Analysis of the Feasibility of using Osmotic Pressure to Power Pepperdine University

Transport II – Professor Chang
Lourdes Brea, Hannah Bruening, Patrick Stepien, Daniel Tawil
May 10, 2017

Abstract

The goal of this project was to generate enough energy to power Pepperdine University using osmotic pressure resulting from the difference in salinity between seawater and freshwater. It was determined that Pepperdine could be powered using this system by building a tank alongside a river and pumping sea water into the tank. The tank will have a membrane wall exposed to the fresh river water in order to facilitate osmotic flow across the membrane. In order to design this process, three different parameters – Q_{in}, total membrane area, and exit pipe area – were optimized, with Q_{in} = 133.8 cubic meters per second, a total membrane area of 20 meters by 262.5 meters, and an exit pipe area of 7.44 square meters.

Introduction and Objective Statement

Energy is one of the most valuable resources in modern society because it is a necessary component for every single process, activity, or action. Supplying energy in a large enough quantity to meet public, commercial, and private demand is one of the great challenges of the 21st century. In particular, there is a societal emphasis on generating clean and renewable energy in order to protect other finite resources such as water, air, and food sources.

This report details a process that is designed to generate energy from osmotic pressure. Placing fresh water from a river and salty water from the sea on opposite sides of a membrane designed for osmosis creates a pressure drop due to the differing concentrations of salt in the two water streams. Essentially, the water from the river flows into the salty water in order to achieve equilibrium. The flowrate resulting from the pressure drop can be used to generate energy from a turbine. The assumptions used in this project are detailed below.

1. The river that the tank is built along is about 20 meters deep, allowing for a 20m high tank to be built.
2. The top of the river is 20 m above sea level, meaning the seawater must be pumped about 20 m high, to the top of the 20 m tank.
3. The pipes through which the seawater is pumped are frictionless, meaning no power is required to pump the water horizontally through the pipes.
4. The system is assumed to be at steady state so that water height is kept constant at 20 m.
5. The pump and turbine are assumed to be 100% efficient.
6. Dilution of salt water in the tank is negligible if the flowrate of seawater into the tank is ten times as much as the osmotic flow rate of river water into the tank.

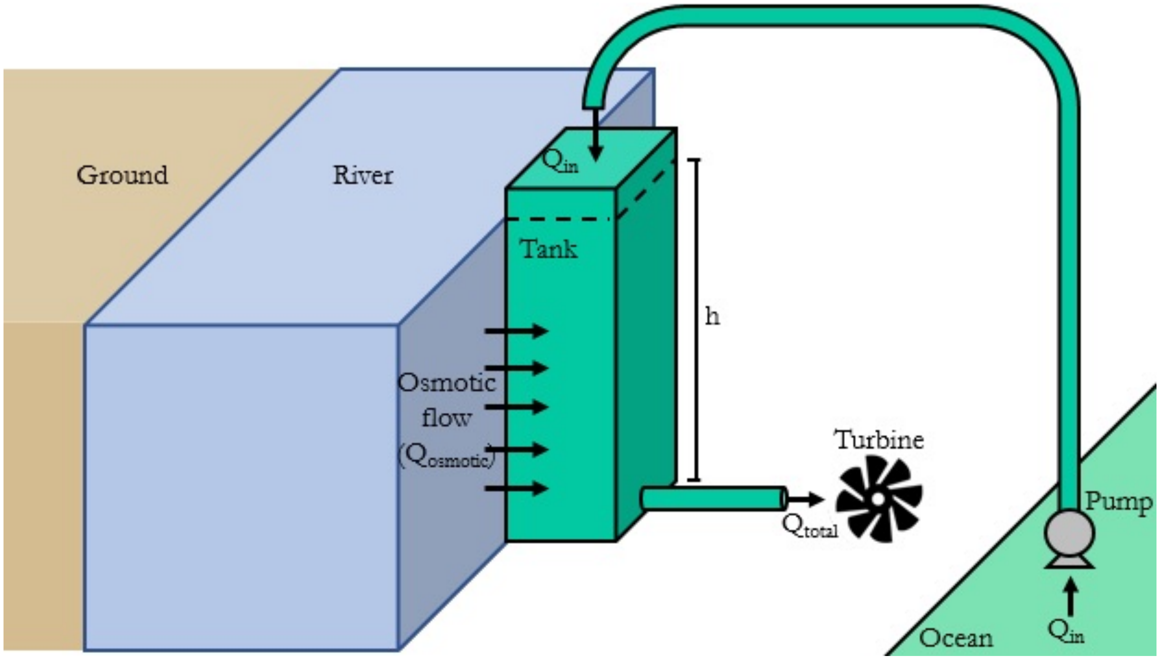

Figure 1: Design for generating energy from osmotic pressure.

Seawater is pumped from the ocean at a flow rate Q_{in}. The river is assumed to have a depth of 20m, and the top of the river is assumed to be about 20 m above sea level. Therefore, it is estimated that the seawater needs to be pumped up about 20 m from the ocean to the top of the river. Friction in the pipes is ignored, meaning it is assumed no power is needed to pump the water horizontally in the pipe leading from the ocean to the tank by the river. The seawater is pumped into a tank constructed along the edge of a river so that one of the tank's walls is exposed to river water.

The side of the tank that is exposed to the river water is a membrane full of nanopores that facilitate osmotic flow. The osmotic flow of the river water into the saltwater tank is driven by the difference in salinity between the fresh river water and the salty seawater. The height of water in the tank is maintained at a height of h. The flowrate of water out of the bottom of the tank is a combination of the osmotic flow of river water into the tank and the gravity-driven flow of seawater down the tank. The combined river and seawater is forced through a pipe out of the bottom of a tank, providing a Q_{total}, which hits the turbine. The turbine can then be used to convert mechanical energy into electrical energy to power a city.

The seawater for this process will be taken from the surface of the ocean, because the surface of the ocean has the highest salinity, as shown in the right graph of the figure, below.

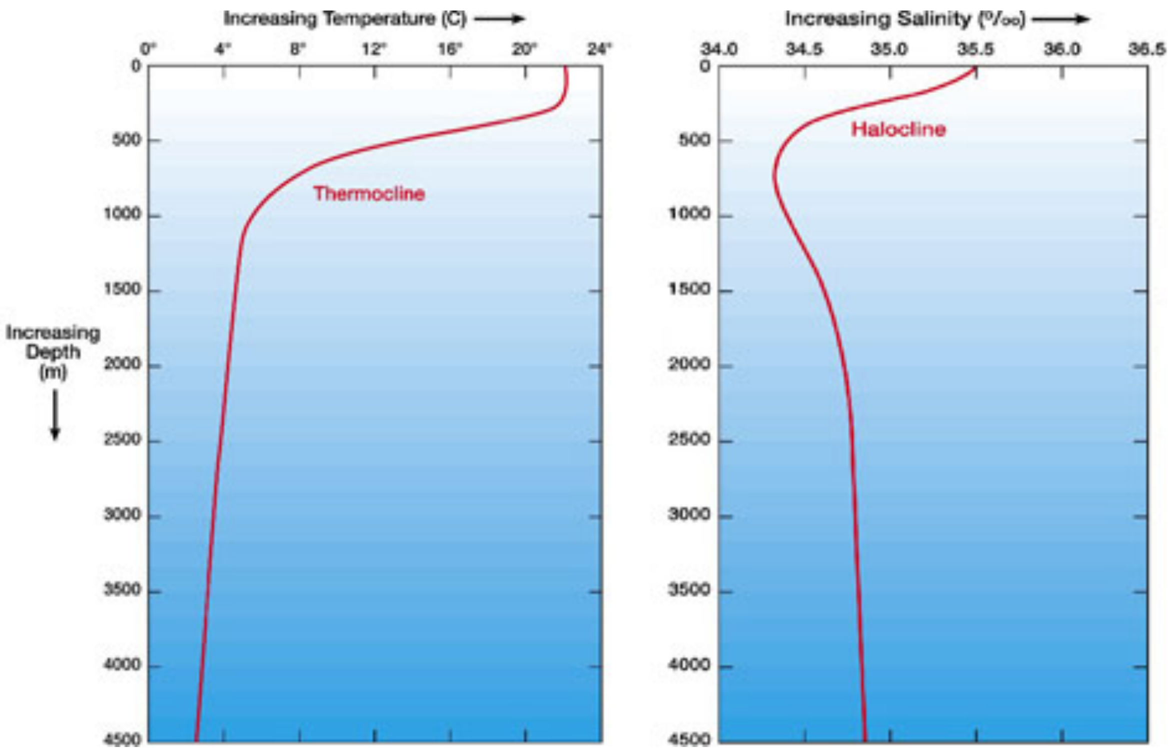

Figure 2: Temperature (left) and salinity (right) gradient of the ocean by depth.

In order for an osmotic system like this to work, the seawater needs to be pumped from the ocean and moved to be near the river water. Thus, there is a required energy input into the system in order to generate energy. An important component of this report is maximizing the process's efficiency, or energy returns on energy invested, in order to demonstrate its feasibility.

This report will determine whether the process described above is feasible for powering Pepperdine University, which uses approximately 22,000,000 kWh of power a year. Thus, the objective of this project is to design a power plant that uses osmotic pressure to provide a constant output of 2,510,000 watts in order to power Pepperdine University.

Theoretical Development

The first step was to determine the osmotic pressure generated from river water flowing past a membrane into seawater. This value was calculated using equation 1.1 below. The temperature and salinity information used in this calculation are representative of open ocean water at the surface, as documented in Figure 2 above.

$$\pi = -\frac{RT}{V_{water}} \ln\left(\frac{x_1}{x_2}\right) \qquad (1.1)$$

Where π is the osmotic pressure, R is the gas constant, T is temperature, V_{water} is the molar volume of water, x_1 is the mole fraction of water in sea water, and x_2 is the mole fraction of water in fresh water

The constant $\frac{RT}{V_{water}}$ was calculated using equations 2.1 and 3.1 below. First, the pressure at surface level was determined by using the ideal gas law and plugging in values for the gas constant, R, temperature, T:

$$P_{atm} = \frac{RT}{V_{air}} = 1\ atm \qquad (2.1)$$

$$\frac{RT}{V_{water}} = \frac{RT}{V_{air}} \times \frac{V_{air}}{V_{water}} \qquad (3.1)$$

Where P_{atm} is the pressure at sea level, R is the gas constant, T is the temperature, V_{air}, is the molar volume of air, and V_{water} is the molar volume of water

The osmotic pressure generated from the river water flowing into the seawater was used to calculate the flowrate through each pore of the membrane, using equation 4.1 below (which can be derived from Navier-Stokes equation for cylindrical coordinates). The membrane used in this design has a pore radius of 100 nm, and is 1 mm thick.

$$P = \frac{8\mu L Q_{pore}}{\pi r^4} \qquad (4.1)$$

Where P is the pressure (osmotic), μ is viscosity, L is thickness of the membrane, Q is water flow rate, π is the constant rather than osmotic pressure, and r is the radius of each pore

The flowrate per pore was converted into the total flowrate using equation 5.1. The area of the pore was calculated from the radius of the pore. The value for total membrane area was altered in this experiment in order to achieve the total power objective.

$$Q_{Osmotic} = Q_{pore} * \frac{1\ pore}{area\ of\ 1\ pore} * Total\ Membrane\ Area \qquad (5.1)$$

According to the tank design discussed in the introduction section, the flowrate generated from osmotic pressure is combined with the flowrate of the seawater pumped into the tank, as shown in equation 6.1. This total flowrate exits through a pipe to a turbine, which was assumed to be 100% efficient for the purpose of this analysis.

$$Q_{Total} = Q_{Osmotic} + Q_{In} \quad (6.1)$$

Where Q_{Total} is the total flowrate that will go exit into the turbine, $Q_{Osmotic}$ is the total flowrate due to osmotic flow, and Q_{In} is the flowrate of seawater.

Since the seawater needs to be pumped into the tank, the flowrate of seawater into the tank was another parameter that needed to be considered when optimizing the process. The total flowrate out of the tank was used to calculate the power generated by the turbine according to the equations for power and velocity, as shown in equations 7.1 and 8.1 respectively.

$$P_G = \frac{1}{2} * \rho * Q_{total} * U^2 \quad (7.1)$$

Where P_G is the power generated, ρ is the density of water (1000 kg/m^3), Q_{Total} is the total flowrate that will exit into the turbine, and U is the velocity of the water in the exit pipe

$$U = \frac{Q_{total}}{A_{pipe}} \quad (8.1)$$

Where A_{pipe} is the area of the exit pipe, Q_{Total} is the total flowrate that will go exit into the turbine, and U is the velocity of the water in the exit pipe

In order to determine whether this process was feasible for generating power, the necessary energy input to add seawater to the top of the tank was also calculated. The power necessary to pump the seawater into the tank was determined using equation 9.1:

$$P_p = Q_{In}\rho g h \quad (9.1)$$

Where P_p is the power generated, ρ is the density of water (1000 kg/m^3), Q_{In} is the flowrate of seawater into the tank, and g is the gravity constant (9.8 m^2/s), and h is the height of water in the tank

The height of the water in the tank is determined by the velocity of water exiting the pipe. This correlation is described using the energy balance shown below, which applies the Bernoulli equation.

$$\frac{1}{2}\rho U^2 = \rho g h \quad (10.1)$$

Where ρ is the density of water (1000 kg/m^3), U is the velocity of the water in the exit pipe, g is the gravity constant, and h is the height of seawater in the tank.

Since water height in the tank is determined by the velocity of the exit pipe, as shown in equation 10.1, and velocity is determined by the area of the exit pipe, as shown in equation 8.1, the area of the exit pipe was also considered when optimizing the power generated. Similarly, the length of the tank, given the height of the water and the total membrane area, was also considered. In addition to meeting the power objective, this project considered the efficiency of the overall process, which is defined as the return on power. The equation for efficiency is seen below in equation 11.1.

$$Efficiency = \frac{P_G - P_P}{P_P} * 100\% \quad (11.1)$$

Calculations and Optimization

Figure 3, below, breaks down salinity into the mass of each type of salt in 1 kilogram of seawater. The total mass of salt adds up to 35.2% salinity, which is a standard amount of salt for surface seawater. The mole fraction of water in surface seawater is x_1.

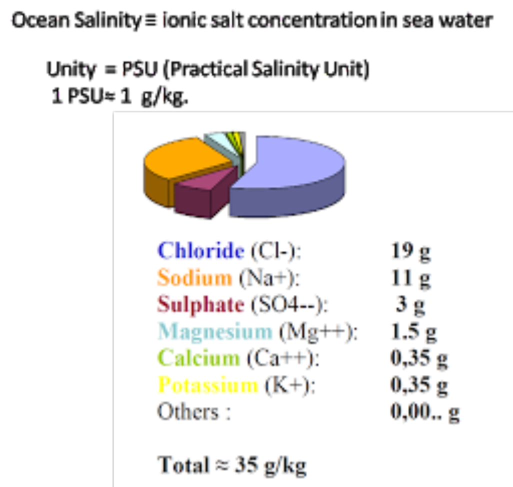

Figure 3: Composition of salts in seawater by mass

The mass of each component of seawater was converted into moles using their respective molecular weights. From this conversion, the mole fraction of water in seawater was calculated, as shown in Table 1 below. The mole fraction of water in fresh water (x_2) was set at .999, which assumes almost no salinity.

Table 1: Calculation of Water for Mole Fractions

	Mass (g)	Molecular Weight (g/mol)	Moles
Chlorine	19	35.45	0.535966
Sodium	11	22.9898	0.478473
Sulphate	3	96.06	0.03123
Magnesium	1.5	24.305	0.061716
Calcium	0.35	40.078	0.008733
Potassium	0.35	39.0983	0.008952
Total Salt	35.2		1.12507

Water	1000	18	55.55556

Mole Fraction of Salt in Seawater	0.019849
Mole Fraction of Water in Seawater	0.980151
Mole Fraction of Water in Fresh Water	0.999

The calculations for the constant $\frac{RT}{V_{water}}$ are shown below:

$$\frac{V_{air}}{V_{water}} = 1242.37 \qquad (3.2)$$

$$\frac{RT}{V_{water}} = \frac{RT}{V_{air}} \times \frac{V_{air}}{V_{water}} = \frac{\left(0.082507 \frac{L*atm}{mol*K}\right) \times (295.15\ K)}{22.4 \frac{L}{mol}} \times 1242.37 \quad (3.3)$$

$$\frac{RT}{V_{water}} = 1350.63\ atm \qquad (3.4)$$

Where P_{atm} is the pressure at sea level, R is the gas constant, T is the temperature, V_{air}, is the molar volume of air, and V_{water} is the molar volume of water

The osmotic pressure was calculated from $\frac{RT}{V_{water}}$, as well as the mole fractions of water in seawater and freshwater. Table 2 and equation 1.2 below summarize the calculation for osmotic pressure.

Table 2: Osmotic Pressure Calculations

Gas Constant (L*atm/mol*K)	0.082507
Temperature (K)	295.15
V_{water} (L/mol)	0.01803
V_{air} (L/mol)	22.4
Mole Fraction of Water in Seawater	0.980151
Mole Fraction of Water in Fresh Water	0.999

Osmotic Pressure (atm)	25.72747
Osmotic Pressure (Pascal)	2606836

$$\pi = -\frac{RT}{V_{water}} \ln\left(\frac{x_1}{x_2}\right) = 1350.63\ atm \times \ln\left(\frac{0.98}{0.9999}\right) = 25.7\ atm \qquad (1.2)$$

The osmotic pressure was used to determine the flowrate through each pore of the membrane, as is discussed in the theory. These calculations are summarized in table 3.

Table 3: Flowrate per Pore Calculation

Radius (m)	1.00E-07
Membrane Thickness (m)	1.00E-03
Viscocity (centiPoise)	1.00E+00
Viscosity (kg*m^-1*s^-1)	1.00E-03
Flowrate per Pore (m^3/s)	1.02E-16

$$Q_{pore} = \frac{P\pi r^4}{8\mu L} = \frac{(25.7\ atm)*(100*10^{-9}\ m)^4*\pi}{8*(1*10^{-3}\ kg*m^{-1}*s^{-1})*(1*10^{-3}\ m)} = 1.02*10^{-16}\ \frac{m^3}{s} \quad (4.2)$$

The flow rate of the pore was converted into the total flow rate due to osmotic pressure, as shown below in equations 5.1 and 6.1. The pore area is constant, as is the flow rate through a pore (equation 4.2).

$$Q_{Osmotic} = Q_{pore} * \frac{1\ pore}{area\ of\ 1\ pore} * Total\ Membrane\ Area \quad (5.1)$$

$$Q_{total} = Q_{Osmotic} + Q_{In} \quad (6.1)$$

Reducing the efficiency equation (equation 11.1) showed that efficiency is the ratio of osmotic flow to seawater flow in. Equations 11.1 to 11.6 below detail the derivation of the reduced efficiency equation.

$$Efficiency = \frac{P_G - P_P}{P_P} * 100\% \quad (11.1)$$

$$= \frac{\frac{1}{2}\rho U^2 Q_{Total} - Q_{In}\rho g h}{Q_{In}\rho g h} * 100\% \quad (11.2)$$

Let $h = \frac{U^2}{2g}$

$$= \frac{\left(\frac{U^2}{2g}\right) Q_{Total} - Q_{In} g \left(\frac{U^2}{2g}\right)}{Q_{In} g \left(\frac{U^2}{2g}\right)} * 100\% \quad (11.3)$$

$$= \frac{Q_{Total} - Q_{In}}{Q_{In}} * 100\% \quad (11.4)$$

$$= \frac{Q_{In} + Q_{Osmotic} - Q_{In}}{Q_{In}} * 100\% \quad (11.5)$$

$$= \frac{Q_{Osmotic}}{Q_{In}} * 100\% \quad (11.6)$$

where $Q_{Osmotic} = Q_{pore} * \frac{1\ pore}{area\ of\ 1\ pore} * Total\ Membrane\ Area$ (5.1)

As shown in equation 11.6, efficiency can be optimized by changing Q_{in} and total membrane area. The effect of varying both total membrane area and Q_{in} are explored in figure 4 below. The data for this figure is given in Appendix A.

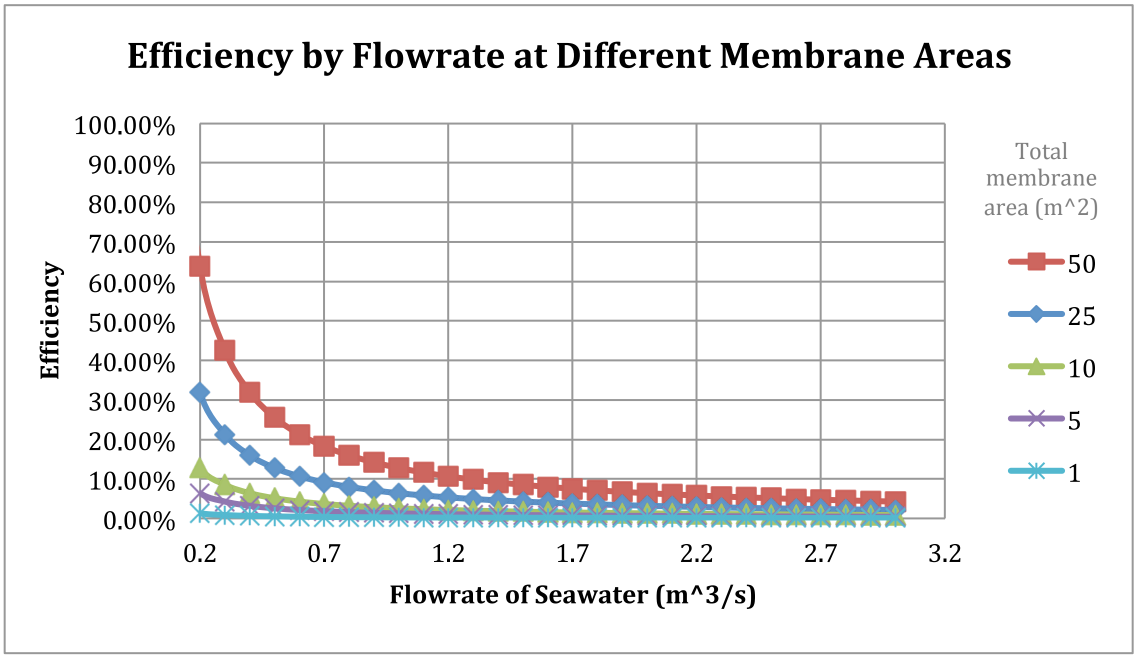

Figure 4: Effect of varying total membrane area and flowrate on efficiency

Graphing energy efficiency over Q_{in} for different total membrane areas revealed that Q_{in} should be minimized at each individual membrane area. Graphing also showed that membrane area should be maximized in order to maximize efficiency.

However, a small seawater flowrate (compared to the osmotic flowrate) would dilute the seawater and decrease the osmotic pressure. As stated in assumption 5, the dilution of salt water in the tank is negligible if the flowrate of seawater into the tank is ten times as much as the osmotic flow rate of river water into the tank. Therefore, the maximum efficiency (defined as the return on energy) for the system is 10%. This value constrained Q_{in} such that maximum efficiency was achieved for any total membrane area. Computationally, the "Goal Seek" function of Excel was used to change Q_{in} to reach an efficiency of 10%.

In addition to achieving the stated return, it was also critical to have enough energy output to reach the objective. Having set the target efficiency at 10%, the two remaining parameters were total membrane area and exit pipe area. Hence, the effect of changing total membrane and exit flow rate on power generated at this maximum efficiency of 10% was graphed. The results are shown below in figure 5. The data for this figure is given in Appendix B.

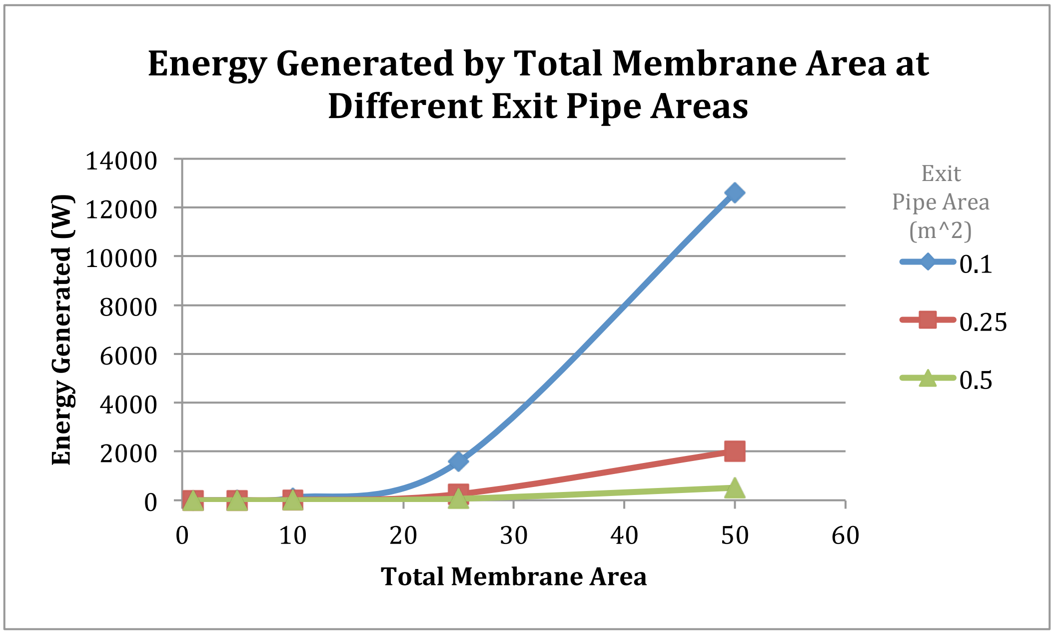

Figure 5: Effect of varying total membrane area and exit pipe area on energy generated

This graph revealed that maximizing total membrane area and minimizing the area of the exit pipe to the turbine results in the most power generated. This result is due to the assumption that frictionless pipes were used. It should be noted that increasing the total membrane area also increased the total flowrate of seawater. Similarly, decreasing the exit pipe flow area would simultaneously increase the velocity through the pipe, according to equation 8.1, above. Due to the energy balance equation explained above in equation 10.1, increasing the velocity through the exit pipe would also increase the height of the column.

Due to the design of this process, where the tank is next to the river, the maximum height of the tank is constrained by the depth of the river and the feasible building depth. Because river depth can vary from 1 meter to several hundred meters, a 20-meter river depth was considered a reasonable depth to use for calculations. 20 meters is also a reasonable building depth. Since velocity, and subsequently height inside the tank, is dependent on the cross sectional area of the exit pipe, the exit pipe area was changed until the height of water in the tank was 20 meters. Computationally, the "Goal Seek" function of Excel was used to change exit pipe area until the height was 20 meters.

The total membrane area does not have any computational or physical constraints other than building constraints, especially since having a large membrane requires a significant amount of sea water pumped in (even when keeping the 10% efficiency) as well as a very long tank. As a result, the total membrane area needed was determined by finding a value that generated enough energy to power Pepperdine, subject to the 10% efficiency constraint and 20 meter high tank. Since a long tank may not be feasible, there is also a calculation for how many 20 meter by 5 meter tanks it would tank to power Pepperdine as well. These calculations are summarized in the appendix C. Table 4 below summarizes the results of optimizing exit pipe area for both one big tank and multiple smaller tanks while maintaining 10% efficiency and a 20 meter water height.

Table 4: Optimized Exit Pipe Area for One Large Tank and Multiple Small Tanks

Total Membrane Area	Qin (m^3/s)	Total Flowrate (m^3/s)	Difference (W)	Tank Length (m)	Exit Pipe Area	Tanks
5250	133.80	147.21	2628210.72	262.50	7.44	1
100	2.54	2.80	50061.89	5.00	0.14	53

Powering Pepperdine with one tank would require a very long tank – 262.5 meters, as well as a very large flowrate of seawater in - 133.8 cubic meters per second. Using multiple tanks gives feasibly small tanks, though there would need to be 53 of them, requiring the same total flowrate of seawater in. Both of these possibilities provide the same amount of total energy of approximately 2.6 million watts at 10% efficiency, or energy returns.

<u>Conclusion and Summary</u>

While it technically would be possible to power Pepperdine with energy generated by osmotic flow, it would not be sensible or practical.

The amount of membrane area needed to generate enough osmotic flow to power Pepperdine is very large, which would mean the material costs to build these tanks would be extremely large. Not only would the price of building the tanks be large, but the probability of finding a stretch of river that is reasonably close to the ocean and remains 20 meters deep over a total of 262.5 meters in length is likely quite low. Considering how difficult it would be to construct a tank that is 262.5 meters, it would be more feasible to construct a series of smaller tanks. For example, constructing 53 tanks that are 20 meters by 5 meters would provide the same amount of power to Pepperdine as the one large tank. Also, it would be extremely difficult to pump 133.8 cubic meters per second of seawater via a single pipe. A more reasonable design would use multiple smaller pipes to deliver the same flow to the tank.

In terms of optimizing the exit pipe area, Figure 5 proves that a smaller exit pipe area increases the velocity of water out of the tank that hits the turbine, thereby increasing power generated. However, there is a limit to how small the exit pipe area could be, due to the large volume of water that needs to exit the pipe in order to maintain steady state. If the pipe is too small, there is likely to be a lot of backpressure that would affect the flowrate out of the tank and consequently the height of water in the tank.

In addition, calculations were made assuming frictionless pipes and perfectly efficient pumps and turbines, meaning the actual power that would be generated would be lower than the calculated generated power. While it is not feasible to use the power generated by osmotic flow as the sole source of energy to power Pepperdine, it could possibly be used to provide some of the University's power in combination with other sources of energy.

Sources

- http://www.statisticbrain.com/hydropower-statistics/
- http://www.salinityremotesensing.ifremer.fr/sea-surface-salinity/definition-and-units
- http://hurricanescience.org/science/basic/water/
- https://community.pepperdine.edu/sustainability/campus/practices/energy.html
- https://www.weatherforyou.com/reports/index.php?forecast=riversnearby&place=malibu%20campus%20pepperdine%20university&state=ca&zipcode=