Analysis of an Ion Trap Quantum Information Processor

Patrick Sinnott

March 2022

Abstract

In this short essay I will offer a short analysis of the trapped ion quantum information processor described in the paper from Schindler et al. (2013) [1]. I will address the key decisions in the design and development of this device and discuss what I consider to be the pros and cons of these choices.

Contents

1 Introduction

1.1 Divencenzo criteria

I feel that the most logical place to start is to outline what factors are important when it comes to trying to objectively assess the success of a quantum computer. The Divencenzo criteria are often discussed as the basis for assessing the viability of a quantum information processor [2].

DiVincenzo's five criteria include:

1) A physical system containing well-defined two-level quantum systems, or qubits (whose computational basis states are usually written as $|0\rangle$ and $|1\rangle$, which can be isolated from the environment

2) the ability to initialize the system into a well-defined and determinate initial state

3) qubit decoherence times much longer than the gate times

4) a set of universal quantum gates which can be applied to each qubit (or pair of qubits, in the case of two-qubit gates)

5) the ability to read out the qubit state with high accuracy.

These criteria give us a good basis for what is important during our discussion.

1.2 Ion traps: pros and cons

I also feel that it is worthwhile to offer a brief word on the choice to use an ion trap system for quantum computation, as opposed to the various other physical implementation possibilities.

Firstly, Trapped ions represent one of only a few qubit technologies which have yet fulfilled all of DiVincenzo's original criteria with high fidelity. Internal electronic states of the ion are used for the qubit states $|0\rangle$ and $|1\rangle$, there are often many different choices of energy levels. These include: Zeeman qubits where the qubit states are magnetic sublevels split by an applied field and typically have tens of megahertz frequencies, or optical qubits, where the qubit states are separated by an optical transition (typically much larger energy scale). The choice of energy levels for the qubit states will be discussed in detail later.

Initialising the system as well as the readout can both be performed with laser manipulation of the ion to high fidelities in short time scales $\sim$ 1ms. [4]

To achieve a universal gate set, we need to achieve high-fidelity single-qubit rotations as well as an 'entangling' operation on two qubits. Laser or microwave drives applied to the ions allow arbitrary and high-fidelity single-qubit rotations to be performed. Several schemes to perform two qubit gates have been performed and we will discuss more about the options for this in a later section. [5]

It has been demonstrated in many cases that Ion coherence times are much greater than gate times which have been observed in literature to be in the degree of seconds for both optical and zeeman qubits. [6]

In terms of our previously outlined criteria for a quantum information processor, ion traps seem to be a very successful physical implementation. However it is worth noting that each of these examples are taken at small scale. Scalability definitely seems to be an issue with these systems. [7] In the article by Monroe and Kim they discuss how when trying to increase the number of qubits in an ion trap system it becomes very difficult to address individual qubits. Also they discuss how interactions between the qubits can lead to mass decoherence.

The main thrust of this essay will now be to discuss how the specific choice of design features for the ion trap quantum processor from Schindler et al. - 2013 [1] contribute to optimising the features described above. I have narrowed it down to the three main choices I believe have the biggest impact on the effectiveness of this quantum computer: 1) Choice of ion 2) Choice of energy levels and 3) Choice of gates

2 Design choices for the Quantum Information Processor

2.1 Choice of Ion

In our given paper. the authors have decided to use $^{40}Ca^{+}$ ions for the quantum information processor. The most common choices of ion used in literature seem to be: Be+, Mg+, Ca+, Ba+, Yb+ and Hg+.

The first obvious benefit to using carbon is the availability and affordability of this element. Considering calcium constitutes 4.5% of all mass on earth and Calcium-40 makes up 97% of carbon atoms, obtaining this element should not be a problem at all, particularly when compared to more exotic elements like ytterbium (2.7 to 8 parts per million).

It is also worth considering the energy cost of ionising each of these elements. This can be deduced from their first ionisation energy. Calcium has a low first ionisation energy of 589.8 $\text{kJ} \cdot \text{mol}^{-1}$. This is comparable to that of barium: 502.9 $\text{kJ} \cdot \text{mol}^{-1}$ or Ytterbium: 603.4 $\text{kJ} \cdot \text{mol}^{-1}$ but is significantly lower than that of Magnesium: 737.7$\text{kJ} \cdot \text{mol}^{-1}$, Berylium: 899.5$\text{kJ} \cdot \text{mol}^{-1}$ or Mercury: 1007.1$\text{kJ} \cdot \text{mol}^{-1}$. [9]

I scoured the literature in an attempt to find reliable information as to the more technical aspects of using specific ions for ion traps and in most cases the authors seem to arbitrarily decide on a choice of ion with little justification.

However I did find a good analysis from Steane, Andrew M. (1996) [8] where they weigh the pros and cons of different ions for quantum information processing.

Ion mass appears to be very important as the RF pseudopotential (potential of the force directed towards the lowest field region) in a Paul trap is mass dependent, and hence larger masses require larger voltages to achieve similar secular trap frequencies(frequency of oscillation). Hence there is a direct impact on achievable speed of operations. Berylium and Magnesium ions are obviously advantageous in this regard.

In the Steane, Andrew M. (1996) paper, a relation is derived for speed of operations (referred to as switching rate) which depends on the recoil energy of the ion (energy of recoil of an ion after emission of a single photon.) eqn.(1)

$$R < \frac{1}{20\pi}\left(\frac{E_R\omega_z}{\hbar N}\right)^{1/2} \tag{1}$$

The derivation of this relation is quite technical but it can be seen in the paper [8]. We can observe here that a high recoil energy E_R leads to larger possibilities for our switching rate R. However, it is also worth noting that a high recoil energy makes it more difficult to acheive the Lamb-Dicke regime which is where our atom's internal qubit states and its motional states is sufficiently small so that transitions that change the motional quantum number by more than one are strongly suppressed. However we do not need to go deep in to the Lamb-Dicke regime for our purposes so we should be okay.

Another consideration for the suitability of an ion seems to be that the chosen ion is sufficiently simple to allow laser cooling without the need for too many different laser frequencies, also very important is the laser frequencies needed to perform the transitions for information processing.

Looking at the switching rate and requirements for cooling we can see that $^9\mathrm{Be}^+$ is an attractive choice, in that it allows the fastest switching rate with a recoil energy of 2×10^2 kHz and requires only one laser wavelength for cooling. Also the hyperfine splitting frequency of 1.25 GHz is easily accessible. However, the wavelength of 313 nm which we need to traverse between the S-P sublevels requires the use of a dye laser (frequency doubled) which is disadvantageous.

We can see that our ion $^{40}\mathrm{Ca}^+$ is a suitable choice but perhaps not as good at $^9\mathrm{Be}^+$. $^{40}\mathrm{Ca}^+$ has a recoil energy of 4kHz giving it a slower switching rate than $^9\mathrm{Be}^+$. $^{40}\mathrm{Ca}^+$ also requires two laser wavelengths for cooling but both can be achieved easily with diode lasers.

2.2 Choice of Energy Levels

We now move on to the choice of energy levels used in our paper for both information processing and information storage. In our setup, the optical qubit is used for state manipulation, eqn.(2) and the ground-state qubit for quantum memory, eqn.(3)

$$\left|4^2S_{1/2}\right\rangle \Longleftrightarrow \left|3^2D_{5/2}\right\rangle \tag{2}$$

$$\left|4^2S_{1/2}\right\rangle \Longleftrightarrow \left|4^2S_{-1/2}\right\rangle \tag{3}$$

The computational basis states are chosen like so: $4\mathrm{S}_{1/2}\,(m_j = -1/2) = |S\rangle = |1\rangle$ and $3\mathrm{D}_{5/2}\,(m_j = -1/2) = |D\rangle = |0\rangle$. The $3\mathrm{D}_{5/2}$ state has a natural lifetime of $\tau_1 = 1.1$ s which puts an upper limit on how long information can be stored here, this would make it an unsuitable state for information storage. The transition connecting the computational basis states we have selected is the least sensitive to fluctuations in the magnetic field which makes them advantageous for computations as we can hopefully avoid errors like bit-flips or phase-errors through noise from the environment.

The two Zeeman substates of the $4S_{1/2}$ ground-state which the authors have chosen as their information storage states are not at all subject to spontaneous decay which immediately presents them as the most rational choice for information storage.

The only small issue I would have with the choice of energy levels is that our two ground-level Zeeman states would be vulnerable to magnetic fluctuations, more-so than the states used for computation. However I still think that overall the choice of energy levels is very sensible.

2.3 Choice of gates

If we have a universal set of quantum gates, any operation possible on a quantum computer can be reduced to a finite sequence of gates from the set. Nearly all choices of universal quantum gates that can be found in the literature consist of gates which can perform an arbitrary single-qubit rotation coupled with some two-qubit entangling gate (eg. C-Not).

In our given setup the entangling gate used by the authors follows the ideas set out by Mølmer and Sørensen. When using this conbination of gates all necessary operations can be performed with a wide laser beam to illuminate globally the entire register uniformly and a second, tightly focused, steerable laser beam to address each ion. These are reasonable requirements for our universal gate set.

This Mølmer-Sørensen gate takes a combination of the computational basis states and turns them in to one of the four possible Bell states as described by eqn. (4)

$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|01\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|10\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|11\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

(4)

This Mølmer-Sørensen gate has the advantage that it does not need to address the ions individually [10] but it should be noted that it is only valid in the Lamb-Dicke regime. Lee et al. have shown that outside of the Lamb–Dicke regime, this gate can be sensitive to changes in the phase of the optical fields due to the change in the position of the ions at different times, leading to decoherence. [11]

It is claimed in our paper that the Mølmer-Sørensen gates have demonstrated the highest fidelities of any two-qubit gates for ion trap systems. From the paper they use to support this claim [12] this does appear to be the case. However, you should definitely bear in mind that this paper was written by the people who proposed the gate and it was published in 1999, better universal gate operations may well have emerged. On the other hand once again, a peruse of the more recent literature shows the Mølmer-Sørensen gate coupled with arbitrary single qubit rotations remains a popular and effective choice for a universal gate set. [13] [14]

References

[1] Schindler, Philipp, et al. "A quantum information processor with trapped ions." New Journal of Physics 15.12 (2013): 123012.

[2] DiVincenzo, David P. "The physical implementation of quantum computation." Fortschritte der Physik: Progress of Physics 48.9-11 (2000): 771-783.

[3] Bruzewicz, Colin D., et al. "Trapped-ion quantum computing: Progress and challenges." Applied Physics Reviews 6.2 (2019): 021314.

[4] Myerson, A. H., et al. "High-fidelity readout of trapped-ion qubits." Physical Review Letters 100.20 (2008): 200502.

[5] Barenco, Adriano, et al. "Elementary gates for quantum computation." Physical review A 52.5 (1995): 3457.

[6] Bermudez, Alejandro, et al. "Assessing the progress of trapped-ion processors towards fault-tolerant quantum computation." Physical Review X 7.4 (2017): 041061.

[7] Monroe, Christopher, and Jungsang Kim. "Scaling the ion trap quantum processor." Science 339.6124 (2013): 1164-1169.

[8] Steane, Andrew M. "The ion trap quantum information processor." arXiv preprint quant-ph/9608011 (1996).

[9] Lide, David R., ed. CRC handbook of chemistry and physics. Vol. 85. CRC press, 2004.

[10] Häffner, Hartmut, Christian F. Roos, and Rainer Blatt. "Quantum computing with trapped ions." Physics reports 469.4 (2008): 155-203.

[11] Lee, Patricia J., et al. "Phase control of trapped ion quantum gates." Journal of Optics B: Quantum and Semiclassical Optics 7.10 (2005): S371.

[12] Sørensen, Anders, and Klaus Mølmer. "Quantum computation with ions in thermal motion." Physical review letters 82.9 (1999): 1971.

[13] Akerman, Nitzan, et al. "Universal gate-set for trapped-ion qubits using a narrow linewidth diode laser." New Journal of Physics 17.11 (2015): 113060.

[14] Roos, Christian F. "Ion trap quantum gates with amplitude-modulated laser beams." New Journal of Physics 10.1 (2008): 013002.