworth filter for $\Omega_c = 1$ rad/sec is $H(s) = \frac{1}{1+s}$.

where $\Omega_c = \Omega_p = 7265$ rad/sec

$$\therefore s = \frac{\Omega_c}{s} = \frac{7265}{s}.$$

The transfer function of highpass filter is

$$H(s) = \frac{1}{s+1}\Big|_{s=\frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation.

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Big|_{s=\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}.$$

~~$H(z) = \frac{0.52}{}$~~

$$\therefore H(z) = \frac{0.5792\left(1-z^{-1}\right)}{1-0.1584\,z^{-1}}$$

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