Project 5 **Last name:** ______________________________

First name: ______________________________

Use the upwind scheme to approximate the solution of the hyperbolic equation

$$\frac{\partial u}{\partial t}(x,t) + (x+1)\frac{\partial u}{\partial x}(x,t) = 0, \tag{1}$$

where $x \in [0,1]$ and $t \in [0,1]$. Equation (1) is supplemented by the initial condition

$$u(x,0) = x+1, \quad x \in [0,1], \tag{2}$$

and the boundary condition

$$u(0,t) = \exp(-t), \quad t \in [0,1]. \tag{3}$$

Build your own Matlab files that will

- allow the user to choose any spatial step size Δx and any temporal step size h and
- print whether or not it is possible to solve the initial-boundary value problem (1)–(3) by the upwind scheme with the chosen step sizes.

Design your Matlab files to compute and graphically illustrate the numerical solution u_i^n and its error $|u_i^n - u(i\Delta x, nh)|$, where $u(x,t) = (x+1)e^{-t}$ is the exact solution of problem (1)–(3). Write supporting documentation describing each part of your files, what each part does, and how the parts work together.

Submit the following items

- a figure with two subplots:
  - subplot(2,1,1) illustrating the numerical solutions to $u(0.5,t)$ and $u(1,t)$ computed with $\Delta x = 0.1$ and $h = 0.01$,
  - subplot(2,1,2) illustrating the corresponding errors of the numerical solutions,
- a figure with two subplots:
  - subplot(2,1,1) illustrating the numerical solutions to $u(0.5,t)$ and $u(1,t)$ computed with $\Delta x = 0.01$ and $h = 0.001$,
  - subplot(2,1,2) illustrating the corresponding errors of the numerical solutions,
- a figure with two subplots:
  - subplot(2,1,1) illustrating the numerical solutions to $u(0.5,t)$ and $u(1,t)$ computed with $\Delta x = 0.001$ and $h = 0.0001$,
  - subplot(2,1,2) illustrating the corresponding errors of the numerical solutions,
- a table presenting the maximum error

  $$err = \max\{|u_i^n - u(i\Delta x, nh)| : 0 \le i \le M,\ 0 \le n \le N\},$$

  (where M and N are such that $M\Delta x = 1$ and $Nh = 1$) for the following step-sizes

	$\Delta x = 0.1,\ h = 0.01$	$\Delta x = 0.01,\ h = 0.001$	$\Delta x = 0.001,\ h = 0.0001$
err			

- all of your Matlab files needed to compute and graphically illustrate the approximate solutions and their errors,
- supporting documentation, described above.