A:

Step 1: Do not modify f(n) = ; But modify this equation g(n) =  by formula logarithm power rule 2\*, use formula logarithm power rule again we can get 2\*

Step 2: The big O notation f(n) = O(log(n)), and for all n ≥ n0 such that C\*f(n) ≥ g(n); also g(n) = O(log(n)) and for all n ≥ n0 such that C\*g(n) ≥ f(n). These two equations have the properties of f(n) = O(g(n)) and f(n) = Ω(g(n)); thus, f(n) and g(n) have the same asymptotic growth rate. Then they should be both, so f(n) = θ(g(n)).

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B:

Given: ; 

Step 1:

Do the logarithm for both functions. ;  we can compare the power of two functions as ; 

Step 2:

Apply the L’Hopital’s rule, then it will be  ==> , if n is approach to infinite, then the value is approaching to infinite as well.

Step 3:

So, 0<= cg(n) <= f(n), for all n >= n0. The increasing rate infinite, g(x) is the upper bound of f(x); which is g(x) = Ω(f(x)).

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C:

Given f(n) = , g(n) = n.

Step 1:

There are two cases, When n is odd number then f(n) = 1, when n is even number f(n) = .

Step 2:

If number n is odd number, f(n) = 1, g(n) = n. Then g(n) is the upper bound of f(n), then g(n) = O(f(n)).

If number is even number, f(n) = , g(n)= n. Then 0 <= c\*g(n) <= f(n), so that, f(n) = Ω(g(n)).

However in a single function f(n), There is none a fixed  could satisfied the condition of such that 0 <= c\*g(n) <= f(n) and 0 <= f(n) <= c\*g(n), for all n ≥ . So, it is neither.