Robot Sensing 1

Conditional Probabilities

The robot's sensors are not perfect. Just because the robot *sees* red does **not** m
red.

$P(\text{see red}|\text{at red}) = 0.8$

$P(\text{see green}|\text{at green}) = 0.8$

Posterior Probabilities

From these prior and posterior probabilities we are asked to calculate the follo
probabilities after the robot sees red:

1. $P(\text{at red}|\text{see red})$
2. $P(\text{at green}|\text{see red})$

and as a reminder, Bayes' rule looks like this:

$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

or, if we want to use our "versions" of A and B (for posterior #1)...
$P(\text{at red}|\text{see red}) = \frac{P(\text{see red}|\text{at red}) \cdot P(\text{at red})}{P(\text{see red})}$

Now, we can read two of those terms from what we already know about our pr
probabilities which means we can rewrite this as...

$P(\text{at red}|\text{see red}) = \frac{0.8 \cdot 0.5}{P(\text{see red})}$

But we still have one unknown! What was the probability that we would see rec
and there are two ways I can convince myself of that. The first is intuitive and th
mathematical.

Why is P(see red) 0.5?

Argument 1: Intuitive

Of course it's 0.5! What else could it be? The robot had a 50% belief that it was i
that it was in green. Sure, its sensors are unreliable but that unreliability is sym
towards mistakenly seeing either color.

So whatever the probability of seeing red is, that will also be the probability of s
these two colors are the only possible colors the probability MUST be 50% for e

Argument 2: Mathematical (Law of Total Probability)

There are exactly two situations where the robot would see red.

1. When the robot is in a red square and its sensors work correctly.
2. When the robot is in a green square and its sensors make a mistake.

I just need to add up these two probabilities to get the total probability of seein

$P(\text{see red}) = P(\text{at red}) \cdot P(\text{see red}|\text{at red}) + P(\text{at green}) \cdot P(\text{see red}|\text{at}$

I can read these quantities from above!

$P(\text{see red}) = 0.5 \cdot 0.8 + 0.5 \cdot 0.2$

$P(\text{see red}) = 0.4 + 0.1$

$P(\text{see red}) = 0.5$