Module-3:
Lecture-19

Graphs Terminology

A **graph** consists of:

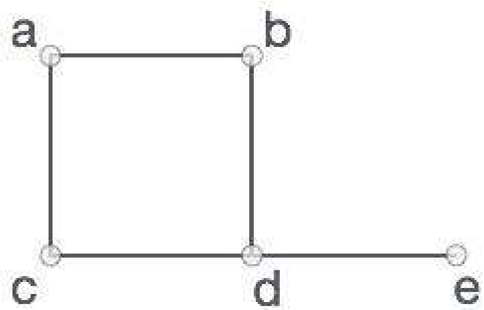
- A set, V, of **vertices** (nodes)
- A collection, E, of pairs of vertices from V called **edges** (arcs)

Edges, also called arcs, are represented by ***(u, v)*** and are either:

Directed if the pairs are ordered ***(u, v)***
u the **origin**
v the **destination**
Undirected if the pairs are unordered

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as **vertices**, and the links that connect the vertices are called **edges.**

Formally, a graph is a pair of sets **(V, E)**, where **V** is the set of vertices and **E**is the set of edges, connecting the pairs of vertices. Take a look at the following graph −



In the above graph,
V = {a, b, c, d, e}
E = {ab, ac, bd, cd, de}

Then a **graph** can be:
Directed graph (di-graph) if all the edges are directed
Undirected graph (graph) if all the edges are undirected
Mixed graph if edges are both directed or undirected

Illustrate terms on graphs

End-vertices of an edge are the **endpoints** of the edge.
Two vertices are **adjacent** if they are endpoints of the same edge.
An edge is **incident** on a vertex if the vertex is an endpoint of the edge.
Outgoing edges of a vertex are directed edges that the vertex is the origin.
Incoming edges of a vertex are directed edges that the vertex is the destination.
Degree of a vertex, *v*, denoted *deg*(*v*) is the number of incident edges.
Out-degree, *outdeg*(*v*), is the number of outgoing edges.
In-degree, *indeg*(*v*), is the number of incoming edges.
Parallel edges or multiple edges are edges of the same type and end-vertices
Self-loop is an edge with the end vertices the same vertex

Simple graphs have **no** parallel edges or self-loops

Properties

If graph, G, has m edges **then** $\Sigma_{v\in G}\ deg(v) = 2m$

If a di-graph, G, has m edges **then**

$\Sigma_{v\in G}\ indeg(v) = m = \Sigma_{v\in G}\ outdeg(v)$

If a **simple** graph, G, has m edges and n vertices:

If G is also directed **then** $m \leq n(n-1)$

If G is also undirected **then** $m \leq n(n-1)/2$

So a simple graph with n vertices has $O(n^2)$ edges at most

More Terminology

Path is a sequence of alternating vetches and edges such that each successive vertex is connected by the edge. Frequently only the vertices are listed especially if there are no parallel edges.

Cycle is a path that starts and end at the same vertex.

Simple path is a path with distinct vertices.

Directed path is a path of only directed edges

Directed cycle is a cycle of only directed edges.

Sub-graph is a subset of vertices and edges.

Spanning sub-graph contains all the vertices.

Connected graph has all pairs of vertices connected by at least one path. **Connected component** is the maximal connected sub-graph of a unconnected graph.

Forest is a graph without cycles.

Tree is a connected forest (previous type of trees are called rooted trees, these are free trees)

Spanning tree is a spanning subgraph that is also a tree.

More Properties

If G is an **undirected** graph with n vertices and m edges:

- **If** G is connected **then** $m \geq n - 1$
- **If** G is a tree **then** $m = n - 1$
- **If** G is a forest **then** $m \leq n – 1$

Graph Traversal:

1. Depth First Search
2. Breadth First Search