

$$[y] = C \cdot [y]_c$$

$$[x] = B \cdot [x]_B$$

$$[y] = A_\varphi [x]$$

$$C^{-1} C [y]_c = C^{-1} A_\varphi \cdot B \cdot [x]_B$$

$$[y]_c = \underbrace{C^{-1} A_\varphi B}_{A_{\varphi(B,C)}} \cdot [x]_B$$

$$A_{\varphi(B,C)} = I$$

$$A_{\varphi(B,C)} = C^{-1} A_\varphi \cdot B$$

$$A_\varphi = C \cdot A_{\varphi(B,C)} \cdot B^{-1}$$

$$A^{-1} = A^T$$

$$C \rightarrow U$$

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$$B \rightarrow V$$

$$\forall A \in \mathbb{R}^{n \times m}$$

$$A = U \Sigma V^T$$

$$\Sigma$$

$$\Sigma = A_{\varphi(r, u)}$$

$$\Sigma = \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_k & & \\ & & & & & 0 \end{pmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_k \geq 0$$

$$A = U \Sigma V^T, n > m$$

$$\begin{pmatrix} A \end{pmatrix}_{n \times m} = \begin{pmatrix} u_1 & u_2 & \dots & u_m \end{pmatrix}_{n \times m} \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_k & & \\ & & & & & 0 \end{pmatrix}_{m \times m} \begin{pmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_m^T \end{pmatrix}_{m \times 1}$$

$$\begin{pmatrix} n & m \\ A \end{pmatrix} = \underbrace{\begin{pmatrix} n & m \\ U \end{pmatrix}}_U \underbrace{\begin{pmatrix} n & m \\ \Sigma \end{pmatrix}}_{\Sigma} \underbrace{\begin{pmatrix} m \\ V^T \end{pmatrix}}_{V^T}$$

$$A = \sigma_1 \begin{pmatrix} n \\ u_1 \end{pmatrix} \begin{pmatrix} m \\ v_1^T \end{pmatrix} + \sigma_2 \begin{pmatrix} n \\ u_2 \end{pmatrix} \begin{pmatrix} m \\ v_2^T \end{pmatrix} + \dots + \sigma_k \begin{pmatrix} n \\ u_k \end{pmatrix} \begin{pmatrix} m \\ v_k^T \end{pmatrix} + \dots + \sigma_m \begin{pmatrix} n \\ u_m \end{pmatrix} \begin{pmatrix} m \\ v_m^T \end{pmatrix}$$

$$A \approx A_{\text{approx}} = \tilde{A} = \sum_{i=1}^K \sigma_i u_i v_i^T, \quad k \leq m < n$$

$$\begin{pmatrix} n & m \\ A \end{pmatrix} = \underbrace{\begin{pmatrix} n & k \\ \tilde{U} \end{pmatrix}}_{\tilde{U} \quad n \times k} \underbrace{\begin{pmatrix} k & k \\ \tilde{\Sigma} \end{pmatrix}}_{\tilde{\Sigma} \quad k \times k} \underbrace{\begin{pmatrix} k & m \\ \tilde{V}^T \end{pmatrix}}_{\tilde{V}^T \quad k \times m}$$

$$\tilde{A} = \tilde{U} \tilde{\Sigma} \tilde{V}^T$$