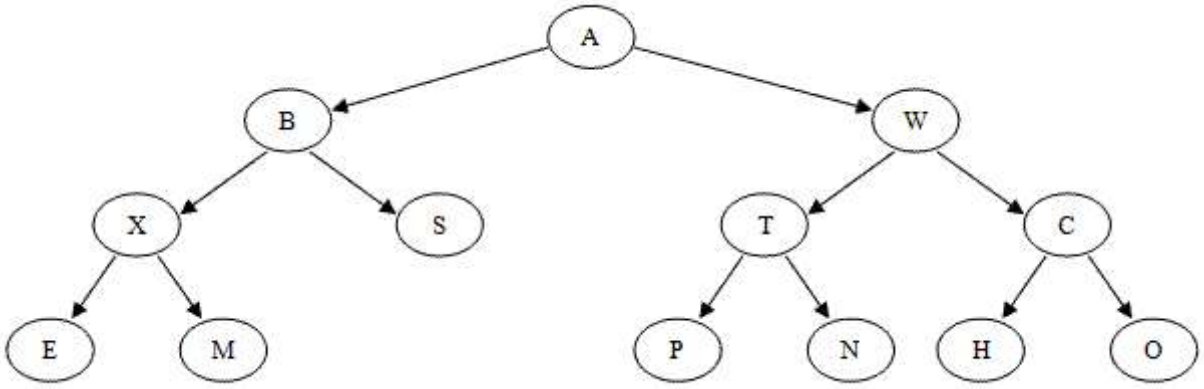
Lecture-13

Binary Tree

A *binary tree* consists of a finite set of nodes that is either empty, or consists of one specially designated node called the *root* of the binary tree, and the elements of two disjoint binary trees called the *left subtree* and *right subtree* of the root.

Note that the definition above is recursive: we have defined a binary tree in terms of binary trees. This is appropriate since recursion is an innate characteristic of tree structures.

Diagram 1: A binary tree



Binary Tree Terminology

Tree terminology is generally derived from the terminology of family trees (specifically, the type of family tree called a *lineal chart*).

- Each root is said to be the *parent* of the roots of its subtrees.
- Two nodes with the same parent are said to be *siblings*; they are the *children* of their parent.
- The root node has no parent.
- A great deal of tree processing takes advantage of the relationship between a parent and its children, and we commonly say a *directed edge* (or simply an *edge*) extends from a parent to its children. Thus edges connect a root with the roots of each subtree. An *undirected edge* extends in both directions between a parent and a child.

- *Grandparent* and *grandchild* relations can be defined in a similar manner; we could also extend this terminology further if we wished (designating nodes as cousins, as an uncle or aunt, etc.).

Other Tree Terms

- The number of subtrees of a node is called the *degree* of the node. In a binary tree, all nodes have degree 0, 1, or 2.
- A node of degree zero is called a *terminal node* or *leaf node*.
- A non-leaf node is often called a *branch node*.
- The *degree of a tree* is the maximum degree of a node in the tree. A binary tree is degree 2.
- A *directed path* from node n_1 to n_k is defined as a sequence of nodes n_1, n_2, ..., n_k such that n_i is the parent of n_i+1 for 1 <= i < k. An *undirected path* is a similar sequence of undirected edges. The length of this path is the number of edges on the path, namely k – 1 (i.e., the number of nodes – 1). There is a path of length zero from every node to itself. Notice that in a binary tree there is exactly one path from the root to each node.
- The *level* or *depth* of a node with respect to a tree is defined recursively: the level of the root is zero; and the level of any other node is one higher than that of its parent. Or to put it another way, the level or depth of a node n_i is the length of the unique path from the root to n_i.
- The *height* of n_i is the length of the longest path from n_i to a leaf. Thus all leaves in the tree are at height 0.
- The *height of a tree* is equal to the height of the root. The *depth of a tree* is equal to the level or depth of the deepest leaf; this is always equal to the height of the tree.
- If there is a directed path from n_1 to n_2, then n_1 is an ancestor of n_2 and n_2 is a descendant of n_1.