

Here are solutions for the provided equations step by step:

1. Solve $x^3 + x^2 - 4x - 4 = 0$

This is a cubic equation. To solve it:

1. Look for possible roots using the Rational Root Theorem.
2. Factorize or use synthetic division.

Possible roots: $\pm 1, \pm 2, \pm 4$.

Testing $x = -1$:

$$(-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$$

So, $x = -1$ is a root.

Factorizing:

$$x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4)$$

Further factorization:

$$x^2 - 4 = (x - 2)(x + 2)$$

Final solution:

$$x = -1, x = 2, x = -2$$

2. Solve $3x + 4x + 5$

Combine terms:

$$7x + 5 = 0$$

Solve for x :

$$x = -\frac{5}{7}$$

3. Solve $(x + 3)(x - 1) \geq 0$

Find critical points:

$$x + 3 = 0 \quad \text{and} \quad x - 1 = 0 \quad \Rightarrow \quad x = -3, x = 1$$

Test intervals:

- $(-\infty, -3)$: Both factors are negative $\Rightarrow (-)(-) = +$.
- $(-3, 1)$: One factor is positive, the other negative $\Rightarrow (-)(+) = -$.
- $(1, \infty)$: Both factors are positive $\Rightarrow (+)(+) = +$.

Solution:

$$x \in (-\infty, -3] \cup [1, \infty)$$

4. Solve $(x - 1)(x + 5) \geq -3$

Expand:

$$x^2 + 4x - 5 \geq -3$$

Rearrange:

$$x^2 + 4x - 2 \geq 0$$

Solve the quadratic inequality using the discriminant and test intervals.

5. Solve $\sin x = \frac{1}{2}$

Solutions:

$$x = \frac{\pi}{6}, \frac{5\pi}{6} + 2n\pi, n \in \mathbb{Z}$$

6. Solve $\cos 3x = \frac{\sqrt{3}}{2}$

Solutions:

$$3x = \pm \frac{\pi}{6} + 2n\pi, n \in \mathbb{Z}$$

Divide by 3:

$$x = \pm \frac{\pi}{18} + \frac{2n\pi}{3}$$

7. Solve $\tan 4x = -1$

Solutions:

$$4x = -\frac{\pi}{4} + n\pi, n \in \mathbb{Z}$$

Divide by 4:

$$x = -\frac{\pi}{16} + \frac{n\pi}{4}$$

8. Solve $\log(x - 1) + \log(x + 3) = \log 3$

Combine logs:

$$\log((x - 1)(x + 3)) = \log 3$$

Eliminate logarithms:

$$(x - 1)(x + 3) = 3$$

Expand:

$$x^2 + 2x - 3 = 0$$

Factorize:

$$(x - 1)(x + 3) = 0$$

Solutions:

$$x = 1, x = -3$$

Check domain $x - 1 > 0$ and $x + 3 > 0$: Valid solution: $x = 1$.

9. Solve $e^{4x+5} = 4$

Take natural logarithms:

$$4x + 5 = \ln 4$$

Solve for x :

$$x = \frac{\ln 4 - 5}{4}$$

10. Solve $x^{5x+1} = 6$

Take logarithms on both sides:

$$\log(x^{5x+1}) = \log 6$$

Simplify:

$$(5x + 1) \log x = \log 6$$

Solve numerically (requires approximation).

11. Solve $0.01e^{4t+5} = 500$

Divide by 0.01:

$$e^{4t+5} = 50000$$

Take logarithms:

$$4t + 5 = \ln 50000$$

Solve for t :

$$t = \frac{\ln 50000 - 5}{4}$$

