Lecture-22

Graph representation

You can represent a graph in many ways. The two most common ways of representing a graph is as follows:

Adjacency matrix

An adjacency matrix is a **VxV** binary matrix **A.** Element Ai,j is 1 if there is an edge from vertex i to vertex j else Ai,jis 0.

Note: A binary matrix is a matrix in which the cells can have only one of two possible values - either a 0 or 1.

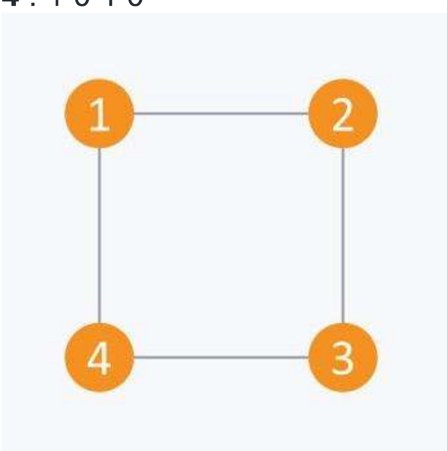
The adjacency matrix can also be modified for the weighted graph in which instead of storing 0 or 1 in Ai,j, the weight or cost of the edge will be stored.

In an undirected graph, if Ai,j = 1, then Aj,i = 1. In a directed graph, if Ai,j = 1, then Aj,i may or may not be 1.

Adjacency matrix provides **constant time access (O(1))** to determine if there is an edge between two nodes. Space complexity of the adjacency matrix is **O(**V2**).**

The adjacency matrix of the following graph is:

i/j : **1 2 3 4**
1 : 0 1 0 1
2 : 1 0 1 0
3 : 0 1 0 1
4 : 1 0 1 0



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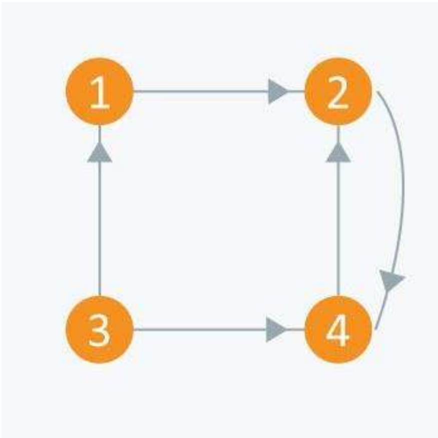


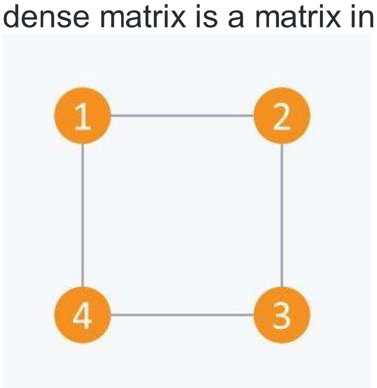
Adjacency list

The other way to represent a graph is by using an adjacency list. An adjacency list is an array A of separate lists. Each element of the array A_i is a list, which contains all the vertices that are adjacent to vertex i.

For a weighted graph, the weight or cost of the edge is stored along with the vertex in the list using pairs. In an undirected graph, if vertex j is in list Ai then vertex i will be in list Aj.

The space complexity of adjacency list is **O(V + E)** because in an adjacency list information is stored only for those edges that actually exist in the graph. In a lot of cases, where a matrix is sparse using an adjacency matrix may not be very useful. This is because using an adjacency matrix will take up a lot of space where most of the elements will be 0, anyway. In such cases, using an adjacency list is better.

Note: A sparse matrix is a matrix in which most of the elements are zero, whereas a dense matrix is a matrix in which most of the elements are non-zero.



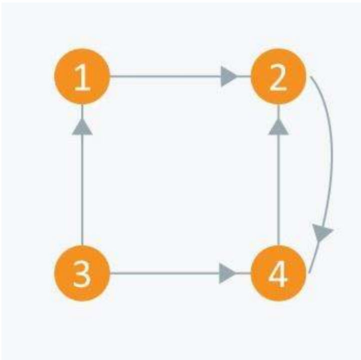
Consider the same undirected graph from an adjacency matrix. The adjacency list of the graph is as follows:

A1 → 2 → 4

A2 → 1 → 3

A3 → 2 → 4

A4 → 1 → 3



Consider the same directed graph from an adjacency matrix. The adjacency list of the graph is as follows:

A1 → 2

A2 → 4

A3 → 1 → 4

A4 → 2