Entanglement Complexity and Frustration

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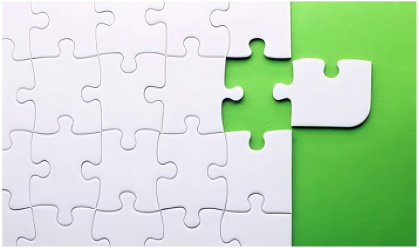
Complexity and frustration

- Global property of an object: relations between the parts of an object
- Frustration: originally a biology term
- Focus on qubits ensembles as complex systems
- Entanglement in qubits ensemble.
- Frustration in Physics.



Entangled states and purity of a subsystem

Entanglement and purity

- Generic pure state of $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
- $|\psi\rangle_{AB}$ is entangled if and only if the number of terms in the Schmidt decomposition is greater than 1
- Purity of the subsystem A

$$\pi_A(\psi) = \mathrm{tr}_A \rho_A^2 \quad \text{where} \quad \rho_A = \mathrm{tr}_B \, |\psi\rangle_{AB\,AB}\langle\psi|$$

Purity of a subsystem of a qubits ensemble

- Hilbert space $\mathcal{H} = \mathfrak{H}^{\otimes n}$ where $\mathfrak{H} = \mathbb{C}^2$ and n is the number of qubits, the system size
- Bipartition of the system: $(A, \bar{A})$, $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$
- Purity of the subsystem A

$$\pi_A = \pi_{\bar{A}} = \mathrm{tr}_A \, \rho_A^2$$

Entangled states and separable states

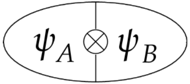
Purity bounds for a bipartition $(A, \bar{A})$ $\frac{1}{N_A} \leq \pi_A(\psi) \leq 1$ where $N_A = \dim \mathcal{H}_A$

A. Lower bound, maximal mixedness of each of the two subsets

$\pi_A = \frac{1}{N_A} \iff \rho_A = \frac{1}{N_A}\mathbb{1}$

B. Higher bound, separable states

$\pi_A = 1 \iff \rho_A$ and $\rho_{\bar{A}}$ are projectors.



- Frustration: the lower bound cannot be reached for every balanced bipartition
- Potential of multipartite entanglement (PME)

  $\pi_{ME}(\psi) = \sum_{|A|=n_A} \pi_A(\psi)$ with $n_A = \lfloor n/2 \rfloor$

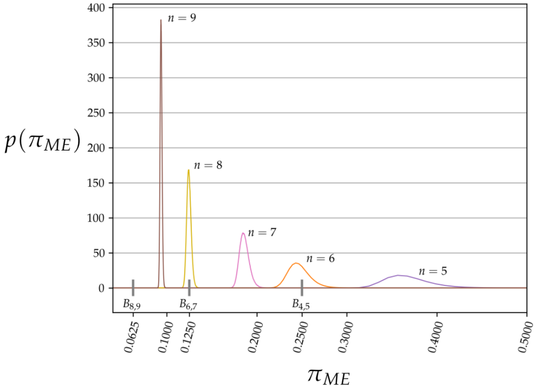
A maximally multipartite entangled state (MMES) is a minimizer of π_{ME}. A state for which the lower bound $1/N_A$ is saturated for every bipartition is a perfect MMES.

n	perfect MMES existence
$2, 3$	Yes
4	No (is frustrated)
$5, 6$	Yes
≥ 7	No (is frustrated)

Probability density function estimates

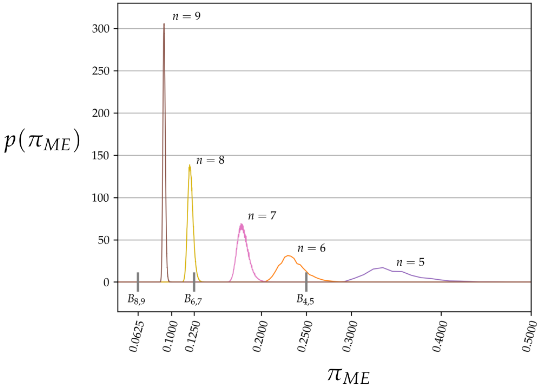
Continuous states

all the states



Discrete states

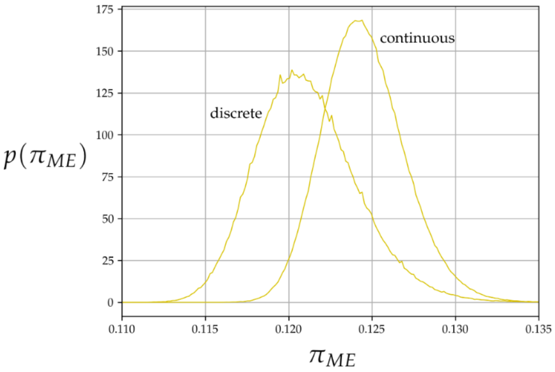
$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{j \in Z_2^n} k_j |j\rangle \text{ with } k_j = \begin{cases} 1 \text{ if } j = 0 \\ \pm 1 \text{ if } j \neq 0 \end{cases}$$

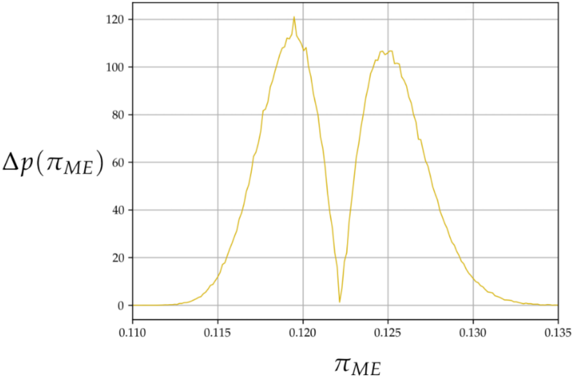


π_{ME} PDF estimates (n=8 case)

Difference between the two PDF estimates

$$\Delta p(\pi_{ME}) = |p_c(\pi_{ME}) - p_d(\pi_{ME})|$$

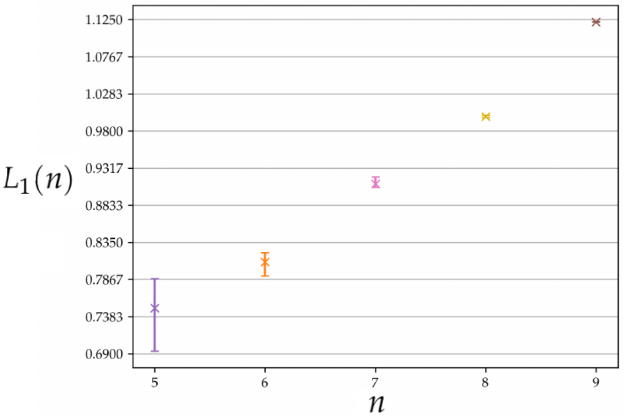




number of bins: $n_b = 7\,127$

Distance between continuous and discrete π_{ME} PDF estimates

$$0 \leq L_1 \leq 2, \quad L_1 = \int \Delta p(\pi_{ME}) \mathrm{d}\pi_{ME} = \int |p_c(\pi_{ME}) - p_d(\pi_{ME})| \mathrm{d}\pi_{ME}$$

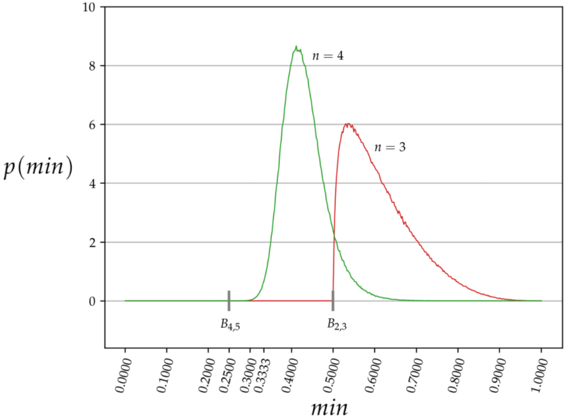


Case $n = 4$, a frustrated ensemble

Probability density function of the minima (continuous states)

$$min(\psi) \equiv \min_{(A,\bar{A})\in BB_n} \pi_A(\psi) \text{ where } BB_n \text{ is the set of all balanced bipartitions}$$

Theoretical prediction:
For $n = 4$, the minimum of the π_{ME} among all states is $\pi_{ME} = \frac{1}{3} > \frac{1}{4} = B_{4,5}$



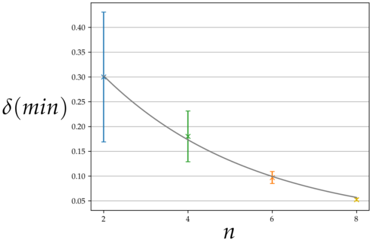
Distance of the minimum from the lower bound, continuous states

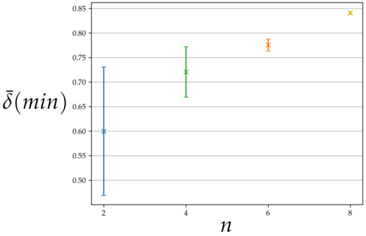
$$y = \alpha \exp(-\beta x)$$

$$\delta(min) = \langle min \rangle - \frac{1}{N_A}$$

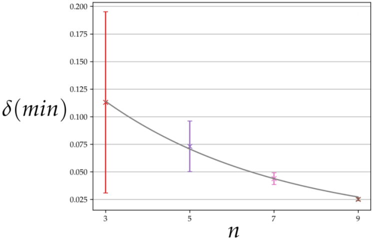
$$\bar{\delta}(min) = \frac{\langle min \rangle - 1/N_A}{1/N_A}$$

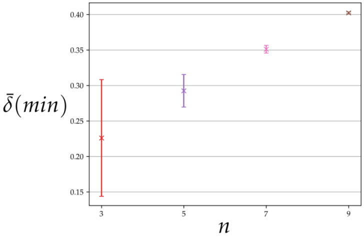
$\alpha = 0.5275, \ \beta = 0.2784$





$\alpha = 0.2333, \ \beta = 0.2387$





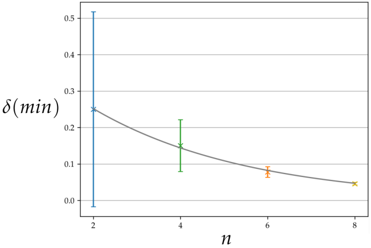
Distance of the minimum from the lower bound, discrete states

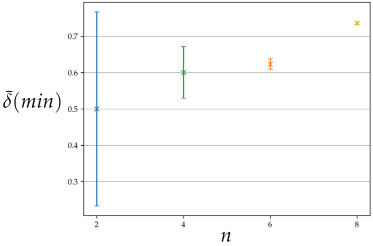
$$y = \alpha \exp(-\beta x)$$

$$\delta(min) = \langle min \rangle - \frac{1}{N_A}$$

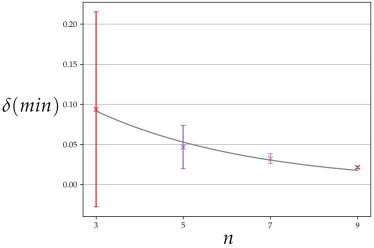
$$\bar{\delta}(min) = \frac{\langle min \rangle - 1/N_A}{1/N_A}$$

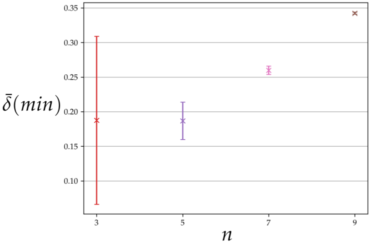
$\alpha = 0.4404, \ \beta = 0.2794$





$\alpha = 0.2092, \ \beta = 0.2752$



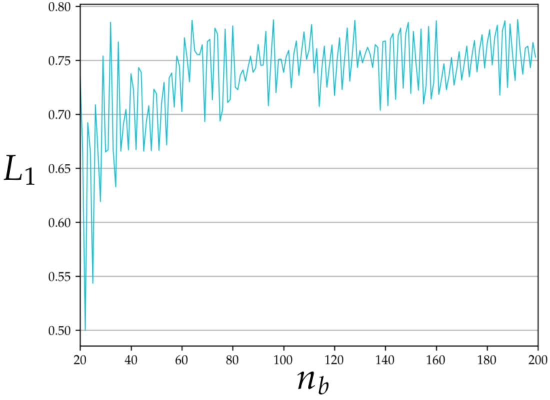


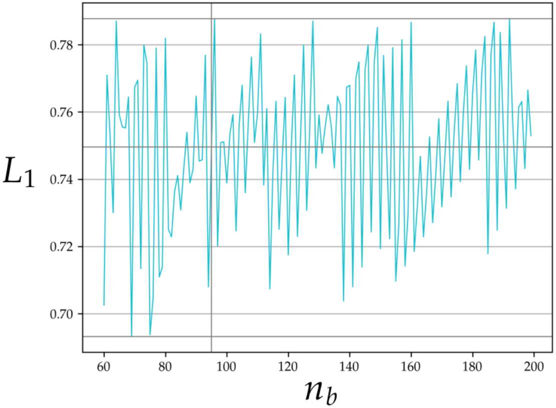
Thank you

Thank you for your attention.

Choice of the number of bins (n=5 case)

$$0 \leq L_1 \leq 2, \quad L_1 = \int \Delta p(\pi_{ME}) \mathrm{d}\pi_{ME} = \int |p_c(\pi_{ME}) - p_d(\pi_{ME})| \mathrm{d}\pi_{ME}$$





Choice of the number of bins for each quantum system

$$0 \leq L_1 \leq 2, \quad L_1 = \int \Delta p(\pi_{ME}) \mathrm{d}\pi_{ME} = \int |p_c(\pi_{ME}) - p_d(\pi_{ME})| \mathrm{d}\pi_{ME}$$

n	interval of n_b	L_1 minimum	L_1 average	L_1 maximum	$\bar{n}_b$
5	$(60, 200)$	0.693268	0.7496023	0.787808	95
6	$(250, 1\,000)$	0.791318	0.8094594	0.821168	291
7	$(1\,500, 10\,000)$	0.90642	0.911284	0.92022	3 399
8	$(3\,500, 8\,000)$	0.99703	0.9988859	1.000162	7 127
9	$(15\,000, 20\,000)$	1.121678	1.1219677	1.122212	18 639