

Question5:

Given: List of Chemical C, List of weight of chemicals W, each day evaporate p, due date K.

Aim: Find the schedule to make the waste chemical to be the least

Solution:

Step 1:

Sort the list W from the lightest to heaviest, then sort the list C according to the list W.

Step 2:

The chemical should be completely made one time in the single day, because if the chemical produced separately then there must be a day that produced more chemicals and a day produce less so that it is not appropriate to the greedy strategy. So that we arrange the heaviest chemical to the last day produce, and put the lightest chemical at the first day. E.g: $A+B = C$, $\{A,B,C\}$ is a subset of Integer; There are K days to be deadline, A produced on K day to deadline, $A/(1-P)^K$; B produced on k-1 day before deadline, $B/(1-P)^{(k-1)}$; C produced on the k day $C/(1-P)$. So that from this calculation we can observe that $A/(1-P)^K + B/(1-P)^{(k-1)} > C/(1-P)^K$. This proved that, if the chemical produced separately then it will waste more than the chemical produce on the same day.

Step 3:

Then we choose the next Chemical in C, and produce it, until produced all the element in C.

Prove to be optimal:

Assume that the greedy method is not optimal. Which means we will have a better solution. Then for the chemical list C, there will be a Chemical C_j weighted W_i which is heavier than C_1 (who are the lightest chemical and weighted W_1). Then we swap the position of C_1 and C_j , let C_j produced earlier than C_1 .

The equation before swap is $W_1(1+p)^k + W_2(1+p)^{k-1} + W_j(1+p)^{k-2} + \dots + W_n(1+p)$

The equation after swap is $W_j(1+p)^k + W_1(1+p)^{k-1} + W_2(1+p)^{k-2} + \dots + W_n(1+p)$

According to the example proof from above Step2. We can know that the Result of before swap equation must smaller than the Result of after swap equation. Which is contradicted to our assumption, So that the greedy method we use is proved to be optimal.