**Quantum Algorithms**

# Solving a Non-Linear Partial Differential Equation

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**16-11-2024**

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# Introduction

There have already been a lot of papers and research that has gone into the solving of linear partial differential equations (PDEs) using quantum computer for an exponential speedup. In the case of linear PDEs quantum algorithms are very well suited and reasonably simple to implement. However, there is somme added complexity for non-linear PDEs. The Navier-Stokes equations are a set of non-linear PDEs that describe the motion of fluid. “They are used in many fields such as weather forecasting, aircraft design and determining the magneto-hydrodynamics of plasmas in space and in nuclear fission” (2). “The Navier-Stokes equations are notoriously difficult to solve on classical computers at large Reynolds numbers” (3) and so a quantum algorithm could offer a significant speedup.

# Problem Definition

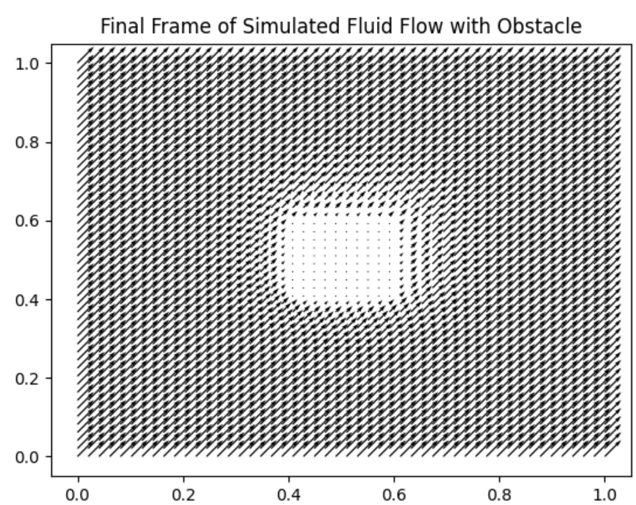
The Navier-Stokes equations are a set of non-linear PDEs. The equations are given by:

$$\nabla u = 0$$

$$\rho \frac{du}{dt} = -\nabla p + \mu \nabla^2 u + F$$

The first equation is the continuity equation, which states that the divergence of the velocity field is zero. The second equation is the momentum equation, which states that the rate of change of momentum is equal to the sum of the pressure gradient, viscous forces and external forces.

The problem that I hope to solve in this project is to develop a quantum algorithm that can solve the Navier-Stokes equations. A common use case for the Navier-Stokes equations is to model air flow around an aircraft. The quantum algorithm developed in this paper will be used to solve the Navier-Stokes and simulate the air flow around an object. The simulation will be very simple and will only consider a 2D flow around some object. The same simulation will be run on a classical computer to have a visual comparison of the results in addition to the numerical comparison. An image of the simulation for a classical computer is shown below:



# Objectives

All of the work in this project will be done using the Qiskit library in Python. The objectives of this project in order are as follows:

1. Develop a quantum algorithm for a trivial case of the Navier-Stokes equations that can be simplified to a linear PDE such as is seen in (2)(1)
2. Implement this algorithm on the IBM quantum computers.
3. Develop a hybrid quantum-classical algorithm for the Navier-Stokes equations such as is seen in (3)
4. Implement this algorithm on the IBM quantum computers.
5. (Time permitting) Explore the possibility of approximating the solution using Variational Quantum Eigensolver

# Conclusion

In conclusion, the Navier-Stokes equations are a set of non-linear PDEs that are notoriously difficult to solve on classical computers. The development of a quantum algorithm to solve the Navier-Stokes equations could offer a significant speedup. The objectives of this project are to develop a quantum algorithm for a trivial case of the Navier-Stokes equations and to develop a hybrid quantum-classical algorithm for the Navier-Stokes equations. The quantum algorithm will be used to simulate the air flow around an object and the results will be compared to a classical simulation.

# Bibliography


1. Nadiga, B., & Karra, S. (2024). Towards Solving the Navier-Stokes Equation on Quantum Computers. eScholarship, University of California. Available at: https://arxiv.org/abs/1904.09033
2. IEEE. (2024). A Quantum Annealing Approach to Fluid Dynamics Problems Solving Navier-Stokes Equations. IEEE Xplore. Available at: https://ieeexplore.ieee.org/document/10612316
3. ArXiv. (2024). Quantum Computing of Fluid Dynamics Using the Hydrodynamic Schrödinger Equation. Available at: https://journals.aps.org/prresearch/abstract/10.1103/PhysRevResearch.5.033182