Insper Instituto de Ensino e Pesquisa

Exercício Aula 17

Ciência dos Dados

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Exercício 1

Item 1)

Variável X:

$$E(X) = \sum_{x} (x \times P(X = x)$$

$$E(X) = (-0.25) \times 0.38 + (0) \times 0.405 + (0.25) \times 0.215$$

$$E(X) = \mathbf{-0.04125}$$

$$Var(X) = \sum_{x} (x - E(X))^2 \times P(X = x)$$

$$Var(X) = (-0.25 + 0.04125)^2 \times 0.38 + (0 + 0.04125)^2 \times 0.405 + (0.25 + 0.04125)^2 \times 0.215$$

$$Var(X) = \mathbf{0.0353}$$

Variável Y:

$$E(Y) = \sum_{y} (y \times P(Y = y)$$

$$E(Y) = (-1) \times 0.25 + (0) \times 0.35 + (1) \times 0.4$$

$$E(Y) = \mathbf{0.15}$$

$$Var(Y) = \sum_{y} (y - E(Y))^2 \times P(Y = y)$$

$$Var(Y) = (-1 - 0.15)^2 \times 0.25 + (0 - 0.15)^2 \times 0.35 + (1 - 0.15)^2 \times 0.4$$

$$Var(Y) = \mathbf{0.6274}$$

Covariância de X e Y:

$$Cov(X,Y) = \sum_x \sum_y (x - E(X))(y - E(Y)) \times P(X = x; Y = y)$$

$$\begin{aligned} Cov(X,Y) = {} & (-0.25 + 0.04125)(-1 - 0.15)(0.05) \\ & + (-0.25 + 0.04125)(0 - 0.15)(0.07) \\ & + (-0.25 + 0.04125)(1 - 0.15)(0.26) \\ & + (0 + 0.04125)(-1 - 0.15)(0.075) \\ & + (0 + 0.04125)(0 - 0.15)(0.21) + (0 + 0.04125)(1 - 0.15)(0.12) \\ & + (0.25 + 0.04125)(-1 - 0.15)(0.125) \\ & + (0.25 + 0.04125)(0 - 0.15)(0.07) \\ & + (0.25 + 0.04125)(1 - 0.15)(0.02) \end{aligned}$$

$$Cov(X,Y) = \mathbf{-0.0726}$$

Correlação de X e Y:

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X) \times Var(Y)}}$$

$$Corr(X,Y) = \frac{-0.0726}{\sqrt{0.0353 \times 0.625}}$$

$$Corr(X,Y) = \mathbf{-0.486}$$

Item 2)

Tabela de juros e câmbio:

Y (câmbio)				
X (juros)	-1	0	1	P(X=x)
-0.25	0.05	0.07	0.26	0.38
0	0.075	0.21	0.12	0.405
0.25	0.125	0.07	0.02	0.215
P(Y=y)	0.25	0.35	0.4	1

Tabela do ganho por juros e câmbio:

Y_G (câmbio)				
X_G (juros)	-0.5	0	0.5	P(X=x)
-0.125	0.05	0.07	0.26	0.38
0	0.075	0.21	0.12	0.405
0.125	0.125	0.07	0.02	0.215
P(Y=y)	0.25	0.35	0.4	1

Probabilidade de cada ganho:

$$P(G) = P(X_G = x; Y_G = y)$$

$$P(G = -0.625) = 0.05 = 5\%$$

$$P(G = -0.5) = 0.075 = 7.5\%$$

$$P(G = -0.375) = 0.125 = 12.5\%$$

$$P(G = -0.125) = 0.07 = 7\%$$

$$P(G = 0) = 0.21 = 21\%$$

$$P(G = 0.125) = 0.07 = 7\%$$

$$P(G = 0.375) = 0.26 = 26\%$$

$$P(G = 0.5) = 0.12 = 12\%$$

$$P(G = 0.625) = 0.02 = 2\%$$

Item 3)

Esperança do gasto:

$$E(G) = \sum_{g} (g \times P(G = g))$$

$$E(G) = (-0.625) \times 0.05 + (-0.125) \times 0.07 + (0.375) \times 0.26 + (-0.5) \times 0.075 + (0) \times 0.21 + (0.5) \times 0.12 + (-0.375) \times 0.125 + (0.125) \times 0.07 + (0.625) \times 0.02$$

$$E(G) = \mathbf{0.0544}$$

Variância do gasto:

$$Var(G) = \sum_{g} (g - E(G))^2 \times P(G = g)$$

$$Var(G) = \mathbf{0.13}$$

Item 4)

Esperança e variância do gasto utilizando propriedades de esperança e variância da soma de variáveis aleatórias:

$$E(X + Y) = E(X) + E(Y)$$
$$E(aX + bY) = aE(X) + bE(Y)$$

$$E(G) = E(0.5X + 0.5Y) = 0.5E(X) + 0.5E(Y)$$
$$E(G) = 0.5 \times (-0.04125) + 0.5 \times (0.15)$$
$$E(G) = \mathbf{0.054}$$

$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$$
$$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$$

$$Var(G) = Var(0.5X + 0.5Y) = 0.5^2 Var(X) + 0.5^2 Var(Y) + 2(0.5)^2 Cov(X, Y)$$
$$Var(G) = 0.5^2(0.0353) + 0.5^2(0.6274) + 0.5(-0.0726)$$
$$Var(G) = \mathbf{0.1293}$$