Given: A string of operator and operand
Aim: Count the number of ways of putting parentheses that make the end equal to true.

Step 1:

Set up: Let the length of String be S symbols and S-1 operations in the string. To count how many ways are there to make the same from lth symbol end at rth symbol but evaluated to be true or to be false.

Step 2:

Solve the problem by solving each Sub-problems:

Base case:

T(i, i) is 1 if symbol i is true, and 0 if symbol i is false.

The reverse applies to F(i, i).

Iterations:

For each sub-problem, we 'split' the expression around an operator m so that everything to the left of the operator is in its own bracket, and everything to the right of the operator is in its own bracket to form two smaller expressions. We then evaluate the sub-problems on each 3 of the two sides, and combine the results together depending on the type of operator we are splitting by, and whether we want the result to evaluate to true or false.

TSplit(l,m,r):

$$= \begin{cases} T(l,m) * T(m+1,r) \\ \uparrow Operator\text{'}AND\text{'} \\ T(l,m) * F(m+1,r) + T(l,m) * T(m+1,r) + F(l,m) * T(m+1,r) \\ \uparrow Operator\text{'}OR\text{'} \\ F(l,m) * F(m+1,r) \\ \uparrow Operator\text{'}NOR\text{'} \\ F(l,m) * F(m+1,r) + T(l,m) * F(m+1,r) + F(l,m) * T(m+1,r) \\ \uparrow Operator\text{'}NAND\text{'} \end{cases}$$

FSplit(l,m,r): =

$$\begin{cases} T(l,m) * F(m+1,r) + F(l,m) * F(m+1,r) + F(l,m) * T(m+1,r) \\ \uparrow Operator\text{''}AND\text{''} \\ F(l,m) * F(m+1,r) \\ \uparrow Operator\text{''}OR\text{''} \\ T(l,m) * T(m+1,r) \\ \uparrow Operator\text{''}NAND\text{''} \\ T(l,m) * T(m+1,r) + T(l,m) * F(m+1,r) + F(l,m) * T(m+1,r) \\ \uparrow Operator\text{''}NOR\text{''} \end{cases}$$

$$T(l,r) = \sum_{m=l}^{r-1} TSplit(l,m,r)$$

$$F(l,r) = \sum_{m=l}^{r-1} FSplit(l,m,r)$$

Operation result depends on TRUTH TABLE below.

Step 3:

Finally,

The table in below is THE TRUTH TABLE

OR	True	False
True	True	True
False	True	False

AND	True	False
True	True	False
False	False	False

NOR	True	False
True	False	False
False	False	True

NAND	True	False
True	False	True
False	True	True