A:

Step 1: Do not modify f(n) = $(\log(n))^2$; But modify this equation g(n) = $\log_2(n^{\log_2 n})^2$ by formula logarithm power rule $2*\log_2(n^{\log_2 n})$, use formula logarithm power rule again we can get $2*\log_2 n*\log_2(n\ \ )$

Step 2: The big O notation f(n) = O(log(n)), and for all n ≥ n0 such that C*f(n) ≥ g(n); also g(n) = O(log(n)) and for all n ≥ n0 such that C*g(n) ≥ f(n). These two equations have the properties of f(n) = O(g(n)) and f(n) = Ω(g(n)); thus, f(n) and g(n) have the same asymptotic growth rate. Then they should be both, so f(n) = θ(g(n)).

B:

Given: $f(n)=n^{10}$; $g(n)=2^{\sqrt[10]{n}}$

Step 1:

Do the logarithm for both functions. $\log f(n)=2^{\log_2 n^{10}}$;

$\log g(n)=2^{\sqrt[10]{n}}$ we can compare the power of two functions as

$\log f(n)=10\log n$; $\log g(n)=n^{0.1}$

Step 2:

Apply the L'Hopital's rule, then it will be $\lim_{x\to\infty}\frac{g'(x)}{f'(x)}$ ==>

$\lim_{n\to\infty}\frac{0.1*n^{-0.9}}{\frac{1}{n}\ln 2}$, if n is approach to infinite, then the value is approaching to infinite as well.

Step 3:

So, 0<= cg(n) <= f(n), for all n >= n0. The increasing rate infinite, g(x) is the upper bound of f(x); g(x) = O(f(x)) which is g(x) = Ω(f(x)).

C:

Given f(n) = $n^{1+(-1)^n}$, g(n) = n.

Step 1:

There are two cases, When n is odd number then f(n) = 1, when n is even number f(n) = n^2.

Step 2:

If number n is odd number, $f(n) = 1$, $g(n) = n$. Then g(n) is the upper bound of f(n), then $g(n) = O(f(n))$.

If number is even number, $f(n) = n^2$, $g(n)= n$. Then $0 <= c*g(n) <= f(n)$, so that, $f(n) = \Omega(g(n))$.

However in a single function f(n), There is none a fixed n_0 could satisfied the condition of such that $0 <= c*g(n) <= f(n)$ and $0 <= f(n) <= c*g(n)$, for all $n \geq n_0$. So, it is neither.