MATH-597-Project1

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1 Interpolation

We observed that using Legendre-Gauss (LG) points and Legendre-Gauss-Lobatto (LGL) nodal points allow us to construct good approximations for the function as we see in Figures 1, 2, and 3. That is because the numerical approximation gets better and better as we increase N, as shown by the error decreasing exponentially, until the errors level off because they reach machine zero (10^{-16}). On the other hand, when we increase the value of N while using equi-spaced points, we noticed that the solution gets better. However, when we increased the value of N from 16 upwards, the solution gets worse, with the error increasing exponentially. Again using equi-spaced points, at $N = 50$, the value of the exact solution equates the analytical solution because of the cardinality property. Because of this, the error at that point was recorded as 0, and hence is not reflected on the log-log plot of the various error norms. The interpolation eventually breaks down

The figures below show the error norms for interpolation of the function using equi-spaced, LGL, and LG points for $N = 1, 2, \cdots, 64$.

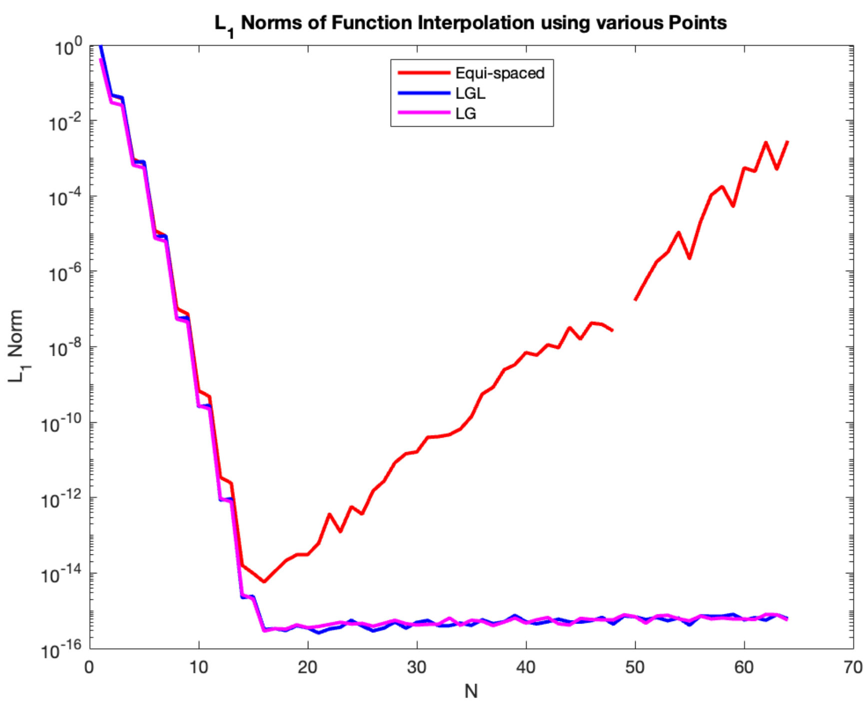


Figure 1: We observe that for lower orders of N (between 0 and 16), the errors decrease for all the various points. But increasing the order of N from 16, the Lagrange interpolation using equi-spaced points fails to approximate the exact solution well, as seen by the increasing errors. The errors for LGL and LG points stay very close to each other as they keep decreasing before leveling out.

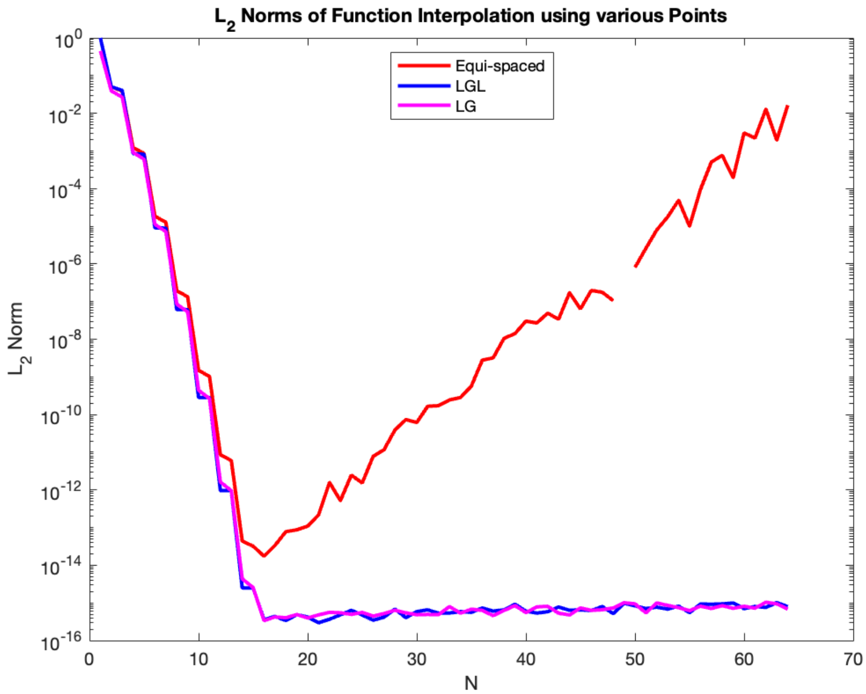


Figure 2: The L_2 error norm is similar to L_1 norm. However, we observe that the errors of the equi-spaced points are more pronounced here than in the L_1 norm. This is seen from when N increases to 6 and continues through 64, unlike in the L_1 norm where the increase in the error using equi-spaced is observed from $N = 7$.

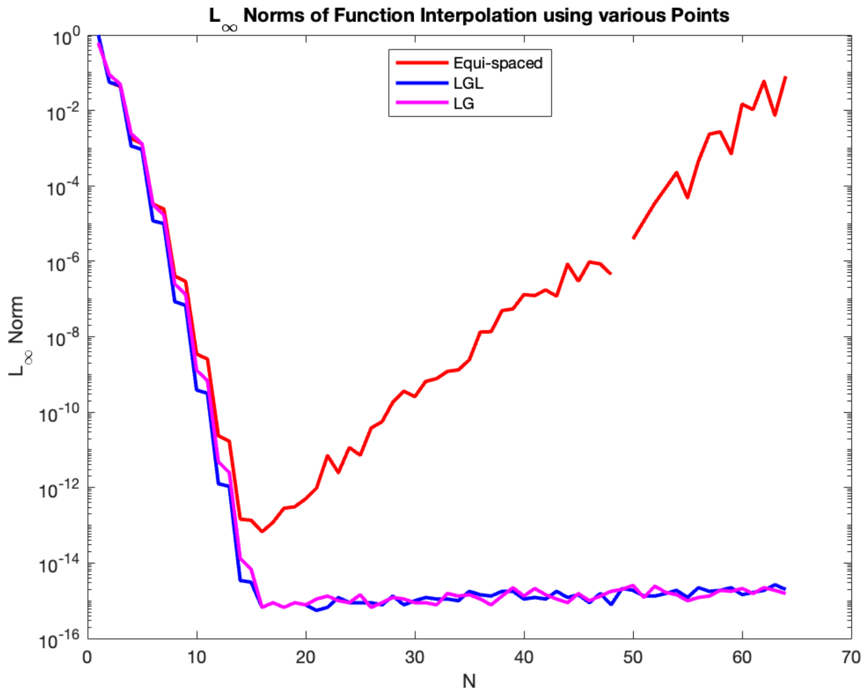
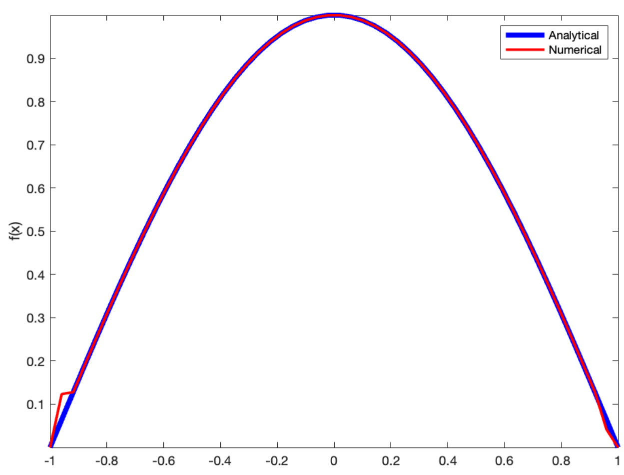
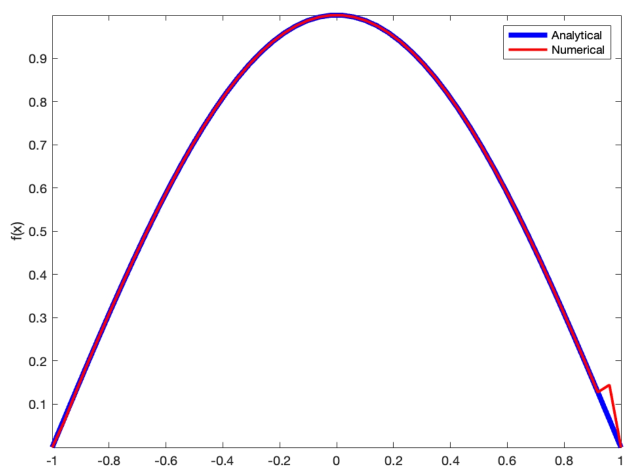


Figure 3: The L_∞ error norm reveals that the interpolation of the function using LGL points outperformed the others as it error was lower than that of the others through out most values of N. It is especially noticed from $N = 1$ to 16. After that, it goes up and down like the error of the LG points. Like in the L_2 norm above, the error of the equi-spaced points was higher than those of the other points.

We also present the plots of the interpolation of the function using equi-spaced points for $N = 62$ and 64 to show how the approximation using equi-spaced points breaks down.



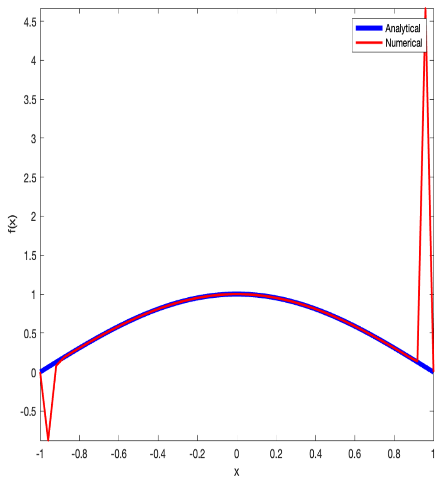
(a) Function interpolation, $N = 62$



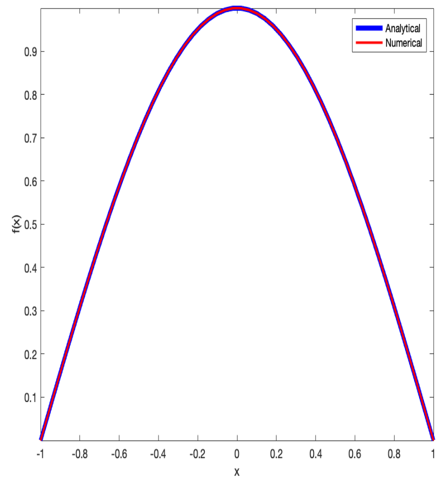
(b) Function interpolation, $N = 64$

Figure 4: Function interpolation using equi-spaced points. Observe how the approximation fails at the left endpoint for $N = 62$, and the right endpoint for $N = 64$, respectively.

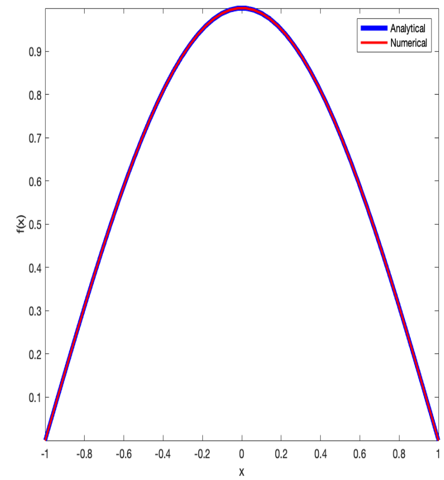
To consolidate our results, we tried it for higher values of N beyond the scope of what we were given and observed how it broke down. Whereas as we increased the value of N, the numerical solution using LG and LGL approximated the exact solution much better.



(a) Equi-spaced points



(b) LGL points



(c) LG points

Figure 5: Function interpolation using equi-spaced, LGL, and LG points for $N = 70$. Observe how the approximation iverges at the endpoints using equi-spaced points. However, LG and LGL points do a good job, and approximate the function well.

2 Derivative

We present the error norms of the derivative of the function using Lagrange basis polynomials.

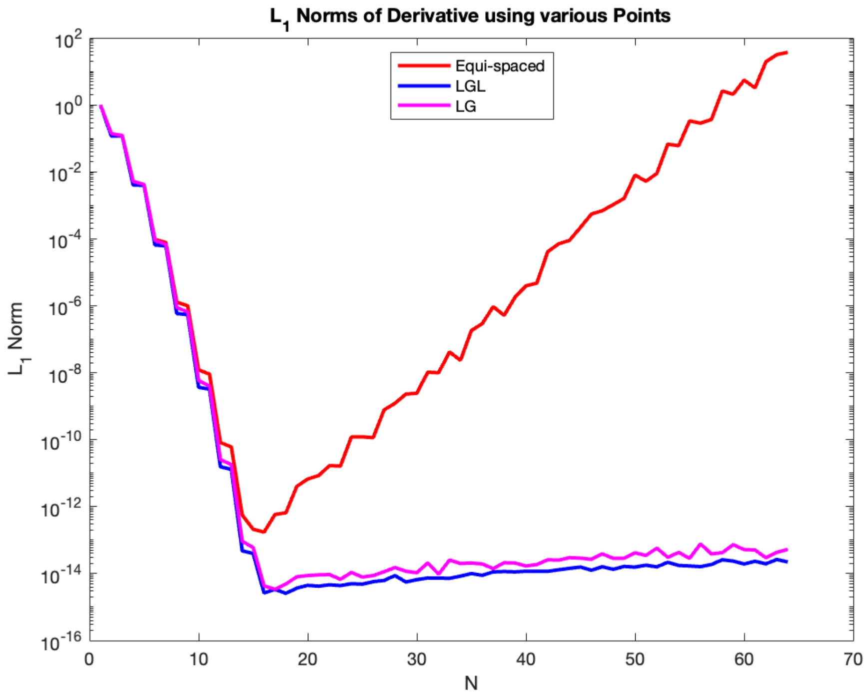


Figure 6: We observe that for lower orders of N (between 0 and 16), the errors decrease for all the various points, and they stay together until N increases to 8. At that point, it becomes obvious that the error of the equi-spaced points is higher than those of the LG and LGL points. Increasing the order of N from 16, the derivative using equi-spaced points fails to approximate the exact solution well, as seen by the increasing errors. The errors for LGL and LG points stay very close to each other, however, it is clear that the error using LGL points is much smaller than that of LG, and it remains so until they both level out.

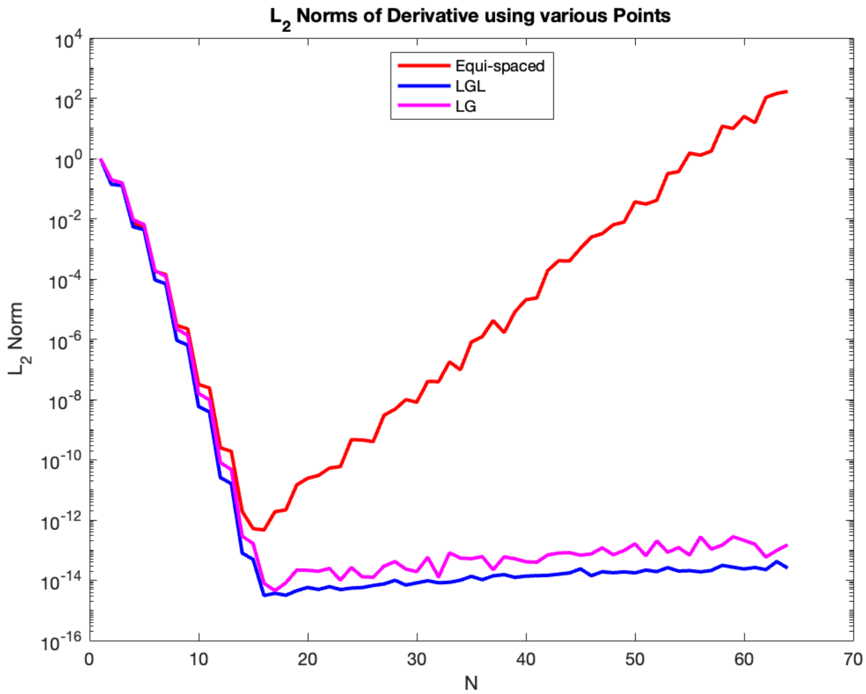


Figure 7: The result of the L_2 error norm is similar to that of the L_1 norm. However, we observe that the errors using LG points are larger here than in the L_1 norm. Once again, the error of the LGL points is the smallest.

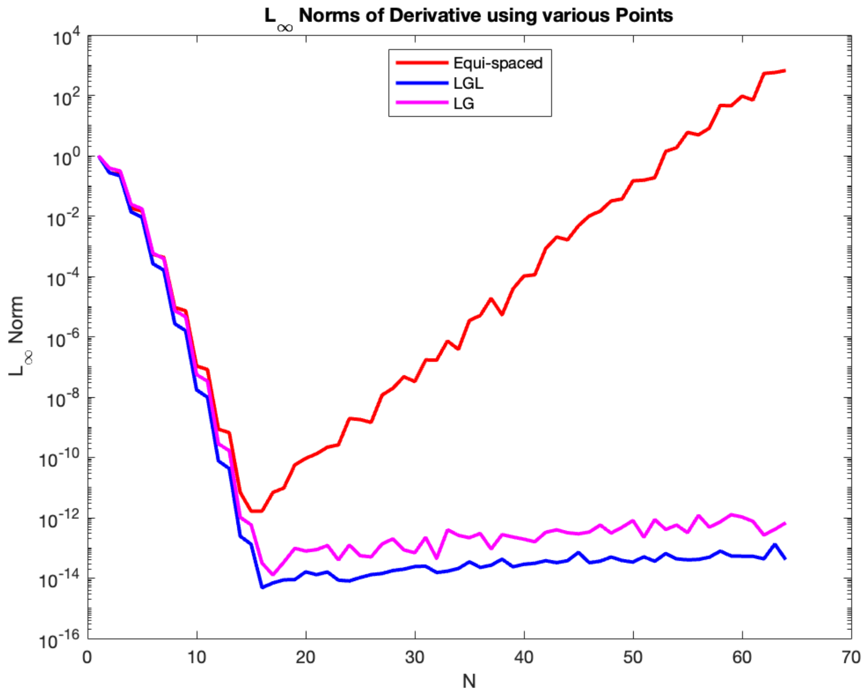


Figure 8: The L_∞ error norm consolidates the fact that the derivative of the function using LGL points outperformed the others as it error was distinctively lower than that of the others for $N = 1, 2, \cdots, 64$. Like in the L_2 norm above, the error of the equi-spaced points was higher than those of the other points.