SATELLITE ATTITUDE CONTROL SYSTEM
EE291 TECHNICAL REPORT

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ABSTRACT

This project presents a mathematical model for a satellite in Earth orbit, focusing on the control of its angular orientation to ensure it continuously faces the planet. The system is governed by a second-order differential equation representing the satellite's rotational dynamics and the required velocity adjustments to maintain orbital alignment.

The project aligns with the United Nations Sustainable Development Goal 13, Climate Action. The satellite is designed to generate heat maps and optical gas imagery to identify industrial regions with thermal emissions, enabling targeted interventions to reduce greenhouse gas output.

We reflect on the development of the simulation, the mathematical techniques implemented and the potential environmental impact of the model. Challenges encountered during the project were addressed through peer collaboration, academic resources and facilitator guidance.

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1 INTRODUCTION

Artificial satellites play a critical role in our contemporary society, supporting systems for global communication, navigation, weather forecasting and earth observation. These satellites emulate the motion of natural satellites which orbit celestial bodies, such as the moon rotating Earth, due to gravitational forces. [1].

This report provides a mathematical, theoretical and experimental overview on artificial satellite dynamics, with a focus on a model designed to monitor thermal and gas emissions from earth. Understanding and controlling the orientation of such satellites is essential for the collecting of accurate environmental data and to support efforts related to climate action.

This report also fulfils the learning outcomes of the EE291 module, System Dynamics and Control PBL Project. The required technical elements are:

- Proportional, Integral, Derivative (PID) Control of dynamic systems.
- State space representation / Modelling of dynamic systems.

Both the theory and application of the two topics are covered.

1.1 Report Layout

The remainder of this report is structured as follows:

- Section 2 outlines research relevant to this project.
- Section 3 details the chosen approach including a theoretical analysis.
- Section 4 covers experimentation and analysis of results.
- Section 5 explores the ethical considerations of this project.
- Section 6 highlights the main conclusions of this project.
- Section 7 provides references to sources of information used throughout this report.

2 LITERATURE REVIEW

The first step to any good project is to research. Research provides a better understanding of a problem and a firm foundation to formulate ideas from. We carried out research in areas we did not have much experience with, including the concept of state space and the simulation of a satellite model, among other topics.

2.1 State Space

State space representation is the mathematical model of a physical system that uses state variables to describe differential equations. The state space model is made up of a set of 1st order ordinary differential equations (ODEs). A state space model can easily describe a system with multiple inputs and outputs, in a matrix form. The state of a dynamic system can be monitored in this form and related to the output. In contrast, a transfer function cannot show internal behaviour [2].

2.2 MATLAB / Simulink

MATLAB and Simulink are both essential to control systems. They provide an environment to work with data, simulate models and provide a range of useful computational techniques. MathWorks, the maker of MATLAB and Simulink describe both as follows:

"MATLAB is a programming and numeric computing platform for engineering and scientific applications like data analysis, signal and image processing, control systems, wireless communications and robotics." [3]

"Simulink is a block diagram environment used to design system with multidomain models, simulate before moving to hardware and deploy without writing code." [4]

2.3 Satellite Attitude Control

The paper 'Satellite Attitude Control System' [5] talks about the design and implementation of discrete-time control loops of a satellite's orientation in space, focusing on PID controllers in a closed-loop system. This article has deepened our understanding of control loops, highlighting concepts like sensor and actuator integration, and discrete-time sampling, which are key aspects of control system. The paper mentions different sensors that can be used in the satellite system. Internally, a gyroscope can be used, while externally, a star sensor can be used.

Thrusters were chosen as the system's actuator as they are "generally the most common solution". In 'Feedback Control of Dynamic Systems' cold-gas thrusters are also chosen as the satellite system's actuator [2]. We learned that thrusters react quickly and accurate enough.

2.4 Ethical Considerations

Although we are not creating a physical satellite build, we still find it important to consider the risks of space debris, especially from satellites and find out if there are active efforts being taken to combat space pollution.

A report published by the European Space Agency (ESA) in 2025 discusses the state of the space environment [6]. It acknowledges that while work is being done in space involving studying the climate, global communication and providing navigation services, the space environment is still impacted by human involvement. It is specifically noted that "satellites that remain in their operational orbit at the end of their mission are at risk of fragmenting into dangerous clouds of debris that linger in orbit for many years". The ESA claims they are working on technology to prevent space debris, including the design of "zero-debris satellites".

We have seen that satellites can be used to study Earth's changing climate as well as contribute to pollution in space. The ESA's "zero-debris satellites" would allow for a better understanding of the global climate while not being a contributing factor itself. The sentiment of harmless innovation is reflected in the call to action in an article by The Global Goals [7]. "Through education, innovation and adherence to our climate commitments, we can make the necessary changes to protect the planet". We are urged to "take urgent action to combat climate change and its impacts". The title of this article 'Goal 13: Climate action', is the Sustainable Development Goal (SDG) we will be exploring in-depth later in this report.

3 PROPOSED APPROACH

After completing the literature review, a detailed study of the attitude control system can now be carried out. To provide a broader understanding of attitude control, this analysis will cover multiple representations of the same problem, as each representation has its own pros and cons. The design of a controller for the system is also included.

3.1 Free-Body Diagram & Equation of Motion

Attitude control provides a way to control the orientation of an object, keeping it pointed to a specific location. To effectively control the attitude of a satellite, the satellite system on its own must first be analysed. The movement of a satellite is based on rotational motion. Rotational motion is the circular movement of a body about an axis. For this report, the satellite is simplified to a rectangular mass to avoid overcomplication. To examine the forces acting on the mass, a free-body diagram (FBD) can be created. An ordinary differential equation (ODE) describing these forces can then be derived from the FBD.

In Figure 1, the control force F_c [8] (from the system's actuators, e.g., thrusters), creates a moment with the distance d about the centre of the mass. Other small disturbances (atmospheric drag, gravitational perturbation, electromagnetic drag, etc.) are expressed by the moment M_D. The summation of these moments gives an output for the system. This is shown in Equation 1. The total input torque is equal to the sum of the external forces and the control force.

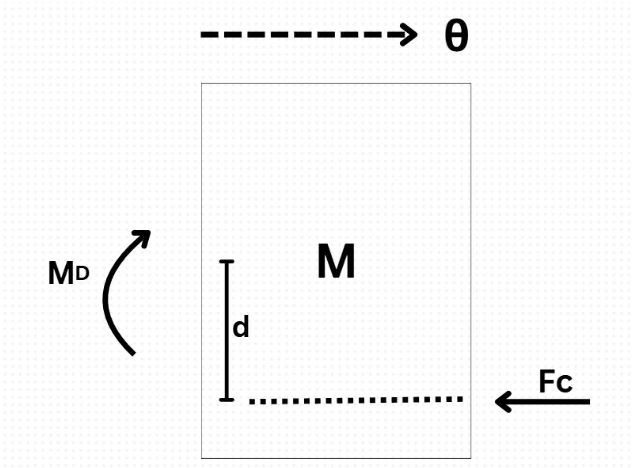

Figure 1 – Satellite FBD

$$I\ddot{\theta} = M_D + M_C \tag{1}$$

Equation 1 – Equation of Motion

3.2 Transfer Function

Looking at an ODE gives a good understanding of the forces acting on a mass. However, it can be easier to work with a model in a different form. A transfer function is an algebraic expression used to understand the relationship between the input and output of a system [7]. Laplace Transforms are used to convert from a continuous-time differential equation to a transfer function. Converting equation 1 to the Laplace domain gives the following open-loop transfer function (OLTF) in equation 2, where Y(s) is the output, U(s) is the input, and I is inertia.

$$\frac{Y(s)}{U(s)} = \frac{1}{Is^2} \tag{2}$$

Equation 2 – Open-Loop Transfer Function

Transforming an open-loop transfer function to a closed-loop transfer function (CLTF) allows control to be added to the system. To achieve this mathematically, the feedback loop equation is used. Equation 3 shows the feedback loop equation, where G(s) is a transfer function and H(s) is the feedback transfer function.

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)} \tag{3}$$

Equation 3 – Feedback Loop Equation

The following shows the conversion of the attitude system transfer function from an OLTF to a CLTF using the feedback loop equation, with the result in equation 4. G(s) is the attitude system transfer function and H(s) = 1 for a unity feedback system.

$$\frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$\Rightarrow \frac{\frac{1}{Is^2}}{1 + \frac{1}{Is^2} \cdot 1}$$

$$\Rightarrow \frac{1}{Is^2 + 1}$$

$$\frac{1}{Is^2 + 1} \tag{4}$$

Equation 4 – Closed-Loop Transfer Function

Transfer functions are easier to work with compared to differential equations because of their algebraic expression form. Calculating the output of a system for a certain input is also easier in this form. Information about the damping and the natural frequency (frequency the system oscillates at with no damping) can be obtained from a transfer function [10].

Equation 4 can be compared to the standard 2nd order closed-loop transfer function shown in equation 5, where ω_n is the natural frequency and ζ is the damping ratio.

$$\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5}$$

Equation 5 – Standard 2nd Order Closed-Loop Transfer Function

By comparing the equations, it is found that $\omega_n = \sqrt{1}$ and $\zeta = 0$, meaning the system does not naturally decrease over time.

3.3 Poles & Zeros Diagram

Figure 2 shows the poles and zeros diagram of the OLTF. The numerator of the transfer function determines the zeros, while the denominator determines the poles. The poles (roots of the transfer function) determine the stability of the system. There are no zeros in the diagram as the attitude transfer function does not contain the s variable in the numerator. The following workings show the poles of the system. As the system is a 2nd order system, there will be two poles, s_1 and s_2.

$$Is^2 = 0$$

$$\Rightarrow s^2 = 0$$

$$\Rightarrow s_1 = 0, s_2 = 0$$

Plotting both poles in the S-plane gives the following diagram. Both poles lie on the origin, therefore proving the system is marginal stability.

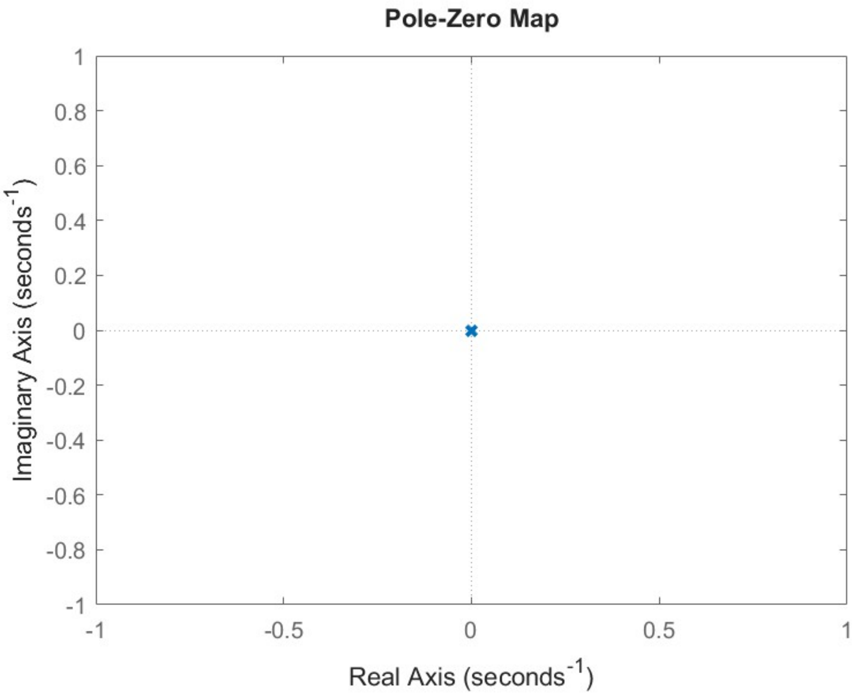

Figure 2 – Poles & Zero Diagram

3.4 Block Diagram

For this project, we used a block diagram to guide us in understanding and designing the control system. The block diagram typically includes components such as a controller, process, actuator and a sensor. Instructions are sent from the controller to be carried out by the actuator. This causes the process to be adjusted to match the desired output.

- Controller – Determines how to respond to the error (difference between the desired and actual output).
- Process/ Plant – The system you are trying to control (motor, pendulum etc.).
- Actuator – Applies the controllers signal to the process/plant.
- Measurement/Sensor – Measures the systems output.
- Desired Output – The target output you would like the system to achieve.
- Error – Difference between the desired output and the actual output.

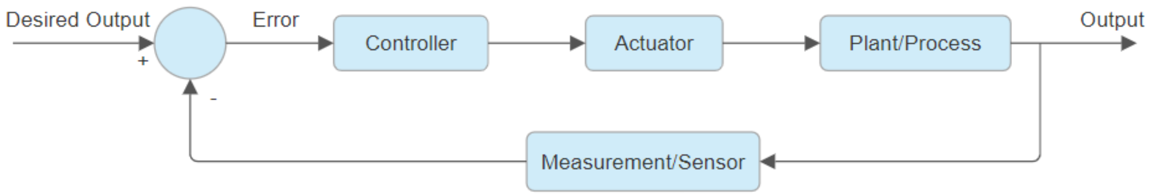

Figure 3 – General Block Diagram

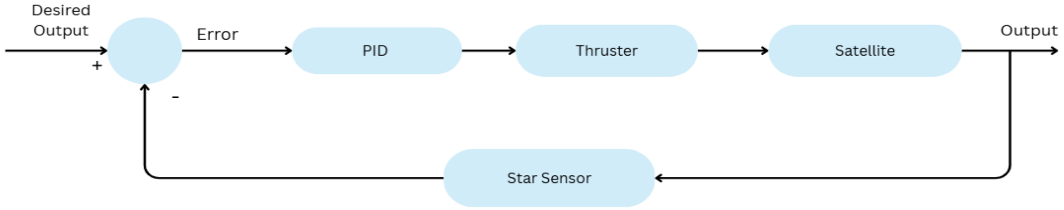

Figure 4 – Attitude Control Block Diagram

In the case of our project-based block diagram we have PID acting as the controller, the motor acting as the actuator and the satellite acting as the plant/process in this system.

- PID - Takes the error and decides how to correct it depending on the desired output.
- Motor - Applies the control signal from the PID to adjust the satellite's turning motion.
- Satellite – The rotational motion of the satellite, it turns in accordance with the control system.
- Measurement/Sensor – Monitors the current angle of the satellite.

3.5 PID Controller

PID stands for Proportional, Integral and Derivative. It works by adjusting a manipulated variable to bring a process variable to a desired set point.

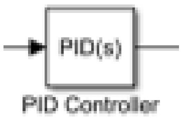

Figure 5 – PID Control Block

- The Proportional responds to the current error by generating a control action proportional to the error magnitude. It provides immediate corrective action to reduce the error. A higher proportional value results in a stronger and quicker response but may lead to overshoot and oscillations.
- The Integral actuates the past error over time and generates a control action to eliminate the accumulated steady-state error. It ensures that even small errors are eventually corrected. It eliminates offset but can lead to slow responses and overshooting if too aggressive.
- The Derivative predicts the future error trend by the rate of change of the error. It adds a damping effect, reducing oscillation and overshooting caused by rapid changes in error. [11]

The PID controller consists of the sum of three parts, namely Proportional, Integral and Derivative as follows [12]:

$$u(t) = K_p e(t) + K_i \int e(t) + K_d \frac{de(t)}{dt} \tag{6}$$

Equation 6 – PID equation

4 IMPLEMENTATION & RESULTS

Simulating the attitude control model in MATLAB/Simulink will present further insights. The system will first be simulated in transfer function form, then state space form. A comparison between the two models can then be made. A controller will be designed based on the transfer function model.

4.1 Open-Loop Transfer Function

Below is a block diagram of the open-loop transfer function (OLTF) created in Simulink. The system is implemented with a step input block, a gain block, two integrator blocks, a scope and a workspace. There is no controller in the system, so the output is the system's natural response.

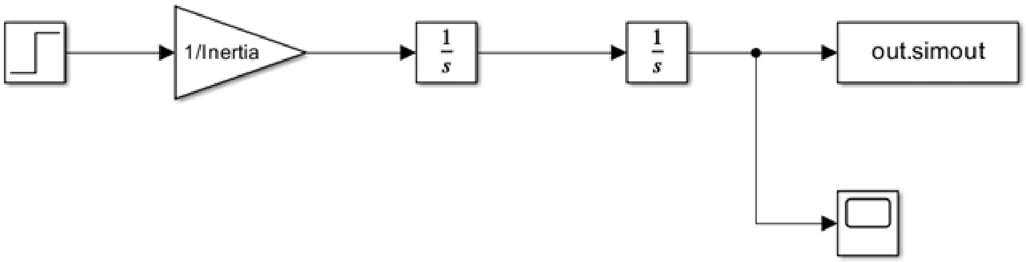

Figure 6 – Simulink Model of OLTF

The model is tested with a unit step input (initial value = 0, final value = 1) and a step time of 1 second. Inertia equals 1 in all simulations.

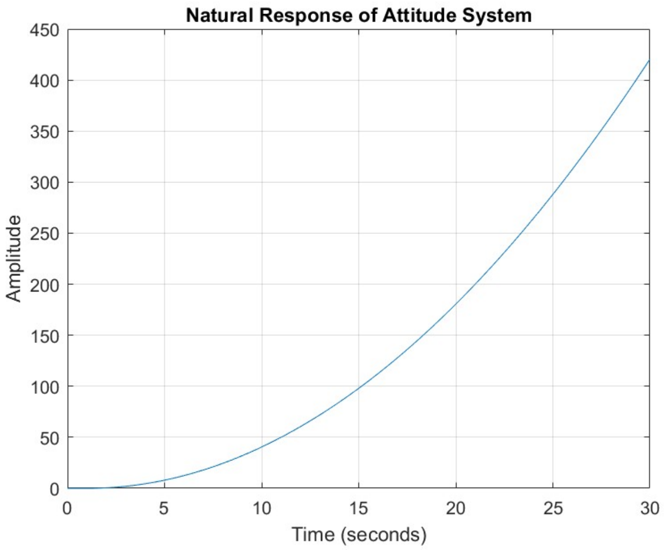

Figure 7 – Natural Response of Attitude System

The above graph shows that the system is unstable. The output is continuously increasing and does not reach a steady state within the simulation time of 30 seconds. This represents the satellite continuously changing the angle without stopping.

4.2 Closed-Loop Transfer Function

Implementing the system with PID control or in a closed-loop transfer function (CLTF) form is more practical than depending on the system's unforced response. A controller can be designed that manipulates the system to reach a desired output at a steady state. There are many methods used to design a suitable PID controller, including Ziegler-Nichols, Root-locus and Frequency domain [13]. Testing the system with various values of P, I and D shows the impact of each parameter on the system.

The block diagram of the system in CLTF form includes the same blocks as in OLTF form with the addition of a feedback loop, a summation block and a PID control block.

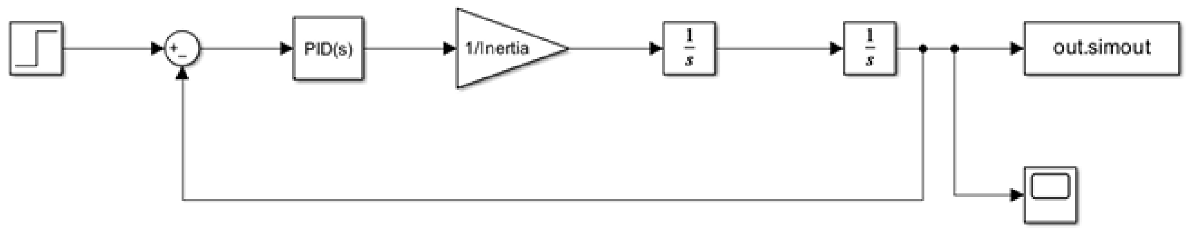

Figure 8 – Simulink Model of CLTF

4.3 Testing P Control

The graphs below demonstrate the change in the output as the value of P increases from 1, to 5, to 10.

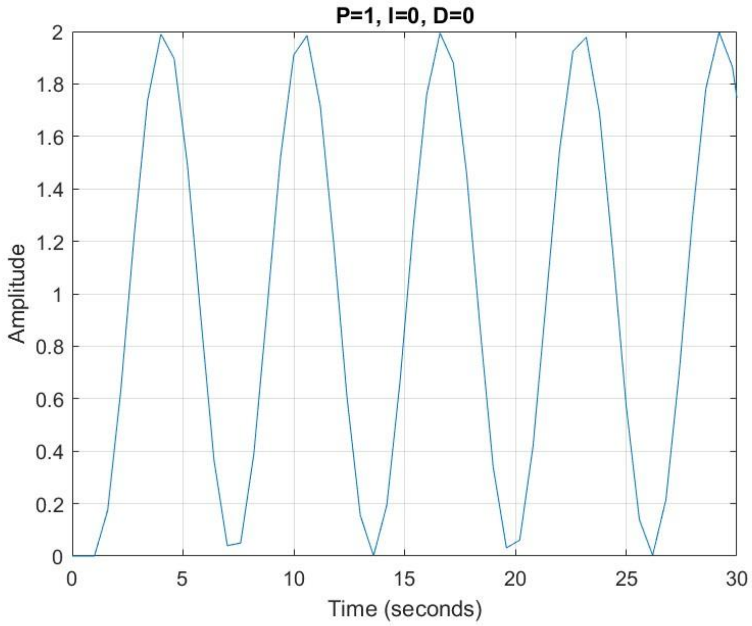

Figure 9 – Plot with P Control (P=1)

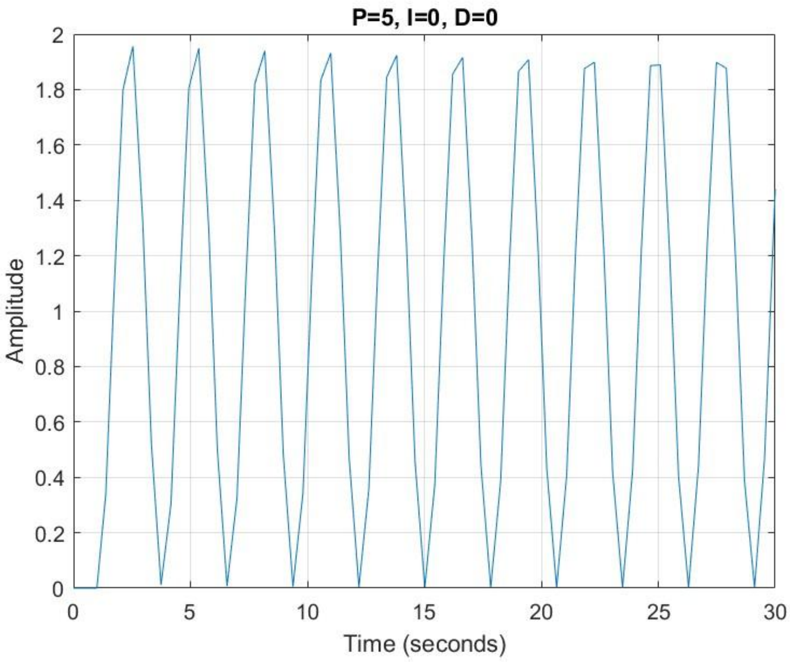

Figure 10 – Plot with P Control (P=5)

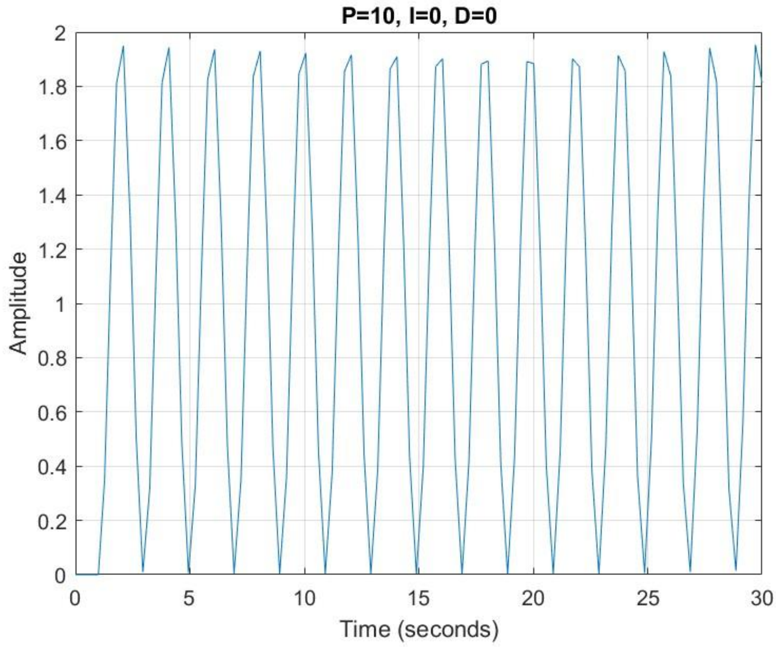

Figure 11 – Plot with P Control (P=10)

Increasing P control increases how quickly the system responds and reaches the final value of 1. This can also cause the system to overshoot past the final value as seen in the above plots. It also increases the number of oscillations within the simulation time. The peak of each oscillation in every plot is about the same, around 2. Without damping, the system will continue to oscillate endlessly. It is marginally stable and will not reach a steady state, as we can prove by finding the poles of the block equation.

4.4 Testing PI Control

The graphs below demonstrate the change in the output as the value of I increases from 1, to 5, to 10. P control is set to 1 for all graphs.

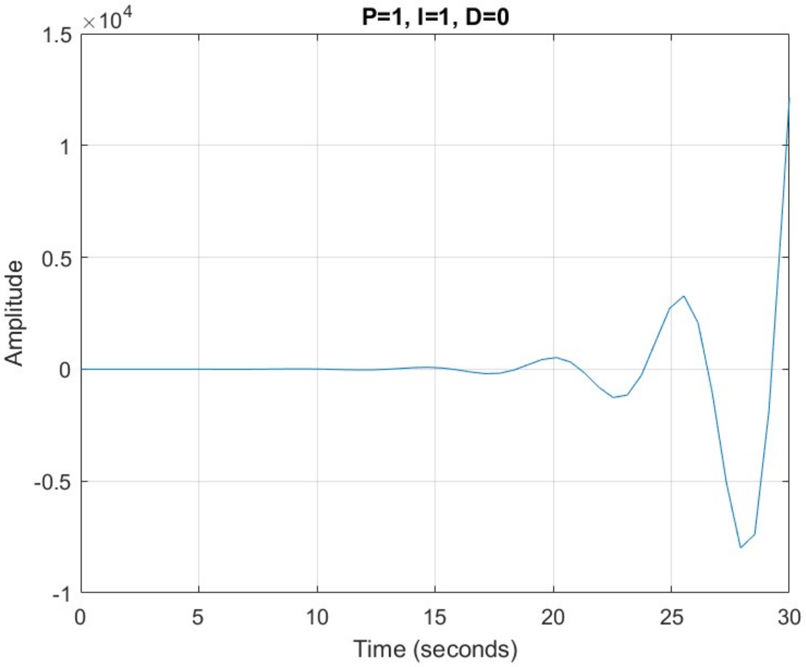

Figure 12 – Plot with PI Control (P=1, I=1)

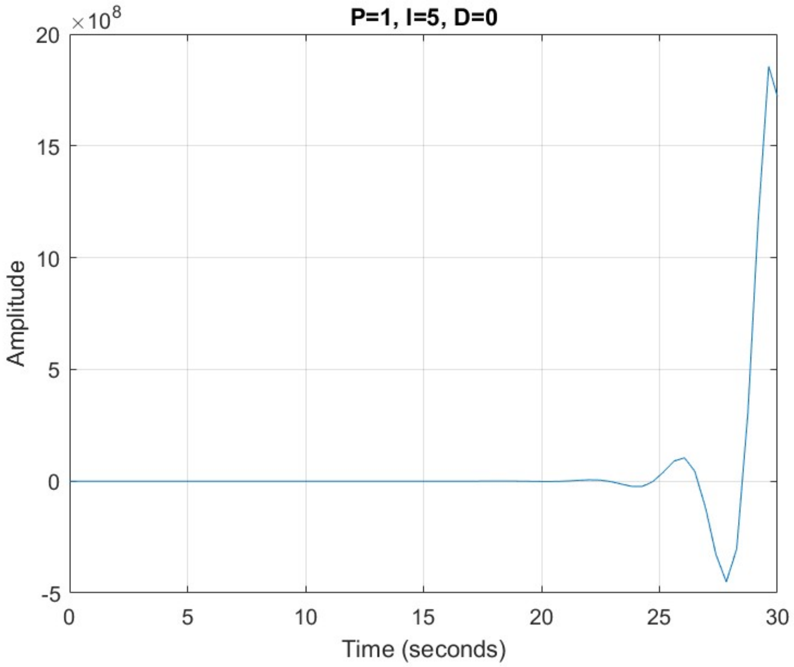

Figure 13 – Plot with PI Control (P=1, I=5)

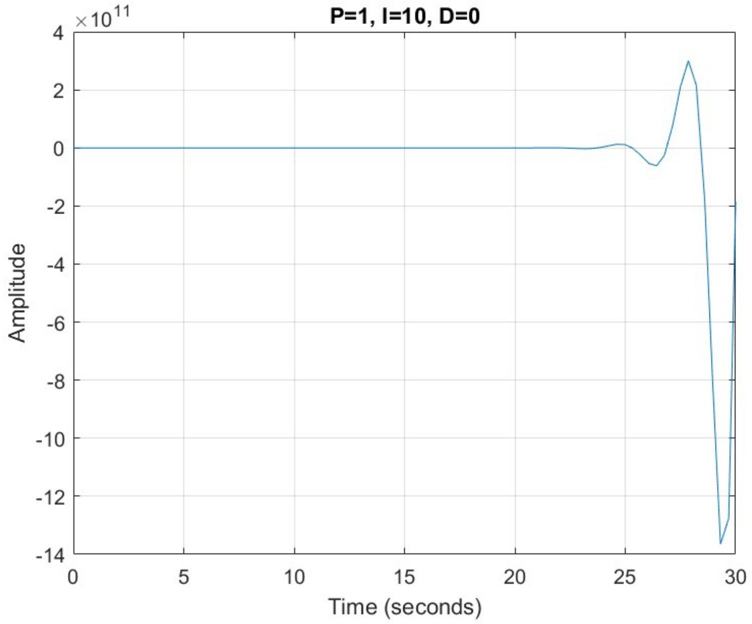

Figure 14 – Plot with PI Control (P=1, I=10)

In the above graphs, introducing I control delays the reaction of the system. As I increases, the delay also increases. The system does not stabilise at the final value of 1 in any of the graphs. It can be concluded from these graphs that I control is not needed to stabilise the system.

4.5 Testing PD Control

The graphs below demonstrate the change in the output as the value of D increases from 1, to 5, to 10. In figure 15, P is set to 0 so there are no oscillations. In this case D control makes the system critically damped, meaning the system will reach the final value without oscillations. The figures after figure 15 include P control, with P set to 1.

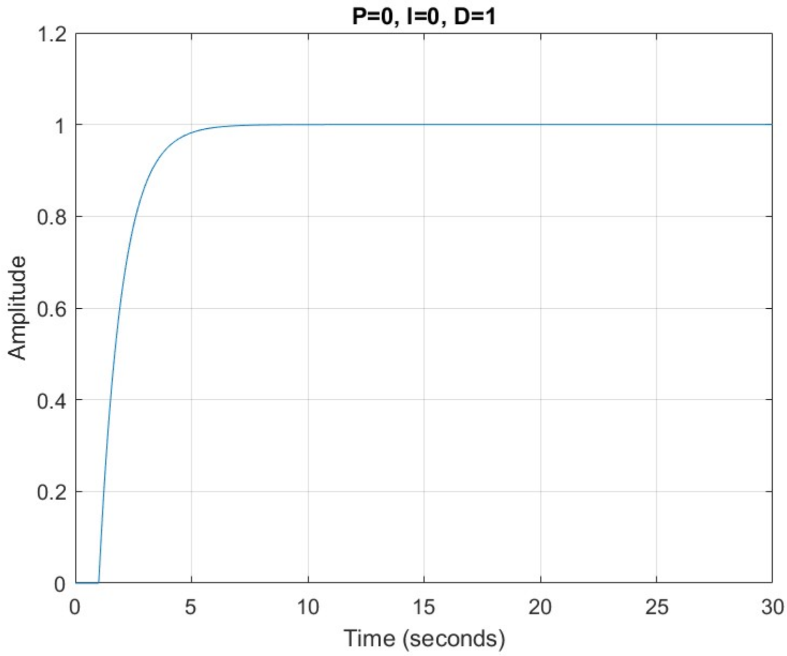

Figure 15 - Plot with D Control (D=1)

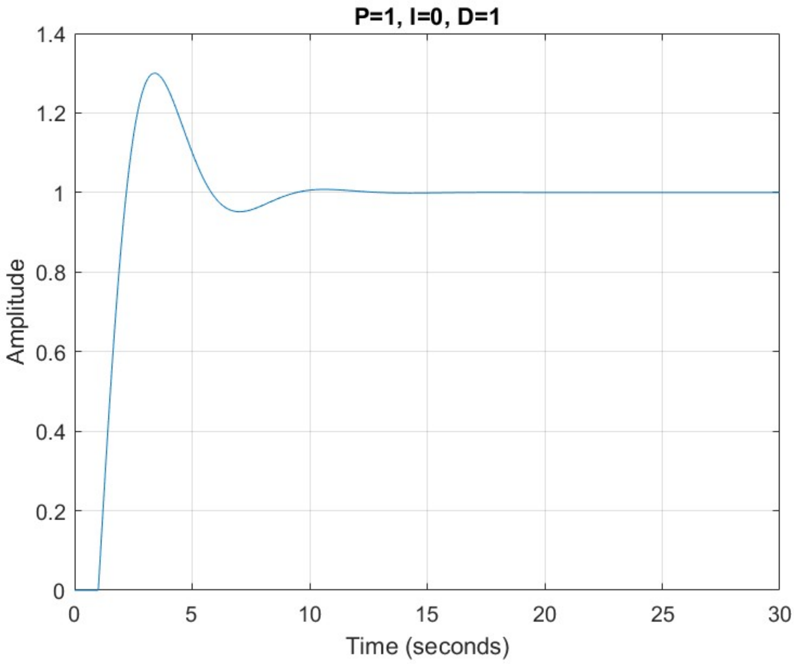

Figure 16 - Plot with PD Control (P=1, D=1)

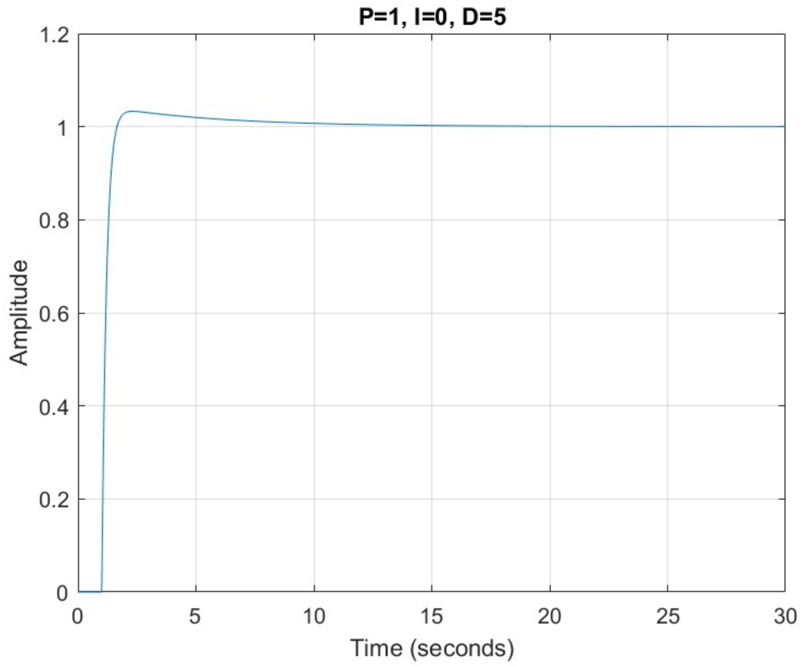

Figure 17 - Plot with PD Control (P=1, D=5)

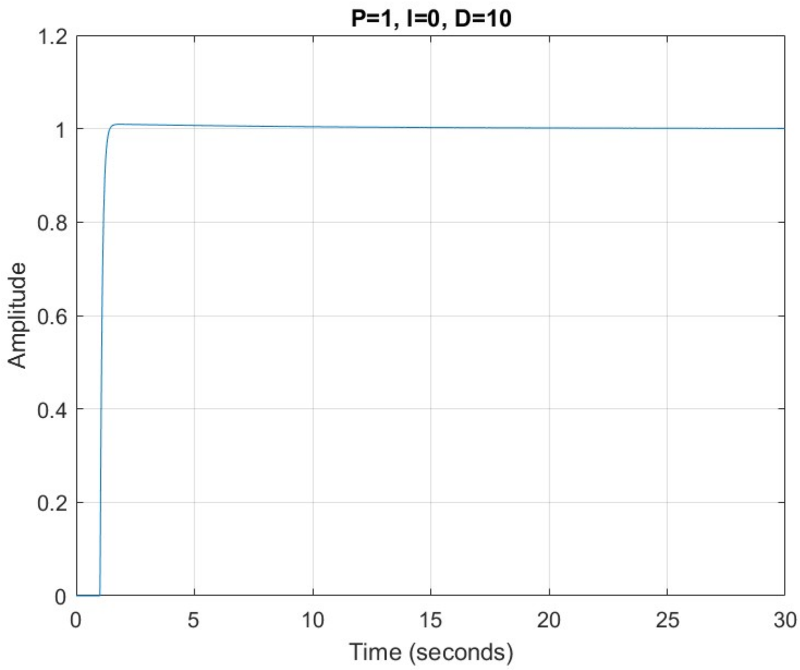

Figure 18 - Plot with PD Control (P=1, D=10)

Increasing D control increases how quickly the system reaches the final value. It also decreases the oscillations in the transient state of the output (before the system reaches steady state).

By testing the CLTF with different PID values in Simulink, it is clear that a form of PD control is the most effective controller. In figures 17 and 18, the system reaches steady state in under 5 seconds, with minimal overshoot. This mirrors the expected behaviour of a unit step input. P control is needed for the system to respond quickly, which makes the system more efficient. D control regulates the oscillatory response of the system and reduces the settling time.

4.6 State Space

State space is another form of mathematical representation. While transfer function can handle only a single input and a single output at a time, state space can handle multiple outputs with 1 input. The following workings show the conversion from transfer function form to state space form, where:

$$\begin{aligned}&\dot{\theta} = Angular\ velocity\\&\ddot{\theta} = \alpha = Angular\ acceleration\\&I = Moment\ of\ Inertia\ about\ the\ axis\ of\ rotation\\&\beta\ =\ Damping\ coefficient\\&u(t) = Input\ torque\end{aligned}$$

There is no damping in the system, however for illustration purposes, the damping coefficient β is included but is set to 0. This changes Equation 2 to Equation 7 shown below.

$$\frac{\mathrm{Y}(s)}{U(s)} = \frac{1}{Is^2 + \beta s} \tag{7}$$

Equation 7 – Transfer Function with Damping Coefficient

Since this is a second order system, two state variables are needed x_1 and x_2.

$$x_1 \ = \ \theta \text{ (Attitude angle)}$$
$$x_2 \ = \dot{\theta}\text{(Angular velocity)}$$

The derivatives of the variables are shown below.

$$\begin{aligned}\dot{x}_1 &= \dot{\theta} = x_2\\ \dot{x}_2 &= \ddot{\theta} = \frac{-\beta}{I}x_2 \ + \ \frac{1}{I}u(t)\\ \Rightarrow \dot{x}_2 &= \frac{1}{I}u(t)\end{aligned}$$

The expanded form of the state equation is shown below, along with the output equation.

$$x_1' = 0x_1 + 1x_2 + 0u(t)$$
$$x_2' = 0x_1 - 0x_2 + \frac{1}{I}u(t)$$
$$y(t) = 0x_1 + 1x_2 + 0u(t)$$

The matrix form of the state equation in the form $\dot{x} = Ax + Bu$ and the output equation in the form $y = Cx + Du$ is shown below.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix} u(t) \qquad (8)$$

Equation 8 – Attitude Control State Equation

$$y(t) = [0 \quad 1] + [0] \qquad (9)$$

Equation 9 – Attitude Control Output Equation

The state space model can now be tested in MATLAB/Simulink. Figure 19 shows the implementation of the system in Simulink using a unit step block, a state space block and a scope.

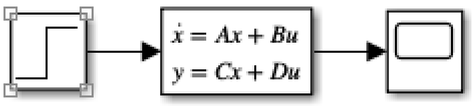

Figure 19 – Simulink Model of State Space

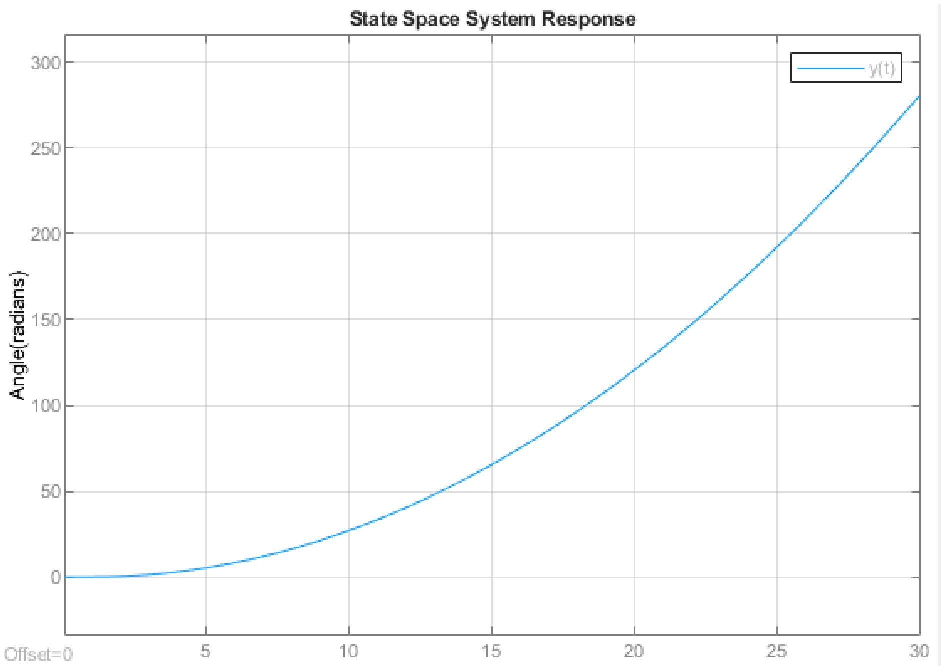

Figure 20 – Natural Response of State Space Model

The natural response of the state space model is identical to the natural response of the transfer function model. This proves the transfer function and state space models were implemented correctly. Figure 20 shows an unstable system with an output that increases forever. The system does not settle at any point. State feedback can be used to make this system stable and reach steady state.

4.7 State Feedback

State feedback is a method of controlling a system modelled with a state space model. It controls the system by monitoring state variables to modify the input [14]. Changing the A matrix in the state equation will change the dynamics of a system. Properties like system stability and the transient response can be changed as desired.

To carry out state feedback, it must first be confirmed that the system is controllable. The workings below prove that the attitude system is controllable as the determinant of C_r, the controllability matrix is not zero.

$$C_r = [B\ AB] = \begin{bmatrix} 0 & \frac{1}{I} \\ \frac{1}{I} & 0 \end{bmatrix}$$

$$\det(C_r) = -\frac{1}{I^2} \neq 0$$

A controller can now be designed for the system with the gain matrix, K. Control law states that the input $u(t)$ of the system is now:

$$u(t) = -Kx$$

$$= -[k_1\,, k_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= -k_1 x_1, -k_2 x_2$$

Substituting the input change into the state equation gives the following:

$$\dot{x} = Ax + B(-Kx)$$

$$\dot{x} = Ax - BKx$$

$$\dot{x} = (A - BK)x$$

Substituting the A and B matrices from the attitude control state equation gives the following:

$$A - BK = \begin{bmatrix} 0 & 1 \\ -\dfrac{k1}{I} & -\dfrac{k2}{I} \end{bmatrix}$$

To get the characteristic equation, the determinant of the above matrix is calculated. Suitable pole placements can then be chosen.

$$\det\big(sI - (A - BK)\big) = 0$$

$$\det\big(sI - (A - BK)\big) = s^2 + \frac{k_2}{I}s + \frac{k_1}{I}$$

$$s^2 + \frac{k_2}{I}s + \frac{k_1}{I} = 0 \tag{10}$$

Equation 10 – Characteristic Equation

Comparing the coefficients in the characteristic equation to the coefficients in the denominator of the standard 2nd order closed-loop transfer function, $s^2 + 2\zeta\omega_n s + \omega_n^2$, will give equations for k_1 and k_2.

$$\frac{k_2}{I} = 2\zeta\omega_n$$

$$\Rightarrow k_2 = (I)2\zeta\omega_n$$

$$\frac{k_1}{I} = \omega_n^2$$

$$\Rightarrow k_1 = (I)\omega_n^2$$

To calculate the poles for a critically damped system, $I = 1$, $\zeta = 1$ and $\omega_n = 2\,rad/s$, we get the following values for the gains.

$$k_1 = 1 \cdot 2^2 = 4 \qquad k_2 = 1 \cdot 2 \cdot 1 \cdot 2 = 4$$

Feedback gain: $K = [4\ \ 4]$

$$s = \sigma + j\omega$$

$$poles(-2, -2)$$

To calculate the poles for an underdamped system, $I = 1$, $\zeta = 0.5$ and $\omega_n = 2\,rad/s$, we get the following values for the gains.

Feedback gain: $K = [4\ \ 2]$

$$poles(-1 \pm j1.732)$$

To calculate the poles for an undamped system, $I = 1$, $\zeta = 0$ and $\omega_n = 2\ rad/s$, we get the following values for the gains.

$$\text{Feedback gain: } K = [4\ \ 0]$$

$$poles(\pm j2)$$

The controllable canonical form (CCF) of the matrices for n=2 is:

$$A_c = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

A_0 a_1 coefficients open-loop characteristics:

$$\det(sI - A)$$

$$= \det\left(\begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}\right)$$

$$= s^2$$

$$= s^2 + 0s + 0 = 0$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\ alligns\ with\ A_c$$

$$B = \begin{bmatrix} 0 \\ 1/I \end{bmatrix}\ B_C \text{ scaled by 1/I}$$

$$k_1 = \alpha_0 - a_0, \quad k_2 = \alpha_1 - a_1,$$

$$K = [a_0\ \ a_1]$$

$$B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$B = B_c \frac{1}{I}$$

$$K = K_c(T)$$

The transformation of CCF is shown below:

$$T = I$$

$$u = -K_c B_c$$

$$(A - BK) = (A - B_c(factor)K)\ want\ to\ equal\ (A_c - K_c B_c)$$

$$K = \frac{K_c}{factor} \quad factor = \frac{1}{I} \quad , \ so\ K = \frac{K_c}{\frac{1}{I}} = \ K_c(I)$$

$$K = [a_0 \quad a_1](I) = \ [a_0\, I \ \ a_1 I]$$

Akerman's Formula simplifies the calculation of the gain matrix K, and is shown below, where C_n = the last row of the controllability matrix:

$$K = \ K_c T = \ C_n A_c(A)$$

$$C_n{}^{-1} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = 2$$

$$\Rightarrow C_2 = [I \quad 0]$$

$$Desired\ polynomial \ \ a_c(S) = \ S^2 + a_1 S + a_0$$

$$Evaluate\ \ a_c(SA) = \ A^2 + a_1 A + a_0$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} zero\ matrix$$

$$a_c(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \ a_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \ a_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a_0 & a_1 \\ 0 & a_0 \end{bmatrix}$$

$$K = \ C_n a_c(A) = [\, I \ \ 0] \begin{bmatrix} a_0 & a_1 \\ 0 & a_0 \end{bmatrix} = \ [\, I a_0 \ \ I a_1]$$

Conclusion: $K = \ [\, I a_0 \ \ I a_1]$

$$\theta = x_1$$

Including reference input is used to track non zero reference $r(t)$ instead or to regulating zero reference The state command matrix N_x which defines the desired value of the state X_{ss} so that the system output $y(t)$ (determined by C) is at the desired reference value. The feedforward matrix N_u which provides a mechanism for faster transient response and/or elimination of steady-state error.

$$\dot{x} = 0 \Rightarrow Ax_{ss} + Bu_{ss} \qquad\qquad y_{ss} = Cx_{ss} + Du_{ss}$$

In order to ensure $y_{ss} = r_{ss}$:

$$x_{ss} = N_x r_{ss} \text{ and } u_{ss} = N_u r_{ss}$$

$$0 = AN_x + BN_u \text{ and } 1 = CN_x + DN_u$$

$$\begin{bmatrix} Nx \\ Nu \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/I \\ 1 & 0 & 0 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -I \\ 1 \end{bmatrix}$$

$$Nx = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad Nu = 0$$

$$Xss = Nxr = \begin{bmatrix} 1 \\ 0 \end{bmatrix} r = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

This sets steady state angle, x_1 at reference r with zero angular velocity, x_2.

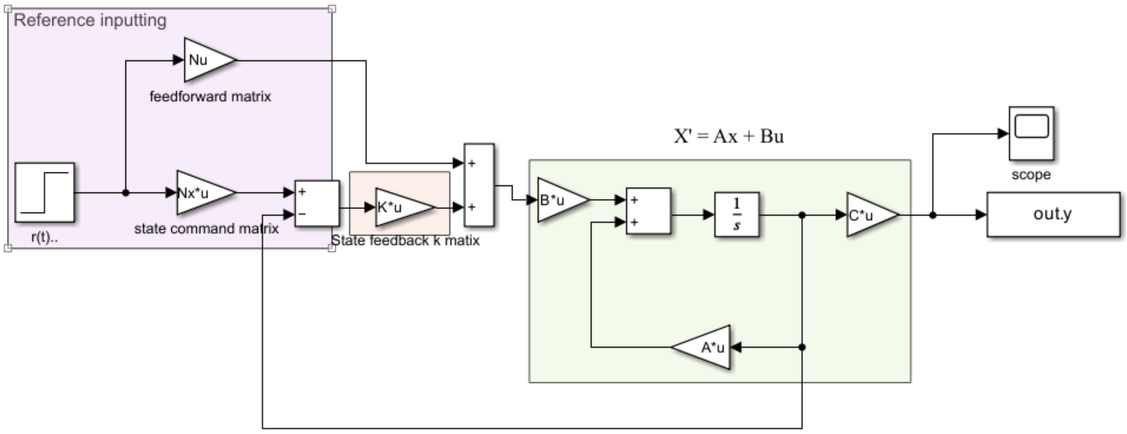

Figure 21– State Feedback System with Reference Input

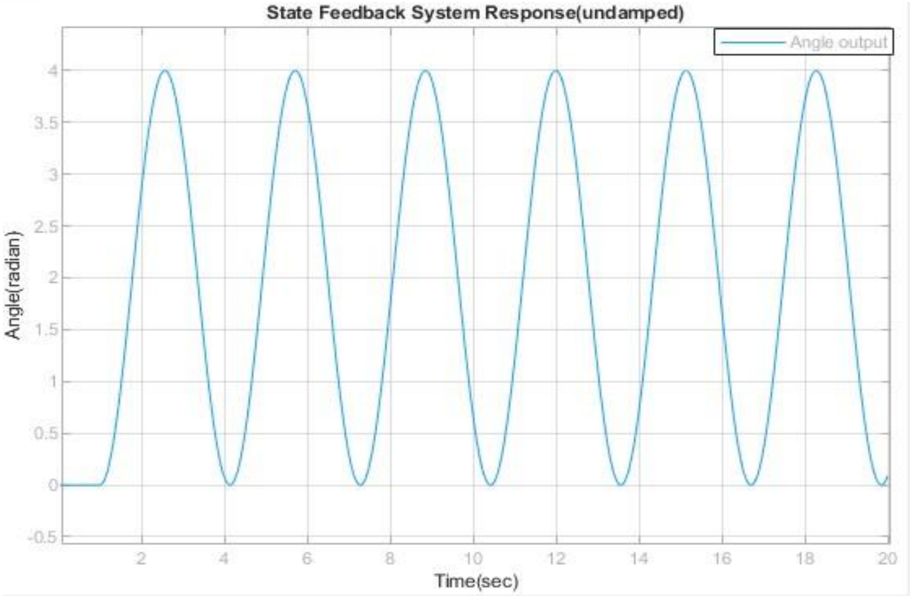

Figure 22– Marginally Stable Angular Response

Figure 22 illustrates the angular response of the system when configured with the state feedback gain of K = [4 0]. The graph clearly shows the angle oscillating around a mean value of 2 radians, with a peak amplitude of 4 radians, causing the angle to swing between 0 and 4 radians. This behaviour is in response to a step input of 0-2 radians. These oscillations maintain a constant amplitude throughout the 20 second simulation period, indicating the satellite is continuously swinging back and forth without the motion decreasing. The period of oscillation is consistently around 3 seconds, which aligns with the natural frequency of 2 rad/s. This behaviour is characterised as marginally stable system where the closed-loop poles lie directly on the imaginary axis of the s-plane, resulting in sustained oscillations and an inability for the system to settle at the specific reference angle.

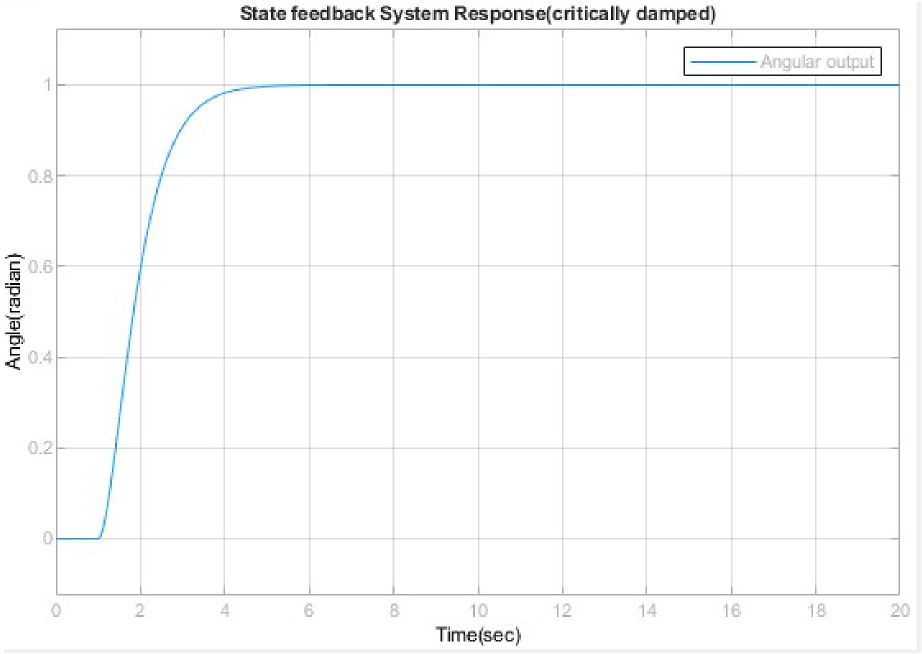

Figure 23 – Stable Angular Response (Critically Damped)

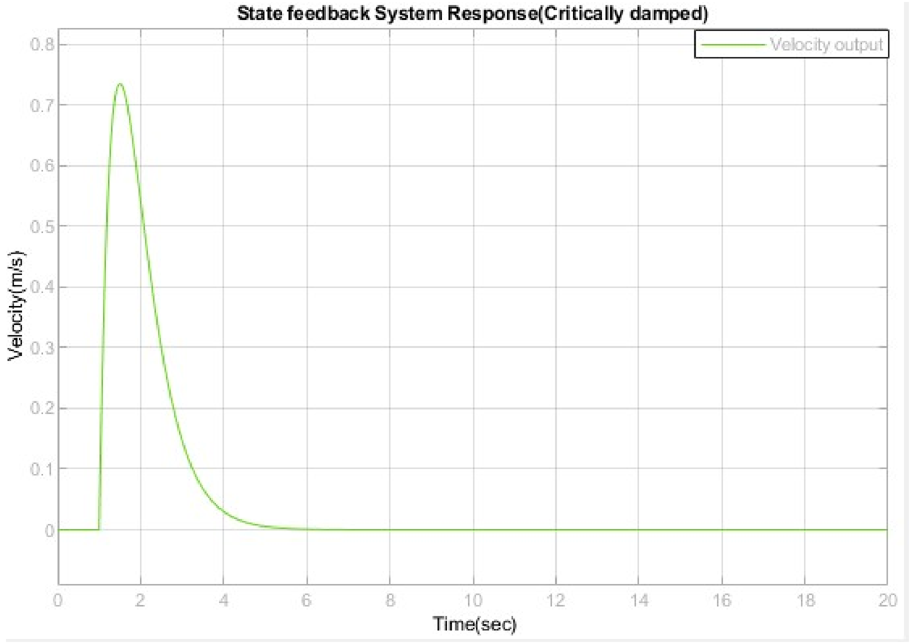

Figure 24– Velocity Response of Critically Damped System

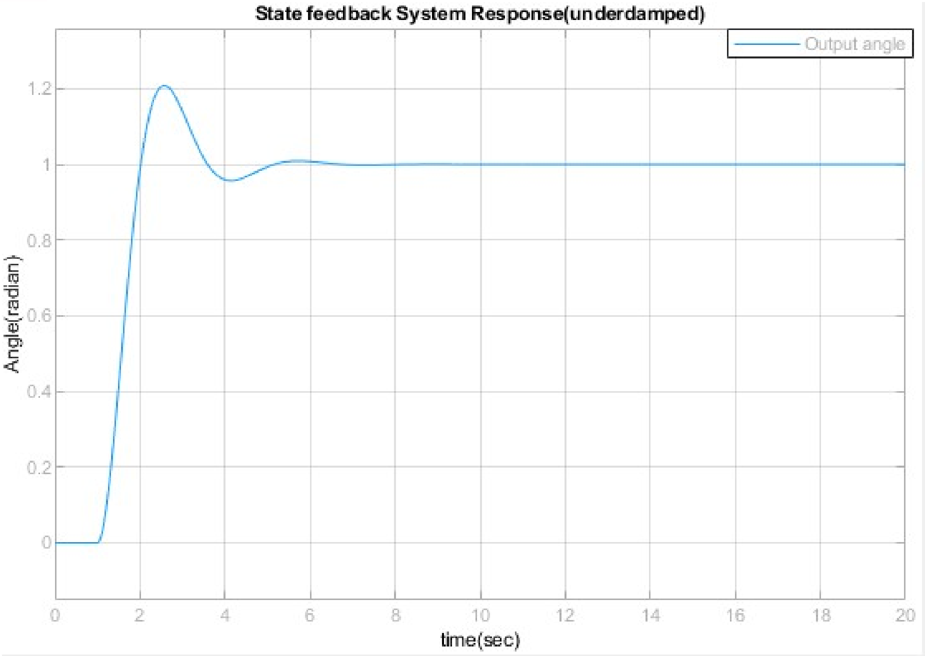

Figure 25–Stable Angular Response (Underdamped)

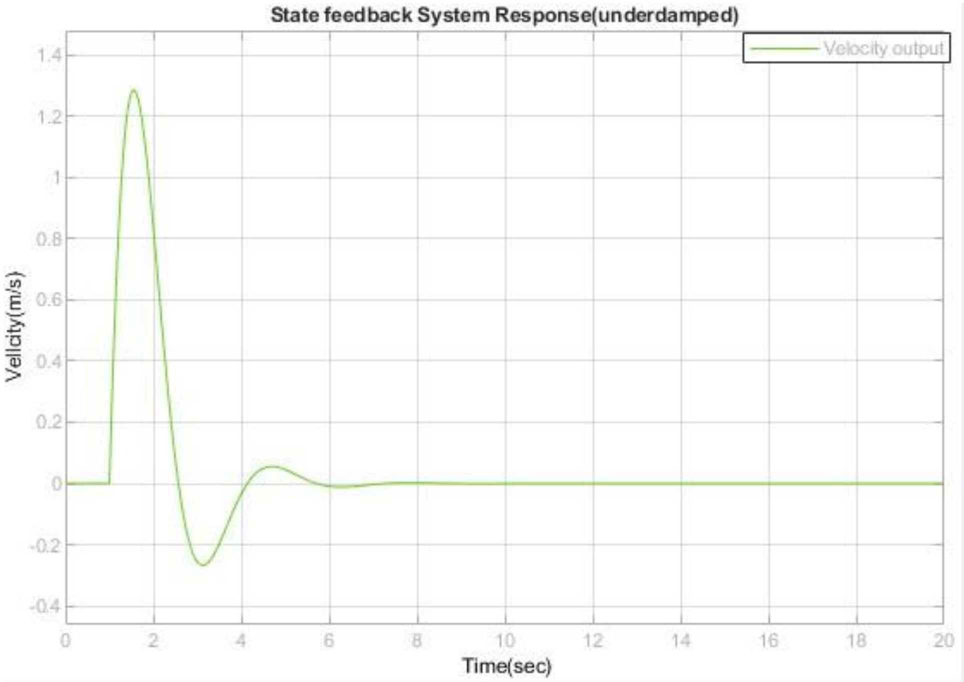

Figure 26– Velocity Response of Underdamped System

Figures 23 and 25 compare the satellite's angular response under critically damped and underdamped control, both targeting a 1 radian reference angle. The critically damped system (figure 23) smoothly reaches the reference in 4-5 seconds without overshoot. In contrast, the underdamped system (figure 25) exhibits a faster initial rise but overshoots around 2 radians, followed by decaying oscillations before settling around 8-10 seconds. While underdamping offers quicker initial approach, critical damping ensures precise, non-oscillatory response critical for satellite attitude. The corresponding velocity response (figure 24 and figure 26) is obtained by changing C matrix to [0 1]. The critically damped velocity smoothly peaks and returns to zero while the underdamped velocity shows a higher peak contributing to overshoot and subsequential oscillations before settling. This oscillating velocity implies more frequent thruster use to correct the overshoots and undershoots, potentially leading to higher fuel consumption and reduced operational efficiency compared to the single well managed acceleration and deceleration phase of the critically damped response suggests a more optimal balance between speed and energy required to stabilize the satellite.

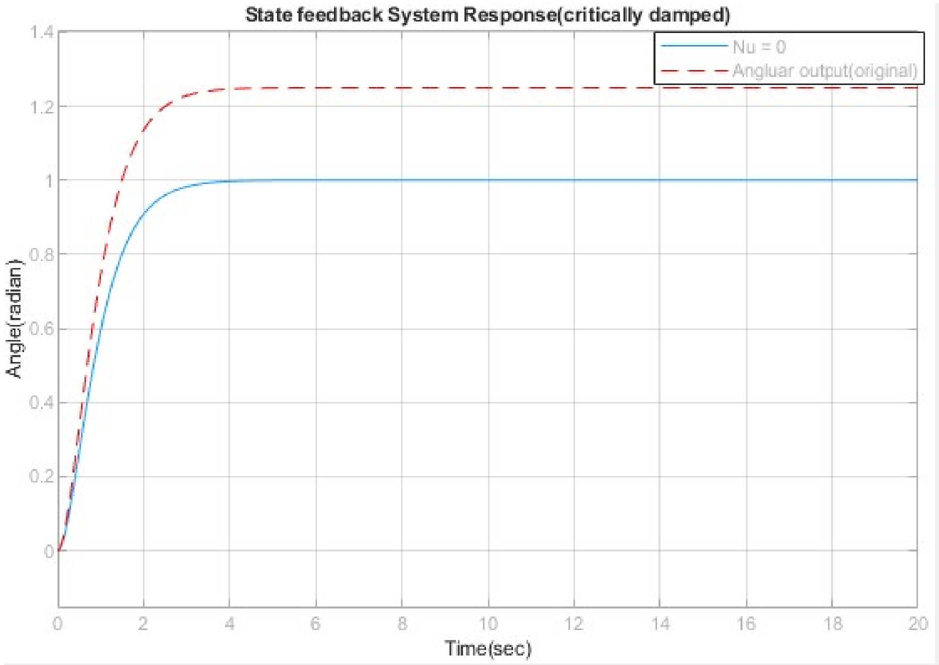

Figure 27 – Stable Angular Response (Critically Damped) with Reference Input

Figure 27 demonstrates the critically damped angular response of the satellite system when tracking a reference input. Here we see the output capture the transient dynamics of the system, but the steady state error is not catered for and hence the incorrect steady state value is obtained and the inclusion of the N_u term corrects the error. In practice this control theory could be employed to enable the satellite to perform earth tracking. The capabilities for precise reference tracking are fundamental for the satellites intention to track emission. The reference input $r(t)$ would be a dynamic signal representing the required pointing angle for sensor targeting and measure emissions from different sources or to scan regions for emissions mapping as the satellite orbits. The controller ability to make the satellite angle x_1

follow this reference as shown here is therefore crucial for acquiring accurate data vital for environmental monitoring.

5 ETHICAL CONSIDERATIONS

Although our project focuses on the modelling of a satellite system using Simulink, it is still important to acknowledge the broader ethical implications such a system could raise if implemented in a real-world scenario. These considerations are essential in guiding the responsible development of future technologies.

Privacy and Security Concerns

Satellite imaging systems must be designed to avoid infringing on individual or group privacy. Although our satellite focuses solely on environmental data, any real-world deployment must limit resolution and data capture to prevent identification of individuals, private property or sensitive locations.

Satellites also face a range of security threats. If used for environmental monitoring, the consequences of breaches can be serious.

- Tampering with thermal or gas emission data could conceal industrial pollution or falsely incriminate low-emission zones. For instance, spoofed infrared signals might mask the environmental impact of a facility.
- Data disruption via jamming or Denial of Service (DoS) attacks could cause data loss during critical moments, such as during a wildfire, pipeline leak or industrial accident, loss of real-time data could hinder emergency response.
- Unauthorised access to satellite systems or processing infrastructure could allow malicious actors to manipulate outputs, delete records or discredit findings, hindering climate policies.
- A satellite in low earth's orbit, which is ideal for high-resolution earth imaging, is more vulnerable to debris. A collision would mean total data loss.
- Political or industrial interference is also a risk. Organisations or regions whose emissions are being monitored might seek to legally or covertly suppress or discredit the satellite's findings such as pressuring providers to block access to key databases or spread misinformation about the satellite's accuracy.

Environmental Trade-offs

While our project does not involve actual satellite construction, we acknowledge that building and launching satellites has environmental impacts, such as emissions and space debris, which must be acknowledged and addressed.

One of the most significant concerns is launch-related emissions. Rockets that deliver satellites into orbit burn large quantities of fuel, releasing carbon dioxide (CO_2), water vapour and black carbon into the upper layers of the atmosphere. These pollutants can have longer-lasting effects than emissions at ground level due to slower chemical breakdown and dispersion at high altitudes. For a satellite system designed to support climate action, this paradox highlights the need for low-emission launch technology or shared launch strategies to mitigate environmental impact.

A second concern is space debris. Each satellite launched into orbit adds to the growing congestion in low Earth orbit (LEO). If not properly decommissioned, satellites can become non-functional "space junk", posing collision risks to other satellites and spacecrafts. These collisions can create even more debris, accelerating the problem in what is known as the Kessler Syndrome [15]. Therefore, sustainable satellite deployment requires detailed end-of-life protocols. For our satellite, this would mean planning a controlled deorbit manoeuvre so that it re-enters the atmosphere and burns up safely or is directed towards a designated spacecraft disposal zone.

In addition, the manufacturing process itself is not impact-free. Building satellites involves resource extraction (e.g., rare earth metals), energy-intensive production and global transportation, all of which contribute to carbon emissions. Although these are "one-time" emissions compared to ongoing pollution from other industries, their environmental cost must be justified by the benefit of the satellite's intended function.

To ensure that satellites are designed for climate monitoring do more good than harm, these trade-offs must be considered early in the development cycle. Ideally, the emissions and environmental risks associated with launch and construction should be offset by the satellite's potential to reduce global emissions through improved monitoring, accountability and disaster response.

Climate Equity and Responsible Interpretation

For satellite systems aimed at monitoring heat and gas emissions, there is an ethical obligation to ensure that the data is used in a way that promotes fairness, equity and justice, particularly when addressing global climate disparities.

Climate equity involves acknowledging that not all regions have the same capacity to respond to environmental challenges. Some countries or communities contribute little to global emissions but are disproportionately affected by climate change. Others may rely on emissions-heavy industries for economic survival. If environmental satellite data is used to impose strict regulations or penalties without providing support, it could deepen existing inequalities.

Our model assumes that emission data would be used to support under-resourced areas through targeted climate aid, infrastructure upgrades or cleaner technology, rather than punish them. For example, identifying high-emission zones in a low-income region could prompt investment in clean energy alternatives rather than fines or sanctions.

Responsible interpretation of data is also critical. Emission heat maps and optical gas imaging data can be misleading if taken out of context. High emissions might reflect industrial activity, population density or temporary environmental conditions rather than reckless pollution. Data should always be paired with socio-economic context to avoid drawing unjust conclusions or crafting unfair policies.

Without careful interpretation, there is also a risk that data will be weaponised in political debates, leading to finger-pointing rather than collaboration. In real-world deployment, it is essential to include climate scientists, local authorities and policy experts in the interpretation and response process.

Transparency

A transparent approach to satellite deployment and data handling is essential to maintain public trust and ensure ethical outcomes.

This begins with clear communication about the purpose of the satellite. Stakeholders, including the public, government bodies and affected industries, should be informed about what the satellite monitors, how it collects data and how that data will be used. This prevents speculation, reduces resistance and fosters a collaborative environment for climate action.

Following this, data governance must be clearly established. This includes defining:

- **Data ownership**: Who owns the data collected by the satellite?
- **Access control**: Who can view, use or download the data?
- **Data protection**: How will the data be secured against unauthorised access or misuse?
- **Accountability**: Who is responsible for managing errors, omissions or breaches?

For our theoretical satellite, we assume responsible and ethical data management. In practice, international standards would be needed to regulate data transparency, especially if multiple countries or agencies are involved in the satellite's operation or funding.

Lastly, satellite data should be made open and accessible wherever possible, especially for researchers, disaster responders and a non-governmental organisation (NGO). This ensures that the benefits of the system are shared widely and that third parties can verify and build upon the findings, contributing to a global effort in combating climate change.

5.1 Sustainable Development Goals

When beginning of this project we came together as a group to look at how this project could impact the environment and how it would have a social impact. For the satellite system we thought it would work best with thirteenth goal - combatting climate change.

We specifically focused on 13.3 - Building knowledge and capacity to meet climate change. This outlines the importance to "improve education, awareness-raising and human and institutional capacity on climate change mitigation, adaptation, impact reduction and early warning". [7] With the satellite system it would allow us to get a greater picture of how emissions truly affect the climate through the heat mapping. With this information we can provide better education to people on where is causing the most damage to the environment and which could then lead to the issues being addressed faster, due to display of early warning signs and reducing any further destruction.

6 CONCLUSION & FURTHER WORK

In conclusion, this report has shown the theory and simulation of a satellite attitude control system. Various aspects of the attitude system have been studied, including PID control for the system and a state space model of the system. A computational approach was investigated with the use of the MATLAB / Simulink environment. This allowed us to visualise theoretical concepts and understand them in a practical way.

The environmental impact of satellites was also examined. Space debris has been identified as a critical problem with satellites. In contrast, this project hopes to combat climate change by highlighting the use of satellites to identify and monitor high levels of emissions.
The above aspects were key points of the EE291 module, their inclusion in this report means we have successfully achieved the aims of the module.

If we had more time to work on this project, we would attempt to implement some orbital control into the satellite model. This would make the satellite orbit around a chosen celestial body, for example, Earth, while maintaining the same distance between the satellite and the body (circular orbit). Some equations which could be used in this idea are mentioned in Appendix A.

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8 APPENDIX

Appendix A: Further Work

Newton's Law of Universal Gravitation

$$F = G \cdot \frac{M \cdot m}{r^2}$$

Equation A – Newton's Law of Universal Gravitation

Where:

- F = Force between M and m
- G = Newton's gravitational constant
- M = Mass of Earth
- m = Mass of satellite
- r = Distance from centre of Earth to the centre of the satellite

Equation A describes the force between two objects. A satellite orbiting Earth would experience a force due to the gravitational pull of Earth.

Speed of Orbit

$$v = \sqrt{\frac{G \cdot M}{r}}$$

Equation B – Orbital Speed

Where:

- v = Orbital speed
- G = Gravitational constant
- M = Mass of Earth
- r = Radius from centre of Earth to satellite

Equation B describes the speed of an object in a circular orbit.

Period of Orbit

$$T = 2\pi \sqrt{\frac{r^3}{G \cdot M}}$$

Equation C – Period of Orbit

Where:

- T = Period of orbit

- r = Radius from centre of Earth to satellite
- G = Gravitational constant
- M = Mass of Earth

Equation C describes the time it takes for the satellite to complete a full orbit around Earth.