1. Check all the problem(s) that admit sparse solution(s):

1 point

- [x] $\min_{\mathbf{x}} \|\mathbf{x}\|_1 \;\; subject\, to\, A\mathbf{x} = \mathbf{b}$
- [ ] $\min_{\mathbf{x}} \|\mathbf{x}\|_2 \;\; subject\, to\, A\mathbf{x} = \mathbf{b}$
- [x] $\min_{\mathbf{x}} \|\mathbf{x}\|_0 \;\; subject\, to\, A\mathbf{x} = \mathbf{b}$
- [x] $\min_{\mathbf{x}} \|\mathbf{x}\|_p \;\; subject\, to\, A\mathbf{x} = \mathbf{b}$ (p is between 0 and 1)

2. Natural image patches (unlike random noise) cannot be sparsely represented over a DCT dictionary.

1 point

- ○ True
- ◉ False

3. In video surveillance applications, background is modeled as a ______ matrix and moving parts are modeled via a ______ matrix.

1 point

- ○ sparse, low-rank
- ○ low-rank, low-rank
- ○ sparse, sparse
- ◉ low-rank, sparse

4. Which one of the greedy algorithms discussed in class is designed to solve the following problem?

1 point

$$\min_{A,X} \|AX - B\|_F$$
$$subject\, to\, \|X(:,i)\|_0 \leq k \;\forall i$$

- ○ Matching Pursuit
- ◉ Method of Optimal Directions
- ○ This problem has a closed form solution and hence doesn't need to be solved in a greedy fashion.
- ○ Orthogonal Matching Pursuit

5. Check all the norms that are convex functions.

1 point

- [ ] The L0 norm ($f(\mathbf{x}) = \|\mathbf{x}\|_0$)
- [x] The L1 norm ($f(\mathbf{x}) = \|\mathbf{x}\|_1$)
- [ ] The Lp norm ($f(\mathbf{x}) = \|\mathbf{x}\|_p$ where p is between 0 and 1.
- [x] The L2 norm ($f(\mathbf{x}) = \|\mathbf{x}\|_2$)

6. Which one of the following statements is true regarding the basis pursuit problem given by
$\mathbf{x}^* = \underset{\mathbf{x}}{argmin} \ \|\mathbf{x}\|_1 \quad subject\, to\, A\mathbf{x} = \mathbf{b}$?

1 point

- ○ If one of the columns in A is identical to $\mathbf{b} \neq \mathbf{0}$, then $\|\mathbf{x}^*\|_1 = 1$.
- ○ If $\mathbf{b} = \mathbf{0}$ and A is full rank, then the problem has no solution since the constraint set becomes empty.
- ○ $\mathbf{x}^*$ also minimizes the corresponding LASSO problem given by $\underset{\mathbf{x}}{min} \ \|A\mathbf{x} - \mathbf{b}\|_2^2 + \lambda \|\mathbf{x}\|_1$.
- ◉ none of the above.

7. The singular value decomposition of the $n \times n$ square matrix A is given by $A = U\Sigma V^T$. Check all correct statements.

1 point

- ☐ The nuclear norm of A (given by the sum of absolute values of entries on the diagonal of Σ) is an upper bound on the rank of A.
- ☑ If Σ has only one non-zero entry, then A is definitely a rank-1 matrix.
- ☐ Only square matrices (such as A) have singular value decomposition.
- ☑ Matrices U and V are orthonormal bases for the n-dimensional space.

8. In this MATLAB assignment you will code-up the orthogonal matching pursuit (OMP) algorithm, as discussed in the lecture, to solve the following optimization problem:

1 point

$$\underset{\mathbf{x}}{min} \ \|A\mathbf{x} - \mathbf{b}\|_2 \quad subject\, to \ \|\mathbf{x}\|_0 \leq S.$$

In this exercise $A = D + I$, where $D_{ij} = sin(i + j)$ for $1 \leq i, j \leq 10$ and I is the 10×10 identity matrix. Use A along with $\mathbf{b} = [-2, -6, -9, 1, 8, 10, 1, -9, -4, -3]^T$, and $S = 3$ to find the solution to the problem given above.

Your solution $\mathbf{x}^*$ uses three columns of A in order to approximate $\mathbf{b}$. Type in the sum of the indices of these three columns. For example, if you find that your solution uses the first, third, and fifth columns of A then you must enter 9. (Hint: $normc(A)$ normalizes the columns of A to a length of 1 in MATLAB.)

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