Senior Fresh Computational Physics Laboratory Report

Laboratory 3: Projectile Motion

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1 Introduction

Aims and overview

The purpose of this lab was to simulate and study the motion of projectiles. However, instead of analysing the simple kinematics of a particle moving without air friction; this lab took a more realistic approach and allowed me to learn how to account for the very important factor of air-resistance. As the lab progressed, the necessity of accounting for air resistance during any reasonable simulation of projectile motion became very clear.

Principles of methods used

Air resistance depends rather clearly on the velocity with which the object moves. You can observe this yourself as when you transition from a walk to a run, you can feel the increased friction force applied to you by the air around you. This force can be represented by the vector **F,** which we write as

$$\mathbf{F} = -\mathrm{f}(\mathrm{V})\mathbf{u}$$

Where **u** is the unit vector in the direction of the velocity. Adding a minus sign to this term inverts the direction to ensure that the friction force always has the opposite direction to the motion. The function f(v) is a quantity that relates the magnitude of the velocity of the projectile to the friction force experienced. It takes in to account both the velocity of the projectile and its diameter to approximate the friction force experienced. It is a reasonable approximation to write f(v) as

$$\mathrm{f}(\mathrm{V}) = \mathrm{bV} + \mathrm{cV}^2$$

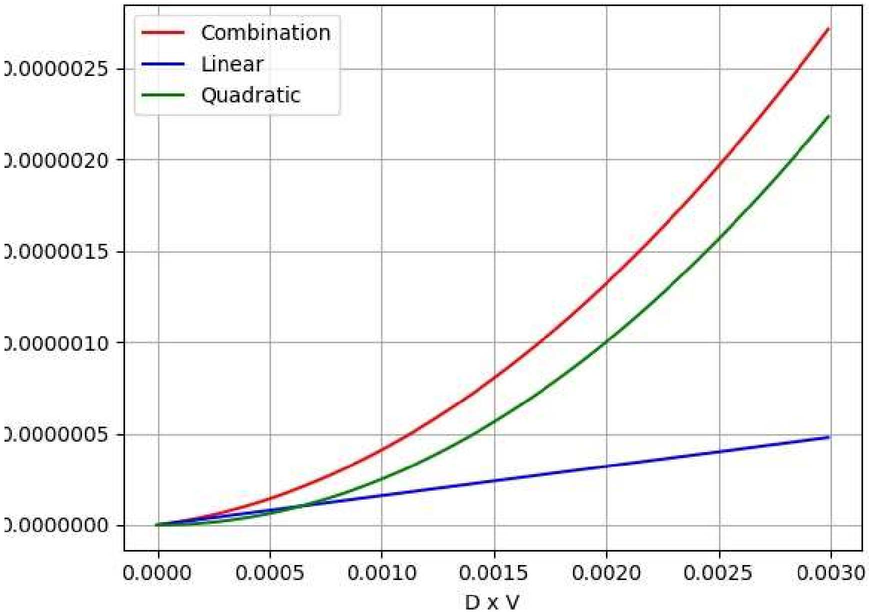
With regards to spherical objects, $\mathrm{b} = \mathrm{BD}$ and $\mathrm{C} = \mathrm{CD\hat{}2}$ where D is the diameter of our projectile and the coefficients C and D depend on the nature of the medium which we base our experiment. However for the purposes of this lab we will be dealing entirely in air so we can fix B and C as constants: $\mathrm{B} = 1.6\mathrm{x}10\hat{} - 4\ \mathrm{N\ s/m\hat{}2}$ and $\mathrm{C} = 0.25\ \mathrm{N\ s\hat{}2/m\hat{}4}$.

2 Methodology

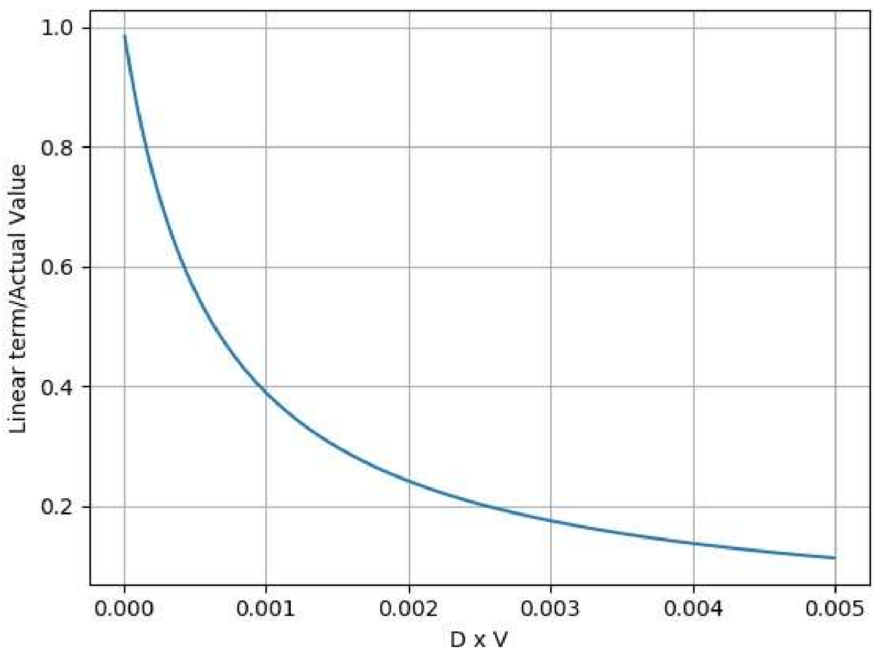
Exercise 1: How does air resistance scale with the velocity?

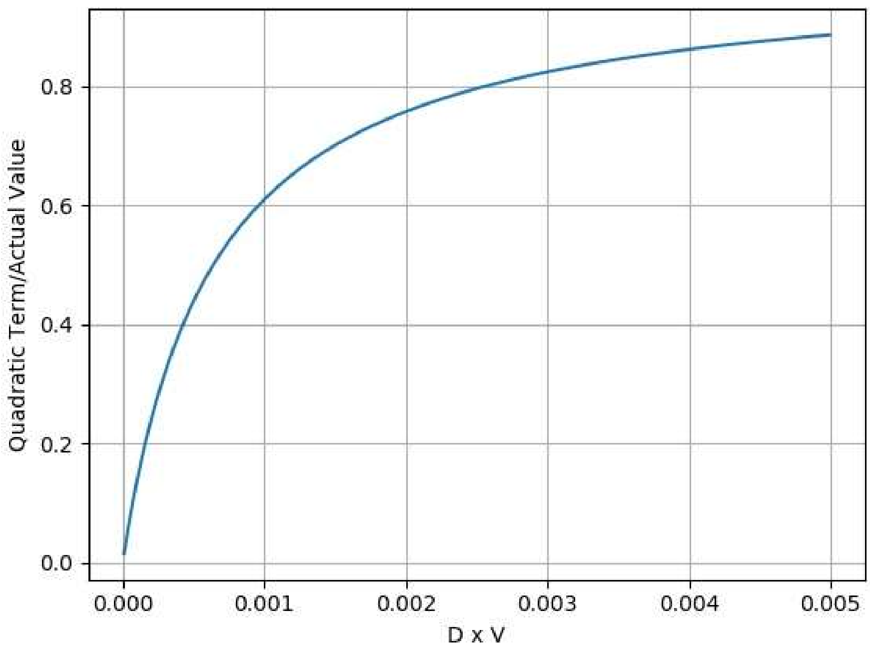
I wrote a python program to plot the function f(v) as a function of the magnitude of the velocity times the diameter, as this is the quantity that decides the magnitude of the friction force experienced.

However, the linear and quadratic terms that make up the function were plotted separately to establish the ranges of values for the following conditions: The range of values for DV where the linear term may be ignored, the range where the quadratic term is of negligible magnitude and the range where both terms should be used to give an accurate approximation.



It can be seen from my graph that for very low values of DV; (DV<0.0003/0.0004) the quadratic term of the friction may be neglected to receive an accurate approximation of the function. However, in the approximate range 0.0004<DV<0.0008 both terms must be used to give an accurate representation of the function. In the range where DV>0.0008, the quadratic term is a good approximation of the function.





From my analysis, it is clear that the ideal form of the friction equation would be just the linear term for the drop of oil and the combination of terms is required for the raindrop.

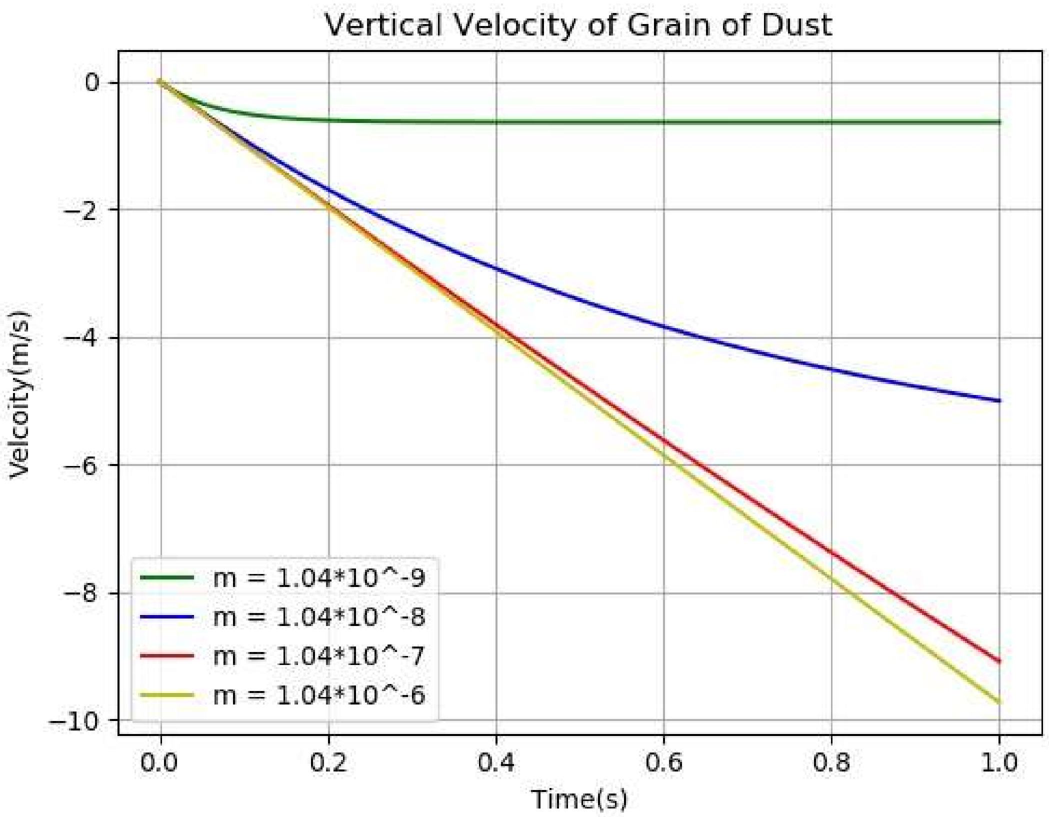
Exercise 2: Vertical motion under the action of air resistance

In this section, I studied a spherical grain of dust which was falling under the influence of gravity. It was acceptable to take the linear term as an approximation as for the particle DV ≈ 3.5x10^-9

I then wrote a simple code which simulated the acceleration of the particle due to gravity. It was based on the formula:

$$\Delta V = -g\Delta t - b/m\, V\Delta t$$

The code read user-input values for m, g, b and Δt then provided a value for V at each time t. I then represented this data graphically as a plot of velocity as a function of time for a number of different masses.

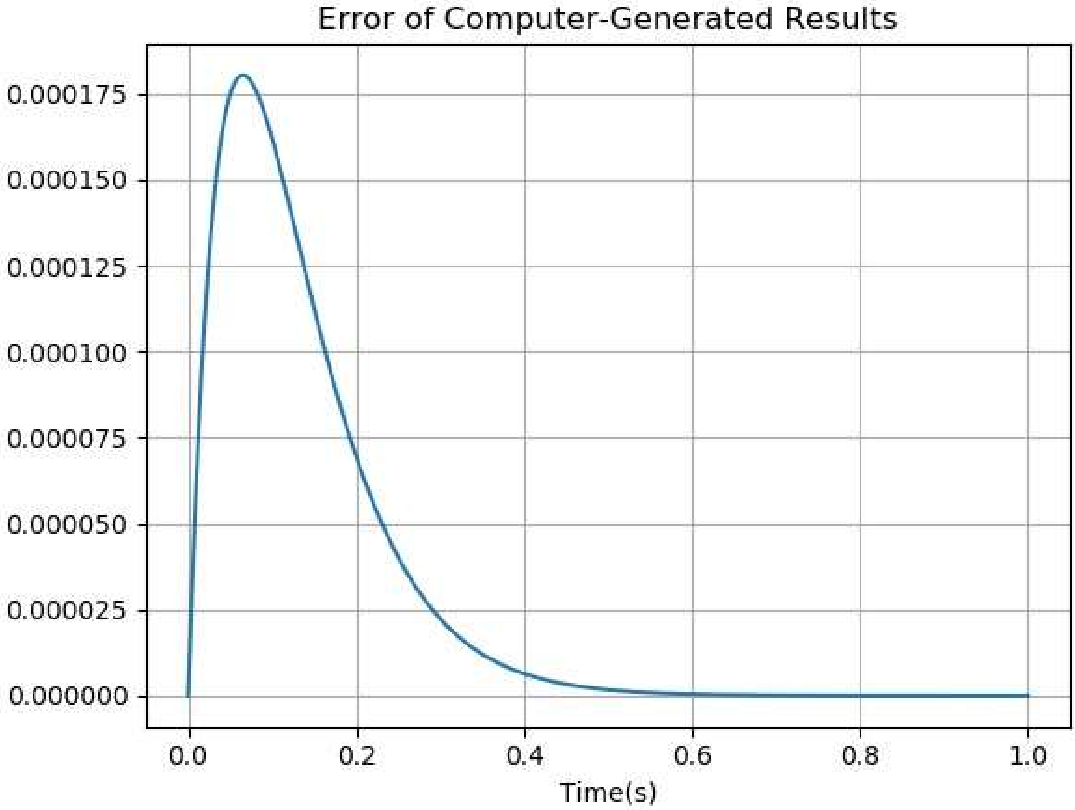


It can clearly be seen from the graph that when the mass becomes longer it is equivalent to very low air resistance.

I then began my investigation of the proposed analytical solution to the current problem:

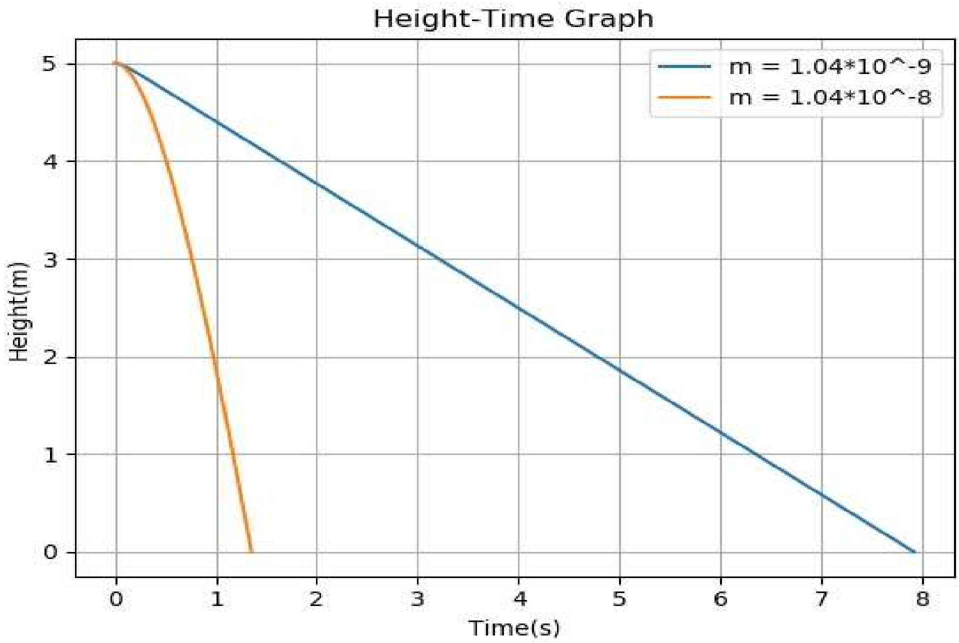
$$V = (mg)/b\left(e^{(-(bt)/m)} - 1\right)$$

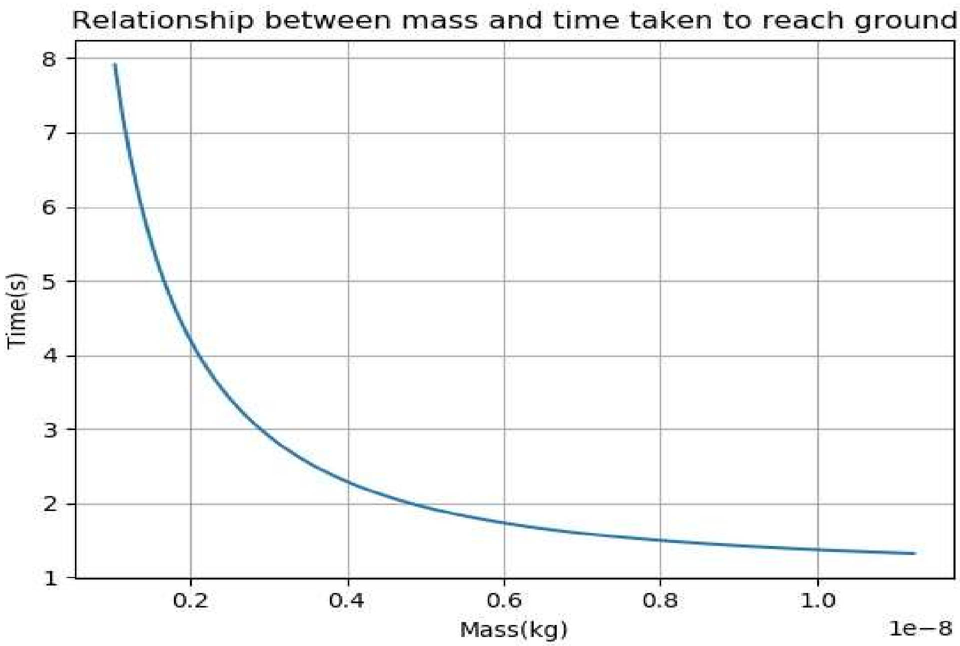
On first observation, this appears to be a reasonable solution however a more thorough investigation is necessary to see how it compares to our computer-generated results. I graphed the error of the computer-generated results over a time period to evaluate its accuracy.



It can be clearly seen that in the beginning of the time interval the approximation has a reasonably large error, however it evolves with time and becomes significantly more accurate. I believe that decreasing the time interval Δt would increase the accuracy of the simulation.

The next part of this investigation involved describing how the position of our particle is changing over time; more specifically, attempting to calculate how long it takes the particle to reach the ground when it is dropped from a height. I examined this property for a number of different masses.





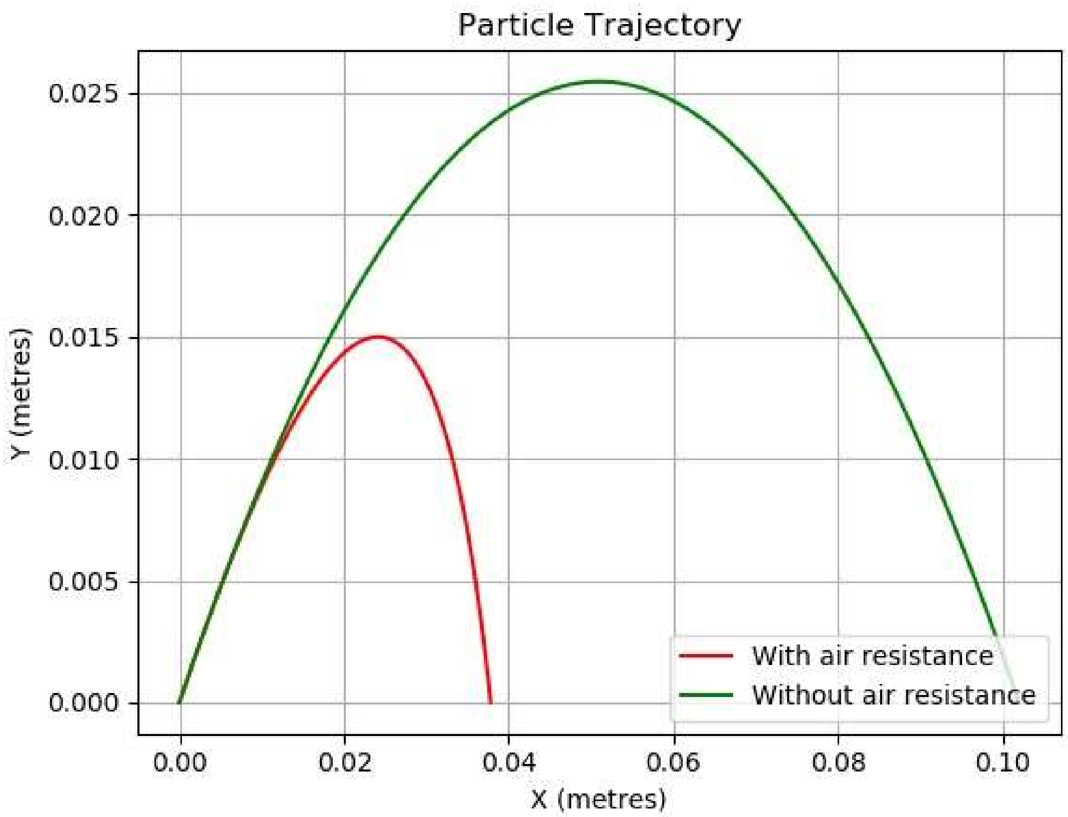
It can be seen rather clearly from these graphs that there is a negative correlation between the mass of an object and the time that it takes to reach the ground, which is contrary to the popular belief that all objects fall at the same rate. However, that assumption would be reasonable for objects that have a relatively high density; as objects with large volumes and low masses feel a huge amount of air resistance which means a change in mass can affect the acceleration due to gravity dramatically.

Exercise 3: Projectile motion under the action of air resistance part 1

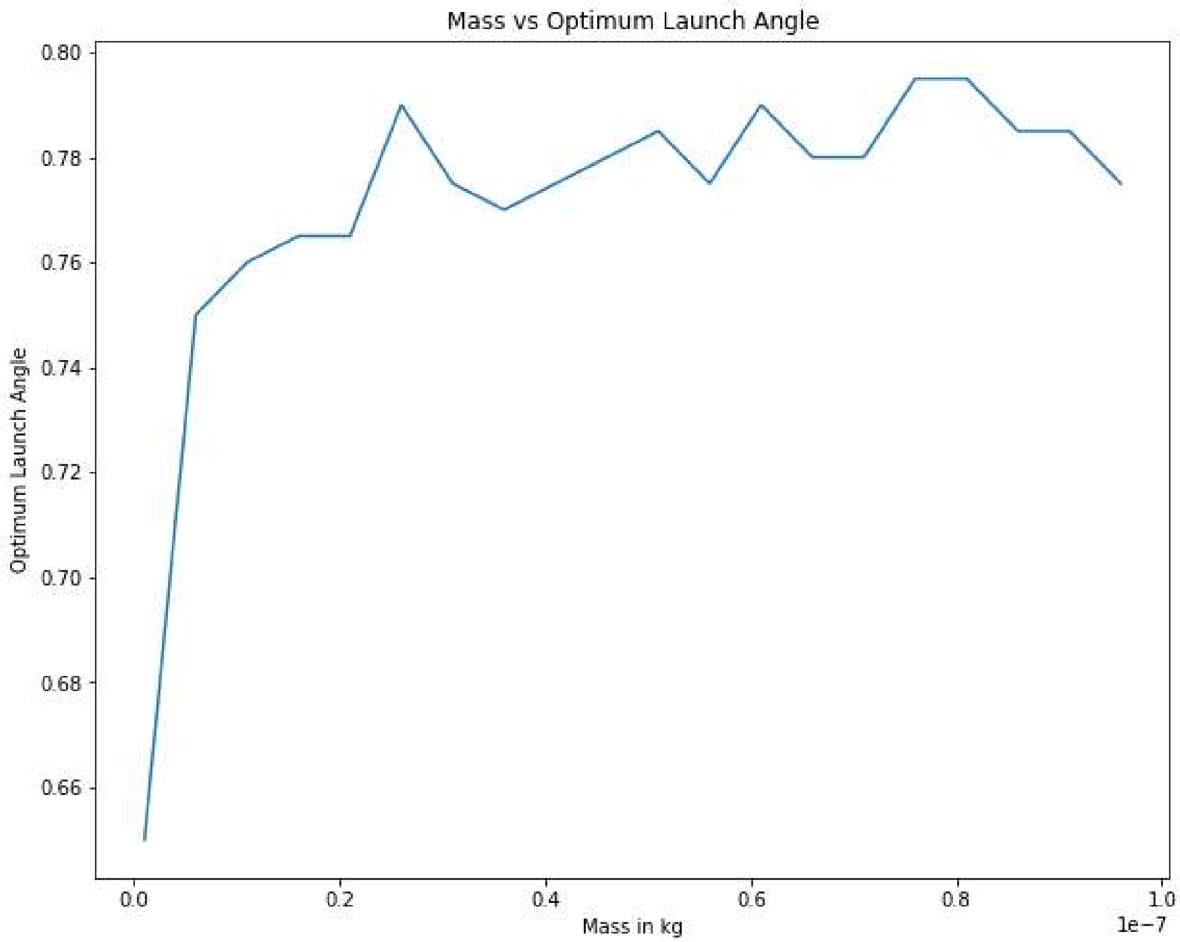
For this exercise I considered a spherical object launched with a velocity V forming an angle θ with the horizontal ground. The trajectory of this object is known to be a parabola without the effect of air resistance however for this simulation we will be taking air resistance into account. This motion can be relatively easily approximated by the following formulae which are derived from Newton's Second Law.

$$(dVx)/(dt) = -b/mVx\,;\ (dVy)/(dt) = -g - b/m\ Vy$$

When the X-coordinate is plotted against the Y-coordinate we can get a clear view of the trajectory of the particle, both with and without air resistance.



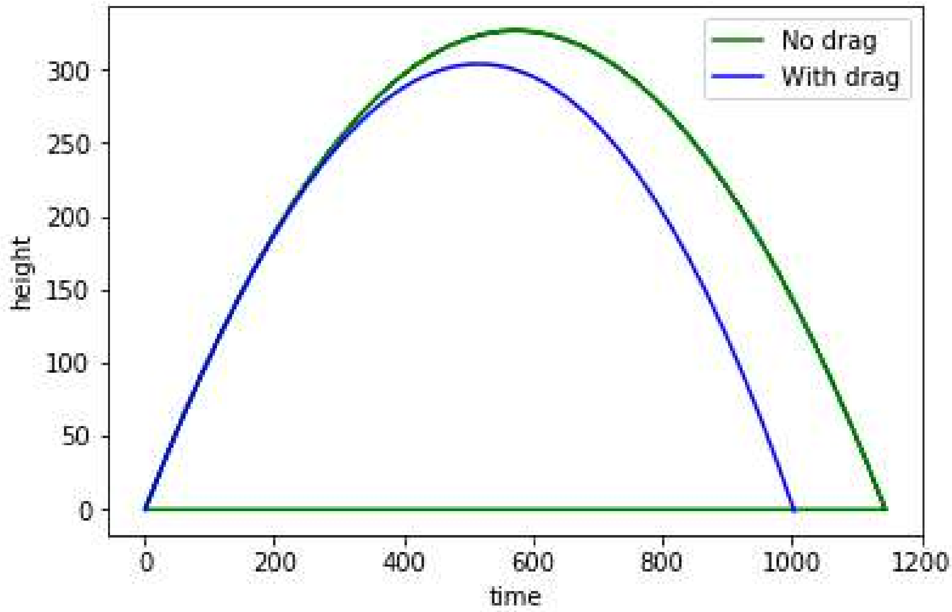
I then turned to the issue of Optimum launch angle. It is a well-known fact that for the simple kinematics of a particle moving without air resistance the optimum launch-angle of a projectile is 45°. However when air resistance is involved the issue becomes more complicated and the launch angle then depends on the mass of the projectile. The dependence is demonstrated in the following graph.



It should be noted that most of the values recorded do not vary hugely from the optimum launch angle in the absence of air resistance which is 45°, however we can distinguish a positive correlation between mass and optimum launch angle up to a point where it appears to level off. I can also note strange fluctuations in the graph which I believe are due to the fact that increasing the mass may function similarly to decreasing the air resistance, but the increase in mass also means that more energy is required to launch the projectile. These two competing properties lead to the fluctuations in the graph.

Exercise 4: Projectile motion under the action of air resistance part 2

For the final exercise, I augmented my code once more to account for the quadratic dependence of the air resistance. I believe that this final program is the most accurate simulation of true projectile motion as it takes all of the aspects of air resistance we have discussed previously into account as well as the inter-dependence of the x and y components of velocity. I graphed this improved simulation against the trajectory of the particle when there is no air resistance.



This graph shows the expected result that the air resistance reduces both the max height reached as well as the distance travelled by the projectile.

3 Conclusions

In conclusion, I feel like throughout this lab I gained valuable insights in to how air resistance affects the motion of moving objects in many different contexts. Throughout this lab I developed an increasingly accurate picture of the effects of air resistance and by the final exercise I had a thorough understanding of the effects of air resistance as well as an accurate computer simulation to represent it.

- The initial program made it clear that the magnitude of the friction force depends mainly on the velocity and diameter of the particle which you are studying. I analysed the linear and quadratic terms which make up the friction force and deduced that only the linear term is required for very small values of DV whereas when you have a large value of DV, the quadratic term works as a suitable approximation.

- In the following exercise I simulated the acceleration of a particle due to gravity. After examining the error of this simulation relative to an analytic solution to the problem. It was clear that I had modelled an accurate representation of the motion. Upon further investigation of this model the dependence of mass on acceleration due to gravity became clear. This result strongly disputes the common misconception that all objects always fall at the same rate which was particularly interesting to me as I was unaware of this fact.

- In the remaining exercises I was concerned with the motion of a projectile which was launched at an initial velocity V at an angle θ to the horizontal. The effect of air resistance on this projectile was dramatic and immediately evident. I then discovered a fascinating dependence of optimum launch angle on the mass of the projectile which was a property I was previously unaware of. Finally, I evaluated the usefulness of my simulation by comparing to a simple kinematic description of a moving projectile and the contrast showed the necessity of taking air resistance in to account when performing any analysis of projectile motion.