## Westfälische Wilhelms-Universität Münster

Übung Modellierung und Analyse von Dynamischen Systemen, WiSe 17/18

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Blatt 9

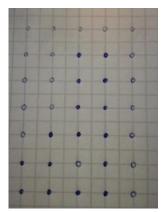
## Aufgabe 1



### Aufgabenteil a:

 $v = \{(2,3), (2,4), (3,3), (3,4), (4,3), (4,4), (5,2), (5,3), (5,4), (6,1), (6,2), (6,4), (7,1), (7,2), (7,3), (7,4)\}$ 

## Aufgabenteil b:



# Aufgabe 2

## Aufgabenteil a:

$$f(x) = \frac{1}{(1-x)^2}$$

$$f'(x) = \frac{2}{(1-x)^3}$$

$$f''(x) = \frac{6}{(1-x)^4}$$

$$f'''(x) = \frac{24}{(1-x)^5}$$

$$P_0(x) = \frac{1}{9}$$

$$P_1(x) = \frac{1}{9} + \frac{2}{27}(x+2)$$
$$= \frac{2}{27}x + \frac{3}{27} + \frac{4}{27}$$
$$= \frac{2}{27}x + \frac{7}{27}$$

$$P_2(x) = \frac{2}{27}x + \frac{7}{27} + \frac{1}{27}(x+2)^2$$

$$= \frac{2}{27}x + \frac{7}{27} + \frac{1}{27}x^2 + \frac{4}{27}x + \frac{4}{27}$$

$$= \frac{1}{27}x^2 + \frac{6}{27}x + \frac{11}{27}$$

$$\begin{split} P_3 = & \frac{1}{27}x^2 + \frac{6}{27}x + \frac{11}{27} + \frac{4}{243}(x+2)^3 \\ = & \frac{4}{243}x^3 + \frac{3}{81}x^2 + \frac{8}{81}x^2 + \frac{18}{81}x + \frac{16}{81}x + \frac{99}{243} + \frac{32}{243} \\ = & \frac{4}{243}x^3 + \frac{11}{81}x^2 + \frac{34}{81}x + \frac{131}{243} \end{split}$$

### Intervalle:

$$g_0(x) = f(x) - P_0(x) = \frac{1}{(1-x)^2} - \frac{1}{9} g_0(-4) = \frac{1}{25} - \frac{1}{9} = \frac{9}{225} - \frac{25}{225} = \frac{-16}{225}$$

$$g_0(0) = 1 - \frac{1}{9} = \frac{8}{9}$$

$$I_0 = P_0 + \left[ -\frac{8}{9}, \frac{8}{9} \right]$$

$$g_1(x) = f(x) - P_1(x) = \frac{1}{(1-x)^2} - \frac{2}{27}x - \frac{7}{27}$$

$$g_1(-4) = \frac{1}{25} + \frac{8}{27} - \frac{7}{27} = \frac{1}{25} + \frac{1}{27}$$

$$g_1(0) = 1 - \frac{7}{27} = \frac{20}{27}$$

$$I_1 = P_1 + \left[-\frac{20}{27}, \frac{20}{27}\right]$$

$$\begin{split} g_2(x) &= f(x) - P_2(x) = \frac{1}{(1-x)^2} - \frac{1}{27}x^2 - \frac{6}{27}x - \frac{11}{27} \\ g_2(-4) &= \frac{1}{25} - \frac{16}{27} + \frac{24}{27} - \frac{11}{27} = \frac{1}{25} - \frac{3}{27} \\ g_2(0) &= 1 - \frac{11}{27} = \frac{16}{27} \\ I_2 &= P_2 + \left[ -\frac{16}{27}, \frac{16}{27} \right] \end{split}$$

$$\begin{array}{l} g_3(x) = f(x) - P_3(x) = \frac{1}{(1-x)^2} - \frac{4}{243}x^3 - \frac{11}{81}x^2 - \frac{34}{81}x - \frac{131}{243} \\ g_3(-4) = \frac{1}{25} + \frac{256}{243} - \frac{176}{81} + \frac{136}{81} - \frac{131}{243} = \frac{1}{25} + \frac{125}{243} - \frac{40}{81} = \frac{1}{25} + \frac{5}{243} \\ g_3(0) = 1 - \frac{131}{243} = \frac{112}{243} \\ I_3 = P_3 + \left[ -\frac{112}{243}, \frac{12}{243} \right] \end{array}$$

### Aufgabenteil b:

$$g_0 = x_0$$

$$g_1 = x_0 + t + x_0 t$$

$$g_2 = x_0 + t + x_0 t + t^2 + x_0 t^2$$

$$g_3 = x_0 + t + x_0 t + t^2 + x_0 t^2 + t^3 + x_0 t^3$$

$$g_4 = g_3$$

Fixpunkt erreicht.