Current Drivers and Control Electronics for the Laser Spectroscopy of Highly Charged Ions

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Current Drivers and Control Electronics for the Laser Spectroscopy of Highly Charged Ions

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Abstract

Recent years have seen an ever increasing range of laser diodes covering the spectral range from ultraviolet to infrared. The classic 780 nm and 830 nm NIR laser diodes have been well established and many laser designs were developed with design parameters for such diodes. Over the years the disparity in development efforts between laser diodes and supporting systems has led to a subpar performance of such systems compared to NIR diode lasers.

The desire for high resolution spectroscopy of highly charged ions having optically accessible transitions in the ultraviolet and blue regime sparked an interest in high precision and compact diode lasers systems adressing these needs. At the same time, other applications like quantum computing, using arrays of neutral atoms, have seen an increasing demand for customized, compact diode laser systems for the addressing and coherent manipulation of hundreds of indivdual quantum systems on the way to even larger systems scaling to thousands of qubits. All of these use cases require state-of-the-art diode laser systems designed for modern laser diodes with unprecedented stability and noise performance surpassing many of the solutions currently available.

This work compares several commercial products and devices developed in academia used as building blocks for diode laser system like laser drivers and temperature controllers. The laser current driver performance is tested in terms of compliance voltage, output noise, stability with respect to both temperature and time and their output impedance, which is a measure for their noise suppression capability. The limitations found with the tested devices are identified and their causes are explained analytically and with simuations. The laser temperature controllers which are inherently closed-loop instruments whose performance is determined by their front end were tested in terms of noise and stability using reference resistors against a reference thermometer.

These results led to the development of a novel fully digital laser diode driver and temperature controller surpassing other solutions in terms of performance by at least an order of magnitude while being open-source and highly customisable to allow adapting to the needs of both high-resolution spectroscopy and coherent control of quantum systems. The laser current driver implements a unique architecture that isolates the current source from the load to combine the high compliance voltage, demanded by modern high performance laser diode, with ultra-low current noise and stability, providing sub-shot noise performance between 20 mA and 500 mA, delivering a performance close to the limits allowed by physics. This is combined with an outstanding noise immunity allowing the use of compact switch-mode supplies to power those laser drivers without impacting their performance.

The digital temperature controller, again an open-source design, provides definitive sub-mK performance with µK resolution. The stability of this system is defined by the performance of the thermistor used, shifting the focus towards the mechanical resonator design as the ultimate limit.

Finally, a data logging system is presented that accompanies these high precision instruments to monitor the environment of the laboratory, the experiment and instrument parameters to give the experimenter real-time information on the state of the system along with user-definable alerts to protect those assets.

All of these developments are in extensive use at several state-of-the-art experiments and are considered essential for their progress.

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1. Changelog

"What I like about photographs is that they capture a moment that's gone forever, impossible to reproduce."

– Karl Lagerfeld

This work is only a momentary snapshot in time and subject to change. Future revisions will be updated as errors are found and lab-gremmlins have been evicted. If an error is found in this work, the reader is encouraged to either send an email to patrick.baus@physik.tu-darmstadt.de or file a bug report over at Github. Do note, the author has mischievously hidden some errors and typos in this work for the observant reader to find. The latest version (and all others) can always be found at: https://github.com/PatrickBaus/phd_thesis.

Current version: v.0.9.0

Version	Date	Comment
0.9.0	2023-05-12	Initial release submitted as the doctoral thesis.

2. Introduction

Highly charged ions offer a unique insight into the very fine details of the world described by quantum electrodynamics (QED). The predictions of the electron's magnetic moment (g-factor) can easily be regarded as the most accurate prediction in all science, matching the experimental value to 10 significant figures [24]. Experimental measurements of the free electron's g-factor have most recently pushed the boundaries as far as an uncertainty of 1.3×10^{-13}. The comparison of experimental values of the g-factor with theory therefore represents the most stringent test of the QED theory.

Extending the application of QED to bound states requires new tests of these calculations. Computing of the g-factor in complex systems such as neutral atoms with many electrons is extremely difficult and currently impossible with decent uncertainty [23]. Using heavy highly charges ions helps both theory and experiment because it reduces the complexity and at the same times scales the QED contribution with the nuclear charge number as $Z\alpha$, reducing the required accuracy [206]. While research has primarily relied on non-optical measurement in Penning traps in the past, laser spectroscopy opens new opportunities [231] as the strong scaling of the fine- and hyperfine-structures with Z^4 and Z^3 brings those transitions into the region of visible and ultraviolet (UV) laser spectroscopy.

Highly charged ions are therefore an interesting field of research with the GSI Helmholtz Center for Heavy Ion Research at the forefront and capable of providing up to bare uranium. The AsymmetRic Trap for the measurement of Electron Magnetic moments in IonS (ARTEMIS) experiment at GSI as part of the Highly charged Ions Trap (HITRAP) platform and Facility for Antiproton and Ion Research (FAIR) aims to perform high-precision measurements of the bound electron's g-factor using a combination of laser- and microwave spectroscopy referred to as laser-microwave double-resonance spectroscopy [169]. The ARTEMIS experiment provides a unique optically accessible Penning trap design [124, 228] with the aim of measurements on hydrogen-like bismuth, $^{209}Bi^{82+}$. Currently the experiment is in its commissioning phase using boron-like Argon, $^{40}Ar^{13+}$, targeting a g-factor measurement at the 10^{-9} level [123].

Both the commissioning and the measurement on hydrogen-like bismuth require a precision laser system consisting of multiple lasers for the targeted closed transition driven by lasers and microwaves. The accessible wavelength of $^{40}Ar^{13+}$ is 441 nm [138], while $^{209}Bi^{82+}$ requires laser radiation at 244 nm [189]. Recent years have seen the development of new laser diodes giving diode lasers access to an increasing part of the spectrum and for both targeted ion species the fundamental wavelengths are now covered by diode lasers. 244 nm can be reached via a quadrupled 976 nm diode laser source and 441 nm is directly accessible due the invention of blue laser diodes. Blue laser diodes were first presented in 1994 by Nakamura et al. [156]. This was followed by the first pulsed UV laser diode developed by Akasaki et al. [18] in 1995 and 1996 the first continuous wave UV laser diode [17]. These developments then warranted the 2014 Nobel Prize in physics. This progress created the opportunity to build a compact and economic laser systems for the spectroscopy of highly charged ions based entirely on diode lasers.

For both applications laser systems were proposed and preliminary tests conducted [19, 32, 141, 166]. These tests have shown severe limitations in the current state of the art in diode laser technology. Most laser diode drivers commercially available are based on the work of Libbrecht et al. presented in 1993 [120] which was designed for near-infrared laser diodes and the characteristics of blue laser diodes could not be foreseen at that time. The rapid development of blue light emitting diodes and laser diodes outpaced the development of the electronics to drive them which led to subpar performance of blue and UV diode lasers in comparison to their near-infrared brethren. Several groups have reported issues with the existing designs [185] without solutions or attempted to include modern digital controls [76].

Such digital control is critical to stay ahead of the ever increasing complexity introduced by modern experiments. Such experiments can, for example, be found in the field of quantum computing. A promising approach is the use of large arrays of individual neutral atoms, captured using optical traps to solve the scaling problem [49]. Having hundreds to thousands of quantum systems that need addressing and manipulation requires dozens compact of laser sources that need to be orchestrated. Such orchestration is no longer feasible by hand. This level of automation requires a high degree of stability over time and temperature from the laser electronics to ensure the repeatability and reliability of the system. These qualities must be paired with an outstanding noise performance to produce the high fidelity of quantum state manipulation necessary for quantum computers. A combination of these digital features with the performance level is currently not available on the market.

This work has has now closed the gap and provides state-of-the-art open-source laser electronics incorporating novel approaches to the design of laser current drivers and temperature controllers for the application in high precision spectroscopy and quantum computing experiments. These solutions include modern remote-controllable digital interfaces for controlling large scale setups.

This work is split into three parts.

The **Preparation** develops the theoretical background by giving a quick introduction into control theory, noise types and current sources. It also presents the requirements for the laser system designed for ARTEMIS.

The **Results** give a detailed comparison of several laser drivers, both commercial and academic to outline the problems discovered during the testing of a blue laser systems. A laser driver design outperforming all solutions currently available is presented along with a high stability temperature controller specifically designed for the stringent needs of high precision laser spectroscopy. Additionally a compact PID controller system for lab application is presented in the context of lab temperature control together with a data monitoring system capable of logging the manifold data accumulated in a modern experiments and environmental monitoring systems. This data can then be accessed in real-time using a graphical web front end.

The **Outlook** summarizes the results developed and, with the sources of electronic noise in diode lasers suppressed, exposes the final barrier imposed by the mechanical design of laser resonators.

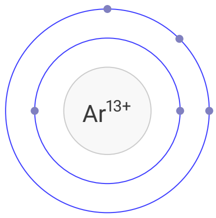
3. Preparation

“Begin at the beginning,” the King said gravely, “and go on till you come to the end: then stop.”

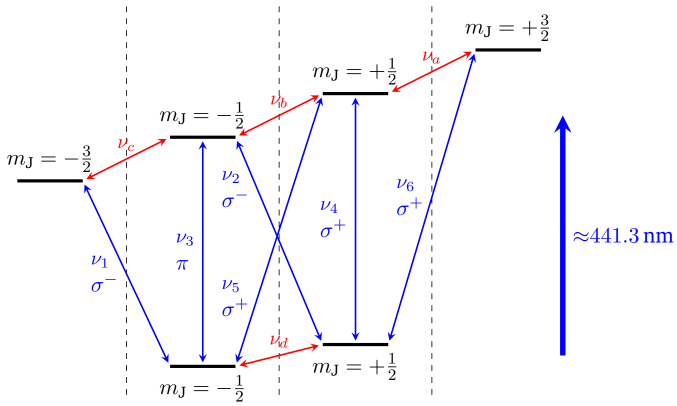
– Lewis Carroll, *Alice in Wonderland*

3.1. Laser System

The ARTEMIS experiment is currently in the process of commissioning. Due to the relative abundance of Argon, the ability to create highly charged ion species using an electron beam along with an interesting transition at $\lambda = (441.255\,75 \pm 0.000\,17)$ nm [138] with a lifetime of (9.573 ± 0.006) ms [111] makes Ar^{13+} an ideal candidate for this purpose. Figure 3.1a shows the simplified electronic configuration of the boron-like Ar^{13+} investigated.



(a) Shell model of boron-like Ar^{13+}.



(b) Level diagram of the Ar^{13+} transitions accessible at 441 nm. Frequencies with numbers denote optical transitions, letters denote microwave transitions.

Figure 3.1.: Electronic configuration and optically accessible transitions of Ar^{13+}.

The optical transitions of highly charged Ar^{13+} around 441 nm shown in figure 3.1b can be used for laser spectroscopy. The transition of interest for commissioning is the transition from the ground state $[(1s)^2(2s)^2 2p]^2\,P_{1/2}$ to the first exited state $[(1s)^2(2s)^2 2p]^2\,P_{3/2}$ [122]. The Zeeman splitting introduced by the 7 T magnet of the ARTEMIS Penning trap results in a frequency shift of the ${}^2P_{1/2}$ ground state by 65 GHz and the exited ${}^2P_{3/2}$ state of 130 GHz. The natural linewidth of these transitions is $\Gamma \approx 2\pi \times (16.63 \pm 0.01)$ Hz which is fairly small, but there is substantial Doppler broadening of

$$\Delta\nu(\lambda = 441\,\mathrm{nm}, T = 4\,\mathrm{K}, m = 39.948\,\mathrm{u}) = \frac{2}{\lambda}\sqrt{2\ln 2\frac{k_B T}{m}} \approx 2\pi \times 150\,\mathrm{MHz}\,, \tag{3.1}$$

seen in the trap which is kept at a temperature of 4 K.

The laser system shown in figure 3.2 was characterised by Martin [141] and the transfer accuracy of the the wavelength was calculated to be 1.2 MHz and, considering the additional long-term drift of the system, led to an upper limit of the absolute wavelength uncertainty of 2.2 MHz. This is more than adequate considering the Doppler broadening of 150 MHz. While being sufficiently accurate, the system has a significant drawback of being fairly complicated to manage if not maintained on a daily basis. The performance and uncertainty of the system relies on the exact knowledge of the tellurium spectrum surrounding both wavelengths of the lasers.

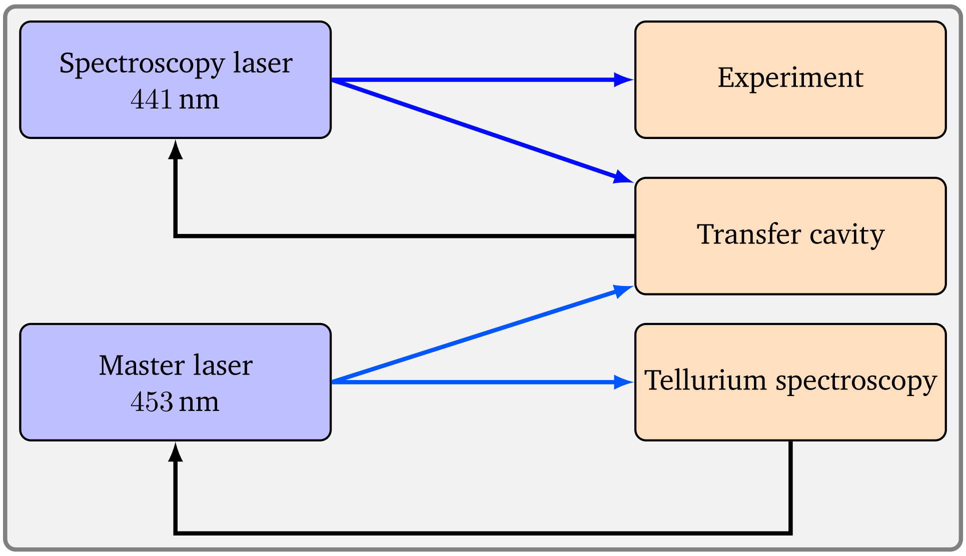


Figure 3.2.: Simplified setup of the laser system in use at ARTEMIS prior to this work. Blue lines are laser beams, black lines are electronic signals delivering feedback to the lasers.

The reason can be seen in figure 3.2 and is founded in the locking scheme. The master laser is first locked to a tellurium reference transition which has to be found manually first using the tellurium map charted by [141]. While not complicated, it requires a trained user. The transfer cavity is then locked to the master laser, a straight forward process. The spectroscopy laser now has to be locked to the correct fringe of the transfer cavity. This is done by observing the tellurium background again to adjust the laser to a frequency close to the desired fringe. This is by no means an automated process and requires a competent operator. Although the locking procedure requires training it does work reliably, but there are more issues that are not readily apparent.

One potential problem lies with the master laser. The whole calibration is geared towards the reference transition at 452.756 nm catalogued by Scholl et al. [183]. Replacing the master laser in case of failure is difficult because blue laser diodes are less flexible when tuning in comparison to their NIR cousins as discussed in [32, 141]. This limits their availability and increases their replacement value.

Another challenge is created by the locking scheme which becomes even more complicated when introducing a third laser to complete the closed transition for the laser-microwave double-resonance spectroscopy. The setup in its present condition is already prepared for the second spectroscopy laser which must also be locked to the transfer cavity. Currently both lasers are sent into the transfer cavity with perpendicular polarisations to separate the beams. Since the reference laser is running at 453 nm and the second spectroscopy laser at 441 nm, it is also possible to separate those two using a dichroic mirror. This scheme requires the close overlap of three beams along with their reflection, because it is a saturated absorption spectroscopy. Such a setup strongly couples the three beam paths and makes replacement of individual lasers more complicated.

The final issue regarding the transfer cavity concerns the required high voltage for the transfer cavity. The cavity contains a piezoelectric actuator to adjust its length. This piezo requires up to 1.25 kV to reach the necessary translation required for scanning the tellurium spectrum. Not only does this pose a risk to the untrained operator, but it was also discovered during the design of the current system that high frequency noise from the voltage supply is radiated and can enter the experiment. It therefore has to be located as far away as possible from the laser setup.

To conclude, the Achilles' heel of the system is the transfer cavity and replacing it can greatly improve the reliability and availability of the whole laser system. During commissioning and testing of the laser system it was also found that there were problems with the laser drivers for the blue laser diodes. The driver used was in-house current source for NIR diodes based on the design of Libbrecht et al. [120]. With these drivers an increasing instability was observed when turning up the current up to the operating point. Other drivers tested either had the same problem or were far noisier and therefore harder to work with. This is due to the modification done by the manufacturers to increase the compliance voltage for the blue laser diodes. The causes are discussed in the next section as it sparked the development of a novel laser driver.

3.2. Laser Current Driver

Laser diodes are current driven devices, because

$$P_{out} \propto I$$

and the diode current I approximately follows the Shockley equation [192]

$$I = I_0 \left(e^{\frac{qV_d}{k_B T}} - 1 \right) . \tag{3.2}$$

k_B is Boltzmann constant, T the temperature, q the electron charge and V_d the diode voltage. The exponential dependence of the current on the supply voltage calls for a current source to drive a laser diode safely without risking thermal damage due to excessive injection currents.

The primary function of a laser driver is therefore to provide a stable, but user adjustable, current. This current can typically be modulated at frequencies up to several MHz to shape the frequency and amplitude of the laser beam. Additional features, like current and voltage limits, aid in protecting the expensive laser diodes and it is not uncommon to have additional safeguards inside the laser head that are under control of the laser driver like a shorting relay to ensure the laser diode is shorted when the driver is disconneted or disabled.

This section deals with the design challenges of such a device used for high precision laser spectroscopy. First the design requirements are established and then technical specifications are developed.

The focus of this work lies on two types of laser diodes, indium gallium nitride (InGaN) and aluminium gallium arsenide (AlGaAs), but is not limited to those two types. The former material is, for example, used for blue laser didoes at around 450 nm, discussed in the previous section, and laser diodes up to green wavelengths, the latter is used for near-infrared (NIR) laser diodes such as 780 nm laser diodes. Both wavelengths are used for experiments in this group. The former type is used in the ARTEMIS experiment for the spectroscopy of highly charged ions, the latter is extensively used to manipulate and control the rubidium atoms used by the quantum computing experiments.

The design requirements are split into three parts which need to be discussed: The ambient environment is one of those parameters and mostly focuses on the ambient temperature, because its effects are the most pronounced. The current source electrical requirements, like the drift, its noise, the output impedance and the modulation bandwidth are discussed as these have a profound impact on the intended application in experiments. Finally, the user interface including the external communication interfaces are defined.

3.2.1. Design Goals: Ambient Environment

The lasers and therefore the laser driver is to be mostly used in a clean laboratory environment. In this particular use case the air is filtered using H14 HEPA filters, but less rigorously controlled environments must be considered as well, because not all fields of application are in optical labs. A mostly dustfree industrial environment is considered acceptable as well. Typical lab temperatures are in the range of $20\,^{\circ}$C to $30\,^{\circ}$C. This temperature range was also encountered in the labs discussed in this work before improvements were implemented as part of this work. The upper end of the range must be considered when operating the devices inside a rack where the temperatures are even higher and the device should therefore be tested for its upper limit.

35 °C is a typical value measured inside the racks used in the lab. Humidity is only controlled with dehumidifiers limiting the upper bound to the range of 15 %rH to 60 %rH.

Figure 3.3 shows a typical one day span of the lab temperature as it was found at the start of this project.

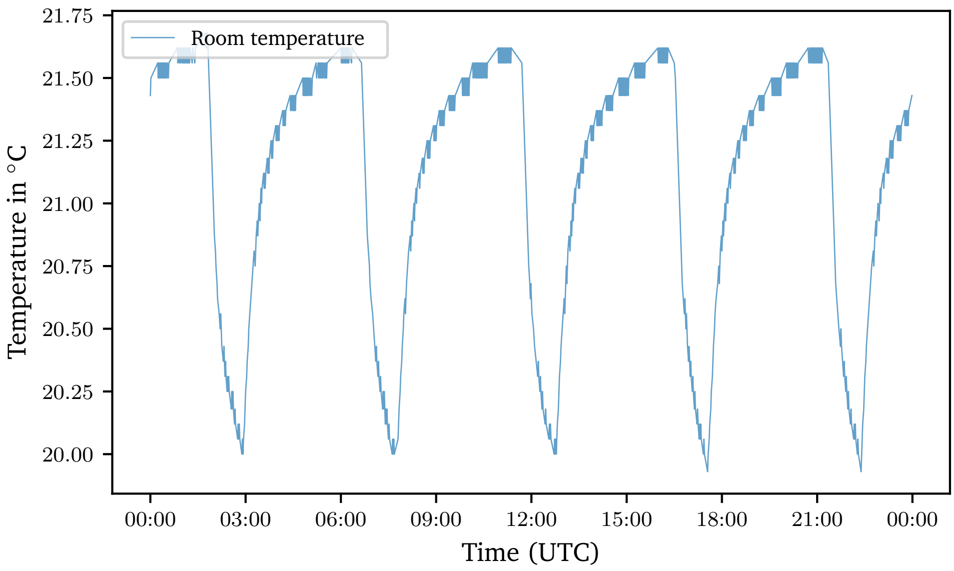


Figure 3.3.: Temperature in Lab 011 on 2016-11-26. Recorded by the LabKraken monitor.

As it can be seen there are strong oscillations of the temperature around the setpoint of 21 °C as a result of the on–off air conditioning temperature controller. The commercial controller initially installed was using an IMI Heimeier `EMO T` thermoelectric actuator [75], which is a two-step valve. Although this solution was later replaced by a custom design described in section 4.3, these type of controllers are found in many other labs and temperature swings of 2 K must therefore be expected.

These environmental parameters can now be used to estimate the design requirements for the laser driver. In comparison to the other laser system used in this group, the 450 nm system [32] required for the spectroscopy of highly charged ions [141] at GSI is the more demanding system. This system was found to be more susceptible to changes of the drive current since the wavelength selective filter element was far broader in comparison to a 780 nm system [142]. This laser is stable over regions of tens of µA and requires a maximum drive current of 145 mA [164].

From these considerations, the requirements for the driver can be inferred. It should be able to supply at least 150 mA and stay well within 10 µA over the whole environmental range. Given a worst-case scenario a tolerance of 3σ (99.7 %) must be met [201].

The environmental parameters that mostly affect current sources are temperature and humidity. Air pressure is typically a matter of concern for high voltage systems [97] and secondary in consideration for this design as it is a low voltage system ($\leq$48 V). Air pressure effects are also the most expensive to test for, as a pressure chamber is required. Humidity affects electronics both directly though corrosion and also indirectly because the epoxy resin used in the FR-4 PCBs and component moulding is hygroscopic and the absorbed humidity leads to swelling and mechanical stress. This effect is very slow at ambient temperature and can easily take days to show [99]. This parameter is therefore handled via the long-term stability and not specified separately.

Given environmental conditions, the relative coefficients can be calculated. This estimation assumes a minimum setpoint resolution of 2 steps within the mode-hop-free region of the laser and calculates the $99.7\,\%$ confidence interval. The steps are given in table 3.1:

Property	Value	Result
Stable range	$10\,\mu\mathrm{A}$	$10\,\mu\mathrm{A}$
2 steps of resolution	$\div 2$	$5\,\mu\mathrm{A}$
1σ	$\div 2$	$2.5\,\mu\mathrm{A}$
Maximum output	$150\,\mathrm{mA}$	$17\,\mu\mathrm{A/A}$
Temperature range	$5\,\mathrm{K}$	$3\,\mu\mathrm{A/(A\,K)}$
Worst case (3σ)	$\div 3$	$1\,\mu\mathrm{A/(A\,K)}$

Table 3.1.: Estimated requirement for the temperature coefficient of the laser driver.

While the requirements look moderate at first sight, tuning a quick estimation shown in table 3.1 leads to a temperature coefficient of $1\,\mu\mathrm{A/(A\,K)}$ or even tighter when using a higher output driver – a rather formidable specification for a current source.

Regarding the long-term stability, a $30\,\mathrm{d}$ number can be estimated. One may be inclined to call for a drift which is smaller than the stable range, but this would be short-sighted, as there are other factors to consider. The laser including the external resonator has its own figure of merit regarding the spectral drift rate. Talvitie et al. [209] reported a drift of $2.9\,\mathrm{MHz/h}$, which was attributed either to the external resonator itself, the piezo or the collimation lens. It is most likely, that this drift was caused by mechanical changes of the external resonator as it defines the output mode of the laser. The mechanical drift limits the required stability of the current source considerably, as a typical frequency change of the internal resonator with the current of $3\,\mathrm{MHz}/\mu\mathrm{A}$ [227] can be assumed. The (linear) ageing drift of the external resonator over $30\,\mathrm{d}$ is equivalent to a $720\,\mu\mathrm{A}$ drift over the same period. For the electronics, the drift is assumed to follow an Arrhenius-like equation resulting from stress, caused during manufacturing. This may eventually change to a slow linear drift after several months of relaxation. The coefficient can either be a positive or negative and leads to

Property	Value	Result
Ageing drift limit	$720\,\mu\mathrm{A}$	$720\,\mu\mathrm{A}$
1σ	$\div 2$	$360\,\mu\mathrm{A}$
Maximum output	$500\,\mathrm{mA}$	$720\,\mu\mathrm{A/A}$
Worst case (3σ)	$\div 3$	$240\,\mu\mathrm{A/A}$

Table 3.2.: Estimated requirement for the long-term stability over $30\,\mathrm{d}$ of the laser driver.

Based on these numbers, it is straightforward to see that the long-term stability of a laser driver is less important than the short-term temperature coefficient since the limiting factor is the mechanical construction of the laser. This necessitates an atomic reference for long-term stability and to compensate for acoustic resonances of the external resonator. Regarding the choice of suitable devices, the tight specification of the temperature coefficient most likely leads to a choice of components, that will pass these long-term criteria as well, alleviating a bit the burden of proof as long-term drift specifications are hard to come by since they need a lot of time to come by and cannot be extrapolated from high temperature burn-in tests [232].

All of this leads to the following design specifications regarding the stability of the current driver:

Design requirements 3.1: Current source, environmental

- Temperature range $20\,°\mathrm{C}$ to $35\,°\mathrm{C}$
- **Temperature coefficient $\leq$1 µA/(A K)**
- Humidity (non-condensing) $\leq 75\,\%\mathrm{rH}$
- Humidity coefficient not specified, but included in the long-term drift
- Maximum altitude not specified
- Long-term drift over $30\,\mathrm{d}$ $\leq 240\,\mu\mathrm{A/A}$

3.2.2. Design Goals: Current Source

The change in output current caused by load impedance should be an order of magnitude less than the drift specification to ensure a negligible effect compared to the drift over time. The load resistance presented by the laser diodes most commonly used in our experiments ranges from $50\,\Omega$ [164] to $30\,\Omega$ [15] and $10\,\Omega$–$15\,\Omega$ for $780\,\mathrm{nm}$ laser diode [86, 110]. It can therefore be estimated as

$$\begin{aligned} \frac{R_{load}}{R_{out}} &= \frac{I_{set}}{I_{out}} - 1 \leq 6.7\,\mu\mathrm{A/A} \\ R_{out} &\geq \frac{50\,\Omega}{6.7\,\mu\mathrm{A/A}} = 7.5\,\mathrm{M\Omega} \end{aligned} \tag{3.3}$$

An output impedance of more than $7.5\,\mathrm{M\Omega}$ for slowly changing loads is a tough requirement, depending on the type of current source, which requires carefully selected components. A high output impedance is, for example, of importance to suppress radiated noise coming from external sources. Especially low frequency components from the mains supply can magnetically couple into the cables, because they are long enough. This noise can be substantial and a high output impedance at low frequencies is therefore important. Other applications will be discussed throughout this work. While a subpar output impedance is more of a limiting factor, the compliance voltage discussed next is a key requirement.

The compliance voltage is the maximum voltage the current source can apply to the load and is another non-ideal component of a real current source. The required voltage strongly depends on the type of laser diode used. The near-infrared laser diodes discussed above have an operating voltage of $1.5\,\mathrm{V}$–$3\,\mathrm{V}$, while the Osram `PL 450B` blue laser diode is specified for $5.5\,\mathrm{V}$–$7\,\mathrm{V}$. The $7\,\mathrm{V}$ required by the Osram laser diode is fairly high for a Fabry–Pérot laser diode and has proven difficult in the past [32] as most laser current drivers available are designed for the much lower forward voltage of the near infrared laser diodes. Even higher voltages of around $12\,\mathrm{V}$–$15\,\mathrm{V}$ are required for quantum cascade lasers, but these are currently neither used nor is their use planned in any experiment in this group.

The maximum output current of the laser driver currently required for laser diodes used in this group is 250 mA for the Thorlabs `L785H1` [110]. Therefore a maximum output current of 300 mA is considered sufficient.

The current noise of the laser driver can be estimated from the laser linewidth sought after as the laser frequency is sensitive to the injection current. At low frequencies, about -3 MHz/µA can be attributed to the thermal expansion of the internal resonator of the diode due to resistive heating [227]. Above 1 MHz this effect starts declining and exposes the change of the refractive index due to the presence of charge carriers. This high frequency effect is an order of magnitude weaker. Since the frequency sensitivity to current variations of the laser diode drops with higher frequencies, the most important range is from DC to 100 kHz.

To estimate the linewidth requirement, it is important to look at the experimental setup. While the spectroscopy of Ar^{13+} at 4 K is limited to around 150 MHz as shown on page 11, the quantum computing experiments in this group have more stringent needs. It was shown in [54, 179, 209] that with reasonable expense a passive linewidth of less than 100 kHz can be achieved. Using the relationship of the frequency sensitivity to a current modulation of laser diodes 100 kHz translates to a current noise of 30 $\mathrm{nA_{rms}}$ from 1 Hz to 100 kHz. The lower 1 Hz limit is chosen fairly arbitrary, but the presence of $\frac{1}{f}$-noise inhibits a definition down to DC. There should be negligible amounts noise below 1 Hz compared to the upper 100 kHz though.

The final aspect of the current source that needs to be specified, is the bandwidth of the current steering input. The bandwidth in these terms define a reasonably flat ($\leq$3 dB) response. As it was discussed above, beyond a frequency of 1 MHz, the frequency sensitivity of the laser diode to current modulation drops by an order of magnitude, altering the transfer function and introducing new challenges for control loops. Therefore a bandwidth of 1 MHz or more is considered sufficient.

Above 1 MHz it is recommended to either use more dedicated solutions like the direct modulation at the laser head presented in [167] or switch to acousto-optic modulators (AOMs) or electro-optic modulators (EOMs).

This leads to the following requirements regarding the current source of the laser driver:

Design requirements 3.2: Current source, electrical

- Maximum output current 300 mA, optionally 500 mA
- **Compliance voltage $\geq$8 V**
- Output impedance $\geq$7.5 MΩ at low frequencies (close to DC)
- **Current noise $\leq$30 $\mathrm{nA_{rms}}$ from DC to 100 kHz**
- 3 dB-bandwidth of the modulation source $\geq$1 MHz

3.2.3. Design Goals: User Interface and Form Factor

The user interface must allow repeatability and reproducibility of the outputs. The reason is that the laser system is intended to be portable to be moved from the university where it is performance tested to the GSI facility. Within the labs, systems are usually moved from test stands to the actual experiment as well. Requiring as little setup effors as possible is a big advantage.

The interface must both be accessible both locally and remotely to allow simple adjustment of the parameters while on the bench and also from within the experimental control software. The local controlls must be directly accessible by humans without tools to give a better user experience.

The remote user interface is strictly required because the Penning trap and the laser system are spatially separated with the laser system being located in a special laser lab for environmental as well as safety reasons. This separation is about 30 m. Ideally this remote interface is computer controlled to give full access to all features of the laser system. USB or Ethernet is preferred as this does not require extra hardware in the lab.

Regarding the application programming interface (API), support for both Python and optionally LabVIEW is favoured, as most of this group has switched from LabVIEW to labscript suite [203] on Python to run the experiments.

The form factor should allow integration into standard 19-inch racks to allow simple transportation from the experiment location at GSI to the university for testing and calibration.

Design requirements 3.3: Current source, user interface

- Local control via the front end without tools
- Remote access via a digital interface
- Software API supporting **Python** and optionally LabVIEW

3.3. Laser Temperature Controller

The external cavity diode laser (ECDL) design employed at GSI and in this group, based on [29], consists of two parts: The laser diode, mounted in an aluminium frame containing a collimator, which is mounted in an external resonator also made of aluminium. The aluminum used for the external resonator is AlZn4.5Mg1, also called alloy 7200 [226]. It has a moderate thermal coefficient of expansion of $23.1\,\mu\mathrm{m}/(\mathrm{m\,K})$, which is an order of magnitude larger than that of Invar, but is significantly easier to machine.

In order to derive the required stability criteria the laser diode and the external resonator must both be considered. The temperature sensitivity of a near-infrared laser at $780\,\mathrm{nm}$ was already calculated by Preuschoff [166] to be

$$\begin{aligned} K_{T,diode} &\approx -3\,\mathrm{GHz/K} \\ K_{T,resonator} &\approx -9\,\mathrm{GHz/K}\,. \end{aligned}$$

As can be seen the external resonator marks the lower bound. Going to a blue $441\,\mathrm{nm}$ laser this criterion becomes even more sensitive, because $K_{T,resonator}$ is proportional to the laser frequency and the frequency almost doubles, this leads to a sensitivity of the resonator on the order of

$$K_{T,resonator} \approx -16\,\mathrm{GHz/K}\,. \tag{3.4}$$

Those numbers imply that in order to match the stability of the laser current driver, the temperate stability should be far better than $1\,\mathrm{mK}$. Temperature stability of better than $100\,\mu\mathrm{K}$ has been demonstrated before [72, 91, 116, 118, 139, 180, 229], but all of these solution have in common, that they use either use multiple layers of shielding and control or elaborate baths into which the subject is submerged. The controller itself is then typically placed inside the controlled environment to shield it from external effects. This type of setup is not feasible in this situation. The laser resonators in use in this group [166] have been set up over the course of several years and there several dozen in use. Taking this design into consideration which does not have an airtight seal are more reasonable requirement can be found. Keeping the temperature deviation well below $1\,\mathrm{mK}$ brings the frequency deviation down to about $10\,\mathrm{MHz}$ or even below that for the near-infrared lasers. Considering the sensitivity of the laser to the barometric pressure which can be estimated using the formula developed by Ciddor et al. [53] to be

$$K_{baro} = -75\,\mathrm{MHz/hPa}\,. \tag{3.5}$$

This leads to a frequency drift of several $100\,\mathrm{MHz}$ due to a typical pressure drift of around $\pm 10\,\mathrm{hPa}$ observed in the lab ($<55\,\mathrm{hPa}$ swing over the year 2022 as recorded by LabKraken), which greatly relaxes the requirements of the temperature controller, because the long term drift is dominated by the barometric pressure affecting the unsealed external laser resonator. To guarantee a stability of $<1\,\mathrm{mK}$ the resolution of the driver should be at least $200\,\mu\mathrm{K}$, preferably $100\,\mu\mathrm{K}$.

The type of temperature transducer used in the laser design is a $10\,\mathrm{k\Omega}$ thermistor, so the design should work with this type of sensor.

Finally, a problem often encountered when temperature controlling larger bodies like optical resonators or cavities is that analog controllers have problems with the long timescales involved. This can be seen in figure 3.4 as oscillations. These oscillations are due the long timescales involved when controlled those well isolated cavities. A digital controller allows very long integration times only limited by the numerical resolution. In addition, a digital system gives more control over the PID tuning parameters which greatly simplifies setting up new laser systems because a common set of PID parameters can be used a starting point before tuning the controller. It is possible to integrate autotuning algorithms the help the user to find a usable set of parameters. Therefore, the new controller should have a digital interface.

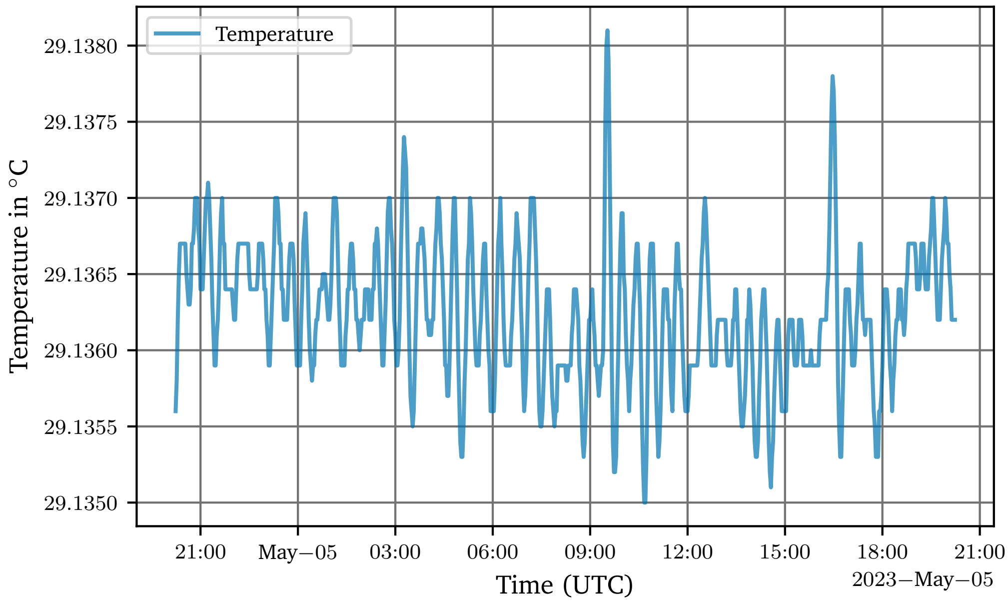


Figure 3.4.: Temperature of a Stable Laser Systems `VH 6020` controlled by a Team Wavelength `HTC1500` temperature controller.

The laser design used in this group uses two Peltier elements to cool both the resonator and the laser diode independently. The driver must therefore integrate two channels. The biggest TECs in use are Laird `CP14,127,06,L1,W4.5` which can draw up to 6 A at 15.4 V [59]. As the laser resonator should not be cooled below the dew point the maximum is not going to be used. Having a driver that can output 4 A–5 A at 12 V is sufficient.

The requirements for the temperature controller can now be summarised as:

Design requirements 3.4: Temperature controller

- Stability: <1 mK
- Resolution: <200 µK, <100 µK preferred
- Temperature sensor: 10 kΩ thermistor
- Two channels
- Output power: 4 A–5 A at 12 V
- Digital interface to work with long timescales and reproducible PID parameters

3.4. LabKraken

3.4.1. Design Goals

LabKraken is a designed to be an asynchronous, resilient data acquisition suite, that scales to thousands of sensors and across different networks.

3.4.2. Software Architecture

LabKraken needs to scale to thousands of sensors which need to be served concurrently. This problem is commonly referred to as the C10K problem as dubbed by Dan Kegel back in 1999 [106] and refers to serving 10 000 concurrent connections via network sockets. While today millions of concurrent connections can be handled by servers, handling 10 000 can still be challenging, especially if the data sources are heterogeneous as is typical for sensor networks of different sensors from different manufacturers.

In order to meet the design goals, an asynchronous architecture was chosen and several different architectures were implemented over time. All in all, four complete rewrites of the software were made to arrive at the architecture presented here. The reason for the rewrites is mostly historic and can be explained by the history of the programming language Python, which was used to write the code. The first first version was written for Python 2.6 and exclusively supported sensors made Tinkerforge. In 2015, Python 3.5 was released, which supported a new syntax for asynchronous coroutines. The software was rewritten from scratch to support this new syntax, because it made the code a lot more verbose and easier to follow. With the release of Python 3.7 in 2018 asynchronous generator expressions where mature enough to be used in productions and the program was again rewritten to use the new syntax. In 2021 a new approach was taken and the program was once more rewritten with a functional programming style. Some of those approaches will be discussed in the next sections to highlight the improvements that were made over time. Each of these sections discusses the same program, but written in different styles to show the differences.

The program does a simple job of opening a network connection then it queries the other side for sensors. When the sensors are returned it looks for a certain sensor, then starts reading data from the sensor to print it.

Threaded Design

The first version of LabKraken used a threaded design approach, because the original libraries of the Tinkerforge sensors are built around threads. The following simplified example shows the code to connect to a temperature sensor over the network and read its data.

```python
ipcon = IPConnection()

# Callback function for the temperature callback
def cb_temperature(temperature: int) -> None:
    """Read the temperature data from the device and print it."""
    print(f"Temperature: {temperature/100.0} °C")

def cb_connected(connect_reason: int) -> None:
    """Query for sensors as soon as the connection is established."""
    ipcon.enumerate()
```

```python
def cb_enumerate(uid: str, *_args) -> None:
    if uid == OUR_KNOWN_DEVICE:
        dev = BrickletTemperatureV2(uid, ipcon)
        # Register temperature callback to function cb_temperature
        dev.register_callback(dev.CALLBACK_TEMPERATURE, cb_temperature)
        dev.set_temperature_callback_configuration(1000, False, "x", 0, 0)

ipcon.connect(HOST, PORT)  # blocking call
# Register enumerate callback
ipcon.register_callback(IPConnection.CALLBACK_ENUMERATE, cb_enumerate)
ipcon.register_callback(IPConnection.CALLBACK_CONNECTED, cb_connected)

input("Press key to exit\n")
ipcon.disconnect()
```

The first thing that stands out with this code, is that the program flow is hard to follow. First the connection is made to the host, then a few callback functions are registered with the connection object. From this point on the program flow looses itself in the callbacks. These functions are called by the connection object and their order can only be guessed from the documentation, but this is only the tip of the iceberg. As the program continues to run it first enter the *cb_connected* callback where it will query the host for its sensors. The answer will be returned through the *cb_enumerate* callback, but within this callback another callback is registered to read the sensor if the correct sensor was found. As the program grows more and more layers of callbacks will be added and in the end, the code will be impossible to read without intimate knowledge.

To untangle this problem, Python introduced so-called generators. This is a type of expression, that will produce a series of values or events which can then be processed in the same function. This is shown next.

Generator Design

```python
async def process_device(device: BrickletTemperatureV2) -> None:
    """Prints the callbacks (filtered by id) of the bricklet."""
    async for temperature in device.read_temperature():
        print(f"Temperature: {temperature} °C")

async def shutdown(tasks: dict[asyncio.Task]) -> None:
    """Clean up by stopping all consumers"""
    for task in tasks:
        task.cancel()
    await asyncio.gather(*tasks)

async def main() -> None:
    """Enumerate the connection, then create workers for each device known."""
    try:
        async with IPConnectionAsync(HOST, PORT) as connection:
            await connection.enumerate()
            async for enumeration_type, device in connection.read_enumeration():
                if device.uid == OUR_DEVICE:
                    asyncio.create_task(process_device(device))
    finally:
        await shutdown(tasks)
```

```
asyncio.run(main())
```

This implementation uses two new language features. The generators mentioned above and a context. The context created using the *async with* command makes sure that the *connection* object is cleaned up as soon as the context is left, greatly simplifying a clean shutdown. Next the host queried again and then a generator is used to read the replies. It is now far easier to follow this style of code because the generator reveals what is happening next. Unfortunately, reading the sensor requires passing it to a new task because the generator should be allowed to generate more sensors in the meantime. Although the code is split into multiple tasks, each task can still be understood and the order of execution can be followed. The only problem is the error handling, because these worker tasks do not communicate with the original task that created them. Implementing this creates a myriad of events that are passed from task to task and failing to pass on a single message can break the program. This is not desirable either as it leads to hard to fix bugs.

The final solution is using so-called streams. A stream is a chain of operators and actions, that are executed in a certain order. This is much like an assembly line where different tasks are executed as the product passes each station.

Stream Design

```
async def main() -> None:
    """Define a stream, then execute it."""
    async with IPConnectionAsync(HOST, PORT) as connection:
        connection.enumerate()
        reader = (
            stream.iterate(connection.read_enumeration())  # read devices
            | pipe.filter(lambda device: device.uid == OUR_DEVICE)  # keep our
                                                                  device
            | pipe.switchmap(lambda device: device.read_temperature())  # read
                                                                  data
            | pipe.print("Temperature: {} °C")  # Print results
        )
        await reader  # start the stream


asyncio.run(main())
```

Using this programming style the intend of the program is revealed immediately, even before starting the stream. The syntax uses was borrowed from the Python library *aiostreams* [150], which is similar to ReactiveX, a library developed by Microsoft to operate on data streams using the so-called Observer pattern. This pipeline does the following: It will again connect to the host using the context manager, then enumerate the host. now the stream is created and every device coming from the connection is going through the pipeline. It will first filter out all device, but the device of interest. The *switchmap* operator will apply the function *read_temperature* and then iterate. It has an interesting property. Should for some reason the same device come up again, the *switchmap* operator will drop the old iterator and start anew. This automatically makes sure that only a single manager or configuration is used for a single object.

This style of programming was found ideal for real-time data processing, as it allows to continuously update configurations or add and remove sensors, or even hosts, without having to worry about what happens along the pipeline.

Device Identifiers

Every sensor network needs device identifiers. Preferably those identifiers should be unique. Typically a device has some kind of internal identifier. Here are a few examples of the sensors used in the authors network:

Device Type	Identifiers	Example
GPIB (SCPI)	**IDN?* returns $manufacturer,$name,$serial,$revision	
Tinkerforge	Each sensor has a base58 encoded integer device id	QE9 (163684)
LabNode	Universal Unique Identifier (UUID)	cc2f2159-e2fb-4ed9-8021-7771890b37ad

As it can be seen above, these identifiers do not guarantee to uniquely identify a device within a network. The Tinkerforge id is the weakest, as it is a 32 bit integer (4.294.967.295 options), which might easily collide with another id from a different manufacturer. The Tinkerforge id is presented as a base58 encoded string. An encoder/decoder example can be found in the TinkerforgeAsync library [34].

The id string returned by a SCPI device is slightly better, but again does not guarantee uniqueness. As it is shown in the example the same device might return a different id depending on its settings. This typically done by manufacturers for compatibility reasons.

The only reasonably unique id is the universal unique identifier (UUID) or globally unique identifier (GUID), as dubbed by Microsoft, used in the LabNodes. Their id can be used for networks with participant numbers going into the millions.

Calculating the probability of a collision between two random UUIDs is called the birthday problem [222] in probability theory. A randomly generated version 4 UUID of variant 1 as defined in RFC 4122 [117] has 122 bit of entropy, that is out of 128 bit, 4 bit are reserved for the UUID version and 2 bit for the variant. This gives the probability of at least one collision in n devices out of $M = 2^{122}$ possibilities:

$$\begin{aligned} p(n) &= 1 - 1 \cdot \left(1 - \frac{1}{M}\right) \cdot \left(1 - \frac{2}{M}\right) \dots \left(1 - \frac{n-1}{M}\right) \\ &= 1 - \prod_{k=1}^{n-1} \left(1 - \frac{k}{M}\right) \end{aligned} \tag{3.6}$$

Using the Taylor series $e^x = 1 + x \dots$, assuming $n \ll M$ and approximating we can simplify this to:

$$\begin{aligned} p(n) &\approx 1 - \left(e^{\frac{-1}{M}} \cdot e^{\frac{-2}{M}} \dots e^{\frac{-(n-1)}{M}}\right) \\ &\approx 1 - \left(e^{\frac{-n(n-1)/2}{M}}\right) \\ &\approx 1 - \left(1 - \frac{n^2}{2M}\right) = \frac{n^2}{2M} \end{aligned} \tag{3.7}$$

For one million devices, this gives a probability of about 2×10^{-25}, which is negligible.

In the Kraken implementation, all devices, except for the LabNodes, will be mapped to UUIDs using the underlying configuration database. It is up to the user to ensure the uniqueness of the non-UUID ids reported by the devices to ensure proper mapping.

Ethernet Bus and Synchronous Buses

There are inherent challenges involved with the Ethernet bus for instrumentation. The Ethernet bus is intrinsically asynchronous and multiple controllers can talk to the device at the same time. Not only that, but different processes within the same controller can talk to the same device. This makes deterministic statements about the device state challenging. A device that is not designed to work asynchronously in the first place may have trouble with multiple requests coming in from different clients. This must be kept in mind when using serial adapters like USB or GPIB to Ethernet.

While it is impossible to rule out the possibility of multiple controllers on a network, care was taken to synchronize the workers within Kraken.

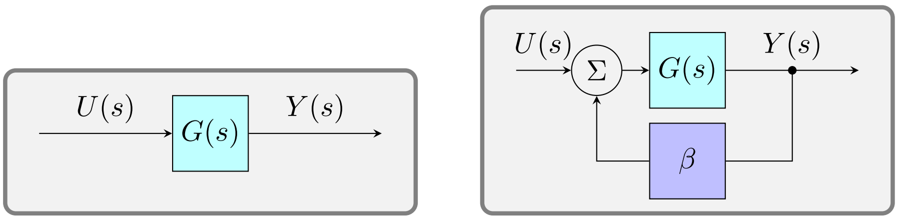
3.5. Short Introduction to Control Theory

This section will give a very brief introduction into some basic concepts of control theory. Many systems require control over one or more process variables. For example, temperature control of a room or a device, or even creating a programmable current from a voltage is one such problem. All of this requires control over a process and is established through feedback, which allows a controller to be aware of the state of the system.

The focus of this section is narrowed down to the concept of feedback and control with regard to developing and understanding PID controllers for temperature control. Simpler feedback loops like those typically used around op-amps will not be primarily considered in this ssction and are discussed in the relevant part of the documentation. In the following sections, first general properties of the Laplace transform and useful relationships are introduced, then, a model for the system and its controller will be developed, finally, using the model, tuning of the control parameters using different tuning algorithms will be discussed.

3.5.1. Introduction to the Transfer Function and the Laplace Domain

There are two types of systems: open- and closed-loop systems. A system is called open loop, if the output of a system does not feed back to its input as in figure 3.5a. On the other hand, if the output influences the input of the system via feedback, it is called a closed-loop system, as shown in figure 3.5b. Although feedback can be treated in static systems, it is more useful to treat it in dynamic systems, either in the time-domain or the frequency-domain. To discuss these system, the terminology used in the following section needs to be defined. $G(s)$ is called the transfer function of the system, while $U(s)$ is the input, $Y(s)$ is the output, s is complex frequency domain variable, β is the feedback parameter, also called feedback fraction, as shown in figure 3.5. In this section, upper case letters are used to denote functions in the Laplace domain, while lower case letters are referring to functions in the time domain.



(a) Open-loop system.

(b) Closed-loop system.

Figure 3.5.: Block diagram of a closed- and an open-loop system.

It is convenient to express the transfer function in its Laplace transform for several reasons that will be explained below. The systems to be discussed are physical system and hence are causal. That means the output only depends on past and present inputs, but not future inputs. For this reason, only the one-sided or unilateral Laplace transform needs to be considered. It is defined as:

$$\mathscr{L}\left(f(t)\right) = F(s) = \int_0^\infty f(t) e^{-st}\, dt. \tag{3.8}$$

with $f : \mathbb{R}^+ \to \mathbb{R}$, that is integrable and grows no faster than $c \cdot e^{s_0 t}$ for $s_0, c \in \mathbb{R}$. The latter attribute is important for deriving the rules of differentiation and integration.

To understand the benefits of using the Laplace representation of the transfer function, a few useful properties should be discussed with regard to the PID controller. First of all, the Laplace transform is linear:

$$\begin{aligned}\mathscr{L}\left(a \cdot f(t) + b \cdot g(t)\right) &= \int_0^\infty (a \cdot f(t) + b \cdot g(t))e^{-st}\, dt \\ &= a \int_0^\infty f(t)e^{-st}\, dt + b \int_0^\infty g(t)e^{-st}\, dt \\ &= a\mathscr{L}\left(f(t)\right) + b\mathscr{L}\left(g(t)\right) \end{aligned} \tag{3.9}$$

Another interesting property is the derivative and integral of a function f. The function f must, of course, be differentiable and grow no faster than the exponential function as defined above:

$$\begin{aligned}\mathscr{L}\left(\frac{df}{dt}\right) &= \int_0^\infty \underbrace{f'(t)}_{v'(t)}\, \underbrace{e^{-st}}_{u(t)}\, dt \\ &= \left[e^{-st} f(t)\right]_0^\infty - \int_0^\infty (-s) f'(t)\, dt \\ &= -f(0) + s \int_0^\infty f'(t)\, dt \\ &= sF(s) - f(0) \end{aligned} \tag{3.10}$$

$$\begin{aligned}\mathscr{L}\left(\int_0^t f(\tau)\, d\tau\right) &= \int_0^\infty \left(\int_0^t f(\tau)\, d\tau e^{-st}\right)\, dt \\ &= \int_0^\infty \underbrace{e^{-st}}_{v'(t)} \underbrace{\int_0^t f(t)\, d\tau}_{u(t)}\, dt \\ &= \left[\frac{-1}{s} e^{-st} \int_0^t f(t)\, d\tau\right]_0^\infty - \int_0^\infty \frac{-1}{s} e^{-s\tau} f(\tau)\, d\tau \\ &= 0 + \frac{1}{s} \int_0^\infty e^{-s\tau} f(\tau)\, d\tau \\ &= \frac{1}{s} F(s) \end{aligned} \tag{3.11}$$

If the initial state $f(0)$ can be chosen to be 0, the differentiation becomes a simple multiplication by s, while the integration becomes a division by s. These three properties greatly simplify the calculations required for studying a proportional–integral–derivative controller in section 3.5.3.

Finally, the most important aspect is the possibility to give a simple relation between the input $u(t)$ and the output $y(t)$ of a system. This relationship between input and output of a system as shown in figure 3.5a is given by the convolution, see e.g. [27]. Assuming the system has an initial state of 0 for $t < 0$, hence $u(t < 0) = 0$ and $g(t < 0) = 0$, one can calculate:

$$y(t) = (u * g)(t) = \int_0^\infty u(\tau) g(t - \tau)\, d\tau \tag{3.12}$$

Applying the Laplace transform, greatly simplifies this:

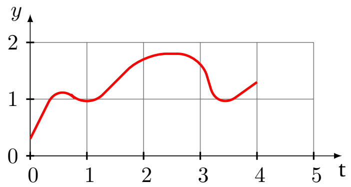
$$
\begin{aligned}
Y(s) &= \int_0^\infty e^{-st} y(t)\, dt \\
&\overset{3.12}{=} \int_0^\infty \underbrace{e^{-st}}_{e^{-s(t-\tau)}e^{-s\tau}} \int_0^\infty u(\tau) g(t-\tau)\, d\tau\, dt \\
&= \int_0^\infty \int_0^t e^{-s(t-\tau)} e^{-s\tau} g(t-\tau) u(\tau)\, d\tau\, dt \\
&= \int_0^\infty e^{-s\tau} u(\tau)\, d\tau \int_0^\infty e^{-st} g(t)\, dt \\
&= U(s) \cdot G(s)
\end{aligned}
\tag{3.13}
$$

This formula is a lot simpler than the convolution of $u(t)$ and $g(t)$, therefore the use of the Laplace transform has become very popular in control theory.

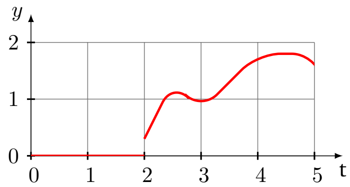
Having derived some of the most useful properties, it is interesting to look at a few functions, which are heavily used in control theory, like a function delayed by the time interval θ. To demonstrate its properties, let $f(t-\theta)$ be

$$
g(t) := \begin{cases} f(t-\theta), & t \geq \theta \\ 0, & t < \theta \end{cases} . \tag{3.14}
$$

The reason for this definition is, that it is mandatory for the system to be causal. This means, it is impossible to get information from the future ($t < \theta$). To satisfy this requirement, any constant other than 0 may be chosen, as is done later in section 3.5.4, when determining tuning parameters and fitting experimental data to a model. An example of such a time delayed function $g(t)$ is shown in figure 3.6b.



(a) Original signal $f(t)$.



(b) Delayed signal $f(t-2)$.

The Laplace transform of a delayed signal $g(t)$ can be calculated as follows:

$$
\begin{aligned}
\mathscr{L}\left(g(t)\right) &= \int_0^\infty g(t) e^{-st}\, dt \\
&\overset{3.14}{=} \int_\theta^\infty f(t-\theta) e^{-st}\, dt \\
&\overset{\tau := t-\theta}{=} \int_0^\infty f(\tau) e^{-s(\tau+\theta)}\, d\tau \\
&= e^{-s\theta} \int_0^\infty f(\tau) e^{-s\tau}\, d\tau \\
&= e^{-s\theta} F(s)
\end{aligned}
\tag{3.15}
$$

To satisfy the causality requirement in the time domain, the Heaviside function $H(t)$ can be used to give a more concise representation of $g(t)$:

$$\mathscr{L}\left(f(t-\theta)H(t-\theta)\right) = e^{-s\theta}F(s) \tag{3.16}$$

Lastly, the Laplace transform of e^{at} is given, which is commonly used in differential equations:

$$\mathscr{L}\left(e^{at}\right) = \int_0^\infty e^{(a-s)t}\, dt = \frac{1}{a-s}\left[e^{(a-s)t}\right]_0^\infty = \frac{1}{s-a} \tag{3.17}$$

Using these tools, it is possible calculate the transfer function of a closed-loop temperature controller, which will be done in the next section.

3.5.2. A Model for Temperature Control

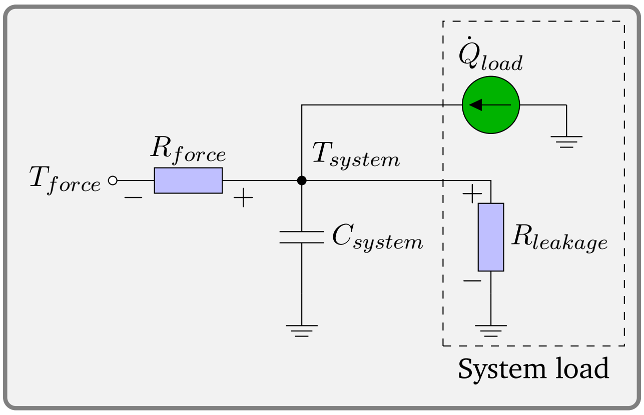


Figure 3.7.: Simple temperature model of a generic system.

In order to describe a closed-loop system using a transfer function $G(s)$, one has to first create a model for the process and the controller involved. This section will derive the simple, but very useful first-order model with dead-time. This model can be derived from the idea, that the system at temperature T_{system} has a thermal capacitance C_{system}, an influx of heat $\dot{Q}_{load}$ from a thermal load and a controller removing heat from the system through a heat exchanger with a resistance of R_{force}. Additionally, there is some leakage through the walls of the system to the ambient environment via $R_{leakage}$. This analogy of thermodynamics with electrodynamics allows to create the model shown in figure 3.7. Since this model is to be used for a temperature controller, more simplifications can be made and a so-called small-signal model can be developed as opposed to the large-signal model shown above. The small-signal model is an approximation around a working point that is valid for small deviations around it, similar to a Taylor approximation. The small-signal model can be used to calculate the system response to small changes of the controller output in order to estimate the control parameters.

Using the small-signal approach, the system response can be split into a constant and a dynamic par: the 0^{th} and 1^{st} order of the Taylor approximation. In order to simplify the system shown in figure 3.7 the assumption can be made that the system load $\dot{Q}_{load}$ and the flux through $R_{leakage}$ is *reasonably stable*. *Reasonably stable* means that it can be treated as small deviations and additionally any changes are within the bandwidth of the controller and well suppressed. This allows to treat them as (almost) constant effects, which result in an offset applied to the output of the controller. This leaves only the room with its heat capacity and the heat exchanger as dominant factors in the dynamic model shown in figure 3.8. Here T_{force} and T_{system} were replaced by T_{in} and T_{out} for better readability:

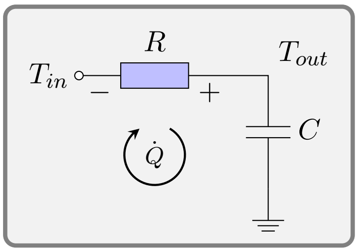


Figure 3.8.: Simplifications of the temperature model of a room lead to this first-order model.

This is the classic RC circuit. To calculate the transfer function, a relationship between T_{in} and T_{out} is required and exploiting the analogy of thermodynamics and electrodynamics again, using Kirchhoff's second law, following the arrow in figure 3.8 one finds:

$$\begin{aligned} \sum T_i &= 0 \\ T_{in}(t) - \dot{Q}(t)R - \frac{1}{C}\int \dot{Q}(t)\,dt &= 0 \end{aligned} \tag{3.18}$$

Taking the Laplace transform, applying equation 3.11, solving for $\dot{Q}(s)$ and using $T_{out} = \frac{1}{sC}\dot{Q}(s)$ to replace $\dot{Q}$, equation 3.18 can be written as:

$$\begin{aligned} T_{in}(s) - \dot{Q}(s)R - \frac{1}{sC}\dot{Q}(s) &= 0 \\ \dot{Q}(s) &= \frac{T_{in}(s)}{R - \frac{1}{sC}} = \frac{T_{out}}{\frac{1}{sC}} \end{aligned}$$

This allows to calculate the transfer function of the process P as

$$\begin{aligned} P(s) = \frac{T_{out}}{T_{in}} &= \frac{\frac{1}{sC}}{R - \frac{1}{sC}} \\ &= \frac{1}{sRC + 1} \\ &= \frac{1}{1 + s\tau} = \frac{K}{1 + s\tau} . \end{aligned} \tag{3.19}$$

with the system gain K and the time constant τ. In case of the RC circuit, the gain is 1, but other systems may have a gain factor of $K \neq 1$. This is generally the case when using any type of sensor that converts the measurand into the input signal. K is there included here for the sake of generality.

Equation 3.19 is called the transfer function of a first-order model, because its origin is a differential equation of first-order. This model describes homogeneous systems like a room very well, as can be seen in section 4.3.3, but in order to derive the transfer function including the controller and the sensor some more work is required to derive the sensor transfer function.

Expanding on figure 3.5a and equation 3.12 the open-loop transfer function of the process and its sensor becomes:

$$G(s) = P(s) \cdot S(s) \tag{3.20}$$

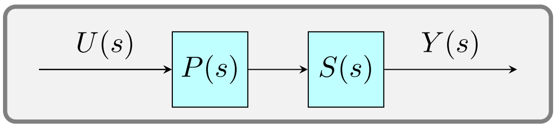


Figure 3.9.: Open-loop system with sensor.

and the block diagram changes to

The transfer function of the sensor, given an ideal linear transducer, can be modeled as a delay line with delay θ and $f(t-\theta) = H(t-\theta)$. A sensor gain of 1 is assumed here, because any system gain is assumed be included in the parameter K of the process transfer funciton. Using equation 3.15, $S(s)$ can be written as

$$S(s) = e^{-\theta s}. \tag{3.21}$$

The full system model including the time delay can now be written as:

$$G(s) = \frac{K}{1+s\tau} e^{-\theta s} \tag{3.22}$$

This is called a first-order plus dead-time model (FOPDT) or first-order plus time-delay model (FOPTD). While the Laplace representation is useful to mathematically explore the mode, in order to fit experimental data to this model it is more convenient to transform the transfer function 3.22 into the time domain. To have a meaningful result, an input $U(s)$ is required, because $G(s)$ is only a transformation. In principal any function can be serve this purpose, but a step function is typically used, for example by Ziegler et al. [237] and many others [27, 145, 163, 196, 197, 198, 221]. The step function is both simple to calculate and to apply to a real system in form of a controller output change. This technique is also explored in more detail in section 4.3.3. Using equations 3.15 and 3.17, the Heaviside $H(t)$ step function transforms into

$$\mathscr{L}\left(u(t)\right) = U(s) = \mathscr{L}\left(\Delta u H(t)\right) = \frac{\Delta u}{s} \tag{3.23}$$

with the step size Δu. The output response $Y(s)$ of the system to the step can then be calculated analytically.

$$\begin{aligned}
Y(s) &= U(s) \cdot G(s) \\
&= \frac{\Delta u}{s} \frac{K}{1+s\tau} e^{-\theta s} \\
&= K \Delta u \frac{1}{s(1+s\tau)} e^{-\theta s} \\
&= K \Delta u \left(\frac{1}{s} - \frac{\tau}{s\tau + 1} \right) e^{-\theta s} \\
&= K \Delta u \left(\frac{1}{s} - \frac{1}{s + \frac{1}{\tau}} \right) e^{-\theta s}
\end{aligned} \tag{3.24}$$

To derive $y(t)$, the inverse Laplace transform of $Y(s)$ is required. Unfortunately, this is not as simple as the Laplace transform. Fortunately though the required equations were already derived in equations 3.11 and 3.17. Making sure causality is guaranteed as shown in equation

3.16, the simple first-order model can be transformed back into the time domain.

$$\begin{aligned}
y(t) &= \mathscr{L}^{-1}\left(Y(s)\right) \\
&= K\Delta u \mathscr{L}^{-1}\left(\frac{1}{s}e^{-\theta s}\right) - K\mathscr{L}^{-1}\left(\frac{1}{s+\frac{1}{\tau}}e^{-\theta s}\right) \\
&\overset{3.17}{=} K\Delta u \cdot 1 \cdot H(t-\theta) - \left(e^{-\frac{t-\theta}{\tau}}\right) H(t-\theta) \\
&= K\Delta u \left(1 - e^{-\frac{t-\theta}{\tau}}\right) H(t-\theta)
\end{aligned} \tag{3.25}$$

The time domain solution of the FOPDT model can now be used to extract the parameters τ, θ and K from a real physical system.

The procedure can be summarised from the above as follows. The controller must be set to a constant output and the room must be given time to reach equilibrium. Once the temperature has settled, an output step of Δu is applied. The system will respond after a time delay and then follow an exponential function. A simulation of the step response applied to a first-order model with time delay is shown in figure 3.10. The gain is $K = 1$. The solid black line shows the response of the transfer function, including the system and the sensor. The dashed lines show the individual components, the Heaviside function governing the delay and the exponential term of the system. The controller output step $\Delta u = 1$ is applied at $t = 0$ and not shown explicitly. From figure 3.10 it can be clearly seen, that the sensor does not register a change until the time delay θ has passed and the Heaviside function changes from 0 to 1. Then the system responds with an exponential decay towards 1.

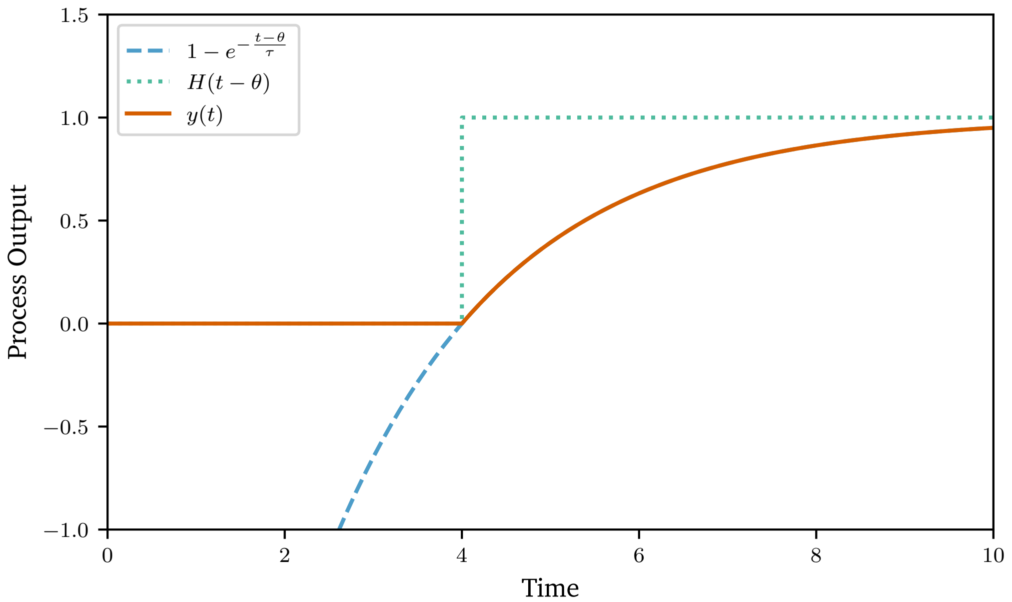


Figure 3.10.: Time domain plot of a first-order plus dead time model showing individual components of the model and the composite function $y(t)$. Model parameters used: $K = \Delta u = 1, \tau = 2, \theta = 4$.

So far, only open-loop systems were discussed. Using the FOPDT model, the system parameters can now be extracted from an existing system using a fit to the time domain reaction of such a system to a step input. Having extracted the system parameters, the next thing to do is to design a controller around the system and close the loop to realize a controlled system. This is shown in the next section.

3.5.3. PID Controller Basics

While there are many different types of controllers, like the bang–bang controller utilized in many temperature controller, which turns on at a certain threshold and turns off at another one producing the saw-tooth shaped room temperature curve shown in figure 3.3, a continuous control system is desired to keep fluctuations to a minimum. The most commonly used controller type for non-integrating systems is the proportional–integral–derivative (PID) controller [69]. A non-integrating system is a system without memory whose steady state does not depend on previous inputs. The advantage of applying a PID controller is that the controller does not need any special knowledge about the system model. A universal PID is simple to implement and can be tuned to control a wide range of different systems. While there are many variations of the PID algorithm [184], this section only introduces the basic, parallel, PID controller commonly used in digital implementation and deals with some of the shortcomings in practical applications.

In order to extend the FOPDT system derived in the previous section 3.5.2, with the PID controller one has to move to a closed-loop system. Adding to figure 3.5b and inserting a new control block into the transfer function yields figure 3.11.

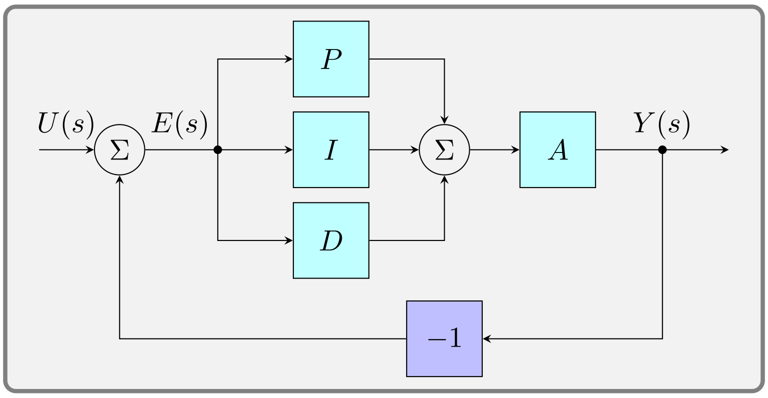


Figure 3.11.: Closed-loop system with a PID controller.

The error signal $E(s)$ used by the PID controller is the difference between the setpoint and the control parameter, in this case the room temperature. The transfer function of the PID controller can be split into three parts. A proportional part that is proportional to the error representing the present state, an integral part that is proportional to the accumulated error, representing the past state, and a derivative part, that is proportional to the change in the error, extrapolating into the future. Analytically, it can be written as

$$c(t) = k_p e(t) + k_i \int_0^t e(\tau)\, d\tau + k_d \frac{\mathrm{d}e(t)}{\mathrm{d}t} \tag{3.26}$$

$$C(s) = k_p + k_i \frac{1}{s} + k_d s\,. \tag{3.27}$$

The following discussion will mostly focus on equation 3.26, because, the time-domain equation is the one that can be implemented in software. As hinted above, there are a few shortcomings with the classic PID equation, when used in a real system which requires dynamic changes of the the setpoint or k_i.

The first problem that needs addressing is occurring when changing the PID parameter k_i, because equation 3.26 is given for a time-independent k_i. Assuming a settled system without external disturbances, the output is fully determined by the integrator value because the error is zero. Now, when k_i is changed, the output immediately changes, due to the change of the integral term. This is unintended. To fix it, the integral term must be changed to

$$k_i \int_0^t e(\tau)\, d\tau \Rightarrow \int_0^t k_i(\tau) e(\tau)\, d\tau \,. \tag{3.28}$$

This way, when adjusting k_i, its new value is applied to future error values only and there is no sudden kick.

The next issue is called derivative kick. When looking at the derivative part of equation 3.26, it can be seen that when instantly changing the setpoint, as in a step function, $\frac{\mathrm{d}e(t)}{\mathrm{d}t} \to \infty$. This behaviour is not intended and to fix this, the derivative part can be modified as follows.

$$\begin{aligned} \frac{\mathrm{d}e(t)}{\mathrm{d}t} &= \frac{\mathrm{d}\left(u(t) - y(t)\right)}{\mathrm{d}t} = \underbrace{\cancel{\frac{\mathrm{d}u(t)}{\mathrm{d}t}}}_{\to\infty} - \frac{\mathrm{d}y(t)}{\mathrm{d}t} \\ &= -\frac{\mathrm{d}y(t)}{\mathrm{d}t} \end{aligned} \tag{3.29}$$

The new derivative term is equal to the unmodified one, except in the case of setpoint changes. Removing the setpoint from the equation, the controller behaves as intended. This solution is sometimes called *derivative on measurement* as opposed to *derivative on error*.

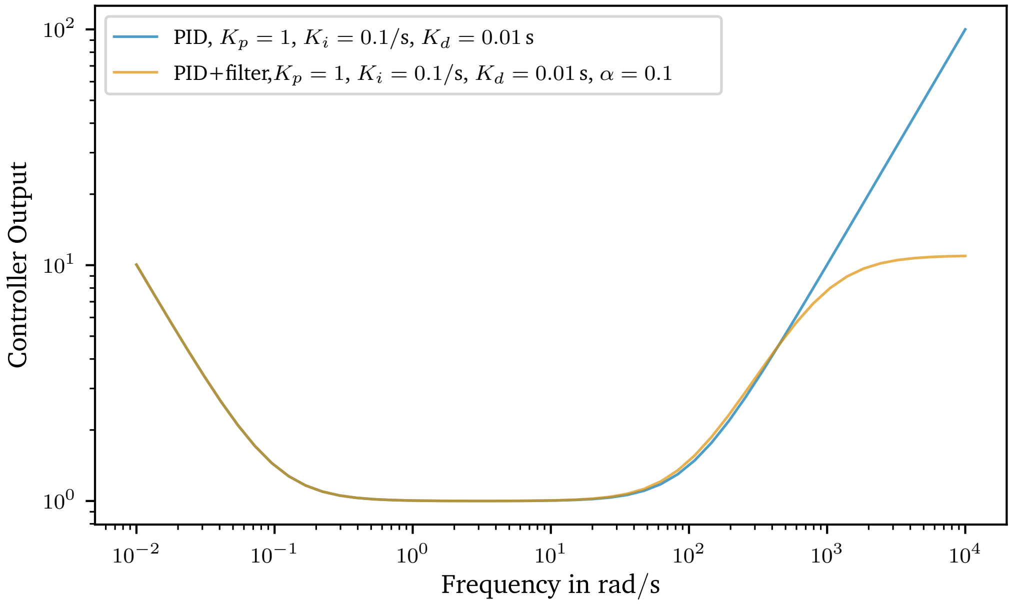


Figure 3.12.: Magnitude plot over frequency of the PID controller transfer function. Both the ideal PID controller and the PID controller with a filtered derivative are shown.

Another issue is caused by the derivative term with noiy inputs. Assuming there is a very short input spike due to noise, the differential of the derivative term will again be sent to very high values, pushing the output away from the correct value and forcing the controller to slowly rebalance.

To further discuss the problem and its solution it is best to visit the frequency domain and visualize the transfer function of the PID controller as shown in figure 3.12. The ideal PID controller without filtering of the derivative displays a very strong response to low frequency inputs. This is due to the integral action, which removes any (constant) offset. It needs to have infinite gain at DC to push the offset to zero. In reality this is limited by input noise. Then follows a plateau with a magnitude of k_p for the proportional term and finally the differential gain starts growing in magnitude and keeps steadily growing with rising frequency, just as expected.

With some knowledge about the process or the sensor it is possible to define an upper frequency, above which inputs become unrealistic and must therefore be unwanted noise. By filtering the derivative term with a first-order filter causes it to roll off and its gain becomes constant as shown in figure 3.12. By adding the filter the PID controller transfer function changes to

$$C(s) = k_p + k_i \frac{1}{s} + \frac{k_d s}{1 + s\alpha k_d} \,. \tag{3.30}$$

Typically α is in the range of 0.05–0.2 [184, p. 129].

An alternative is to filter the whole input. Depending on the filter cutoff, there is not much difference to equation 3.30, because the filter will not touch the proportional and integral part of the transfer function if both are well within its passband.

From figure 3.12 it also be seen, why in some publications, the gain k_p is applied to all three terms and k_i and k_d are replaced with T_i and T_d to accommodate for that.

$$C(s) = k_p \left(1 \frac{1}{T_i s} + \frac{T_d s}{1 + s\alpha T_d} \right) \tag{3.31}$$

Using this form allows to shift the curve up and down keeping its shape instead of just the k_p part, thus changing the corner frequencies. The alternative form is only given here for the sake of completeness. The author uses the ideal form shown in equation 3.27 with the parameters k_p, k_i, and k_d wherever possible.

This concludes the discussion of the PID controller and the introduction of the basic terms. It now begs the question how the controller interacts with the system and how to derive the optimal PID parameters from a given system or model. The next section discusses controller tuning rules and their effect on the system performance.

3.5.4. PID Tuning Rules

While many PID tuning rules can be found in the literature, their application depends on the underlying system and the desired system response. This section will discuss several proposed solutions and compare them to the authors use case. The sections aims to give a simple method to determine decent PI(D) parameters for the applications found in the lab. Among the methods discussed are the most classic set of tuning rules developed by Ziegler et al. [237], an improved version by Skogestad [198] which promises better performance for non-integrating systems. These rules all include simple instructions to extract the necessary parameters using pen and paper. Using a computer and fitting algorithms, the bar for *simple* has been raised considerably, so more complex approaches can be undertaken which extract more parameters from the system. Using these additional parameters more precise control is promised by Åström et al. [27, 28] with a method called AMIGO. Finally, it is possible to shape the control loop to result in a desired transfer function. This technique is mostly used in motor control [28, 184] and also requires the model parameters.

All of these rules will be compared against a demo model of a room to explain the details. It is the first-order model with delay which was derived in equation 3.22. The discussion is limited to the FOPDT model, because the systems treated in this work could be modelled very well using this equation. Higher order models are discussed in more details for example in [28, 184, 198], in case the reader encounters such a system and feels the need to extract the model parameters.

$$G(s) = \frac{Ke^{-\theta s}}{1 + s\tau} \tag{3.32}$$

The following parameters were extracted from Lab 011, using the techniques shown in section 3.5.2 using equation 3.25. The details are discussed in section 4.3.3. The system gain K was scaled to the full scale output ($4095\,\mathrm{bit}$) of the controller, hence the somewhat strange unit $\mathrm{K\,bit\,bit^{-1}}$.

Gain K	Lag τ	Delay θ
$13.07\,\mathrm{K\,bit\,bit^{-1}}$	$395\,\mathrm{s}$	$187\,\mathrm{s}$

Table 3.3.: Example paramters extracted from lab 011 using the techniques shown here and as applied in section 4.3.3.

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Before detailing the tuning parameters, the loop shaping method will be explained first, because it cannot only be used to derive custom rules but was also used to create the SIMC rules proposed by Skogestad [198]. The aim of this method is to derive a controller, that shapes the model in such a way, that a desired system response to setpoint changes is achieved. A general closed-loop system with a controller C and a system G is shown in figure 3.13. This will be used as a basis to find the required controller for a desired transfer function $\frac{Y(s)}{U(s)}$.

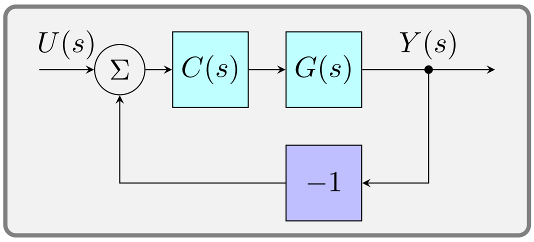


Figure 3.13.: Closed-loop system G with a controller C.

Starting with the transfer function of the controlled system, made up of the controller and the system, most experimenters would, at least in a feverish dream, prefer a transfer function of the following divine form

$$\frac{Y(s)}{U(s)} = 1\,,$$

but unfortunately life is more profane and there is no controller, that will always (and with warp speed) force a system to a certain setpoint. One may therefore settle for the second-best choice, a first-order low pass with a slow roll-off and a small delay, which must be added to ensure causality. One therefore arrives at

$$\frac{Y(s)}{U(s)} = \frac{e^{-\theta s}}{1 + s\tau_c}\,, \tag{3.33}$$

where τ_c is the closed-loop time constant and a measure for the aggressiveness of the controller. A small τ_c results in a more aggressive controller with faster response.

For the system shown figure 3.13 the closed-loop transfer function is found to be

$$\begin{aligned} \frac{Y(s)}{U(s)} &= \frac{C(s)G(s)}{C(s)G(s)+1} \\ \Rightarrow C(s) &= \frac{1}{G(s)} \frac{1}{\frac{Y(s)}{U(s)} - 1} \end{aligned}$$

This loop now needs to be shaped into the desired transfer function given in equation 3.33, so substituting $\frac{Y(s)}{U(s)}$ yields

$$\begin{aligned} C(s) &= \frac{1}{G(s)} \frac{e^{-\theta s}}{s\tau_c + 1 - \underbrace{e^{-\theta s}}_{\approx 1-\theta s}} && (3.34) \\ &\approx \frac{1}{G(s)} \frac{e^{-\theta s}}{s(\tau_c + \theta)}\,. && (3.35) \end{aligned}$$

$e^{-\theta s}$ was approximated using a first-order Taylor expansion. The desired controller response now only depends on the system (including the sensor) to be controlled. So, substituting the system equation 3.32 results in

$$\begin{aligned} C(s) &= \frac{1}{K} \frac{s\tau + 1}{(\tau_c + \theta)s} \\ &= \underbrace{\frac{1}{K} \frac{\tau}{\tau_c + \theta}}_{k_p} + \underbrace{\frac{1}{K} \frac{1}{\tau_c + \theta}}_{k_i} \frac{1}{s}\,. && (3.36) \end{aligned}$$

This is a PI controller with $k_p = \frac{1}{K}\frac{\tau}{\tau_c+\theta}$ and $k_i = \frac{1}{K}\frac{1}{\tau_c+\theta}$. From these calculations, it can be seen that a first-order model can be fully treated using a PI controller. Second-order (and higher order) models typically necessitate a PID or more sophisticated controller for optimal control. The problems discussed in this work mainly focus temperature control of (mostly) homogeneous objects, so the focus lies on the PI controller for most of the remaining section but the ideas and simulations can similarly be applied to the PID controller as well. Any caveats to be expected when treating a PID instead of a PI controller will be discussed.

Using the loop shaping technique, it is fairly easy to derive custom rules in case the model parameters can be extracted. As mentioned above, one such loop-shaped tuning rule is the SIMC rule set and the authors of those rules give advice for an ample variety of different models and also investigate the parameter choice regarding stability, load, and setpoint disturbances. Before attempting a custom approach, it is therefore recommended to check [198] for an appropriate set of rules for more complex models in order to save time and effort.

Tuning Rule	k_p	T_i	T_d	Source
Z-N PI	$\frac{0.9\tau}{K\theta}$	$\frac{\theta}{0.3}$	–	[237]
Z-N PID	$\frac{1.2\tau}{K\theta}$	2θ	$\frac{\theta}{2}$	[237]
SIMC PI	$\frac{\tau}{K(\tau_c+\theta)}$	$\min\left(\tau, 4(\tau_c+\theta)\right)$	–	[198]
SIMC PID	$\frac{\tau_1}{K(\tau_c+\theta)}$	$\min\left(\tau_1, 4(\tau_c+\theta)\right)$	τ_2	[198]
AMIGO PI	$\frac{0.15}{K} + \left(0.35 - \frac{\tau\theta}{(\tau+\theta)^2}\right)\frac{\tau}{K\theta}$	$0.35\theta + \frac{13\tau^2\theta}{\tau^2+12\tau\theta+7\theta^2}$	–	[28, p. 228]
AMIGO PID	$\frac{1}{K}\left(0.2 + 0.45\frac{\tau}{\theta}\right)$	$\frac{0.4\theta+0.8\tau}{\theta+0.1\tau}\theta$	$\frac{0.5\tau\theta}{0.3\theta+\tau}$	[28, p. 233]

Table 3.4.: PI/PID parameters for different tuning rules. The PI controllers assume a first-order model, the PID rules are required when dealing with a second-order model.

For reasons of brevity, in table 3.4, the PID parameters are given as k_p, T_i and T_d as introduced in equation 3.31. k_i and k_d can be calculated from

$$k_i = \frac{k_p}{T_i}$$
$$k_d = k_p T_d \,.$$

Regarding the SIMC PI/PID algorithm, Skogestad [198] and [221, ch. 5] suggests using $\tau_c = \theta$ for "*tightest possible subject to maintaining smooth control*". Following this recommendation, the minimum can be calculated from the parameters given in table 3.3 on page 37 as $\min\left(\tau, 4(\tau_c+\theta)\right) = \min\left(\tau, 8\theta\right) = \tau$.

Using the rules above, the full system can be simulated now. This was done using Python. The simulation source code can be found in data/simulations/sim_pid_controller.py as part of the online supplemental material [42]. The simulation can be used to model arbitrary PI(D) controller and arbitrary models can be used as well. It allows to compare different settings before applying them to a real system. It also considerably shortens deployment times because especially for systems with long timescales, it becomes difficult to test several parameter sets on the fly, thus a simulation can reduce deployment time to a few minutes instead of hours.

The simulation emulates the PID controller developed for the lab temperature controller. By default it has a sampling rate of 1 Hz. The simulation will apply a setpoint change of 1 K 10 s into the simulation. After the simulation, it will plot the time domain response of the controlled system. The setpoint change in this scenario is very similar to the load disturbances that are expected. Typically a noise source is used here instead, but in contrast to the statistical noise, which could be used to test for disturbance rejection, the situation in labs are different and cannot be modelled with stationary noise. While there is some noise coming from the sensor and the lab, the major disturbances are usually caused by the experimenters instead of the lab itself. These are events like a device being switched on or off for an extended period of time, longer than the controller needs to settle. This is equivalent to a setpoint change in terms of the error term in equation 3.26, since there is no difference in the error term between a setpoint and a process variable change. Do note, that this is not true for the PID controller, whose derivative term directly works on the measurement (or process variable) as this was explicitly implemented above. For PID controllers, there is a difference between the setpoint change behaviour and system noise rejection. This must be kept in mind and tested accordingly.

Simulating the model above and using the PI parameters derived from table 3.4, gives the plot shown in figure 3.14.

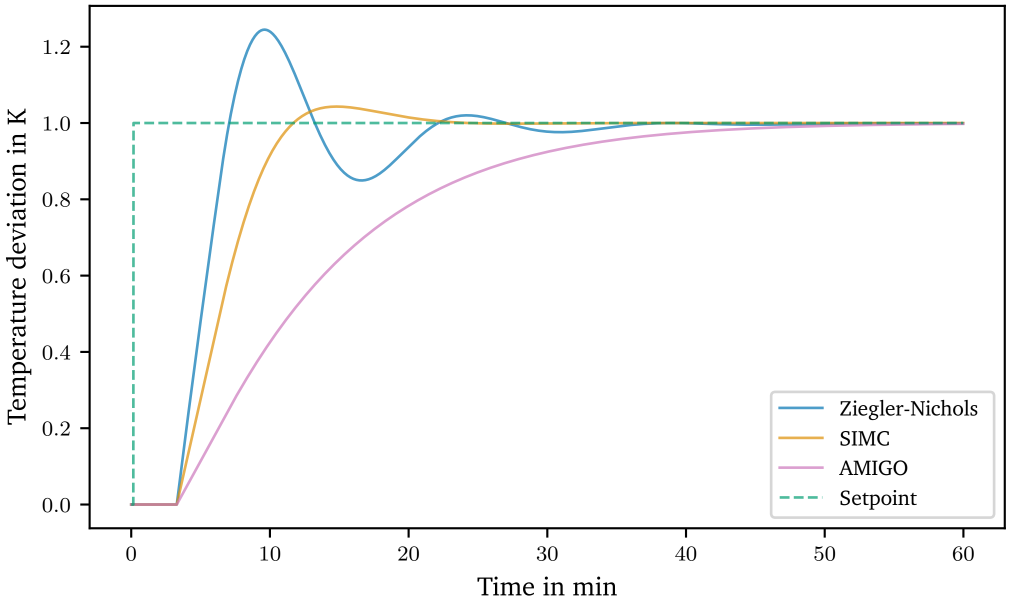


Figure 3.14.: Different PI Controllers tuned with parameter derived using the following methods: Ziegler-Nichols, SIMC and AMIGO. The system model is the FOPTD model for room 011.

As it can be seen in figure 3.14, the Ziegler-Nichols tuning rule produces a very aggressive PI controller, that shows quite a bit ringing, which is undesired for this application. The AMIGO rules are rather conservative, but do not produce any overshoot. The SIMC rules have proven the most useful for this application so far. This experience is in line with the results from Liebmann [121], who tested different PID tuning algorithms for their viability for temperature control in the labs discussed here.

To conclude, several PID tuning rules were presented and using a Python simulation tool it is possible to test a set of PID parameters before implementation. Using an example based on parameters extracted from a real environment, the different tuning rules were applied to a model for a real lab and the SIMC tuning rules were found to give the best results for this application. The reader should now be able to extract the model parameters from physical systems and have the tools to choose an optimal set of tuning parameters for the PID controller. Further reading recommendations are for a broad overview [184], and for more details [28].

3.6. Noise and Allan Deviation

The Allan variance [20] $\sigma_A^2(\tau)$ is a two-sample variance and used as a measure of stability. The Allan deviation $\sigma_A(\tau)$ is the square root of the variance. Originally, the Allan variance was used to quantify the performance of oscillators, namely the frequency stability, but it can be used to evaluate any quantity. In order to define the Allan variance, a few terms need to be defined first. A single measurement value of a time series $y(t)$ can be written as

$$\bar{y}_k(t) = \frac{1}{\tau} \int_{t_k}^{t_k+\tau} y(t)\, dt. \tag{3.37}$$

This is the k-th measurement with a measurement time or integration time τ. The latter term is frequently used for DMMs. t_k is the start of the k-th sampling interval including the dead time θ

$$t_{k+1} = t_k + T \tag{3.38}$$

with

$$T \coloneqq \tau + \theta. \tag{3.39}$$

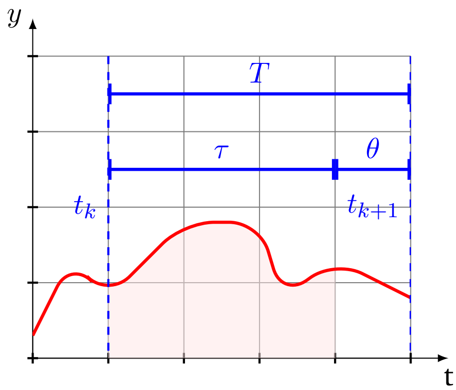


Figure 3.15.: Measurement interval according to equation 3.37. The shaded region is the signal acquisition period.

Using this, the deviation over N samples is defined as [20, 31]

$$\sigma_y^2(N,T,\tau) = \left\langle \frac{1}{N-1} \left(\sum_{k=0}^{N-1} \bar{y}_k^2(t) - \frac{1}{N} \left(\sum_{k=0}^{N-1} \bar{y}_k(t) \right)^2 \right) \right\rangle \tag{3.40}$$

The $\langle\,\rangle$ denotes the (infinite time) average over all measurands y_k or, simply put, the expected value.

The Allan variance is a special case of this definition with zero dead-time ($\theta = 0$) and only 2 samples:

$$\sigma_A^2(\tau) = \sigma_A^2(N=2, T=\tau, \tau) \tag{3.41}$$

$$= \left\langle \frac{\left(\bar{y}_{k+1} - \bar{y}_k\right)^2}{2} \right\rangle \tag{3.42}$$

It can be shown [31], that 3.43 is indeed more useful than $\sigma_A^2(N \to \infty, T = \tau, \tau)$, because $\sigma_A^2(N = 2, T = \tau, \tau)$ converges for processes, that do not have a convergent $\sigma_A^2(N \to \infty, T = \tau, \tau)$.

In practice, no experiment can take an infinite number of samples, so typically the Allan variance must be estimated using a number of samples m:

$$\sigma_A^2(\tau) \approx \frac{1}{m} \sum_{k=1}^{m} \frac{\left(\bar{y}_{k+1} - \bar{y}_k\right)^2}{2} \tag{3.43}$$

This estimation can lead to artifacts in the results as discussed later. In order to derive the Allan variance from a set of data points, the different values of τ are usually obtained by averaging over a number of samples as there is no dead time (by definition of the Allan variance).

Additionally, the Allan variance is mathematically related to the two-sided power spectral density $S_y(f)$ [31]:

$$\sigma_A^2(\tau) = 2 \int_0^\infty S_y(f) \frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2} \, df \tag{3.44}$$

and therefore all processes, that can be observed in the power spectral density plot can also be seen in the Allan deviation. The inverse transform however, is not always possible as shown by Greenhall [89].

Distinguishing different noise processes using the Allan deviation will be elaborated in the next section.

3.6.1. Identifying Noise in Allan Deviation Plots

It was already mentioned by Allan in [20], that types of noise, whose spectral density follows a power law

$$S(f) = h_\alpha \cdot f^\alpha \tag{3.45}$$

can be easily identified in the Allan deviation plot. The constant h_α is called the power (intensity) coefficient. The most common types of noise encountered in experimental data and their representations can be found in table 3.5, which serves as a summary of this section. Since those types of noise are present in any measurement or electronic device, it warrants a further discussion to understand their root causes and ideas to minimize them. While not a type of noise, linear drift can also be easily identified in the Allan deviation plot. It is therefore included in table 3.5 as well.

Amplitude noise type	Power-law coefficient α	Allan variance σ_A^2
White noise	0	$\frac{1}{2} h_0 \tau^{-1}$ [21]
Flicker noise	-1	$2 \ln 2\, h_{-1} \tau^0$ [21]
Random walk noise	-2	$\frac{3}{2} \pi^2 h_{-2} \tau^1$ [21]
Burst noise	0 and -2	$y_{rms}^2 \frac{\overline{\tau}^2}{\tau^2} \left(4e^{-\frac{\tau}{\overline{\tau}}} - e^{-\frac{2\tau}{\overline{\tau}}} + 2\frac{\tau}{\overline{\tau}} - 3\right)$
Drift	–	$\frac{1}{2} D^2 \tau^2$ [90]

Table 3.5.: Power law representations using the Allan variance.

In order to arrive at a good understanding of the features seen in an Allan deviation plot, this section will provide the reader with examples of each type of noise and the corresponding time domain, power spectral density and Allan deviation plot. Since a complete overview is not available in current literature, all required mathematical descriptions and simulation tools will be discussed here. The simulations were done using Python and the source code is linked to in the discussions. The files are found in the online supplemental material found at [42]. Using these scripts, all the graphs shown can be recreated and explored further.

White Noise

White noise is probably the most common type of noise found in measurement data. Johnson noise found in resistors, caused by the random fluctuation of the charge carriers, is one example of mostly white noise up to a bandwidth of $100\,\mathrm{MHz}$, from where on quantum corrections are required [79]. Amplifiers also tend to have a white noise spectrum at higher frequencies.

For the latter reason, white noise typically makes up for a considerable amount of noise in measurements, unless one works at very low frequencies. White noise is a series of uncorrelated random events and therefore characterised by a uniform power spectral density, which means there is the same power in a given bandwidth at all frequencies up to infinity. White noise therefore has infinite power (variance). In reality a measurement is always limited in bandwidth and hence the above property of a constant power spectral density only holds within that bandwidth. Those bandlimited samples of white noise thus have a finite variance. Since white noise is so common, a few of its properties should be mentioned. One such property is, that the variance σ^2_{x+y} of two uncorrelated variables x and y adds as:

$$\sigma^2_{x+y} = \sigma^2_x + \sigma^2_y + \underbrace{2\,\mathrm{Cov}(x,y)}_{\text{uncorrelated}\,=\,0} = \sigma^2_x + \sigma^2_y \tag{3.46}$$

This results in simple addition rules for variances from different sources, but it must be stressed here, that this property is only valid for uncorrelated sources like white noise, although it is usually incorrectly applied to all measurements which unfortunately obscures rather than clarifies the uncertainties involved.

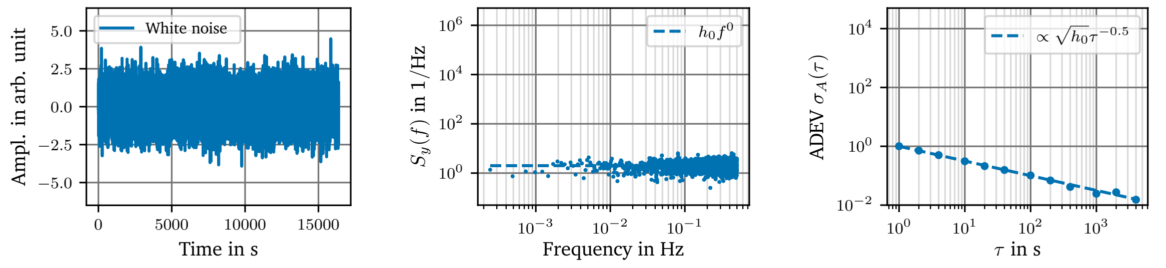
In order to demonstrate the effect of white noise in Allan deviation plots, it was simulated using the *AllanTools* library written by Wallin [224]. The noise generator is based on the work of Kasdin et al. [102]. The full Python program code is published online [42] and found in data/simulations/sim_allan_variance.py. To allow better comparison, all noise densities are normalized to give an Allan deviation of $\sigma_A(\tau_0) = 1$, with τ_0 being the smallest time interval.

Figure 3.16 shows a sample of white noise in its three different forms. Figure 3.16a is the time series representation from which the power spectral density was calculated and is shown in figure 3.16b. The dashed line shows the expectation value of the power spectral density and the Allan deviation.

From this simulation, several features can be observed. First of all, the power spectral density is flat and constant with $h_0 = 2$, which is in accordance with table 3.5 and the normalization mentioned earlier. Figure 3.16c shows the typical $\tau^{-\frac{1}{2}}$ dependence of white noise in the Allan deviation plot. This immediately explains, why filtering white noise scales with $\frac{1}{\sqrt{n}}$ with n being the number of samples averaged.

Burst Noise

Burst noise, popcorn noise, or sometimes referred to as random telegraph signal is a random bi-stable change in a signal and is caused by generation-recombination processes. This happens, for



(a) Time domain (b) Power spectral density (c) Allan deviation

Figure 3.16.: Different representations of white noise.

example, in semiconductors if there is a site, that can trap an electron for a prolonged period of time and then randomly release it. Impurities causing lattice defects are discussed in this context [55, 104, 105, 168]. Such lattice defects can also be introduced by ion implantation during doping. Fortunately, this type of noise has become less prevalent in modern manufacturing processes, because the quality of the semiconductors has improved. But if a trap site is located very close to an important structure, for example a high precision Zener diode, its effect might be so strong that it can be clearly seen.

The discussion is split into two parts. First the power spectral density is calculated and then the Allan variance is calculated using that result.

The spectral density of burst noise caused by a single trap site was derived in [137] by Machlup. Machlup used the autocorrelation function of the burst noise signal and applied the Wiener-Khinchin (Wiener-Хи́нчин) theorem, which connects the autocorrelation function with the power spectral density. A more detailed derivation can be found in [235], in this paper the preconditions like stationarity of the process, are also discussed. The burst noise signal consists of two energy levels, called 0 and 1, split by Δy. Multiple burst noise signals can be superimposed in a real device. This would then result in multiple levels, but they can be treated separately. The measurement interval over an even number of transitions, so that one ends in the same state as the measurement has started, is the time T. The mean lifetime of the levels is called $\bar{\tau}_0$ and $\bar{\tau}_1$:

$$\bar{\tau}_0 \approx \frac{1}{N}\sum_i^N \tau_{0,i} \qquad \bar{\tau}_1 \approx \frac{1}{N}\sum_i^N \tau_{1,i} \tag{3.47}$$

Figure 3.17 shows a burst noise signal along with the definitions above.

Using these definitions, one can then derive [137]:

$$R_{xx}(T) = \Delta y^2 \cdot \frac{\bar{\tau}_1\bar{\tau}_0 e^{-\left(\frac{1}{\bar{\tau}_1}+\frac{1}{\bar{\tau}_0}\right)T}}{\left(\bar{\tau}_1+\bar{\tau}_0\right)^2} \quad \text{and} \tag{3.48}$$

$$S(\omega) = 4R_{xx}(0)\frac{\frac{1}{\bar{\tau}_1}+\frac{1}{\bar{\tau}_0}}{\left(\frac{1}{\bar{\tau}_1}+\frac{1}{\bar{\tau}_0}\right)^2+\omega^2} \qquad \omega > 0. \tag{3.49}$$

Note, that the power spectral density is the one-sided version, hence an additional factor of 2 is included. The constant term was omitted here and can usually be neglected, because it is not relevant for calculating the power spectral density as it only contributes a single peak at

$\omega = 0$. Using the following definitions of the average time constant and the duty cycle

$$\frac{1}{\bar{\tau}} = \frac{1}{\bar{\tau}_1} + \frac{1}{\bar{\tau}_0} \quad \text{and} \tag{3.50}$$

$$D_i = \frac{\bar{\tau}_i}{\bar{\tau}_1 + \bar{\tau}_0} \qquad i \in \{0; 1\} \tag{3.51}$$

equations 3.48 and 3.49 can be rewritten to give a more intuitive form.

$$R_{xx}(T) = \Delta y^2 D_1 D_0 \, e^{-\left(\frac{1}{\bar{\tau}_1} + \frac{1}{\bar{\tau}_0}\right)T} \tag{3.52}$$

$$S(\omega) = 4R_{xx}(0)\frac{\bar{\tau}}{1 + \omega^2\bar{\tau}^2} \tag{3.53}$$

The special case $\bar{\tau}_0 = \bar{\tau}_1$ with $D_i = \frac{1}{2}$ is the previously mentioned case of random telegraph noise.

$R_{xx}(0)$ can be identified as the mean squared value of y:

$$y_{rms} = \sqrt{R_{xx}(0)}\,. \tag{3.54}$$

Equation 3.53 is a Lorentzian function and from this, it can be easily seen that a single trap site has a power spectral density, which is proportional to $\frac{1}{f^2}$ at high frequencies and is flat at low frequencies.

With the spectral density in hand, it is now possible to calculate the Allan variance as it was done by Van Vliet et al. in [218] for the classic example of random telegraph noise where $\bar{\tau}_1 = \bar{\tau}_0$. Do note that table I given by Van Vliet et al. shows the total number of events instead of the instantaneous number of events typically given. Hence their notation must be multiplied by $\frac{1}{\tau^2}$ (or $\frac{1}{T^2}$ in their notation). For the generic case with $\bar{\tau}_1$, $\bar{\tau}_0$ and the definition of $\bar{\tau}$ given in equation 3.50 one finds for the Allan variance of burst noise:

$$\sigma_A^2(\tau) = R_{xx}(0)\frac{\bar{\tau}^2}{\tau^2}\left(4e^{-\frac{\tau}{\bar{\tau}}} - e^{-\frac{2\tau}{\bar{\tau}}} + 2\frac{\tau}{\bar{\tau}} - 3\right) \tag{3.55}$$

Having arrived at equations 3.53 and 3.55 of the power spectral density and Allan variance, it it now possible to model it. For this purpose, parts of the Python library *qtt* [74] was used. This algorithm written by Eendebak et al. implements continuous-time Markov chains to simulate the burst noise signal. The result can be see in figure 3.18. For these simulations one of the time

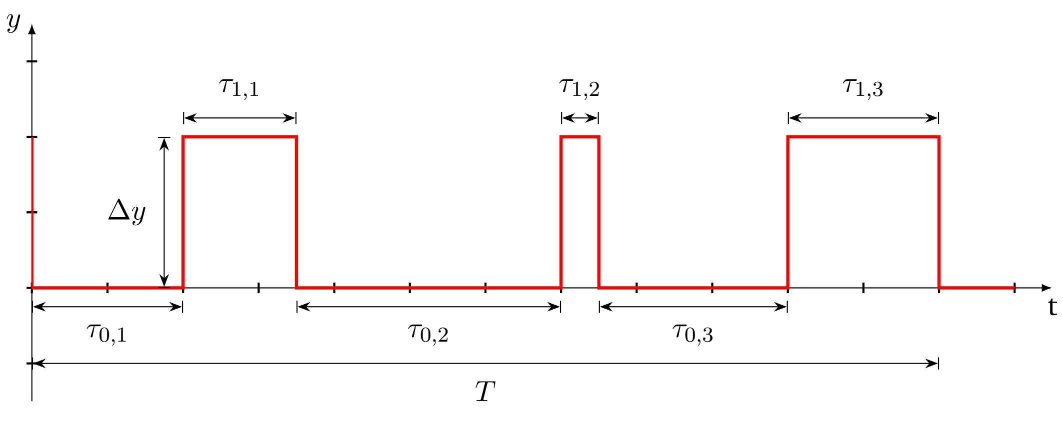


Figure 3.17.: A random burst noise signal.

constants, namely the lifetime of the lower state $\bar{\tau}_0$ was held constant, while the lifetime of the upper state was varied to show the effect of different $\bar{\tau}$. By looking at the time domain in figure 3.18a it can be seen, that the maximum average number of state changes can be observed, when $\bar{\tau}_1 = \bar{\tau}_0$. If $\bar{\tau}_1 > \bar{\tau}_0$ the system will favour the upper, while if $\bar{\tau}_1 < \bar{\tau}_0$ it will favour the lower state instead. This explains why the noise is strongest for random telegraph noise when $\bar{\tau}_1 = \bar{\tau}_0$, which can also be seen in the power spectral density plot in figure 3.18b. Looking at the Allan deviation in figure 3.18c confirms this, but also shows another interesting implication as it shows an obvious maximum. If the application allows a choice over the sampling interval τ, the effect of the burst noise can be mitigated by staying well clear of the maximum.

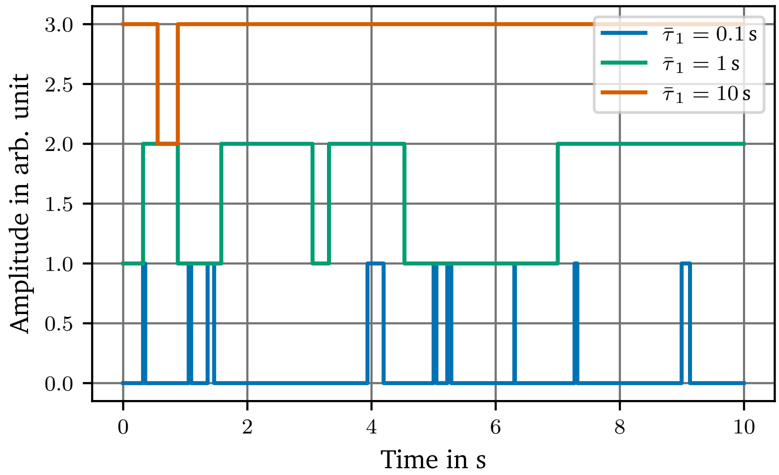
The small deviation from the analytical solution in figure 3.18c suggesting an upwards trend at large τ is a typical so-called end-of-data error. As it was discussed above, the Allan deviation can only be estimated given a limited number of samples using equation 3.43 and going to longer τ means there are fewer samples to average over.

The burst noise equations can be used to gain further insight into other types of noise. The first one is Shot noise, which is commonly found in photodetectors and lasers. Here, electrons or photons are created at discrete intervals resulting in an instantaneous signal. This means, that the lifetime of the upper level is very short in comparison to the lower level ($\tau_1 \ll \tau_0$) equation 3.49 becomes:

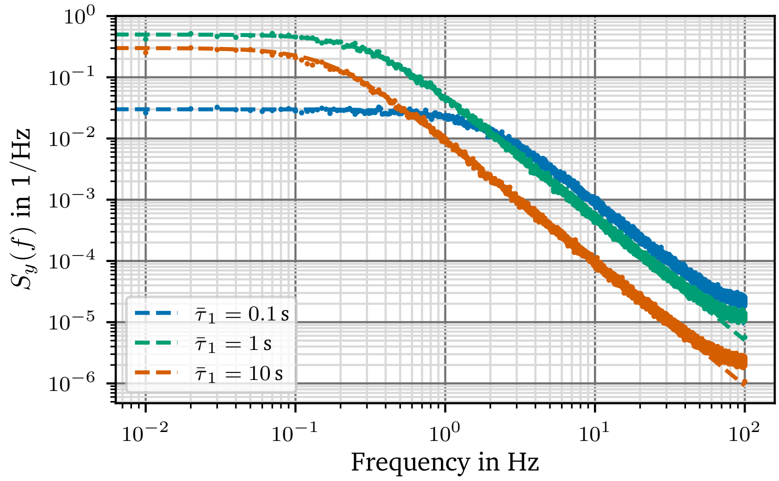
$$
\begin{aligned}
S_{Shot}(\omega) = S_{\tau_1 \ll \tau_0}(\omega) &= 4\Delta y^2 \frac{\tau_1}{\tau_0} \frac{\frac{1}{\bar{\tau}_1}}{\left(\frac{1}{\bar{\tau}_1}\right)^2 + \omega^2} \\
&= 4\Delta y^2 \frac{1}{\tau_0} \frac{1}{\frac{1}{\tau_1^2} + \omega^2} \qquad (3.56) \\
&\overset{\omega \ll 1/\tau_0}{\approx} 4\Delta y^2 \frac{\tau_1^2}{\tau_0} = \text{const.} \qquad (3.57)
\end{aligned}
$$

Typically, a very large number of such events happen. When not counting single events, but rather a stream, the relation $\omega \ll 1/\tau_0$ is valid and hence the result is a white spectrum as $S_{Shot}(\omega)$ is constant with respect to ω — just as observed in photodetectors and lasers.

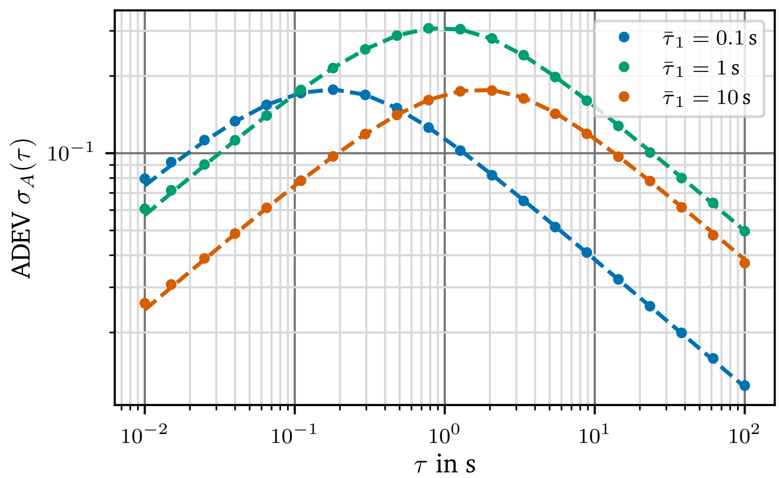
The other interesting occurrence is a case where many trap sites with different time constants are contributing to the noise. This can change the shape of the spectrum from f^{-2} to f^{-1} and is discussed in the next section.



(a) Time domain



(b) Power spectral density



(c) Allan deviation

Figure 3.18.: Different representations of burst noise for different $\bar{\tau}_1$ and fixed $\bar{\tau}_0 = 1\,\mathrm{s}$.

Flicker Noise

Flicker noise is also called $\frac{1}{f}$-noise and it can be observed in many naturally occurring phenomena. Its origin is not clear, even though there have been many explanations. An overview can be found in [77, 153, 165]. This section concentrates on flicker noise in electronic devices. In thick-film resistors, for example, it was shown to extend over at least 6 decades without any visible flattening [162]. In transistors, flicker noise is caused by the existence of generation-recombination noise or burst noise discussed in the previous section [77]. If there are many uncorrelated trap sites which contribute to the total noise, the envelope of the noise spectral density changes from $\frac{1}{f^2}$ to $\frac{1}{f^1}$ as shown in figure 3.19

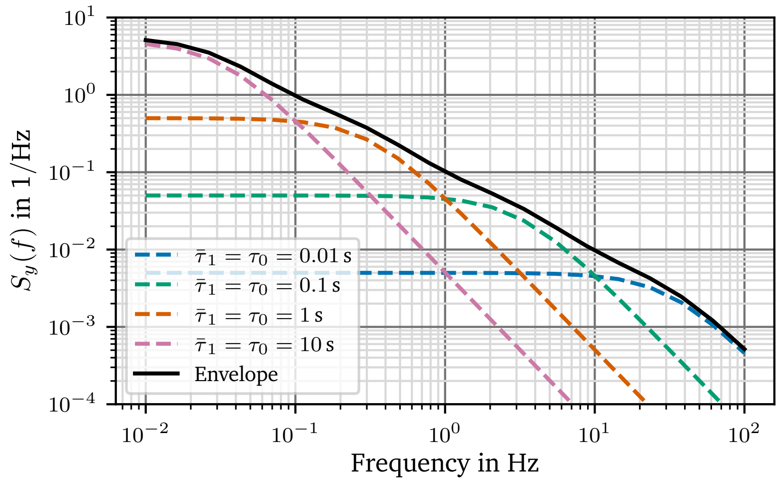


Figure 3.19.: Multiple overlapping Lorentzian noise sources forming a $\frac{1}{f}$-like shape.

Given that no trap site can store an electron indefinitely, the number of trap sites N with a certain time constant $\frac{1}{2}\bar{\tau} = \bar{\tau}_0 = \bar{\tau}_1$ must decline when going to longer time scales. Assuming N is inversely proportional to the time constant $\bar{\tau}$

$$N(\tau) \propto \frac{1}{\bar{\tau}}\,, \tag{3.58}$$

which can be motivated if the trapping process is thermally activated [73] and using equation 3.53 from the previous section, multiplying the weight function 3.58 and integrating over all possible storage times gives:

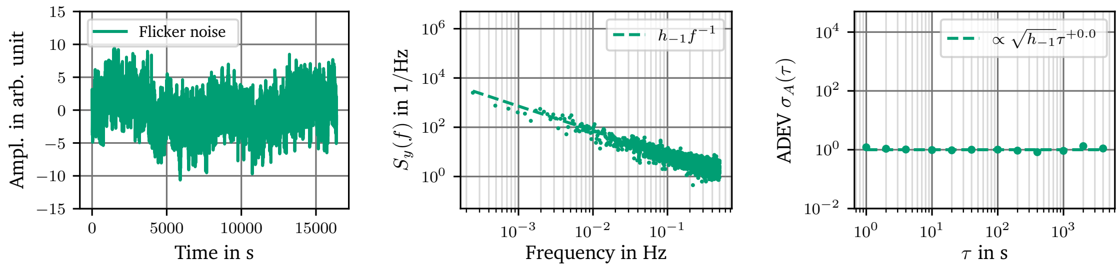
$$\begin{aligned}
S(\omega) &= \lim_{t\to\infty} \int_0^t N(\bar{\tau})\, 4R_{xx}(0) \frac{\bar{\tau}}{1+\omega^2\bar{\tau}^2}\, d\bar{\tau} \\
&\overset{\bar{\tau}_0=\bar{\tau}_1}{=} 4R_{xx}(0)\, C_N \lim_{t\to\infty} \int_0^t \frac{1}{1+\omega^2\bar{\tau}^2}\, d\bar{\tau} \\
&= \frac{4R_{xx}(0)\, C_N}{\omega} \lim_{t\to\infty} \arctan \bar{\tau}\omega \Big|_{\bar{\tau}=0}^{t} \\
&= \frac{4R_{xx}(0)\, C_N}{\omega} \cdot \frac{\pi}{2} \\
&= \frac{2\pi R_{xx}(0)\, C_N}{\omega} && (3.59) \\
S(f) &= h_{-1} f^{-1} && (3.60)
\end{aligned}$$

C_N is the proportionality constant of 3.58 and h_{-1} is the power coefficient introduced in 3.45. This shows, that for a large number of distributed trap sites, a noise spectrum of f^{-1} is found.

Using equation 3.44, the Allan variance can be calculated from the power spectral density:

$$\begin{aligned}\sigma_A^2(\tau) &= 2h_{-1}\int_0^\infty \frac{1}{f}\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}\,df \\ &= 2\ln 2\, h_{-1}\end{aligned} \tag{3.61}$$

Again, using the *AllanTools* library [224], flicker noise was simulated to give an impression of its properties.



(a) Time domain (b) Power spectral density (c) Allan deviation

Figure 3.20.: Different representations of flicker noise.

While it is not immediately evident from the power spectral density, the Allan deviation plot explains very well, why additional filtering does not affect flicker noise. No matter how long the integration time, the variance will still be the same.

The small wiggles at longer τ are typical end-of-data errors caused by spectral leakage, because there are insufficient samples to average over [95]. As it was discussed above, the Allan deviation can only be estimated using equation 3.43 given a limited number of samples. Therefore, at $\frac{\tau}{2}$ there are only 2 samples left, so there is no averaging possible to improve the estimate of the Allan deviation, which causes the oscillations at low frequencies or large τ.

As a last remark, a commonly used definition in combination with flicker noise is the corner frequency f_c. The corner frequency appears in situations where there is both flicker and white noise present. It is the crossover point in frequency, where the flicker noise is equal compared to the white noise.

$$f_c = \frac{h_{-1}}{h_0} \tag{3.62}$$

It can be graphically extracted from the power spectral density plot by drawing a line trough the flicker noise and the white noise and finding the intersection. This can be seen in figure 3.25 on page 54. The corner frequency can be found where the horizontal dashed blue and green line meet.

Random Walk

Random walk noise can be attributed to environmental factors such as temperature [219] and diffusion processes, the latter contributing to the ageing effect seen in semiconductors. It is a process, where in each time step the change is randomly determined to be either a positve or negative step with equal probability and a fixed step size. Its mean is

$$\langle y_n \rangle = \langle e_1 + e_2 + \dots e_n \rangle = \underbrace{\langle e_1 \rangle}_{=0} + \langle e_2 \rangle + \dots + \langle e_n \rangle = 0\,, \tag{3.63}$$

but its variance

$$\sigma_y^2 = \langle y_n^2 \rangle - \underbrace{\langle y_n \rangle}_{=0} = \sigma_{e_1}^2 + \sigma_{e_2}^2 + \dots \sigma_{e_n}^2 = n\sigma_e^2 \tag{3.64}$$

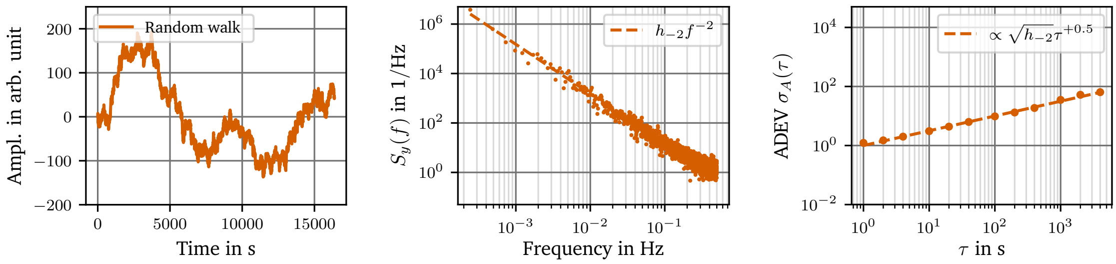
goes with n (or t). It therefore not a stationary process as can also be seen in figure 3.21c.
The power spectral density can be calculated [31, 102] to

$$S(f) = h_{-2}\frac{1}{f^2} \tag{3.65}$$

and the Allan deviation can again be calculated from the spectral density

$$\begin{aligned}\sigma_A^2(\tau) &= 2h_{-2}\int_0^\infty \frac{1}{f^2}\frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2}\,df \\ &= \frac{2}{3}\pi^2 h_{-2}\,\tau \end{aligned} \tag{3.66}$$

The *AllanTools* library [224] can then be used to simulate the random walk.



(a) Time domain (b) Power spectral density (c) Allan deviation

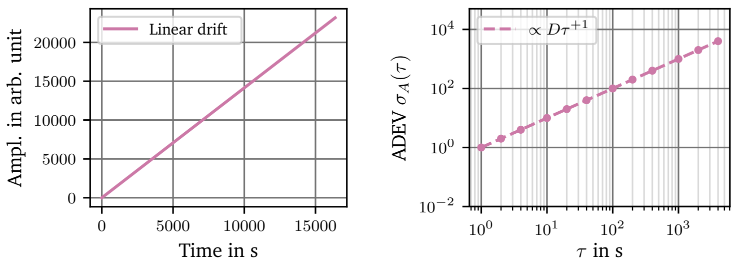
Figure 3.21.: Different representations of random walk noise.

Drift

Finally, the last feature of the Allan deviation plot that needs to be discussed is drift. Drift happens at very long time scales and describes a linear dependence of the measurand on time. This is also part of the ageing effect. Greenhall discussed the effect of drift [90] on the Allan variance and found the following relationship:

$$\sigma_A^2(\tau) = \frac{D^2}{2}\tau^2 \tag{3.67}$$

with slope of the drift D.



(a) Time domain (b) Allan deviation

Figure 3.22.: Different representations of linear drift.

Dead Time

The coefficients given in the previous examples were derived using the assumption, that all samples in a measurement are continuous with a dead time $\theta = 0$. Unfortunately, measurements sometimes have a dead time, that is non negligible. This problem was extensively discussed by Barnes et al. [31]. Dawkins et al. even developed special models to account for the algorithms of modern frequency counters [66]. While some frequency counters support gapless measurements, the situation is entirely different for digitizers and digital multimeters. Several settings commonly used affect the dead time, which can be considerable. It is therefore important to discuss typical measurement settings for voltmeters to estimate the errors that arise from those settings. The focus of this discussion lies on the dead time introduced by digital multimeters, but the application is not limited to this field.

The most commonly used settings that affect the dead time of a voltmeter are auto-zeroing and line synchronization. Auto-zeroing is done by adding additional measurements to the normal input integration cycle. To correct for the zero offset drift a zero measurement is added where the ADC is switched to the low terminal. Additionally, some devices add a reading of the reference voltage to correct for gain errors. The implementation details and type of measurements are manufacturer dependent.

The other setting, which can be enabled in voltmeters, is the line synchronization to increase the noise rejection of the instrument. This setting synchronizes the start of a measurement to the zero crossing of the power line. Depending on the instrument, this might cause a delay of one power line cycle (PLC) after each measurement if the instrument is not capable of processing the previous measurement while at the same time recording another one.

A simple measurement with dead time is shown in figure 3.15 on page 41. That model assumes that the dead time is constant and is always added after the actual integration time

τ. This is rarely true for real measurement data as many devices and even ADCs use internal averaging and auto-zeroing to produce a measurement. The actual dead time is therefore spread over the whole measurement and not limited to the end of the measurement. An example is the Keysight 3458A DMM that automatically switches to averaging when selecting integration times greater than 10 PLC. The reason is simple as for longer integration times, more and more flicker noise starts contributing to the measurement. The measurement is therefore split into single measurements of 10 PLC and, using auto-zeroing, the flicker noise is suppressed. This is discussed in more detail as an example in section 3.7. The mathematical problem of a distributed dead time was already noted by Allan [21] and it is distinctively different from the calculations made by Barnes et al. [31] for a single dead time at the end of the measurement. The exact mathematical treatment is complex and is beyond the scope of this work, especially considering that auto-zeroing does a lot more than just adding dead time at the end of the measurement. Fortunately, using a few assumptions, the problem can be greatly simplified.

An interesting observation can be made for white noise. Since it is uncorrelated, it makes no difference whether it is sampled in full, or only partially, and therefore the Allan deviation for a white noise process with or without dead time is the same:

$$\sigma^2(N,T,\tau) = \sigma^2(N=2,T=\tau,\tau) = \sigma_A^2(\tau)\frac{1}{2}h_0\tau^{-1} \tag{3.68}$$

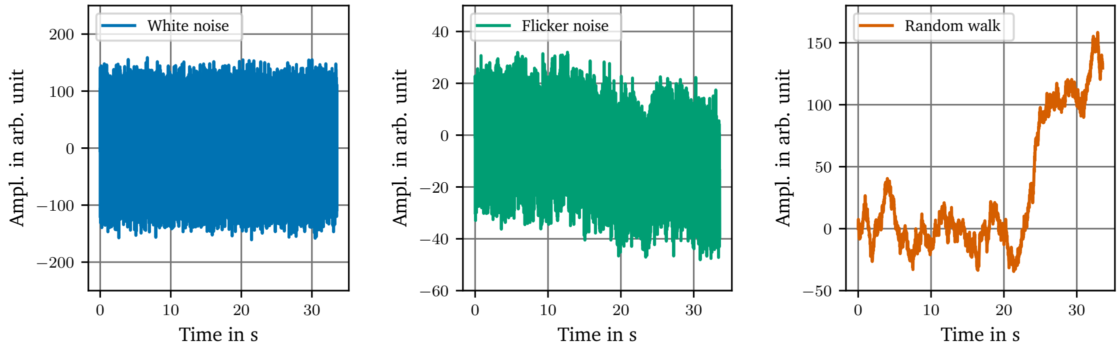
Consequently, if the dead time is added at a frequency high enough, so that the input amplifier output is dominated by white noise, the dead time will have no influence on the Allan variance.

Finally, Barnes et al. [31] notes that for measurement durations or averaging times $T \gg T_0$, the Allan variance with respect to T shows an asymptotic behaviour of $\sigma_A^2(T) \rightarrow \sigma_A^2(\tau)$.

3.6.2. Example

Using the results from the previous sections, it is possible to simulate a typical measurement sample containing white noise, flicker noise and random walk behaviour. The simulation was written in Python using the *AllanTools* library [224] to generate the time domain data, which was then converted to a power spectrum using the algorithm of Welch [225]. The Allan deviation was calculated using the *AllanTools*. The full Python source is available at [42] and found in data/simulations/sim_allan_variance_example.py. The time domain data shown here were downsampled from 2^{25} data points to 2000 points for faster plotting, using the Largest-Triangle-Three-Buckets (LTTB) algorithm created by Steinarsson [204]. The downsampling algorithm chosen is optimal for this application because it aims to visually keep the result the same, by favouring parts of the data where there is more dynamics. The only difference noticeable to the author is that the edges of the white noise plot are a slightly rougher. The full data set can be obtained using the source code given above if one desires. The power spectrum and the Allan deviation were always calculated from the full dataset. The data of the power spectrum were additionally binned to be evenly spaced on a logarithmic scale. This considerably reduced the high frequency noise and made the plot easier to read while not negatively impacting the shape.

The three time series shown in figure 3.23 were sequentially generated using a fixed seed for the random number generator to ensure repeatability as long as the order of creation is kept the same. For generating the noise, the algorithm presented by Kasdin et al. [102], implemented in the *AllanTools* library was used. The noise strength parameters were deliberately chosen in



(a) White noise (b) Flicker noise (c) Random walk

Figure 3.23.: Three separate noise components that were summed together to simulate a typical noise source.

such a way that both the white noise and the random walk part, have more noise power than the flicker noise. This allows to distinguish them in the plots at both extremes of the frequency scale. Finally, the three types of noise data were summed together to give the combined signal, which is shown in figure 3.24, again downsampled using LTTB. The summed series clearly shows the white noise content and it is possible to deduce some flicker or random walk noise, but it is highly obscured due to the amount of white noise. Using only the time domain plot makes it very hard to distinguish the type of noise present, let alone estimate the individual noise power of the three sources. Therefore, a different analysis tool is called for.

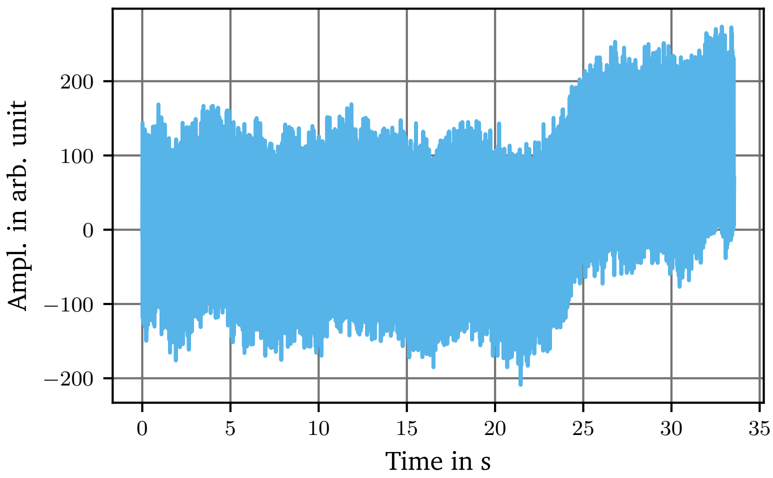


Figure 3.24.: A simulated time series containing white noise, flicker noise and random walk behaviour.

A common approach to identify noise sources is the power spectrum. It is easily accessible, even in real-time using spectrum analysers and, utilizing the computational power of modern computers, large time-domain data sets can be converted making this the method of choice in the lab. The power spectrum of figure 3.24 is shown in figure 3.25. It allows to clearly separate the white noise part from the other f^{α} components. The dashed lines representing the individual components were plotted using the h_{α} values calculated from the input parameters of the simulation. The noise spectral density h_0 of the white noise signal can be easily extracted even by hand without resorting to a fit. This yields $h_0 = 2 \times 10^{-3}/\text{Hz}$. h_{-1} and h_{-2} can be extracted as well using a fit to

$$S(f) = \sum_{\alpha=-2}^{0} h_\alpha f^\alpha \,. \tag{3.69}$$

The noise corner frequency f_c can either be calculated from h_0 and h_{-1} using equation 3.62 or determined graphically by constructing a tangent with a slope of -1 to the spectral density. From the intersection of the blue h_0 line and the green h_{-1} line the corner frequency is found to be $f_c \approx 1.8\,\text{kHz}$.

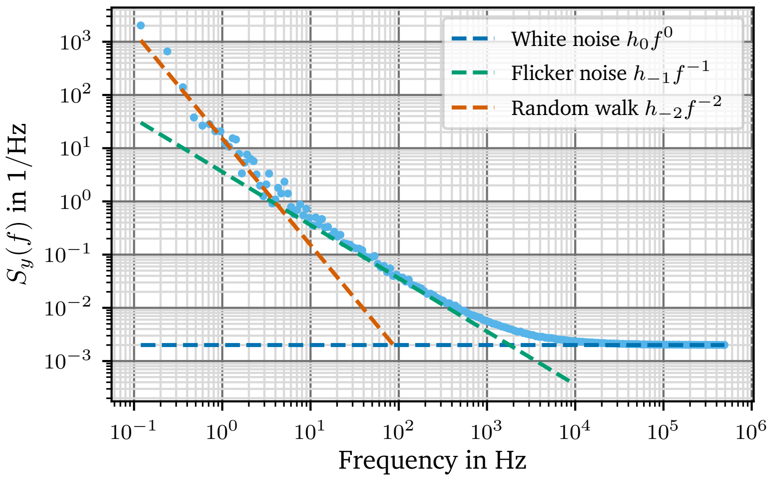


Figure 3.25.: A simulated power spectrum containing white noise, flicker noise and random walk behaviour.

To get an even better representation of the individual noise contributions, the Allan variance or Allan deviation can be used. The Allan deviation plot shown in figure 3.26 gives very clean results and all noise components can be clearly identified. The individual components were plotted using dashed lines as well.

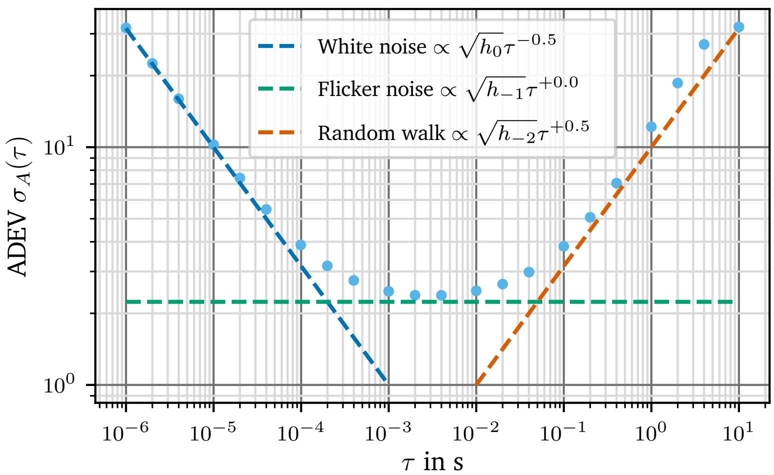


Figure 3.26.: A simulated Allan deviation containing white noise, flicker noise and random walk behaviour.

The Allan variance was calculated using the overlapping Allan variance algorithm [174] and only Allan deviation values for frequency values of $(1, 2, 4)$ per decade were plotted. The overlapping Allan variance gives a better confidence at longer intervals or lower frequencies, allowing to identify very low frequency noise like the random walk shown here. Reference [174] also gives a very good comparison of other algorithms to identify even more noise types in data sets like phase noise. Plotting only three values per decade improves the clarity of the plot, because at longer τs, even though the overlapping Allan variance is used, some oscillations inevitably show up. Using fewer values of τ causes less distractions in this case. From the figure 3.26, the Allan deviation of the flicker noise can be estimated from the flat minimum to be around 2.3 or $\sqrt{5}$. Using table 3.5 the Allan variance can be converted to

$$h_{-1} = \frac{5}{2\ln 2} \approx 3.6$$

Using the previously found h_0, this corner frequency is calculated using equation 3.62 to be:

$$f_c = \frac{5}{2 \times 10^{-3}/\text{Hz} \cdot 2\ln 2} \approx 1.8\,\text{kHz}$$

This is obviously the same result as the one from the geometric approach above.

This concludes the examples section for different noise types. The reader should now be able to identify different types of noise in measurement data and have learnt to appreciate the value that the Allen variance brings to the table. An example was presented that applied all techniques shown in this section to extract information about the noise sources in a dataset. Additionally, Python source code is provided to further explore the topic.

3.7. Auto-Zeroing

Auto-zeroing (AZ), sometimes called zero-drift or dynamic offset compensation, is such an important concept, that it must be discussed in its own right. The need for auto-zeroing comes from the typical behaviour of amplifiers. Every amplifier has some offset, be it small or large, and especially at high gains, this offset becomes a problem for high precision measurements. To make matters worse, this offset is not stable over time and drifts with both time and temperature. It can therefore not be calibrated out once, it must be permanently adjusted during operation, depending on environmental conditions. This procedure is called auto-zeroing.

There are many different ways to implement auto-zeroing and regarding operational amplifiers a good overview can be found in [94]. As an example, the auto-zero cycle for the Keithley `Model 2002` and the Keysight `3458A` Multimeter is shown in figure 3.27. Keithley uses a more complex and slower algorithm, while HP implemented a simpler but faster algorithm. The most simple (digital) approach is to regularly switch the input from the signal to zero, take a reading, then subtract this reading from all subsequent readings until a new zero reading is taken. An alternative approach adds another measurement of the reference voltage to apply a gain correction as well. This is done by the Keithley `Model 2002` and works very well to suppress gain drift in the input amplifier due to temperature changes but increases the time between samples by another $50\,\%$. The Keysight/HP `3458A` in the other hand calculates those gain corrections only during the manual auto-calibration (ACAL) routine to maintain a higher throughput.

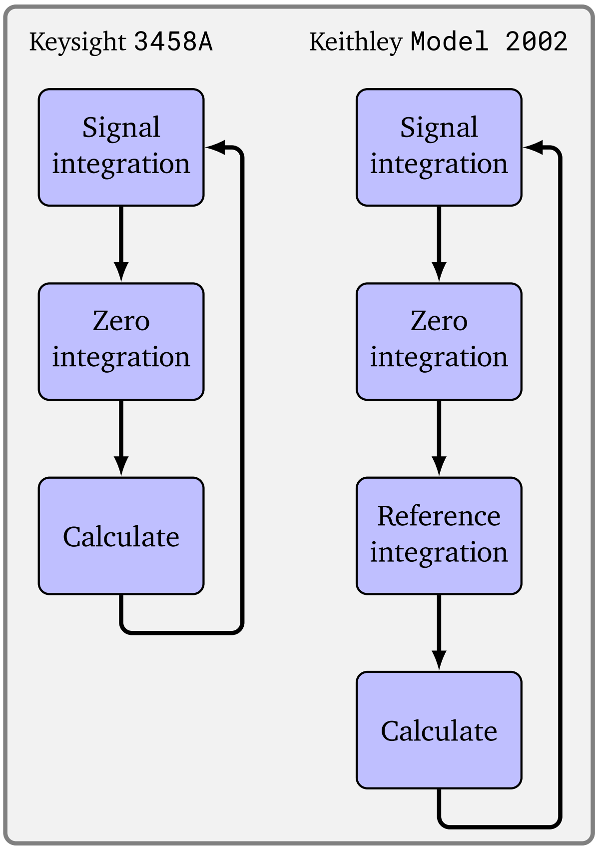


Figure 3.27.: Auto-zero phases of the Keysight `3458A` and Keithley `Model 2002`.

3.7.1. Offset Nulling

Offset nulling is the most basic approach to auto-zeroing. It aims to remove the offset drift of an amplifier. Especially at high gains, the offset, which is multiplied by the gain, can be substantial. In order to explain how offset nulling works and how it shapes the spectrum, it is best to discuss it based on an example. While this technique can also be found in many integrated circuits, it is more noticeable in DMMs, because it is a switchable option. Therefore, the example data set simulated is based on the parameters of the aforementioned Keysight `3458A` multimeter. The corner frequency and the white noise floor is modeled after the $10\,\mathrm{V}$ range of the `3458A` [112, 114] with the values given below. Do note that both references [112, 114] contain a typographical error. The corner frequency of the noise floor is erroneously given as $0.5\,\mathrm{Hz}$, but should be $1.5\,\mathrm{Hz}$. This can be seen in figure 2.35 in [112, p. 116], where the noise spectral density is plotted and it was also confirmed with the author [113]. The data used in this section is generated using the Python *AllanTools* library [224] and the simulation source code can be found in data/simulations/sim_auto-zero.py as part of the online supplemental material [42].

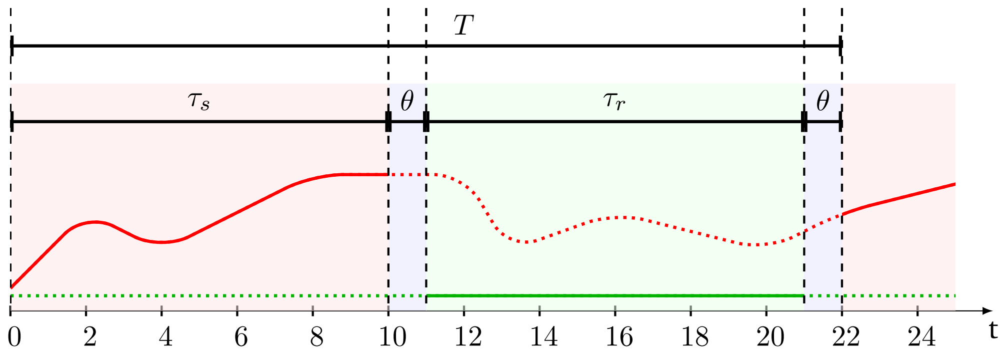


Figure 3.28.: Integration sequences of the offset nulling algorithm. Solid lines denote sampled data. Red is the input signal, green is the zero reading and blue is the dead time required for switching inputs.

For this simulation, a noise-free and arbitrarily chosen $10\,\mathrm{V}$ input is assumed to be sampled by the device at a sampling rate of $10\,\mathrm{PLC}$ at $50\,\mathrm{Hz}$, the same rate discussed previously on page 51. As it will be shown, the actual mean value of the input signal has no bearing on the outcome of the calculation when considering offset nulling, but its value must be considered for other types of auto-zeroing as discussed in section 3.7.2 and is included here only for the sake of completeness.

Figure 3.28 shows the individual sequences of the offset nulling algorithm. First, the source is sampled for $\tau_s = 10\,\mathrm{PLC}$, then the input is switched to the LO terminal. While this operation is very fast and takes less than $1\,\mathrm{ms}$ [173], if the instrument is synchronized to the line frequency the zero measurement will nonetheless be delayed until the next zero crossing, hence the dead-time $\theta = 1\,\mathrm{PLC}$. Finally, the zero reference is measured for another $\tau_r = 10\,\mathrm{PLC}$ and then the instrument switches back to the HI terminal.

The data is simulated in the following way: First, two sets of noise data are generated, a white noise spectrum with a noise spectral density of $165\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ and a flicker noise spectrum with an intensity scaled to result in a final spectrum with a corner frequency of $1.5\,\mathrm{Hz}$. The required flicker noise intensity is calculated using equation 3.62. To get a good low frequency estimate, $2^{20} \approx 10^6$ values were generated. Finally, the two noise data sets are summed with the noise-free input source to give the final result. Other effects, such as power-line hum are neglected in this simple simulation because it would needlessly overcomplicate the example and limit the educational value. The same goes for higher order random-walk f^{-2} noise components, which can be introduced by temperature fluctuations and other environmental effects and would be present in a real measurement.

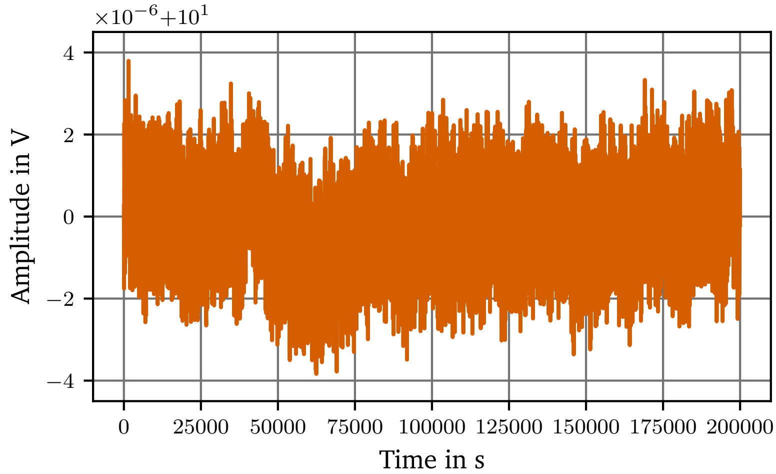


Figure 3.29.: Time series data with white noise and flicker noise.

The time domain plot of the simulation is shown in figure 3.29. The white noise component is clearly visible, while the f^{-1} flicker noise can be recognized, but its strength can hardly be estimated. It was already shown in section 3.6.2, that different types of noise have different frequency components and can be be distinguished in the frequency domain, which leads to the next approach.

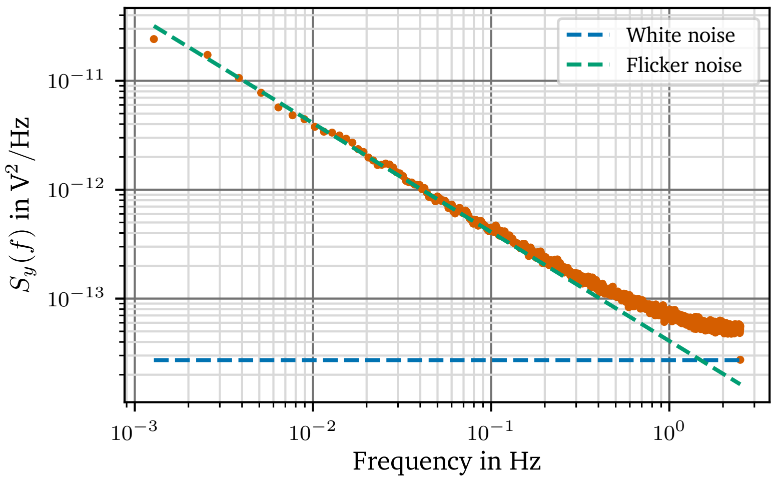


Figure 3.30.: Simulated power spectrum of a Keysight 3458A containing white noise and flicker noise. The line frequency is $50\,\mathrm{Hz}$.

The noise power spectral density shown in figure 3.30 is calculated from the time series and confirms the flicker and white noise content. The theoretical white noise floor is shown as a horizontal dashed blue line and the flicker noise as a dashed green line. The $1.5\,\mathrm{Hz}$ corner frequency, which is defined as the intersection between the f^{-1} noise and the white noise floor easily identified using those lines. It is evident that the 5 Hz sampling frequency with a 2.5 Hz bandwidth does not allow the spectral density to fully settle to the noise floor.

From the power spectral density is can be seen, that higher frequencies have a significantly lower noise spectral density than low frequencies. It is therefore most beneficial to do measurements at higher frequencies. To discuss the optimal measurement interval, the Allan deviation is an excellent tool.

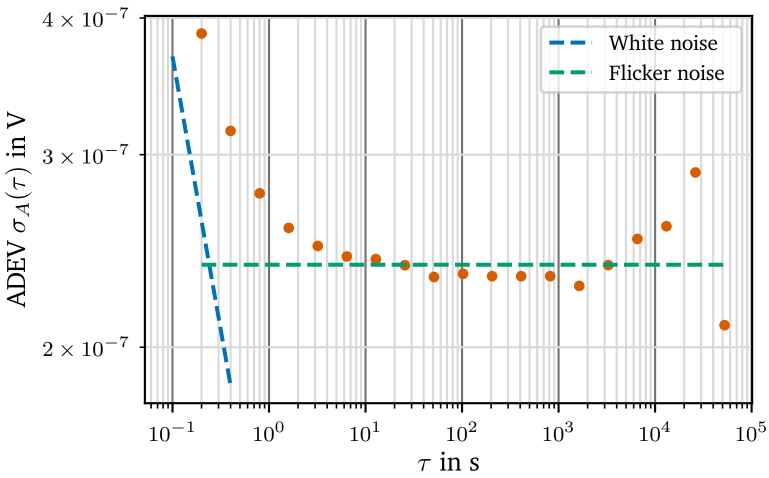


Figure 3.31.: Simulated Allan deviation of the input amplifier of a Keysight 3458A containing white noise and flicker noise. The line frequency is 50 Hz.

The Allan deviation it plotted in figure 3.31 and shows two distinct regions. Short τ display an asymptotic behaviour towards white noise with a $\tau^{-0.5}$ dependence and at longer τ the constant flicker noise region can be identified. At very long τ typical end-of-data oscillations can be seen, which are the result of the limited confidence of the Allan deviation estimator as previously discussed and can therefore be safely ignored. The Allan deviation clearly demonstrates the performance of the device at longer integration times and it is obvious, that beyond an integration time of about 1 s or 50 PLC no additional information can be extracted from the measurement and the variance is constant. This leads to the need for auto-zeroing to remove the flicker noise. It can be shown [182] that subtracting a reference measurement from the actual measurement data removes all correlated effects. Since flicker noise is autocorrelated, it can be removed by subtracting a zero measurement.

To demonstrate auto-zeroing, two cases will be discussed. Going back to figure 3.28 it can be seen that between switching inputs, a dead time θ is added. For a first discussion, this dead time is neglected and then the effect of adding a dead time is discussed in a next step.

Using figure 3.31 it was shown, that integrating over flicker noise, does not reduce the variance. In order to have as little flicker noise content in the final measurement value as possible, it is clear that the auto-zeroing should be done as fast as possible to keep the flicker noise content out. This allows to calculate the expected variance of the auto-zeroed measurement. The noise of the input measurement x and the reference measurement y are the same, because in this model the only noise source comes from the input amplifier, as the input signal is assumed to be noise-free. The zero level is, by definition, noise-free. As discussed above, the auto-zero interval is chosen, in such a way that its variance is dominated by white noise. The variance σ^2 of the combined measurement of $x - y$ can then be calculated using equation 3.46:

$$\sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 \tag{3.70}$$

By subtracting the zero reading the amplifier noise is effectively added twice to the final result, once for the input measurement and once for the zero measurement. Additional noise from the input signal noise would simply be added to this as it is uncorrelated as well.

Do note, that the number of samples is now half the number before applying auto-zeroing. This leads to an interesting effect. Taking for example a data set containing only white noise with a variance σ^2 and removing half the samples obviously does not change the variance as white noise is not correlated, but subtracting the samples is effectively decimating the data set and since the sampling rate is halved, the Nyquist band is halved as well. Unfortunately the input noise bandwidth stays the same. The second Nyquist band is then folded back into the first, thus doubling the noise power density.

To conclude, it is expected that the variance doubles and the power spectral density quadruples.

These considerations can be compared to the simulated data. Applying the auto-zeroing algorithm to the simulated data set, the constant $10\,\mathrm{V}$ input signal was nulled for every odd value and then the residual noise was subtracted from the signal value. The result in the time domain is shown in figure 3.32.

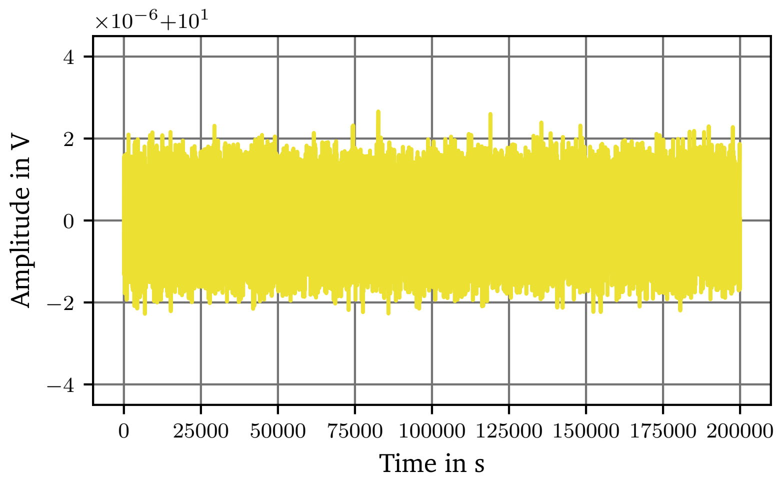


Figure 3.32.: Simulated measurement with auto-zeroing applied.

When comparing figure 3.32 to figure 3.29 it is immediately evident, that the f^{-1} component is no longer present. The difference in white noise strength is difficult to compare and it must be turned gain to the power spectral density. When calculating the spectral density it is important to remember, that the sampling rate is now halved. The result is shown in figure 3.33 along with dashed lines showing the noise content prior to applying the auto-zero algorithm as before in figure 3.30.

The power spectral density in figure 3.33 confirms an increase in the white noise power as discussed above and it can be determined that the white noise power $\sqrt{h_{-1}}$ has increased from $165\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ to $489\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$, an increase by a factor of $\sqrt{8.8}$, which is more than estimated from equation 3.70, including the factor of 2 for the decimation, which gauged the increase of $\sqrt{h_{-1}}$ to be $\sqrt{4}$. The reason for the additional noise was already mentioned above. There is still some substantial f^{-1} noise present at the auto-zero frequency of $5\,\mathrm{Hz}$. This type of noise is not uncorrelated and therefore the covariance is not zero, hence equation 3.46 does not strictly hold and additional correlated noise is leaking into the result. The hypothesis can be confirmed by increasing the sampling frequency by an order magnitude. Doing this, the white noise floor of the auto-zero measurement now only increases by a factor of $\sqrt{4.5}$, which is close to the expected factor of $\sqrt{4}$. The means, that the auto-zeroing frequency should be at least a decade above the noise corner frequency to be most effective.

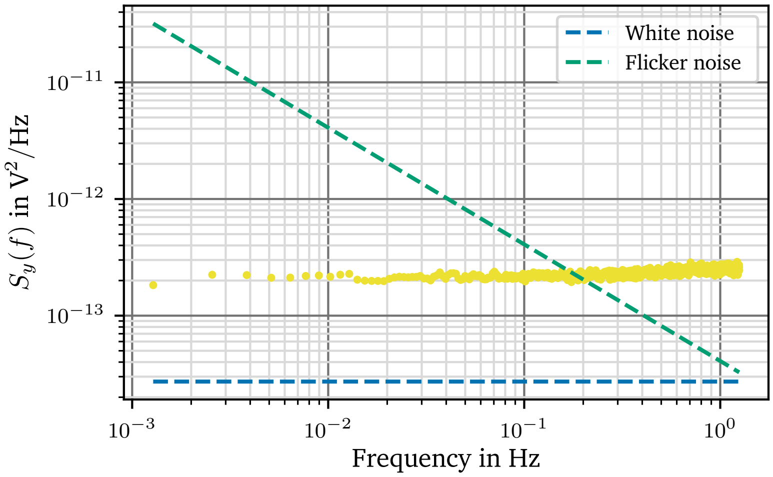


Figure 3.33.: Simulated power spectrum of a Keysight 3458A with auto-zeroing applied. The dashed lines denote the noise present prior to applying the auto-zero algorithm. The line frequency is $50\,\mathrm{Hz}$.

Nonetheless, down to very low frequencies, f^{-1} noise is effectively suppressed and the spectral density is almost perfectly flat. For reference, the dashed lines show the noise content that was in the dataset prior to auto-zeroing, which is less white noise, but far more flicker noise.

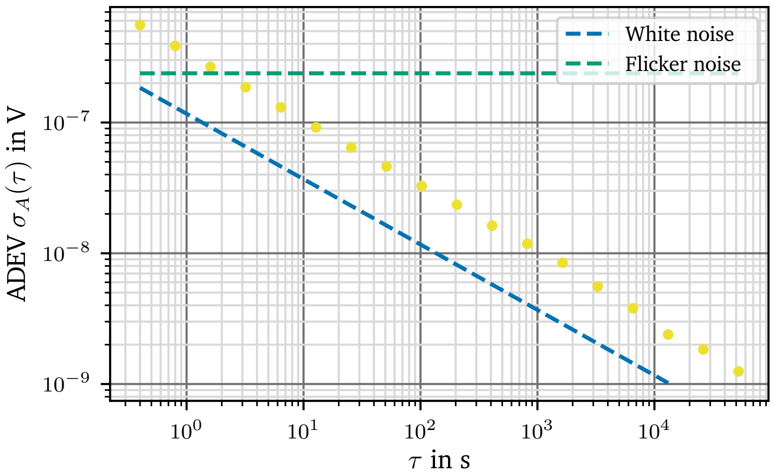


Figure 3.34.: Simulated Allan deviation of a Keysight 3458A with auto-zeroing applied. The dashed lines denote the deviation prior to applying the auto-zero algorithm. The line frequency is $50\,\mathrm{Hz}$.

The Allan deviation plot in figure 3.34 also confirms that white noise is the only component and shows a $\tau^{-\frac{1}{2}}$ dependence for the full range of integration times.

From this plot it can be seen, that for measurement times longer than about 2 s or 100 PLC, auto-zeroing has a clear benefit over a measurement without auto-zeroing. It must be noted though that, judging from this simulation, the device would reach a noise floor of 0.01 µV/V only at integration times of slightly more than 10 s, while the datasheet claims 2 s. Do note, that this simulation is for the 10 V range of the DMM and therefore 0.01 µV/V is 0.1 μV_{rms}. It is therefore likely that the noise parameters of a real device are better than the numbers used in the simulation. Additionally, the datasheet likely refers to an instrument that is synced to a 60 Hz power line frequency which shifts the sampling frequency up by 20 % and, as discussed, reduces the noise floor because more noise content is white noise at the auto-zero interval. In this simulation the 0.01 µV/V (0.1 μV_{rms}) noise level would be reached at exactly 10 s when using a line frequency of 60 Hz. For the purpose of demonstrating the auto-zeroing algorithms these subtleties are irrelevant.

For the comparison of different integration times before applying auto-zeroing figure 3.35 can be consulted. Using the Allan deviation makes it is very simple to compare noise figures for identical measurement times τ, yet different integration times, before auto-zeroing is applied. The simulation source code can be found in data/simulations/sim_optimal_autozero.py as part of the online supplemental material [42].

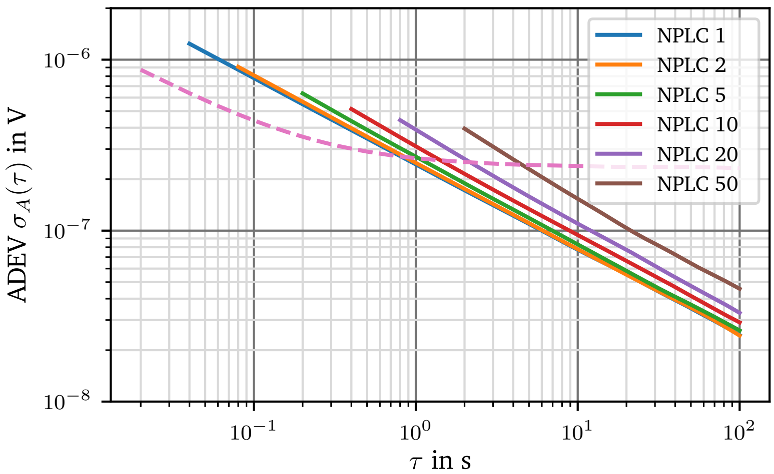


Figure 3.35.: Allan deviation for different integration times before applying the AZ algorithm. Dead time $\theta = 0$ s. The dashed line denotes the Allan variance without auto-zeroing. The line frequency is 50 Hz.

It can be seen, that with increasing integration times before applying the AZ algorithm more uncertainty is accumulated due to the f^{-1} content, which cannot be filtered. As a result, after removing the f^{-1} content using auto-zeroing more time is required for filtering until the same Allan deviation can be reached. From these simulations it can be concluded that when there is negligible dead time θ involved when switching the inputs, it is advantageous to switch early, while white noise is still dominating the noise content.

Finally, the case of a non-negligible dead time shall be treated. When the dead time has to be considered, it is clear, that the auto-zero frequency cannot be arbitrarily increased, because an increasing proportion of sampling time is lost to the dead time. This effective loss in sampling time then increases the noise spectral density due to aliasing as discussed above. To show this effect, the simulation above is modified to include a dead time of 1 PLC as detailed in figure

3.28. The dead time is added once after each measurement because the input is switched after each measurement. There are also alternative switching patterns like the one proposed by Schieder, R. et al. [182] splitting the measurement interval in two and instead of measuring HI-LO-HI-LO-HI-LO, to measure HI-LO-LO-HI-HI-LO. This scheme has both advantages and disadvantages, because the f^{-1} flicker noise is correlated and its autocorrelation function decays with e^{-t}. Therefore constantly changing the order of subtracted samples is not as efficient in removing the noise as the normal auto-zero procedure. Only when the dead time is large in comparison to the measurement time, this method yields an advantage. Some measurements also allow for another scheme. If the measurement is differential, the HI and LO input can be inverted without incurring the noise penalty of equation 3.70 because both measurements taken contain the desired data. This puts the auto-zeroing closer to a synchronous detection scheme, but this is outside the scope of this discussion. For the sake of simplicity, only the case of a HI-LO-HI-LO measurement mentioned first is treated here. To compare the zero dead time case with the non-negligible dead time case, the Allan deviation for different integration times is again evaluated in the same way as it was in figure 3.35. The results are shown in figure 3.36.

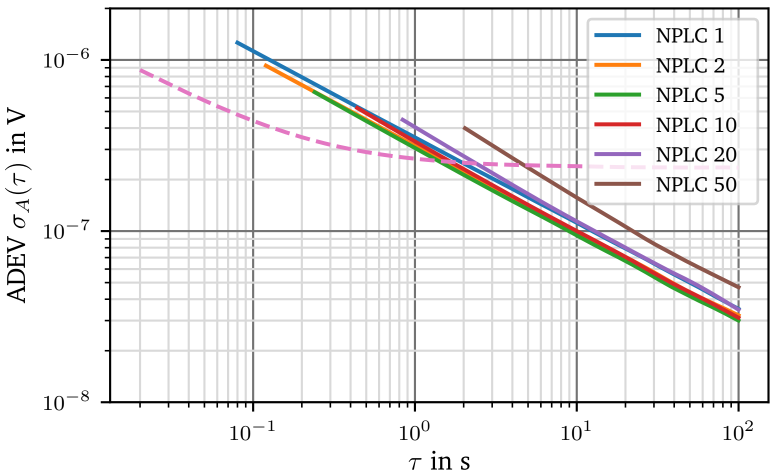


Figure 3.36.: Allan deviation for different integration times before applying the AZ algorithm. Dead time $\theta = 1\,\text{s}$. The dashed line denotes the Allan variance without AZ. The line frequency is $50\,\text{Hz}$.

Figure 3.36 demonstrates that the effectiveness of the AZ scheme no longer keeps increasing with an ever rising switching frequency. Instead, there is an optimal auto-zero interval. Above this optimal frequency, the portion of time spent with dead time is getting too large and too little information is collected. For the parameters chosen for this simulation ($f_c = 1.5\,\text{Hz}$ and $165\,\text{nV}/\sqrt{\text{Hz}}$), $5\,\text{PLC}$ at $50\,\text{Hz}$ is the optimal interval. If the corner frequency is shifted to a lower frequency, the optimum shifts more towards $10\,\text{PLC}$. The same goes for a higher line frequency of $60\,\text{Hz}$. This explains, why HP chose $10\,\text{PLC}$ as the maximum integration time. For integration times higher than that, software averaging is used delivering the performance shown in figure 3.36 along the $10\,\text{PLC}$ line.

It should be stressed here that the dead time is not the only factor to consider when choosing the auto-zero interval. For example, in case of an amplifier, switching the input also adds an error current due to the charge injection of the switching transistors. This may negatively impact the measurement of a high impedance source. These additional drawbacks are implementation specific and must considered already during the design phase.

3.7.2. Gain Correction

The effect of the gain correction, where the input value x is scaled by a scaling factor y to adjust the gain error, can be calculated, assuming white noise, as follows:

$$\begin{aligned}
\sigma_{x\cdot y}^2 &= \langle x^2 y^2\rangle - \langle xy\rangle^2 \\
&= \langle x^2\rangle\langle y^2\rangle + \underbrace{2\,\mathrm{Cov}\left(x^2, y^2\right)}_{\text{uncorrelated}\,=\,0} - \left(\langle x\rangle\langle y\rangle + \underbrace{2\,\mathrm{Cov}\left(x, y\right)}_{=\,0}\right)^2 \\
&= \left(\sigma_x^2 + \langle x\rangle^2\right)\cdot\left(\sigma_y^2 + \langle y\rangle^2\right) - \langle x\rangle\langle y\rangle \\
&= \sigma_x^2\sigma_y^2 + \sigma_x^2\langle y\rangle^2 + \sigma_y^2\langle x\rangle^2
\end{aligned} \tag{3.71}$$

With respect to the gain correction, equation 3.71 can be further simplified. The scaling factor is derived from the reference voltage V_{ref} and normalized using $\frac{V_{ref,measured}}{V_{ref}}$. The expected value, therefore is $\langle y\rangle \approx 1$, as the ADC full scale gain should not drift much. Furthermore, σ_y^2 is scaled by the constant $1/V_{ref}$ and $\sigma_x^2\sigma_y^2 \ll \sigma_x^2$. The latter should be true for any measurement of significance.

$$\sigma_{x\cdot y}^2 \approx \sigma_x^2 + \sigma_y^2\langle x\rangle^2 \tag{3.72}$$

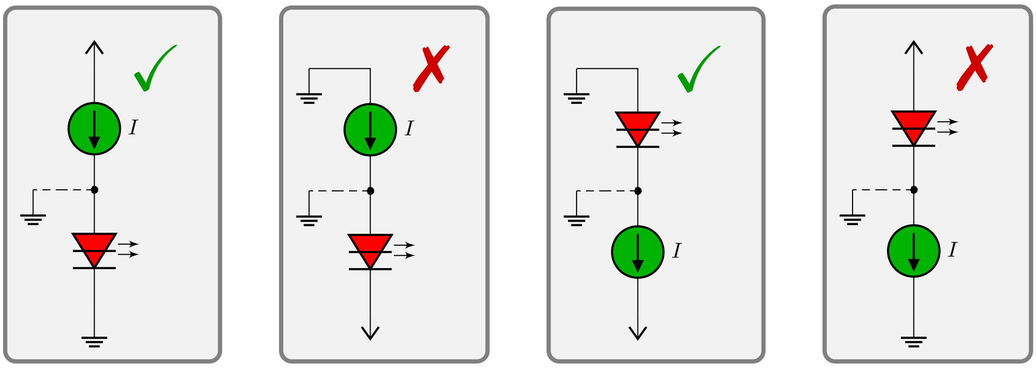
The gain correction noise therefore behaves similar to the offset correction case, except that it scales with the input voltage x and has no effect with a shorted input, while fully introducing its additional noise when a full scale input is applied.

3.8. Current Sources

Throughout this work the concept of current sources is widely used, for example section 3.2 discusses a current source to drive laser diodes and the temperature controller discussed in section 4.4 uses a current source to measure the resistance of a temperature sensitive resistor. While there are many more use cases, this section will limit the discussion to a few examples used by the devices presented in this work. Namely, a unidirectional transconductance amplifier with an operational-amplifier in conjunction with a field-effect transistor and a bidirectional Howland current pump invented by Bradford Howland in 1962 and first published in 1964 by Sheingold [190]. The discussion will start with the properties of the ideal current source and, based on this, develop a more accurate model. The models developed typically represent the static, time-independent case unless explicitly stated. First, the unidirectional current source is treated, then the bidirectional Howland current pump is discussed.

3.8.1. Current Sink and Current Source

The question whether to use a current source or a current sink is elemental for the design of a laser driver. Figure 3.37 shows different configurations of current sinks and sources with respect to the laser diode.



(a) Source with grounded LD.

(b) Source with floating LD.

(c) Sink with grounded LD.

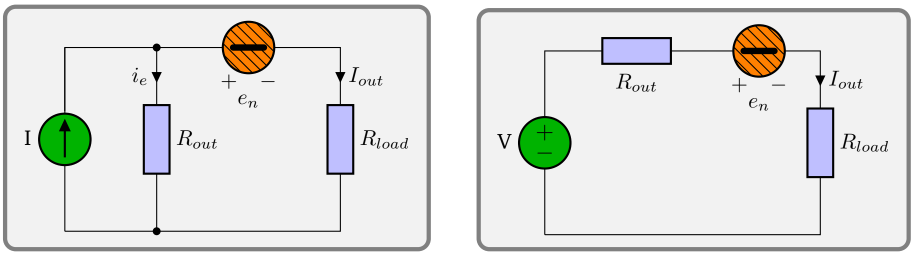
(d) Sink with floating LD.

Figure 3.37.: Different configurations of current sinks and sources with respect to the laser diode. A green check mark denotes a fail-safe configuration when accidentally shorting one or more pins of the diode to the laser chassis, illustrated by a dashed connection.

The optimal configuration depends on the laser diode and safety aspects in terms of protecting the laser diode. The protection of the laser diode is discussed first. The laser resonator is assumed grounded in the setup. This is not the design case, but incorrect assembly can facilitate this condition. While not intended, there are numerous ways to also accidentally short the diode to ground and since there are no immediate consequences arising from it, when the controller is disconnected, it might easily be overlooked. This blunder should not bear the risk of destroying an expensive laser diode. To ensure this, a configuration where the laser diode is shorted out, instead of the current source or sink, must be chosen. That way, the laser diode is automatically removed from the circuit in case of an error condition. Choosing between a current sink and a current source is more subtle. If the other shell of the laser diode is connected to the anode, a current sink can be considered to keep the diode can at ground potential. This is not an issue with the laser design in this group though, because the laser diode mount is floating. Another aspect is the electronics. A current source is typically implemented using p-channel field-effect transistors, while current sinks are using n-channel transistors and additionally the input of a current source is referenced to the positive supply, while the sink is referenced to the negative supply. Using the negative supply as a reference for control signals brings more challenges than vice versa, because typically integrated components like digital-to-analog converters prefer working with positive voltages and would need additional support to be floated to a negative reference. This makes a current source simpler to implement in this scenario and this work focuses on the current source. In principle all methods that will be discussed can be applied to a current sink as well.

3.8.2. Ideal Current Source

The ideal current source as shown in figure 3.38 has two major properties besides the output current I_{out}, the output impedance R_{out} and the compliance voltage, which are best understood when looking at the two equivalent representations of a current source separately. On the left in figure 3.38a, the Norton representation can be seen. Norton's theorem reduces any linear circuit to a current source, shown in green, with a parallel resistance R_{out}, usually called output resistance or impedance. On the right, the Thévenin representation can be seen, which simplifies a circuit as a voltage source, also shown in green, with a series resistance.



(a) Norton representation. (b) Thévenin representation.

Figure 3.38.: An ideal current source with output impedance R_{out} and noise e_n.

First, the output impedance is discussed. Ideally, R_{out} is infinite and all current is forced to flow through the load. Given a finite output impedance leads to a decreased accuracy of I_{out}, because it is influenced by the load impedance as

$$I_{out} = I_{set} \cdot \frac{R_{out}}{R_{load} + R_{out}} \,. \tag{3.73}$$

In addition to a decreased accuracy, inserting a noise voltage source between the current source and the load as shown in figure 3.38 in orange, has the same effect as a changing load resistance and due to the finite output impedance R_{out}, any voltage noise e_n translates to current noise i_n through the load as

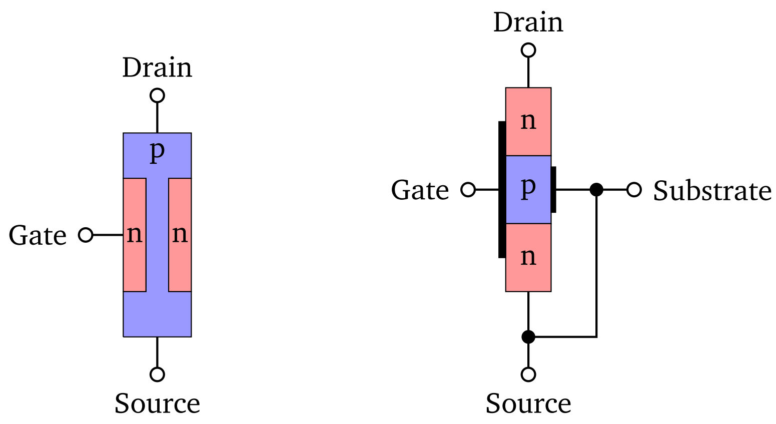
$$i_n = \frac{e_n}{R_{load} + R_{out}} \approx \frac{e_n}{R_{out}} \,, \tag{3.74}$$

again making a high output impedance desirable to suppress noise sources between the current source and the load.

Going to figure 3.38b of a current source in the Thévenin representation allows discussing the compliance voltage property. As it was said above, the output impedance of an ideal current source is infinite and so is the maximum output voltage of said current source. A finite output impedance immediately implies a finite supply voltage to keep the current to a finite limit, which dictates a maximum output voltage. This is called the compliance voltage.

3.8.3. The Field-Effect Transistor Current Source

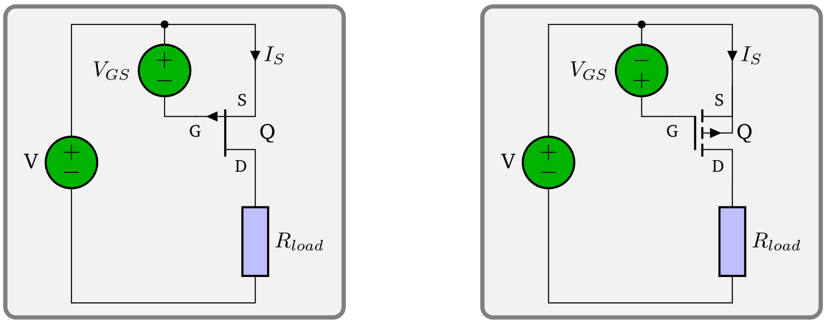
Given the limited supply voltage of a real current source drives the need for a resistive element that has a finite resistance and infinite, or very high, frequency dependent dynamic impedance to react to load changes. One such pass element, having these properties, is a field-effect-transistor (FET). A junction-gate field-effect transistor (JFET) or metal–oxide–semiconductor field-effect transistor (MOSFET) can be used either as a current source or sink, depending on its doping. A p-channel FET, which uses a positive doping of the channel, is a current source, while an n-channel FET works as a current sink. This discussion is focussing on the p-channel FET with MOSFETs at its centre, because it covers the bulk of the laser current driver design in section 3.2.



(a) P-Channel JFET. (b) P-Channel MOSFET.

Figure 3.39.: The simplified semiconductor structure of a JFET and a MOSFET.

The difference between a JFET and a MOSFET is the gate structure as illustrated in figure 3.39. While a MOSFET has an insulated gate, the JFET does not. This reduces the gate leakage current, typically by about three orders of magnitude and allows to forward bias the device since there is no diode, resulting in larger current handling capacity. So for low currents up to a few mA or low noise applications, JFETs are preferred, while MOSFETs can handle several hundred ampere. The same mathematical approach can be applied to both types of FETs though. The other difference between a JFET and a MOSFET is the fact that JFETs are only available as depletion-mode (normally-on) devices, while MOSFETs are available as both depletion and enhancement (normally-off) devices. The reason is the gate structure as mentioned above. An enhancement-mode device does not conduct when the gate-to-source voltage $V_{GS} = 0\,\mathrm{V}$, so V_{GS} must be decreased or the junction enhanced for the device to allow conduction. This is not possible with an uninsulated gate like a simple n-p junction of a JFET, which would then start conducting. A p-channel depletion-mode device on the other hand conducts at $V_{GS} = 0\,\mathrm{V}$ and V_{GS} must be increased and the junction depleted to reduce the current, which is possible with the uninsulated gate, because the n-p junction is reverse biased. The annotated circuit symbol and the quantities used to discuss the device properties are shown in figure 3.40.



(a) P-channel JFET. (b) P-channel MOSFET.

Figure 3.40.: Basic p-channel FET circuit.

A p-channel FET has its source (S) connected to the positive supply and the drain (D) is connected to a more negative voltage, typically the load. For the MOSFET the gate (G) is biased below the source to allow conduction. The source is usually connected to the substrate for solitary devices as shown in figure 3.39b. This will be assumed in all further discussions and the consequences of a substrate that is biased differently are omitted here. The interested reader may look up these details in [22].

As it was hinted above, if appropriately biased, a FET can be considered a voltage controlled current source. This property can be seen in figure 3.41.

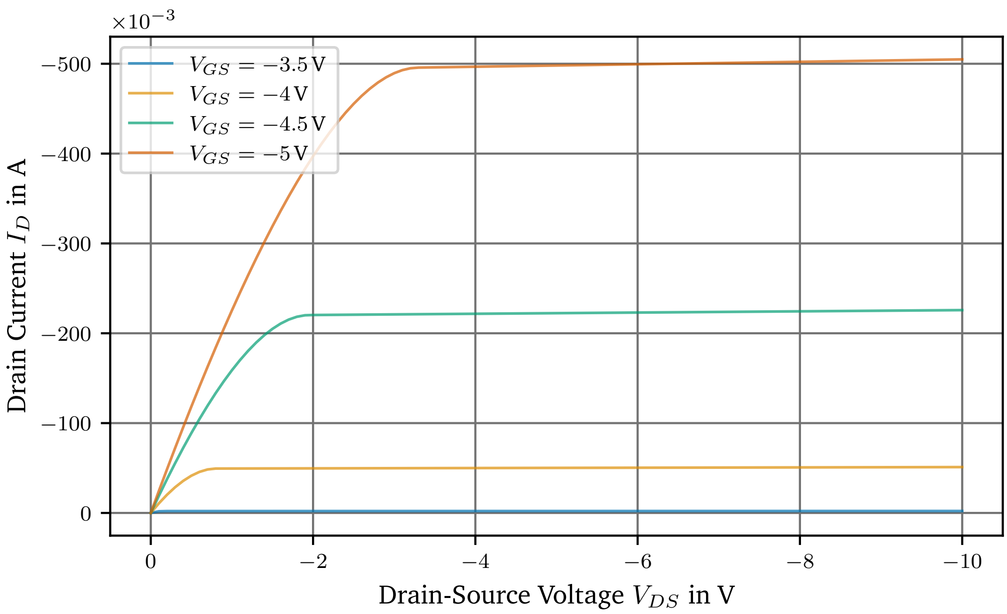


Figure 3.41.: Simulated drain current for different gate bias voltages of an `IRF9610` p-channel MOSFET.

Figure 3.41 shows the current I_D flowing out of the drain of a p-channel MOSFET over the drain-to-source voltage V_{DS} which is applied across the FET. For illustrative purposes an example p-channel MOSFET was chosen and its *Simulation Program with Integrated Circuit Emphasis* (SPICE) model [45, 93] was used to generate the data, yet the overall shape is the same for all FETs. For more information on modelling MOSFETs in SPICE, [56, p. 442] can be consulted. There are two regions, the first region, where $V_{DS} > V_{GS} - V_{th}$, demonstrates an almost linear correlation of the channel current and the voltage across the device. This is called the ohmic region, where the MOSFET behaves much like a (gate-) voltage controlled resistor and can be described [193] as

$$I_{D,ohmic} = \underbrace{\kappa(V_{GS} - V_{th})V_{DS}}_{\text{ohmic}} - \underbrace{\frac{1}{2}\kappa V_{DS}^2}_{\text{pinch off}} . \tag{3.75}$$

For small voltages V_{DS} the output current is proportional to the applied voltage V_{DS} across the channel just like a normal resistor, giving rise to its name ohmic region. As the voltage increases further I_D starts leveling off because V_{DS} starts affecting the channel conductivity. The channel is slowly getting pinched off at one end and becomes tapered. The reason is, that the voltage V_{DS} is dropped across the length of the channel. This voltage drop is linear with V_{DS}, resulting in a $-V_{DS}^2$ dependency of the current, reducing the conductivity of the channel. V_{th} is called the threshold voltage of a MOSFET or pinch-off voltage V_p in case of a JFET and is the voltage at which a current starts flowing.

The parameter κ is a device specific parameter and depends on process parameters and the geometry of the device.

$$\kappa = \kappa' \frac{W}{L} = \mu C_{ox} \frac{W}{L} \tag{3.76}$$

μ is the electron mobility, which is about $1350\,\text{cm}^2/\text{V}$ for n-channel MOSFETs and about $540\,\text{cm}^2/\text{V}$ for p-channel MOSFETs [187]. C_{ox} is the gate-oxide capacitance per unit area and determined by the thickness t_{ox} of the silicon dioxide layer of the gate

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \approx \frac{3.9 \cdot \epsilon_0}{t_{ox}} \approx \frac{3.45 \times 10^{-11}\,\text{F/m}}{t_{ox}} , \tag{3.77}$$

W is the width of the channel, and L is the length of the channel.

The letter κ is used here instead of the usual k as it is used by Sedra et al. [187] to avoid confusion with the Boltzmann constant k_B. Unfortunately, κ is not well controlled [94], because it is not just determined by the size, but also the doping of the material. While the size of the structure can be well controlled to within a few nm using lithography masks, the doping is a matter of temperature and time in a diffusion furnace. The ohmic mode of operation is, for example, used in switches or linear voltage regulators to control the output voltage of the regulator, forming a low impedance voltage source and not the desired current source. This brings up the next region to discuss.

Once the voltage V_{DS} has reached $V_{GS} - V_{th}$, the channel is fully pinched off, any further increase in V_{DS} will not lead to an increase in I_D, in other words the output resistance becomes infinite. The MOSFET is said to be pinched-off or in saturation. In practice there still is a small influence of V_{DS} on the channel. While the depth can no longer decrease as its length is 0 at one end already, the channel will retract a small amount in length with increasing V_{DS}. This is taken into account by the factor λ, called channel-length modulation. The drain current in saturation can now be described [193] as

$$I_{D,sat} = \underbrace{\frac{1}{2}\kappa\,(V_{GS} - V_{th})^2}_{\text{ideal FET}}(1 + \lambda V_{DS}) . \tag{3.78}$$

The parameter λ is the first-order Taylor expansion of the length dependence of κ and typically is small and on the order of $0.01–0.05\,\mathrm{V}^{-1}$ for p-channel MOSFETs [172, p. 23]. It mainly depends on the length of the channel to which it is inversely proportional, since the channel length defines the slope of the tapered channel. Sometimes the value $\frac{1}{\lambda}$ is also referred to as the Early voltage V_A. It is noteworthy, that more modern processes choose a smaller channel length to reduce the on-state resistance of the MOSFET because the main application of a MOSFET nowadays is as a switch. The reduced channel length makes the MOSFET more susceptible to the channel length modulation effect. This will be discussed in more detail in section 3.8.7, when choosing a suitable MOSFET.

Going back to figure 3.41 the effect of the channel-length modulation can be seen as a small slope of I_D in the saturation region.

Combining the previous equations, the FET drain current behaviour can be summed up as

$$I_D = \begin{cases} 0 & \text{if } V_{GS} - V_{th} < 0 \\ \kappa(V_{GS} - V_{th})V_{DS} - \frac{1}{2}\kappa V_{DS}^2 & \text{if } V_{GS} - V_{th} >= 0 \text{ and } V_{DS} < V_{GS} - V_{th} \\ \frac{1}{2}\kappa\left(V_{GS} - V_{th}\right)^2\left(1 + \lambda V_{DS}\right) & \text{if } V_{GS} - V_{th} >= 0 \text{ and } V_{DS} \geq V_{GS} - V_{th} \end{cases} \tag{3.79}$$

The saturation region is the region of interest for building a high output impedance current source, because for a wide range of V_{DS} the current remains almost constant and can be adjusted using the gate voltage V_{GS}. As a reminder, for the p-channel MOSFET, all voltages are reversed. V_{GS}, V_{th}, V_{DS}, κ and I_D are negative. Some datasheets therefore only give the magnitude of those quantities. The important aspect to remember is that for the p-channel enhancement-mode MOSFET the gate must be biased negative with respect to the source pin by a least the threshold voltage ($V_{GS} < V_{th}$ or $|V_{GS}| > |V_{th}|$) to turn the transistor on and allow current to flow.

Before proceeding to the precision current source in section 3.8.4, the concept of conductance and transconductance must be explored. The transconductance describes the relationship of the input voltage with the output current. The conductance is a measure for how well current flows from input to output. The transconductance g_m and the channel conductance g_{DS} are defined as

$$g_{m,sat} \coloneqq \left.\frac{\partial I_{D,sat}}{\partial V_{GS}}\right|_{V_{DS}=const} = \kappa\left(V_{GS} - V_{th}\right)\left(1 + \lambda V_{DS}\right), \tag{3.80}$$

$$= \sqrt{2\kappa I_D\left(1 + \lambda V_{DS}\right)} \approx \sqrt{2\kappa I_D} \tag{3.81}$$

$$g_{DS,sat} \coloneqq \left.\frac{\partial I_{D,sat}}{\partial V_{DS}}\right|_{V_{GS}=const} = \frac{1}{2}\kappa\left(V_{GS} - V_{th}\right)^2\lambda \tag{3.82}$$

$$= \frac{I_D}{\frac{1}{\lambda} + V_{DS}} = \frac{1}{R_o} \approx I_D\lambda\,. \tag{3.83}$$

The transconductance g_m, as a measure of the current gain with respect to the gate-source voltage of the MOSFET, is proportional to the square root of the drain current I_D. The inverse of the channel conductance g_{DS} is called output resistance R_o and discussed below. Typically the V_{DS} term in the denominator of the output resistance in equation 3.83 can be neglected.

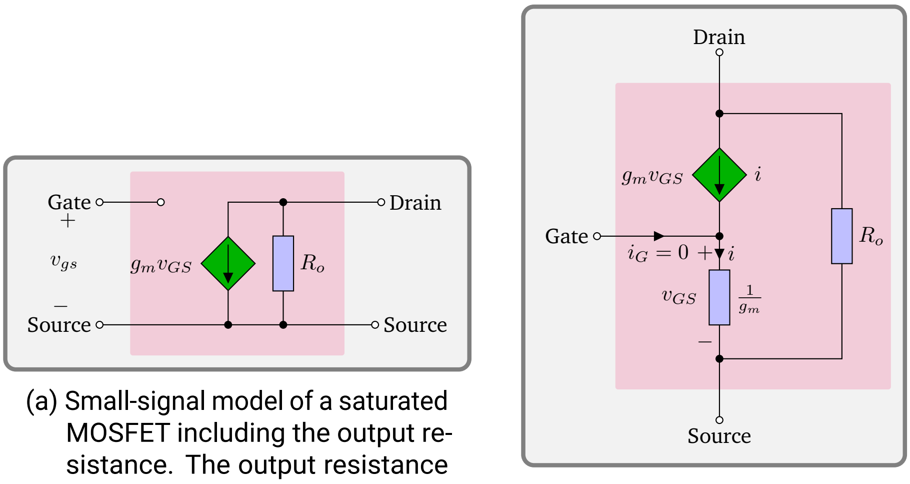
The meaning of g_m and g_{GS} can be best understood when looking at a mathematical model of the MOSFET. These models come in varying complexity and either as a large-signal or small-signal model. Only the latter is used here. The small-signal model, is a first-order Taylor approximation around the working point, for a constant gate-source voltage V_{GS} and constant drain-source V_{DS}, hence both g_m and g_{GS} are constants.

$$I_D \approx \frac{\partial I_D}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial I_D}{\partial V_{DS}} \Delta V_{DS} \tag{3.84}$$

$$= g_m \Delta V_{GS} + g_{DS} \Delta V_{DS} \tag{3.85}$$

$$= g_m v_{GS} + \frac{1}{R_o} v_{DS} = i_D \tag{3.86}$$

The lower case letters denote the variables of the small-signal model as they only change very little compared to the working point parameters. From 3.86 it can be seen, that the g_{DS} term adds to the output current and is proportional to v_{DS}. Comparing this with figure 3.38a, the proportionality constant can be identified as $\frac{1}{R_o}$ like proposed above. Just like the ideal current source in figure 3.38, the model can be given in the Norton or Thévenin representation, both shown in figure 3.42.



(a) Small-signal model of a saturated MOSFET including the output resistance. The output resistance models the channel-length modulation as given by equation 3.86.

(b) MOSFET model in Thévenin representation.

Figure 3.42.: Equivalent MOSFET models in Norton and Thévenin representations.

A detailed graphic derivation of the Thévenin representation can be found in [187]. The Thévenin representation will prove especially valuable when treating circuits with a resistance in the source leg. The small-signal model now shows that the output impedance is dependent on the channel-length modulation λ and v_{DS}. Typically, $\frac{1}{\lambda} \gg v_{DS}$, so λ is the most important factor governing the output impedance of a MOSFET.

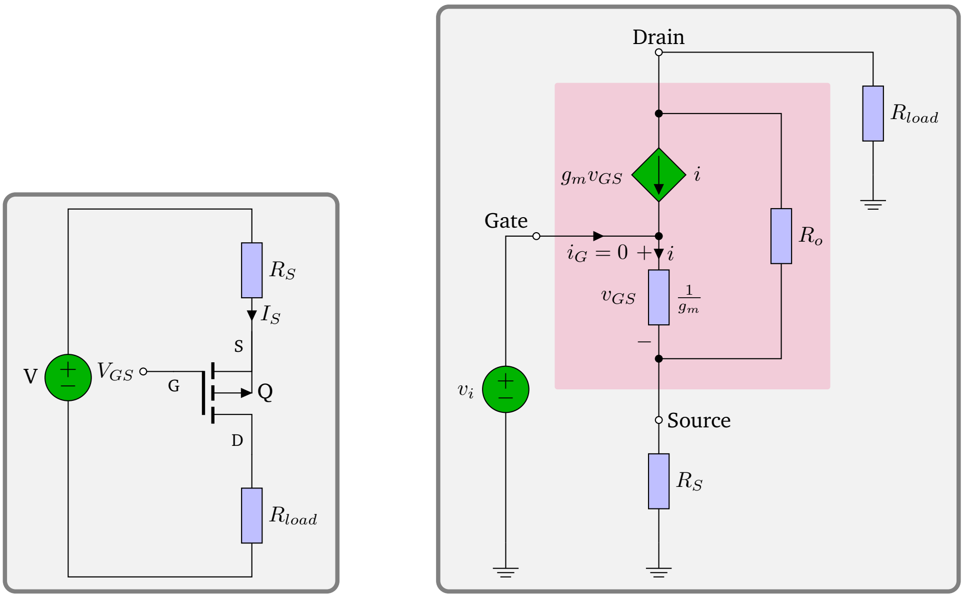
To give an example of the output impedance of a MOSFET, parameters were taken from the aforementioned SPICE model of the `IRF9610`. Do note that these parameters of the model are tuned to match certain operating conditions by their creators and only present an estimation of the real MOSFET. Using the example parameters from table 3.10, $I_D = 250\,\text{mA}$, $\lambda = 4\,\text{mV}^{-1}$, $V_{DS} = 3.5\,\text{V}$ equation 3.83 yields

$$R_{out} = R_o \left(I_D = 250\,\text{mA}, \lambda = 4\,\text{mV}^{-1} \right) = 1014\,\Omega \overset{V_{DS}=0}{\approx} 1\,\text{k}\Omega\,, \tag{3.87}$$

which is not very convincing as a current source. The insignificant impact of V_{DS} on the output impedance can be seen when dropping the V_{DS} term, which leads to an output impedance of $1\,\text{k}\Omega$. In textbooks this dependence is therefore usually neglected. To improve R_{out}, the focus thus lies on the λ dependence. The model derived from equation 3.86 can be used to do so, leading to the precision current source presented next.

3.8.4. Precision Current Source

In the previous section 3.8.3 it was shown in equation 3.86 that the output impedance of a MOSFET depends on the channel-length modulation λ and is too low for practical purposes. On the quest to improve the output impedance of the MOSFET circuit in figure 3.42a, the most obvious solution would be to simply add a source resistor R_S into the circuit as shown in in figure 3.43a. At first glance this may seem to only add a series resistance to R_o, but the attempt is more intriguing and will lead to an even better solution.



(a) MOSFET with source resistor R_S to improve the output impedance R_{out}.

(b) Small-signal Thévenin model.

Figure 3.43.: Circuit of a MOSFET with source degeneration resistor and equivalent Thévenin model.

Before calculating the output impedance, we shall have a look at v_{GS} and the input signal v_i derived from it. With the introduction of the source resistor R_S, v_i no longer equals v_{GS}, because $\frac{1}{g_m}$ now forms a voltage divider with R_S and it follows

$$v_{GS} = v_i \frac{\frac{1}{g_m}}{R_S + \frac{1}{g_m}} = v_i \frac{1}{1 + g_m R_S} . \tag{3.88}$$

This implies a reduction in gain by the factor $\frac{1}{1+R_S g_m}$ compared to the previously discussed approach. The cause of this reduction is negative feedback. To understand this, imagine, that with a constant v_i and hence a constant current I_D flowing, a changing load resistance is trying to modulate I_D. Any increase in I_D will cause the voltage across R_S to rise, reducing v_{GS}, because v_i is still constant. The decreasing v_{GS} will then reduce I_D, thus introducing negative feedback. Having realized there is negative feedback present, it can be postulated, that the reduction in input sensitivity, or effective transconductance, will be passed on to the output impedance. This very interesting relationship will now be derived.

To calculate the output impedance, figure 3.43b can be simplified by grounding v_i, because there is no AC component as there is no current flowing through the insulated MOSFET gate and is not modulated. The load R_{load} resistance must is is replaced by an AC test voltage v_{load} to modulate I_D. These changes result in the small-signal model shown in figure 3.44. This configuration is also called a common-gate amplifier.

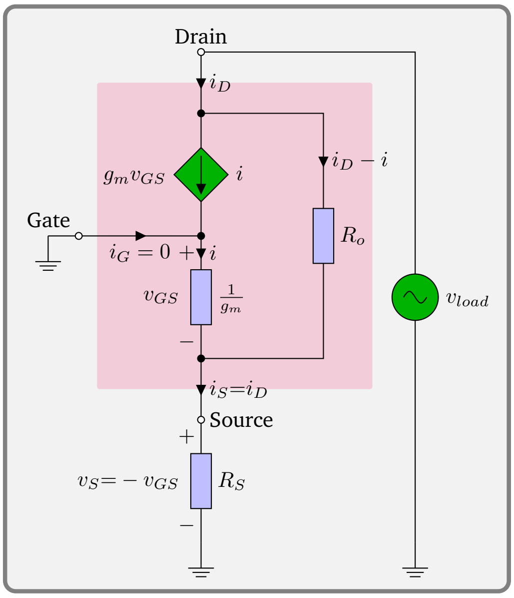


Figure 3.44.: Small-signal model of the common-gate amplifier with source resistance R_S.

The (dynamic) output impedance is given by

$$R_{out,cg} = \frac{v_{load}}{i_D}\,, \tag{3.89}$$

with $i_D = i_S$, since there is no gate current. v_{load} can easily be calculated by looking at figure 3.44 and equals the total voltage across R_o and R_S. v_{GS} can also be found, because the gate is grounded. With the resistance $\frac{1}{g_m}$ at one end, the voltage at the source pin must be $-v_{GS}$.

$$\begin{aligned} v_{load} &= (i_D - i)\,R_o + i_S R_S \\ &= (i_D - g_m v_{gs})\,R_o + i_D R_S \\ &= (i_D + g_m i_D R_S)\,R_o + i_D R_S \end{aligned} \tag{3.90}$$

Using equations 3.89 and 3.90 gives

$$R_{out,cg} = (1 + g_m R_S)\,R_o + R_S \tag{3.91}$$

for the output impedance.

This result is interesting, as it can be be immediately seen, that the output impedance scales very quickly with the transconductance g_m and R_S. As it was already speculated above, the reduction in the transconductance $\frac{1}{1+g_m R_S}$ of the MOSFET is transferred to the output impedance, which is increasing by the inverse of the loss in transconductance.

Going back to the quest for an increased output impedance, it is apparent that increasing R_S quickly raises the output impedance, as it scales with $gm_m R_o$, but it would come at the cost of a significantly reduced compliance voltage. Therefore, other means need to be explored. As we have seen, the scale factor $gm_m R_o$ is explained by feedback and this leads to another solution.

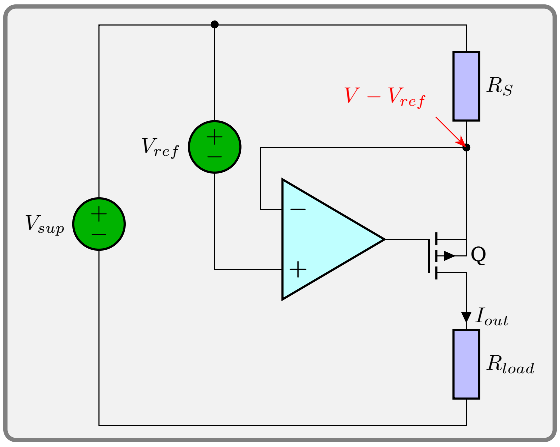


Figure 3.45.: Transconductance amplifier with a p-channel MOSFET.

The amount of feedback can be increased further using an operational amplifier (op-amp) as shown in figure 3.45.

The output impedance of this transconductance amplifier is amplified by the open-loop gain of the op-amp as shown in appendix A.3, while the transfer function greatly simplifies to

$$\begin{aligned} R_{out} &\approx A_{ol}\left(g_m R_o R_S + R_o + R_S\right) \\ I_{out} &\approx \frac{V_{ref}}{R_S} \end{aligned} \tag{3.92}$$

In addition to the increased output impedance, the current $I_D = I_{out}$ can now steered by adjusting V_{ref} and is, given sufficient loop gain of the op-amp, no longer dependent on the MOSFET but rather only on the sense resistor R_S.

This has the added benefit that it is possible to leverage the tight accuracy and precision of a resistor over the poor specifications of a MOSFET. Resistors can be manufactured with tolerances of less than $100\,\mu\Omega/\Omega$, which is orders of magnitude better than FETs, which can be matched to low % values with patience.

Using the example parameters from table 3.10, the output impedance in saturation can now be calculated again for $I_{out} = 250\,\text{mA}$ and the ideal `IRF9610` model with the addition of an idealized `AD797` op-amp using the worst-case specifications.

$$R_{out} \approx 2\,\text{V}\,\mu\text{V}^{-1}\left(0.64\,\text{S}\cdot 1014\,\Omega\cdot 30\,\Omega + 1014\,\Omega + 30\,\Omega\right) \approx 40\,\text{G}\Omega \tag{3.93}$$

From these consideration, it can be seen, that the open-loop gain and the unity-gain bandwidth of the op-amp essentially determine the properties of the current source, given that $R_{id} \gg R_S$ and $R_o \gg R_S$. This will be important for selecting an operational amplifier later.

The next section will focus on the MOSFET and discuss the compliance voltage of the current source, which was only briefly touched during the introduction. It will give rise to criteria for selecting a MOSFET for the precision current source.

3.8.5. Compliance Voltage

The compliance voltage of a current source is the maximum voltage it can output to maintain the requested output current. For an ideal current source, the compliance voltage is infinite, but it is obviously limited in the physical world.

The precision current source discussed in section 3.8.4 has several limiting factors of the compliance voltage, which shall be discussed now. The compliance voltage is taxed most at the maximum output current $I_{out,max}$. Thus for the following discussion, the output is always treated as set to maximum.

Looking at figure 3.45 of the precision current source it is immediately evident that the output voltage can be calculated by subtracting the voltage across the source resistor V_{R_S} and the MOSFET V_{DS} from the supply voltage V_{sup}

$$V_{out} = V_{sup} - V_{R_S} - V_{DS} = V_{sup} - V_{ref} - V_{DS}\,.$$

The voltage V_{R_S} is given by equation 3.92 and equal to the setpoint voltage and hence given by the system parameters. This leads to the question of the minimum working point voltage V_{DS} at $I_{out,max}$. As a reminder, from equation 3.79 and figure 3.41 one can see, that the drain current is almost constant over V_{DS} in the saturation region, and in the ohmic region is proportional to V_{DS}. The transition point from the ohmic region to the saturation region is at $V_{DS} = V_{GS} - V_{th}$ and putting this into equation 3.79 yields for the drain current

$$\begin{aligned} I_D &= \frac{1}{2}\kappa V_{DS}^2 \left(1 + \lambda V_{DS}\right) \\ \Rightarrow V_{DS} &\approx \sqrt{\frac{2I_D}{\kappa}} \qquad (3.94) \\ &\approx 784\,\mathrm{mV} \qquad (3.95) \end{aligned}$$

The latter result was calculated using the example parameters from table 3.10. At this point it can already be postulated, that the MOSFET will severely change in its function as a current source for $V_{DS} < 0.78\,\mathrm{V}$. To quantify this, one has to look at the output impedance of the transconductance amplifier once again. In the last section, the output impedance was only treated for the saturation region, but this time, R_{out} must be considered over a wide range of V_{DS}, thus not only in the saturation region but also in the ohmic region. Instead of using the small-signal model as before, which assumed only small changes of V_{DS}, a large-signal model must be applied, which also includes the non-linear nature of the piece-wise defined equation 3.79 of the drain current.

For the sake of simplicity, a SPICE simulation of figure 3.45 was carried out in LTSpice [152]. Solving this analytically bears no educational value over the numerical solution shown below as will be seen. Additionally, the SPICE simulation also offers the opportunity to add additional, parasitic elements to the model to evaluate their effect, for example, the capacitive nature of the MOSFET gate.

The simulation itself is numerically challenging and the typical approaches will lead to the limits of the numerical precision. To make the simulation feasible, the large-signal model is broken down into several small segments. For each of these segments, the small-signal model at its respective working point is evaluated and then the result joined back together to reconstruct the large-signal model sought. How this is done in detail, is shown in appendix A.4 as it is beyond the scope of this section. The final result was calculated for two different frequencies, one frequency was deliberately chosen so low ($1\,\mu\mathrm{Hz}$) that it is well below the dominant pole of the op-amp, meaning that the full open-loop gain applies and the other frequency chosen was $1\,\mathrm{MHz}$, were the gain had dropped to $10\,\mathrm{V/V}$. This is shown in figure 3.46.

Looking at figure 3.46 clearly shows the effect of entering the ohmic region of the MOSFET. Over a range of about $100\,\mathrm{mV}$ below the $0.78\,\mathrm{V}$ calculated above, the output impedance drops by two orders of magnitude and then keeps dropping at an exponential rate with decreasing

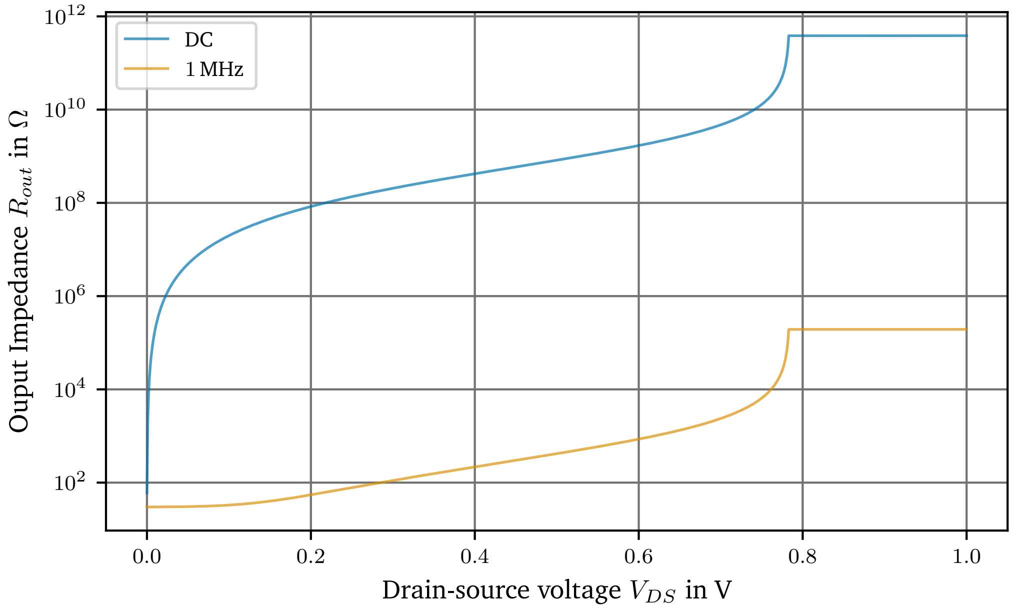


Figure 3.46.: Simulated output impedance for the precision current source from figure 3.45 at DC and 1 MHz over the drain-source voltage.

V_{DS}. The same effect applies to the output impedance at 1 MHz, although the starting value is around 200 kΩ due to the reduced gain from the op-amp at 1 MHz. It can also be seen, that R_{out} levels off at 30 Ω, the value of the sense resistor.

This overall effect of leaving the saturation region is so drastic, that the compliance voltage must be defined in such a way that the MOSFET remains in saturation and this leads to

$$V_{comp} = V_{sup} - V_{ref} - \sqrt{\frac{2I_D}{\kappa}}\,. \tag{3.96}$$

Now turning to the supply voltage, it is limited by the op-amp which must drive the gate of the MOSFET all the way up to the supply to turn off the current source. The reference voltage is, unless one divides it down dictated by the reference chosen. This, unfortunately, leaves only little room for the MOSFET and it must be carefully chosen not limit the compliance voltage too much.

At this point a fallacy the author has observed multiple times must be addressed. In order to address the limited compliance voltage, one may be tempted to use multiple MOSFETs in parallel to divide the current between the MOSFETs and thereby reduce the voltage that needs to be dropped across the FET proportional to $\frac{1}{\sqrt{N}}$, where N is the number of MOSFETs paralleled.

Imagine the following modified circuit of the precision current source shown in figure 3.47 with two MOSFETs in parallel. For clarity the gate resistors required are not included.

While at first this seems like a solution to the limited V_{DS}, it is not recommended for a number of reasons given here.

The first reason is, MOSFET specifications are very loose, notably the threshold voltage V_{th}, the transconductance g_m and the capacitances, but the latter is of little concern here. These tolerances limit the usefulness of paralleling MOSFETs to certain conditions, for example, when

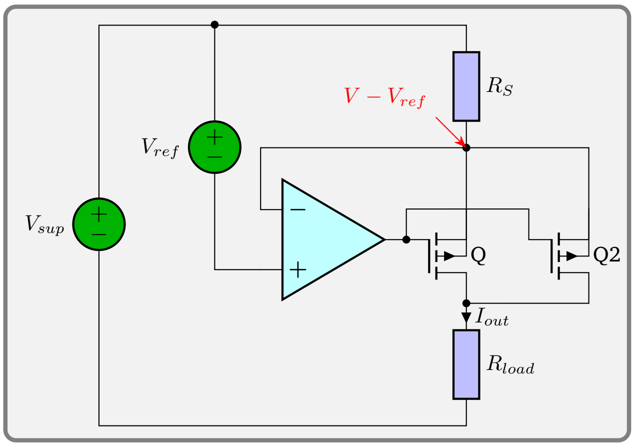


Figure 3.47.: Transconductance amplifier with two p-channel MOSFETs in parallel.

using the MOSFETs as a switch ad not as a current source. The difference between its use as a switch and a current source is the thermal load, which is a lot higher when using the MOSFET as current source. In this respect it seems to be a common misunderstanding, that MOSFETs are immune to thermal runaway. This is mostly true when using them as a switch, fully turned on and in the ohmic region. In this case, there are two effects occurring, the first is, that the (absolute) value of V_{th} decreases with temperature, thus increasing I_D and the second effect is, $R_{DS,on}$ is rising with temperature [48]. The latter effect is, depending on the physical design, stronger, but it depends on the specific MOSFET. A detailed analysis of paralleling MOSFETs as switches can be found in [80]. In case the MOSFET is operating in pinch-off and not the ohmic region, $R_{DS,on}$ has no influence on the current, therefore, the only effect at work is the decreasing V_{th}. Depending on the difference in V_{th} between the paralleled MOSFETs, one MOSFET will take most of the current and power. Adding source resistors can compensate for this by pushing down the source voltage as the current goes up. This will then reduce V_{GS}. The size of the resistor depends on the transconductance g_m and the temperature coefficient of V_{GS}, which is around 1.5–$2\,\mathrm{mV/K}$ [83]. Unfortunately, $1\,\Omega$ or $2\,\Omega$, will already use up, most of the benefits gained in compliance voltage as will be shown below.

The second reason why paralleling MOSFETs is not desirable can be seen when remembering equation 3.79. It is known that the transition from the unwanted ohmic region to the saturation region is

$$V_{DS} \geq V_{GS} - V_{th} \,. \tag{3.97}$$

Looking at 3.47, it can be seen that V_{GS} is set by the op-amp and is the same for both MOSFETs because their gates and sources are connected. However, V_{th} is device specific and according to the datasheet of the example `IRF9610` [98] V_{th} values can show a spread of as much as -2 to $-4\,\mathrm{V}$, although [160] suggests that MOSFETs from the same reel show a spread of only $\pm 125\,\mathrm{mV}$ of V_{th} within the same batch for consecutive devices. The $125\,\mathrm{mV}$ was found for the `BUK7S1R5-40H` [50], which was sampled in this report. The number given in the report is for 3σ and, assuming the datasheet values for the spread are also referring to 3σ, the spread found in the report is about twice as high as the datasheet value of 2.4–$3.6\,\mathrm{V}$. Assuming similar numbers for `IRF9610` MOSFET used in our examples, this leads to $\pm 208\,\mathrm{mV}$ for the `IRF9610`, again applying 3σ. Using this number, a Monte Carlo simulation (not quite, because the dice were biased to yield a Gaussian distribution instead of equal probabilities) was run using LTSpice, simulating the circuit shown in figure 3.47 and also the original circuit using only one MOSFET. For this simulation the current source was set to $250\,\mathrm{mA}$ as per table 3.10. The load voltage was set to

$$V_{DS,parallel} = \sqrt{\frac{2\frac{I_d}{2}}{\kappa}} \approx 555\,\mathrm{mV}\,, V_{DS,single} = \sqrt{\frac{2I_d}{\kappa}} \approx 784\,\mathrm{mV}\,.$$

$\frac{I_D}{2}$ was used to calculate $V_{DS,parallel}$ for the parallel configuration to show the effect assuming perfect current sharing between the MOSFETs. Additionally, a configuration with an increased safety margin of $1\sigma = 70\,\mathrm{mV}$ added to $V_{DS,parallel}$ was investigated. 4000 samples were drawn and the spread of the output impedance was calculated for each circuit. The results are shown as a histogram in figure 3.48. The counts give the number of cases for each bin of the output impedance.

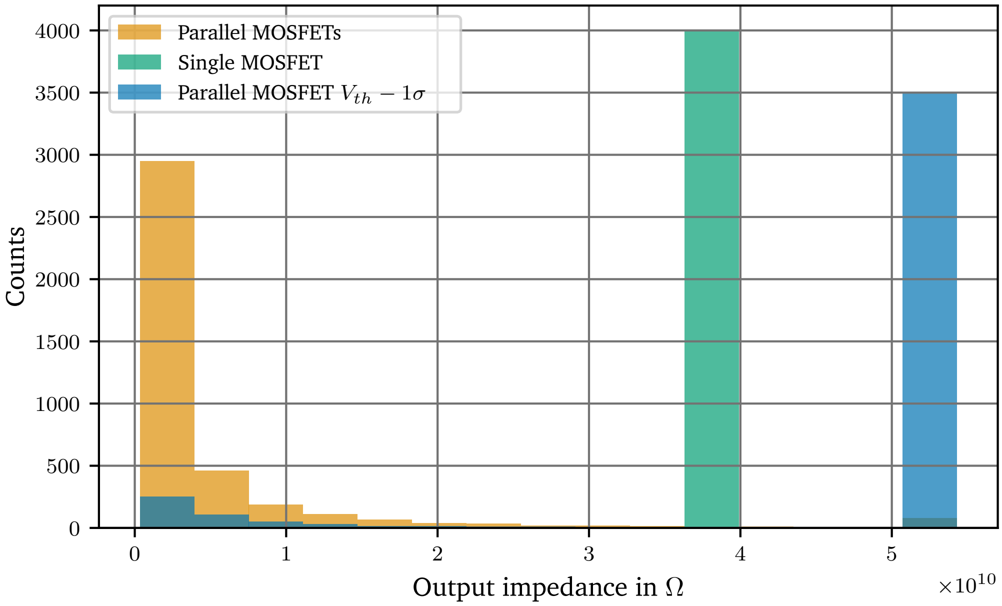


Figure 3.48.: Results of a Monte Carlo simulation of the output impedance for different configurations of MOSFETs.

Unsurprisingly, there is no variance of the output impedance in the single MOSFET case in accordance to what can be seen in appendix A.3. The op-amp gain simply suppresses all device properties of the MOSFET. The slight variation of g_m for different samples was not simulated, because this variation stems from the variation of κ and goes as $\frac{1}{\sqrt{\kappa}}$, so its effect is not as pronounced as the threshold.

In case of two MOSFETs, the output impedance varies over an order of magnitude from about 1.8–$52\,\mathrm{G\Omega}$. Even when increasing the drain-source voltage by $1\sigma = 70\,\mathrm{mV}$ to $625\,\mathrm{mV}$ on average, the spread is still an order of magnitude. Only when increasing V_{DS} to around $700\,\mathrm{mV}$, the situation stabilises, but then the net gain from this measure has shrunk to a meager $84\,\mathrm{mV}$. It can be seen from this simulation, that the system-to-system spread becomes very unstable in tough situations. Such instability can also be brought into the system by temperature effects as V_{th} is temperature dependent as discussed above. Additionally, it may suffer from thermal runaway unless each individual MOSFET is laid out to carry the full current.

3.8.6. Noise Sources

The fundamentals of different types of noise were already introduced in section 3.6. Here, a subset of these noise types is treated. It is expected, that the dominant noise observed in this circuit is $\frac{1}{f}$-noise at low frequencies and white wideband-noise. All noise components will be converted to the so-called input referred notation to make the noise sources comparable. This can be easily understood, when looking at two amplifiers with different gain. If both of them add a fixed amount of noise to the output signal, the absolute amount of noise may be the same, but the signal to noise ratio shows a different picture. To compare these amplifiers it is useful to divide the noise by the transfer function (gain) of the amplifier. This is called input-referred noise, since it treats the noise in relation to the input signal. Additionally, when calculating noise figures, the noise bandwidth is always considered to be 1 Hz.

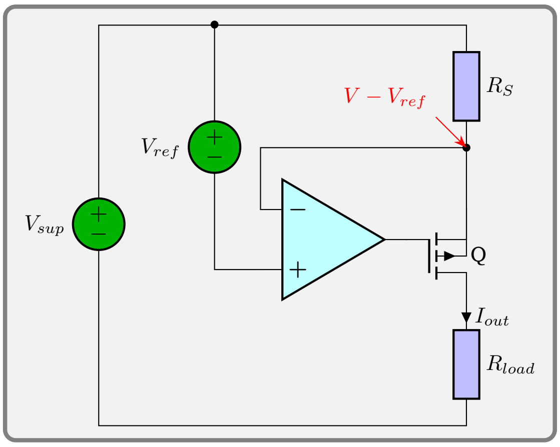


Figure 3.49.: Transconductance amplifier with a p-channel MOSFET. Repeated from page 74.

Noise sources are ubiquitous in the circuit in figure 3.45 on page 74, repeated here as figure 3.49 for clarity. The resistor R_S, the MOSFET, the op-amp, the setpoint voltage V_{ref} and the supply voltage V_{sup} can all contribute noise to the output current. Fortunately, some of those noise contributions are either very small or are well suppressed in this design, so each component must be briefly discussed to see if this is the case.

Starting with the supply voltage V_{sup}, it can be seen, that any change of this voltage affects the string R_S-Q-R_{load}. From equation A.8, we know that if the op-amp gain is high (true within the bandwidth of the op-amp) all disturbances of the voltage across R_S will be suppressed and the output current is only defined by the reference input and R_S. Looking closer, the supply noise is present at the inverting and non-inverting input of the op-amp with the same magnitude. If there is no current flowing into the op-amp pins, which is true for low frequencies, the noise is affecting both pins equally and it will be suppressed by the common-mode-rejection ratio (CMRR) of the device. Fortunately, this is a strong quality of precision op-amps and values of more than $1\,\mu\mathrm{V}\,\mathrm{V}^{-1}$ are not uncommon. The op-amp will therefore take care of the supply noise at low frequencies. At high frequencies the parasitic capacitance of the input pins and the reduced gain and CMRR come into play, reducing the CMRR and the gain also drops going to high frequencies. To take care of this, it is therefore prudent to filter the supply for high frequency noise.

The next noise source is the reference voltage. The reference is directly connected to the input and its noise dictates most of the circuit noise. While the high-frequency noise can again be filtered to some extend, the low frequency noise, which is mostly $\frac{1}{f}$-noise can not be filtered as was shown in section 3.6.1, so it must be kept low from the start and the reference selected for low flicker noise.

The MOSFET as a noise source is considered in appendix A.5 and the interested reader may find the derivation of the MOSFET noise component there. The two types of noise that need to be considered are the flicker noise of the MOSFET and its wideband thermal noise as calculated in equation A.20

$$i_n = \sqrt{\underbrace{4k_BT\frac{2}{3}g_m}_{\text{thermal}} + \underbrace{\frac{K_fI_D}{C_{ox}L^2}\frac{1}{f}}_{\text{flicker}}}\,.$$

To calculate the input referred noise, which shows that the MOSFET noise will be suppressed by the op-amp, the current noise needs to be divided by the open-loop gain derived as equation A.7

$$e_{n,FET} = \frac{i_n}{A_f} = \frac{\sqrt{4k_BT\frac{2}{3}g_m + \frac{K_fI_D}{C_{ox}L^2}\frac{1}{f}}}{\frac{A_{op}}{R_S}\frac{g_m(R_o||R_S||R_{id})}{g_m(R_o||R_S||R_{id})+1}}\,. \tag{3.98}$$

Looking at the parameters from table 3.10, we find $(R_o||R_S||R_{id}) \approx R_S$ and e_n can be simplified to

$$\begin{aligned} e_{n,FET} &\approx \frac{\sqrt{4k_BT\frac{2}{3}g_m + \frac{K_fI_D}{C_{ox}L^2}\frac{1}{f}}}{A_1\frac{1}{R_S+\frac{1}{g_m}}} \\ &\approx \frac{R_S+\frac{1}{g_m}}{A_1}\sqrt{4k_BT\frac{2}{3}g_m + \frac{K_fI_D}{C_{ox}L^2}\frac{1}{f}} \\ &\overset{A_1\to\infty}{=} 0 \end{aligned}$$

Unless the MOSFET transconductance g_m or the gain of the op-amp A_1 become very small, the noise of the MOSFET is very well suppressed. This means, that if the wideband thermal noise contribution is small (it is, see A.5) and the flicker noise corner frequency is within the bandwidth of the op-amp, the noise contribution from the MOSFET can be neglected.

The noise contribution from the sense resistor R_S is the (approximated) Johnson–Nyquist noise, which when transformed to its Norton representation can be written as a current noise

$$i_{n,R} = \sqrt{\frac{4k_BT}{R_S}}\,. \tag{3.99}$$

Additionally, it was shown, that depending on the material of the resistive element, a flicker noise component can also be present. This is especially prevalent in carbon and tick-film resistors [78, 162]. While thin-film resistors are less noisy, their performance varies greatly between different models [188], so their make and model must be carefully selected for the application. Foil and wirewound resistors were shown to perform best and have almost no flicker noise [44, 188]. Using a high quality resistor, the flicker noise can be neglected and only the thermal noise must be taken into account.

The sense resistor is part of the feedback network and therefore it contributes fully to the noise of the transimpedance amplifier. Input referred, the current noise must be divided by the closed-loop gain A_f given by A.2.

$$e_{n,R} = i_{n,R}\cdot\beta \approx i_n \cdot R_S = \sqrt{4k_BTR_S} \tag{3.100}$$

The final component to be discussed is the operational amplifier. Although the op-amp is a rather complex device, its noise can be modeled by a small number of noise sources. This noise model of the op-amp is shown in figure 3.50.

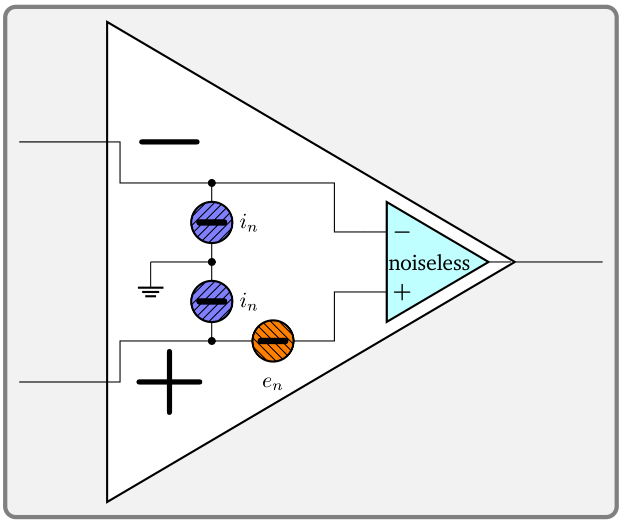


Figure 3.50.: Noise model of the operational amplifier.

In figure 3.50 we can see, that there are three noise sources required to treat the op-amp. The input voltage noise e_n and two input current noise sources i_n. The current noise noise sources i_n are assumed to be mostly uncorrelated. This assumption will lead to an upper bound as can seen from figure 3.51, which shows the the input differential amplifier, that is the first stage of a typical bipolar op-amp.

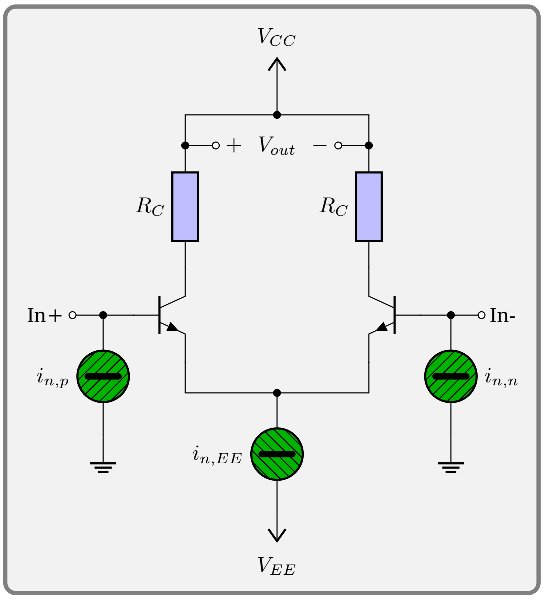


Figure 3.51.: Bipolar op-amp input stage with noise sources.

Of the three noise sources $i_{n,p}$ and $i_{n,n}$ are uncorrelated, because it is the input bias current of the individual transistors, and only the effect of $i_{n,EE}$ is correlated, because the current of the emitter bias current source is equally distributed between the two input transistors. Since effects of equal magnitude and sign cancel out due to the differential nature of the input stage, correlated effects are suppressed. An equal magnitude can be assumed, because the gain of the two transistors is well matched, due to their close proximity on the semiconductor die. Therefore assuming all noise is uncorrelated presents an upper bound. A more detailed analysis can be found in [87], if interested. Due to the matching of the transistors, the magnitude $i_{n,p}$ and $i_{n,n}$ are also closely matched, hence in our model, they are assumed equal and only called i_n.

As we have done before in this section by referring all noise sources to the input, the same was done by the manufacturer and the noise figures given in the datasheet are typically input referred and combine all noise sources in the op-amp. The two current noise sources can not be combined with the input voltage noise, because they depend on the external impedance connected to the op-amp, so this final step must be done by the circuit designer. Given the complete circuit as in figure 3.49 it is possible to calculate the full noise contribution of the op-amp as

$$e_{n,op} = \sqrt{e_n^2 + e_{n,+}^2 + e_{n,-}^2}\,, \tag{3.101}$$

assuming the noise sources are uncorrelated. The input referred noise $e_{n,-}$ of the inverting input can be calculated in a similar fashion as e_{n_R} in equation 3.100. It is likewise part of the feedback network and must therefore be divided by the closed-loop gain A_f as before.

$$e_{n,-} \approx i_n \cdot R_S \tag{3.102}$$

The current noise of the non-inverting input can be translated by looking at its input impedance. This is be determined by the filter of the reference voltage, which is required to remove the high frequency noise as discussed above. Assuming an RC-filter of first-order, the output impedance can be calculated from the transfer function of the low-pass filter, derived in equation 3.19

$$R_{out,filt} = R_{filt} \cdot A = \frac{R_{filt}}{1 + sR_{filt}C} \tag{3.103}$$

$$\lim_{s\to 0} R_{out,filt} = R_{filt}$$

$$\lim_{s\to\infty} R_{out,filt} = 0\,.$$

$$e_{n,-} \approx \frac{i_n R_{filt}}{1 + sR_{filt}C} \tag{3.104}$$

Looking at the output impedance of the filter, it can be seen, that for high frequencies, the output impedance goes to 0, while for low frequencies it is R_{filt}. If the filter corner frequency $\omega_0 = \frac{1}{RC}$ is close to or at the flicker noise corner frequency of the reference voltage, it means that there is almost no wideband current noise contribution as well. Only the $\frac{1}{f}$ component of the op-amp current noise multiplied with R_{filt} is left. This should be lower than the reference noise to have negligible impact and must be kept in mind when selecting an op-amp. This leads to the total noise of the op amp

$$e_{n,op} = \sqrt{e_n^2 + (i_n R_S)^2 + \left|\frac{i_n R_{filt}}{1 + sRC}\right|^2}\,. \tag{3.105}$$

To conclude, table 3.6 is given as a reference for the noise contributions in the low-frequency and also the wideband domain. From this table, it can be seen, that the only wideband-noise contributors are the reference resistor and the op-amp. The low-frequency contributors are the voltage reference and the op-amp, since they have a strong flicker noise component. A low-noise, precision op-amp typically has far less low frequency noise than a voltage reference and the dominant low frequency contributor remains the voltage reference.

Noise component	Low frequency	Wideband
V_{sup}	≈ 0	≈ 0
MOSFET	≈ 0	≈ 0
V_{ref}	$\sqrt{e_{n,ref}^2 + 4k_BTR_{filt}}$	≈ 0
R_S	$\sqrt{4k_BTR_S}$	$\sqrt{4k_BTR_S}$
Op-amp	$\sqrt{e_n^2 + i_n^2(R_S^2 + R_{filt}^2)}$	$\sqrt{e_n^2 + i_n^2 R_S^2}$

Table 3.6.: Input referred noise components of the transimpedance amplifier. Multiply by $\frac{1}{R_S}$ to get the output referred current noise.

To summarize the findings, the most important choices regarding the noise contributions are a good quality metal-foil or wirewound sense resistor R_s, a low noise voltage reference and a low noise op-amp. Regarding the low-noise op-amp it is important to decide between low voltage noise or low current noise. This choice depends on the value of R_S. For typical values of R_S below $1\,\mathrm{k\Omega}$ voltage noise is dominating. The reference should be chosen for low flicker noise.

3.8.7. Component Selection

This section deals with selecting the right components for the precision current source presented in section 3.8.4. The focus lies on the requirements defined in section 3.2, notably specifications 3.1 and 3.2. Most attention will be on the MOSFET, the operational amplifier and the voltage reference. First is the voltage reference, because this will define several parameters down the road. Then, the op-amp is discussed, for which several examples from scientific publications and other alternatives are shown and the best solution is presented. Finally, the selection parameters for the MOSFET will be elaborated. The reader must warned though, that the lineup of p-channel MOSFETs in production is decreasing, with more and more products being discontinued in favour of n-channel MOSFETs. The examples may therefore already be outdated.

Numerous laser driver designs for different applications and laser diodes can be found in literature [76, 92, 120, 185, 210, 211, 216]. While Libbrecht et al. were not the first to present their circuit and a similar solution can already be found in [51], their design stands out for its simplicity. The designs mentioned can be divided into two groups. High power drivers for quantum cascade lasers (QCL) typically featuring a compliance voltage V_c of more than $10\,\mathrm{V}$ and output currents of up to several ampere based on the work of Taubman and medium power devices for laser diodes having a lower compliance voltage of around $2\,\mathrm{V}$ and capable of driving a few hundred mA based on the work of Libbrecht et al. The requirements of this work mostly fall into the latter category, except for the compliance voltage, which is targeted for blue laser diodes and $V_c \geq 8\,\mathrm{V}$. All these drivers share one common aspect, though, the type of voltage reference. Most laser drivers in literature and also commercial products are designed around low-noise, low-drift buried Zener diode voltage references, namely the Analog Devices

(ADI) LM399 [125] or ADI LTZ1000 [134].

Component	Voltage	Temperature coefficient	Stability	Package
LT1021	$7\,\mathrm{V}$	$2–5\,\mu\mathrm{V}/(\mathrm{V\,K})$	$15\,\mu\mathrm{V}/(\mathrm{V}\sqrt{\mathrm{kh}})$	SO-8
LT1027	$5\,\mathrm{V}$	$1–2\,\mu\mathrm{V}/(\mathrm{V\,K})$	not specified	SO-8
LM399	$7\,\mathrm{V}$	$0.3–1\,\mu\mathrm{V}/(\mathrm{V\,K})$	$8\,\mu\mathrm{V}/(\mathrm{V}\sqrt{\mathrm{kh}})$	TO-46
ADR1399	$7\,\mathrm{V}$	$0.2–1\,\mu\mathrm{V}/(\mathrm{V\,K})$	$7\,\mu\mathrm{V}/(\mathrm{V}\sqrt{\mathrm{kh}})$	TO-46
LTZ1000	$7.2\,\mathrm{V}$	$0.05\,\mu\mathrm{V}/(\mathrm{V\,K})$	$0.3\,\mu\mathrm{V}/(\mathrm{V}\sqrt{\mathrm{kh}})$	TO-99
ADR1000	$6.6\,\mathrm{V}$	$<0.2\,\mu\mathrm{V}/(\mathrm{V\,K})$	$0.2\,\mu\mathrm{V}/(\mathrm{V}\sqrt{\mathrm{kh}})$	TO-99

Table 3.7.: List of commercially available buried Zener diodes and selected properties.

The buried types of voltage references are so-called Zener diodes, that are created within the bulk silicon using ion implantation. This reduces noise caused by surface contamination [191]. These diodes are not true Zener diodes though, but called Zeners nonetheless, and use a mix of Zener and avalanche breakdown to compensate the temperature coefficient.

The Zener effect is the tunneling of electrons through the barrier formed between the valence band and conduction band. It has a negative temperature coefficient, because an increase in temperature reduces the size of the band gap.

Avalanche breakdown, on the other hand describes a mechanism that free electrons (due to temperature) are accelerated to such energies that they knock out other electrons, causing an avalanche of electrons. This effect has a positive temperature coefficient, because a higher temperature results in more free carriers, that cause leakage but less avalanches due to the reduced free length of path. While the zero temperature coefficient point is around $5\,\mathrm{V}$, this operating point implies a high susceptibility to changes in the reverse current. Typically, the Zener voltage is shifted slightly upwards to result in a net positive coefficient, which is then compensated by the negative temperature coefficient of a forward biased diode [191]. This results in the common Zener diode voltage of around $6.2\,\mathrm{V} + 0.7\,\mathrm{V} = 6.9\,\mathrm{V}$. In comparison to other types of diodes, buried Zeners have the best stability and lowest noise. In order to achieve high stability and low noise, $V_{ref} \approx 7\,\mathrm{V}$ is therefore pretty much set in stone for the reasons given above. Table 3.7, lists some commercially available buried Zener diodes. All of these diodes are manufactured by ADI as they are the sole manufacturer left on the market to produce this kind of diodes.

Choosing a voltage reference can be done according to specification 3.1. A temperature coefficient of $\leq 1\,\mu\mathrm{A}/(\mathrm{A\,K})$ rules out any non-hermetic unheated voltage reference. Using a hermetic package improves the stability against humidity as the epoxy used for an SO-8 package is hydrophilic and swells when exposed to water vapour causing pressure on the die, resulting in a change of the output voltage. The hermetic voltage references can be divided into two groups, the LM399 and the newer ADR1399 in one group and the LTZ1000 and its newer counterpart ADR1000 in another. While the LM399 requires very few external components, the external circuit for the LTZ1000 is far more elaborate requiring more parts and space. Additionally, the LTZ1000 is more than four times the price of the LM399 in quantities of 10 at the time of writing. Last but not least, the stability and temperature coefficient of the LTZ1000 cannot be matched by the performance of the sense resistor, so the sense resistor gives a lower bound of about $0.5\,\mu\mathrm{A}/(\mathrm{A\,K})$. Unless the better low frequency noise performance is absolutely required, the LM399 and ADR1399 are the more economical parts. The performance of those two references will be discussed in section 4.1.13.

With the maximum reference voltage of $7\,\mathrm{V}$ known, a sense resistor between $14\,\Omega$ ($500\,\mathrm{mA}$) and $28\,\Omega$ ($250\,\mathrm{mA}$) is required. The Vishay VPR221Z are high power low drift metal foil resistors in a TO-220 package and a solid choice here. The low value resistors, combined with the requirement for a low noise output, limits the choice of op-amps to bipolar low-noise devices or discrete implementations. Table 3.8 lists some choices compiled from the literature sources, which will now be discussed.

Component	Wideband-noise	Low frequency noise	Temperature coefficient
LT1028	$0.85\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$	$35\,\mathrm{nV_{p-p}}$	$0.2\,\mathrm{\mu V/K}$
AD797	$0.9\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$	$50\,\mathrm{nV_{p-p}}$	$0.2\,\mathrm{\mu V/K}$
ADA4898	$0.9\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$	not specified	$1\,\mathrm{\mu V/K}$
ADA4004	$1.8\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$	$150\,\mathrm{nV_{p-p}}$	$0.7\,\mathrm{\mu V/K}$
AD8671	$2.8\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$	$77\,\mathrm{nV_{p-p}}$	$0.3\,\mathrm{\mu V/K}$

Table 3.8.: List of low-noise precision bipolar operational amplifiers with typical performance properties.

The low value of the sense resistor makes a bipolar op-amp the preferred choice, because they have a very low voltage noise and their current noise and input bias current do not interfere with such a low value resistor. While a discrete solution using matched JFETs or bipolar transistors may push the input noise even lower, the temperature stability, circuit complexity and again the size speaks against this option, so the discussion will be limited to integrated solutions only.

To find a reference point for the choice of op-amp, the thermal noise of the sense resistor must looked at. The $28\,\Omega$ sense resistor has a thermal noise of

$$e_n\,(23\,^\circ\mathrm{C}) = 0.67\,\mathrm{nV}/\sqrt{\mathrm{Hz}}\,.$$

This means, that even the lowest noise op-amp from table 3.8 dominates the wideband-noise. With this said, the component choices made in literate can be discussed. The AD8671 chosen by [76] only makes sense, because they have chosen a very large filter resistor R_{filt} of $2 \times 3\,\mathrm{k\Omega}$. The ADA4004 was used by Moglabs in the DLC-102, again likely due to the high values of R_{filt} used. The ADA4898 might seem like a good choice at first sight but the very limited (in terms of precision op-amps) open-loop gain of $0.14\,\mathrm{V/\mu V}$ makes this op-amp a cheap, but poor choice. The final choice is between the AD797 and the LT1028, both op-amps have very similar specifications, but there is a peculiarity in the datasheet of the LT1028 [127]. The *High Frequency Voltage Noise vs Frequency* plot shows a noise bump at around $400\,\mathrm{kHz}$. The original application of the LT1x28 is for audio frequencies, which is well away from that bump, but in this case poses a problem. The publication by Libbrecht et al. also blames the LT1028 for the noise peak around $400\,\mathrm{kHz}$. This peak is the reason why Seck et al. found the LT1028 to have a higher integrated noise than the AD797. Additionally to the superior noise performance, the AD797 (B-grade) has excellent specifications overall. The open-loop gain is between 2–$20\,\mathrm{V/\mu V}$, the supply rejection is greater than $1\,\mathrm{\mu V/V}$, the bias current is almost constant between 20–$100\,^\circ\mathrm{C}$ and the unity gain bandwidth is around $10\,\mathrm{MHz}$. Finally it does have a very high output drive capability of $50\,\mathrm{mA}$, which allows to drive fairly large MOSFETs. These features make the AD797 the ideal op-amp for those low-value sense resistors, although it puts limits on the maximum filter resistor to limit the low frequency current noise contribution.

Finally, the choice of MOSFETs can be discussed. As it was shown in section 3.8.3 in equation 3.78, the channel length modulation plays an important role in increasing the channel conductance g_{DS} and limiting the output impedance. To reduce the channel length modulation a longer channel is preferred. Manufacturers do not give these numbers, nor the manufacturing process. Older technologies like the planar (lateral) FET is better suited for operating in the saturation region than the modern trench (vertical) FET. Trench MOSFETs are geared towards a low on-state resistance $R_{DS,on}$, which is important for switching applications, but their lower resistance comes from a shorter channel. One of the few planar MOSFETs still available on the market is the HEXFET, which was designed for switching applications, but proves useful nonetheless as we will see. High voltage MOSFETs also have longer channels than low voltage MOSFETs, so browsing for MOSFETs, that are rated for 60–100 V or more can narrow down the candidates. While the output impedance is a factor worth keeping in mind, the most important aspect is, whether the MOSFET can drive the load regarding the compliance voltage. To outline the problem, we can again refer to the example parameters from table 3.10.

Assuming a supply voltage of 15 V and the `AD797` op-amp, the current source supply voltage V_{sup} is then limited to about 11–12 V, because the `AD797` is no rail-to-rail op-amp and its output only swings to within 3 V of the rail (minimum) and the input is limited to within 2 V of the rail (minimum). Considering the maximum V_{ref} at full output of 7 V and a load voltage of 3 V in case of the `L785H1` [110] laser diode used as an example in this section leaves only

$$V_{DS,min} = V_{sup} - V_{ref} - V_{load} = (11\text{–}12)\,\mathrm{V} - 7\,\mathrm{V} - 3\,\mathrm{V} = (1\text{–}2)\,\mathrm{V} \tag{3.106}$$

for the MOSFET – a serious challenge.

To find a suitable MOSFET, one has to consult the *Typical Output Characteristics* graph in the datasheet. Using the maximum output current specification it is possible to estimate the minimum drain-source voltage V_{DS} to keep the MOSFET in saturation at the given maximum output current. This again narrows down the list of candidates.

The final aspect is the capacitive nature of the MOSFET gate. This property was brushed in appendix A.5 and the parasitic capacitances can be found in figure A.8. The `AD797` can drive fairly large capacitive loads and several hundred pF are possible. It is best to keep the input capacitance C_{iss} below 500 pF. Do remember the output impedance of the `AD797`, is about 10 Ω at 1 MHz and rising by an order of magnitude at 10 MHz. The 500 pF results in an impedance of around 300 Ω dropping by an order of magnitude at 10 MHz, so keeping capacitance low, allows for a higher bandwidth of the current source.

Using these guidelines, searching a MOSFET across a lot of manufactures can still be tedious, but, for example, the distributor Digikey allows filtering and sorting by voltage and input capacitance. The following MOSFETs in table are given as an example and can be chosen for their respective current ranges.

MOSFET	Maximum V_{DS}	Input capacitance C_{iss}	Current range
`IRF9610`	200 V	170 pF	100–250 mA
`IRF9Z10`	50 V	270 pF	250–500 mA
`IRF9Z14`	60 V	270 pF	250–500 mA

Table 3.9.: Example MOSFETs for a current source and recommended current ranges.

The current range of the MOSFETs in table 3.9 is given based on the datasheet, making sure, that the MOSFET can be biased into saturation for the estimated minimum V_{DS} according to

3.106. The IRF9Z10 is a lower voltage version of the IRF9Z14 and the IRF9Z14 should be preferred if available. Those MOSFETs starting with *IRF* are all HEXFETs formerly made by International Rectifier, whose MOSFET business was bought by Vishay in 2007.

To summarize the component selection. The ADI AD797 op-amp is a highly recommended choice for being low-low noise with enormous gain, high bandwidth and a strong drive current. The MOSFET to accompany it is the Vishay IRF9Z14 for medium power applications. The reference must be a buried Zener diode and the ADI LM399 or ADR1399 is recommended. Regarding the sense resistors, they must be able to dissipate up to $500\,\mathrm{mA} \cdot 7\,\mathrm{V} = 3.5\,\mathrm{W}$ with minimal drift, making the Vishay VPR221Z a very good choice.

3.8.8. Current Source Example Parameters

Throughout this section, example calculations are performed to give the reader an idea of real-life parameters that can be applied to the theoretical models. These parameters are summarised in table 3.10, including their origin.

Parameter	Value	Source
MOSFET drain current I_D	$250\,\mathrm{mA}$	L785H1 [110]
MOSFET κ	$0.813\,\mathrm{A\,V^{-2}}$	IRF9610 SPICE model [45]
MOSFET channel length modulation λ	$4\,\mathrm{mV^{-1}}$	IRF9610 SPICE model [45]
MOSFET source voltage	$3.5\,\mathrm{V}$–$4\,\mathrm{V}$	section 3.8.7
Source/Sense Resistor R_S	$30\,\Omega$ or $50\,\Omega$	section 3.8.7
Op-amp differential input impedance R_{id}	$7.5\,\mathrm{k\Omega}$	AD797 [10]
Op-amp open-loop gain A_{ol}	$2\,\mathrm{V\,\mu V^{-1}}$	AD797 [10]
Op-amp gain bandwidth product GBP	$10\,\mathrm{MHz}$	AD797 [10]

Table 3.10.: Parameters used throughout this section and their sources.

3.8.9. Howland Current Pump

This section will discuss two versions of the popular Howland current source (HCS) [190]. Both the traditional Howland current pump and the so-called *improved* Howland current pump, which is similar, but changes some properties for good and bad. They are both bidirectional current sources and can be used for high frequency current modulation or the generation of a precision floating current source. The circuit is shown in figure 3.52.

As can be seen in figure 3.52, this type of current source requires a set of either 4 or 5 resistors depending on the desired configuration as the classic Howland current source does not need R_{2a}. The output impedance of the Howland current source is derived in appendix A.7 and the result was equation A.40 repeated here,

$$R_{o,m,A} = \left(\frac{R^2 - R_{2a}^2}{R}\right) \frac{(AR + R - \epsilon + 1)}{A\,(R + \epsilon - 1) + 2R - 2\epsilon + 2}\,, \tag{3.107}$$

where A is the frequency dependent gain of the op-amp as discussed in equation A.11, $R = R_1 = R_2 = R_3 = R_4$ and ϵ is the resistor mismatch factor introduced in A.31. The mismatch factor can be used to make worst-case calculation for a given resistor tolerance T using equation A.33.

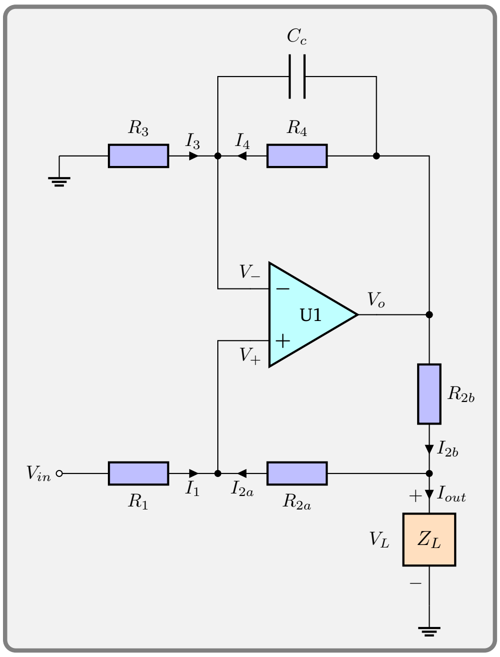


Figure 3.52.: The Howland current source. Using $R_{2a} = 0\,\Omega$ is the classic version, while $R_{2a} \neq 0\,\Omega$ is the *improved* version. C_c is a compensation capacitor to improve stability.

Using the formulae above it is possible to calculate the output impedance for a modulation current source for a laser driver. Since the the output impedance of the *improved* Howland current source is worse than that of the classic Howland current source according to equation 3.107, the discussion will start here. The past has shown, that an input sensitivity of $1\,\mathrm{mA/V}$ is a reasonable choice, giving enough headroom to steer the laser and apply a modulation. The resistor value can be calculated from the input sensitivity to be $1\,\mathrm{k\Omega}$. To reduce manual labor it is recommended to use an array of 4 matched resistors like the Vishay `MORN` [155] or the ADI `LT5400` [129], both quad resistor arrays offering a ratio matching between $0.5\,\%$ and $0.01\,\%$. Commonly available is the `MORN` array with $0.05\,\%$ tolerance and the `LT5400` with $0.01\,\%$ tolerance and even $0.005\,\%$ between direct neighbours. For such arrays and an amplifier with $A = 10^7$, the worst case output impedance can be calculated

$$R_{out}(R = 1\,\mathrm{k\Omega}, T = 0.05\,\%) \approx 499\,\mathrm{k\Omega} \qquad R_{out}(R = 1\,\mathrm{k\Omega}, T = 0.01\,\%) \approx 2.5\,\mathrm{M\Omega}$$

R_{out} is proportional to $\frac{1}{T}$, since the amplifier gain is very high and the tolerances are small, so the Taylor expansion shown above is valid. The upper limit imposed by the op-amp gain can be calculated according to equation A.39 as

$$R_{out,max}(A = 10\,\mathrm{V/\mu V}) \approx 2.5\,\mathrm{G\Omega}\,.$$

Comparing these values with the requirements from specification 3.2, $\geq 7.5\,\mathrm{M\Omega}$, it is clear that the Howland current source can meet them, but not without extra trimming or a tighter matched resistor array (T $0.01\,\%$) which is hard to come by and expensive. In order to get a better understanding of the output impedance and to show the effect of different tolerances, a Monte Carlo simulation was conducted. The parameters assumed a Gaussian distribution of the resistor values with $3\sigma = T$ due to the tolerance. The Monte Carlo simulation was then run 10^5

times and the output impedance was extracted for each run. Finally, the absolute value (there are positive and negative values, see below) was binned into 100 bins of $500\,\mathrm{k\Omega}$ width and plotted as a histogram. The LTSpice simulation file can be found at source/spice/howland_current_source_ideal.asc and the result, limited to bin values $\leq 50\,\mathrm{M\Omega}$ for better readability, is shown in figure 3.53.

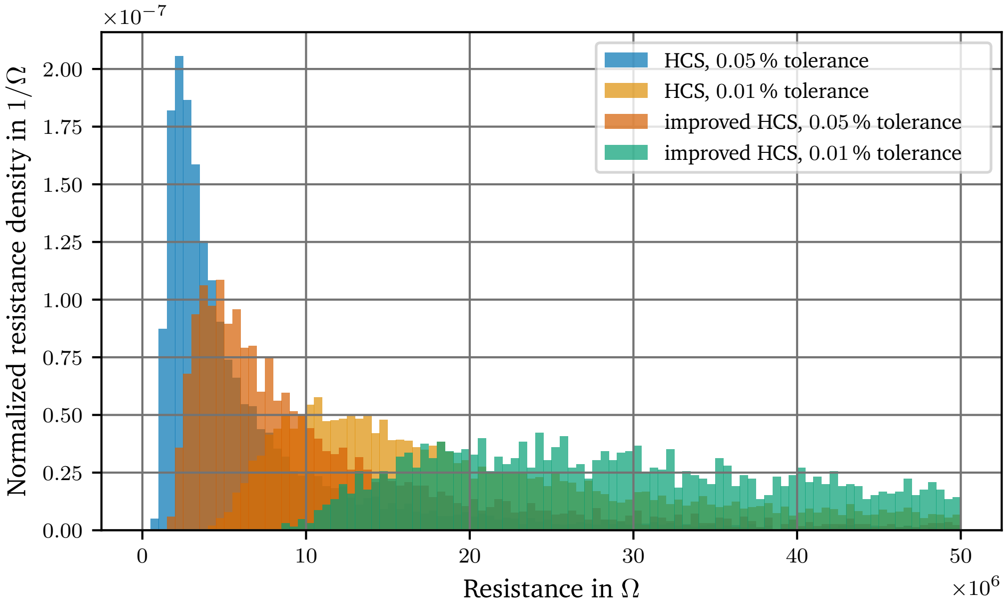


Figure 3.53.: Histogram of Monte Carlo simulations of the output impedance for the classic Howland current source with different resistor tolerances, assuming $A = \infty$, $3\sigma = T$, $R = 1\,\mathrm{k\Omega}$. The improved Howland current sources uses $R = 10\,\mathrm{k\Omega}$ and $R_{2b} = 1\,\mathrm{k\Omega}$. The number of simulation runs was 10^5.

Figure 3.53 gives a more complete picture of the expected output impedance distribution when implementing an (improved) Howland current source without trimming. Do note, that figure 3.53 only shows the absolute value of the output impedance. The impedance can be either positive or negative, depending on the resistor ratios. The probability is evenly distributed between negative and positive impedance, producing a left-handed negative copy of the plot in figure 3.53. For the purpose of improving readability of the plot, only the absolute value is plotted. A negative impedance means, that with increasing load voltage, the output current increases as well. For the purpose of of driving laser diodes, this distinction is irrelevant as the output impedance mostly determines the noise immunity.

First, regarding the basic Howland current source, using $0.05\,\%$ tolerance resistors the lower limit of $500\,\mathrm{k\Omega}$ can be easily identified as the leftmost bin of the histogram, that contains a non-zero number of counts. The maximum probability is reached with around $2\,\mathrm{M\Omega}$. Integrating over the probability density of the values gives a $31.5\,\%$ probability to end up with an output impedance of at least $7.5\,\mathrm{M\Omega}$. This is not nearly enough to meet the specif action. Going to $0.01\,\%$ tolerance resistors, the lower limit of $2.5\,\mathrm{M\Omega}$ can again be identified by the absence of counts in the lower bins. The maximum probability is around $10\,\mathrm{M\Omega}$–$20\,\mathrm{M\Omega}$ and the chance of getting an output impedance of more than targeted $7.5\,\mathrm{M\Omega}$ is $95.6\,\%$, a number far closer to the desired 3σ value ($99.7\,\%$) and can be considered acceptable.

The improved Howland current source was simulated with $R = 10\,\mathrm{k\Omega}$ and $R_{2b} = 1\,\mathrm{k\Omega}$. It is therefore expected that according to equation A.36 the minimum output impedance is about a factor of two higher than compared to the basic Howland current source. Looking at figure 3.53, this assumption holds. For a resistor tolerance of $T = 0.05\,\%$, the minimum bin is $1.5\,\mathrm{M\Omega}$, with the maximum probability at $4.5\,\mathrm{M\Omega}$. The probability of getting a more than $7.5\,\mathrm{M\Omega}$ is $79.7\,\%$, which is quite an improvement over the $31.5\,\%$ of the basic current source, but still not usable. Using $T = 0.01\,\%$ resistors, the improved Howland current source has a minimum impedance of $8.5\,\mathrm{M\Omega}$, which is sufficient to meet the requirements and the maximum is between $20\,\mathrm{G\Omega}$ and $30\,\mathrm{G\Omega}$. The results are summarised again in table 3.11.

Configuration	Tolerance T	$R_{out,min}$ Bin	P($R_{out} \geq 7.5\,\mathrm{M\Omega}$)
HCS	$0.05\,\%$	$500\,\mathrm{k\Omega}$	$31.5\,\%$
	$0.01\,\%$	$4\,\mathrm{M\Omega}$	$95.6\,\%$
Improved HCS	$0.05\,\%$	$1.5\,\mathrm{M\Omega}$	$79.7\,\%$
	$0.01\,\%$	$8.5\,\mathrm{M\Omega}$	$100\,\%$

Table 3.11.: Summary of the Monte Carlo simulations for different current source configurations.

In comparison to the FET based precision current source, for which an output impedance of $40\,\mathrm{G\Omega}$ was calculated in equation 3.93, the Howland current source is the weak spot, when both are combined. To improve this situation the resistors can either to be trimmed or a JFET or MOSFET cascode can be added to improve the output impedance. Do note a cascode is not bidirectional though. Several trimming options were discussed by Pease [161], but trimming at this level will prove difficult. The desired trim for $R = 1\,\mathrm{k\Omega}$ can be calculated as

$$\begin{aligned} R_{out} &\approx \frac{R}{\epsilon} \approx \frac{R}{4T} \geq 7.5\,\mathrm{M\Omega} \\ \Rightarrow T &\approx 33\,\mathrm{\mu\Omega/\Omega}\,. \end{aligned}$$

The final aspect that needs to be discussed is the compliance voltage. The compliance voltage of the HCS and improved HCS was calculated in equations A.42 and A.43 as

$$V_{c,HCS} \leq \frac{1}{2} V_{o,max} \qquad\qquad V_{c,iHCS} \leq V_{o,max} - V_{in}$$

From those equations it can be seen that there is a significant difference between the the basic Howland current source and the improved HCS. The compliance voltage of the basic howland current source only depends on the maximum output voltage of the op amp and hence the supply voltage. The improved HCS depends on both the supply voltage and the input voltage. This makes it unsuitable for a laser driver modulation source, because the maximum modulation input typically is roughly the same voltage as the maximum output of a typical op-amp, since the laser driver is modulated or steered by another box using the same power rail. The improved Howland current source can only by employed in situations, where the input is well defined and a lot lower than the power supply rail of the HCS op-amp. For a laser driver modulation current source, the basic HCS is more suitable, since it is independent of the input. To achieve a high output impedance, a high quality matched array is necessary or alternatively a difference amplifier like the ADI LT1997 [128] can be used. Those amplifiers contain both the amplifier and a resistor network that is tightly matched. In case of the LT1997, the matching is better than $60\,\mu\Omega/\Omega$. Unfortunately, the choice of resistor values in this case is even more limited than the range of resistor arrays.

The problem of a limited choice of resistor values as arrays or integrated resistors can be addressed by a circuit that combines both the basic HCS and the improved HCS. This circuit is shown in figure 3.54.

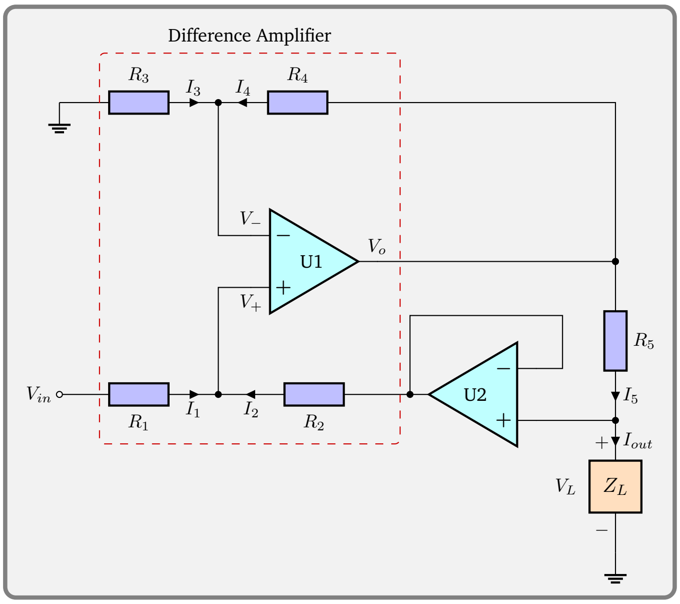


Figure 3.54.: A buffered Howland current source combining the improved HCS and the basic HCS.

By adding another op-amp R_2a is now connected to a low impedance node, just like in case of the basic Howland current source. This leads to the same formulae regarding the output impedance and the compliance voltage as the improved HCS, but removes the matching requirement of R_2a and R_2b. The full derivation can be found in the Python notebook at data/simulations/howland_current_source.ipynb as part of the online supplemental material [42].

With the buffered HCS it is now possible to use either 4 matched resistors or a difference amplifier with integrated resistors. The single resistor R_5 is used to set the output current and can be chosen independently to result in

$$I_{out} = \frac{V_{in}}{R_5}\,. \tag{3.108}$$

4. Results

"It's still magic even if you know how it's done."

– Sir Terry Pratchett, *A Hat Full of Sky*

4.1. Laser Current Driver

For this project several commercial and publicly available laser current drivers were evaluated for their performance. The following devices were all tested for the requirements listed in 3.1 and 3.2.

- Moglabs `DLC-102`
- Newport `TLB-6800-LN`
- SISYPH `SMC11 Puy Mary`
- Toptica `DCC 110`
- Vescent `D2-105`
- LQO `LQprO` [85]
- A driver based on the work of Erickson et al. [76]

As a disclaimer, Moglabs, Vescent and SISYPH provided demo units, free of charge to the author and without any obligations regarding this work. The opinions and measurements in this work are in no way biased by this service. All of these drivers claim low-noise in various comparative forms, but vary in features. The `DLC-102`, the `TLB-6800-LN` and the `D2-105` additionally include a peltier controller. The `DLC-102` and the `TLB-6800-LN` have a modulation source. The `TLB-6800-LN`, the `DCC 110` and the design by Erickson et al. also feature a digital interface.

The drivers were all compared to our requirements. While not all drivers feature a remote accessible interface, their performance was assesed nonetheless to have a broader range of choices. A performance comparison can be found in the sections 4.1.9, 4.1.10 and 4.1.11 . Unfortunately, none the drivers was able to properly drive the high compliance voltage required by the blue laser diode `PL 450B` of about $6–7\,\mathrm{V}$ [164]. As it was discussed in section 3.8.5, the compliance voltage of all laser drivers based on the design of Libbrecht et al. [120] is limited to around $2–3\,\mathrm{V}$ at full output (compare 3.106 for details). Since the compliance voltage is rougly proportional to the reciprocal of the output current current, limiting the maximum output current to about $30–40\,\%$ increases the compliance voltage to the required level. Not only does this limit the choice of drivers, but also requiring a $500\,\mathrm{mA}$ driver for a $150\,\mathrm{mA}$ laser diode seems excessive and does not help with the noise requirements, because the output noise of those drivers scales roughly with I_{max} as detailed in section 3.8.6, since the op-amp noise is the limiting factor. This lead to the decision to design a current source that meets all of our requirements, while surpassing all available alternative and tackling the compliance voltage limit. This design and its individual components are discussed in the following sections. First, the state of the art is presented, then the problems we encountered are outlined and finally our design, that resolves these issues is presented and the caveats and technical challenges are discussed.

4.1.1. The State of the Art in Laser Current Drivers

Prior to this work, all laser drivers for scientific purposes, were more or less strictly following the design proposed by Libbrecht et al. [120]. This design was presented in 1993 and back then, blue laser diodes were were not available and only developed in 1996. See [143] for an interesting historic summary. Finally, the efforts of Isamu Akasaki, Hiroshi Amano and Shuji Nakamura were rewarded with the Nobel Prize in Physics in 2014. The original laser driver design was therefore created for laser diodes requiring a low current and low compliance voltage compared to modern laser diodes. While the design remains useful for many low power near-infrared (NIR) laser diodes, these shortcommings were never addressed or even acknowleged by commercial alternatives. Sadly, the topic of the compliance voltage is usually not even mentioned in the datasheets – the Moglabs `DLC-102` and SISYPH `SMC11` are notable exceptions, but it is unclear from the data sheet to which version and/or currents of the devices the numbers relate. The Newport `TLB-6800-LN` is a bit different to the rest of the drivers tested, because it comes with Newport laser heads and reads its configuration data from the laser head. Ours came with a `Vantage TLB-7100` and the laser head needs to be connected for it to work. Without some reverse engineering, these drivers can only be used with certain Newport laser heads. The `TLB-6800-LN` is included in the list of devices anyway to give an idea of its performance in existing systems.

Laser driver	Output current	Compliance voltage	Additional features
Moglabs `DLC-102`	**100 mA**, 250 mA, 500 mA	**6 V**, 3.2 V	TEC, PID, Piezo
Newport `TLB-6800-LN`		–	TEC, Piezo, Digital
SISYPH `SMC11`	210 mA, **470 mA**	5 V	
Toptica `DCC 110`	**100 mA**, 500 mA, 3 A, 5 A	–	
Vescent `D2-105`	200 mA, **500 mA**	–	TEC
LQO `LQprO`	**140 mA**, 400 mA	–	

Table 4.1.: Overview of laser current drivers tested. Marked in bold is the version tested in this work. A dash denotes that no official information is available.

The drivers shown in table 4.1 will now be discussed in a little more detail to familiarize with them.

Starting with the Moglabs `DLC-102`, which is fully integrated unit, that leaves little functions to be desired, it includes the current driver, a pizo driver, a temperature controller for a thermoelectric cooler (TEC) and finally a lock-in amplifier with a PID controller. It is the most integrated solution, that was tested and brings all features required to set up an ECDL locked to an atomic transition. Some of its features are accessible via pin headers. The current source can be remotely enabled and the PID controller can be manipulated to allow relocking the laser or deliberately taking it out lock remotely. There is no way directly adjust the output current over the whole range. It also features a broad range of protection mechanisms for the laser diode, for example disconneting the driver in case of a short or open condition. All relavant quantities can be adjusted and read back from the front panel display. The manual is fairly comprehensive and gives a lot of examples to set up a laser system.

The Vescent `D2-105` also features more than just the laser driver and includes a TEC controller as well. Unfortunately, adjusting this temperature controller proved fairly cumbersome, because the driver has to be opened to adjust trimpots inside the driver. When the unit is placed in a rack, this might even be impossible to realize. The laser current is normally adjusted via a 10-turn potentiometer and can also be steered via an external input, but the stability of the current then depends on the external control voltage. It does not feature any protection features like open or short detection, so disconnecting and reconnecting the cable to the laser will most likely damage the laser diode. The display can be switched to show all relavant quantities. Moreover, the manual is not as comprehensive as the manual of the `DLC-102`, but covers all relavant settings of the driver.

The SISYPH `SMC11` does not include any additional features, but covers the most important protection features like an open detection and shuts down the driver accordingly. It is fully rackmountable, unlike the drivers discussed so far. The setpoint of the driver is adjusted via a recessed trimpot using a screwdriver, which has proven troublesome to adjust in the lab when not directly in front of the unit. The current can be externally adjusted using an input connector, but again this limits the stability to that of the external source. The driver does not have any display and all setpoints must either be adjusted blindly or a volmeter must be attached to the monitoring connector limiting the usefulness in a lab environment. The user manual covers only the basic settings and gives no details regaring the layout of the pin headers or external connectors making it hard to understand without having the device at hand.

The Toptica `DCC 110` is also only a current source and is rack mounted. It comes with a separate display module, which connect via the backplane. The setpoint is adjusted via 10-turn potentiometer and can additionally be adjusted via the backplane with an external signal, again limiting the stability of the driver to the external source. The manual is fairly comprehensice and covers all essentials.

The Newport `TLB-6800-LN` is the only driver, that has a digital interface and supports Standard Commands for Programmable Instruments (SCPI) commands. It incorporates a TEC controller and a piezo controller. Unfortunately, it only works with a limited number of lasers, because it reads some parameters, like the maximum output current from the laser head at startup. The user manual covers all device functions, but gives little detail about the hardware, making this a closed system.

The results of the performance tests conducted prior to building our own solution will be presented on the following pages, including problems typically encountered and their solution. These tests include the stability, current noise and output impedance of the drivers. Not all drivers were put through the full test, if it was already clear that they could not perform in our setup for a laser system for the spectroscopy of highly charged ions driving a blue laser diode.

4.1.2. Laser Driver: Design Concept

In order to interpret the results in sections 4.1.10, 4.1.12, it it helpful to discuss in more detail the design concept of the current state of the art, which is based on the driver design presented by Libbrecht et al. [120]. The design can be split into the four building blocks shown in figure 4.1. A supply voltage input filter, a reference voltage to create the setpoint, a unidirectional current source and some form of bidirectional current source used for modulating the laser current at high frequency.

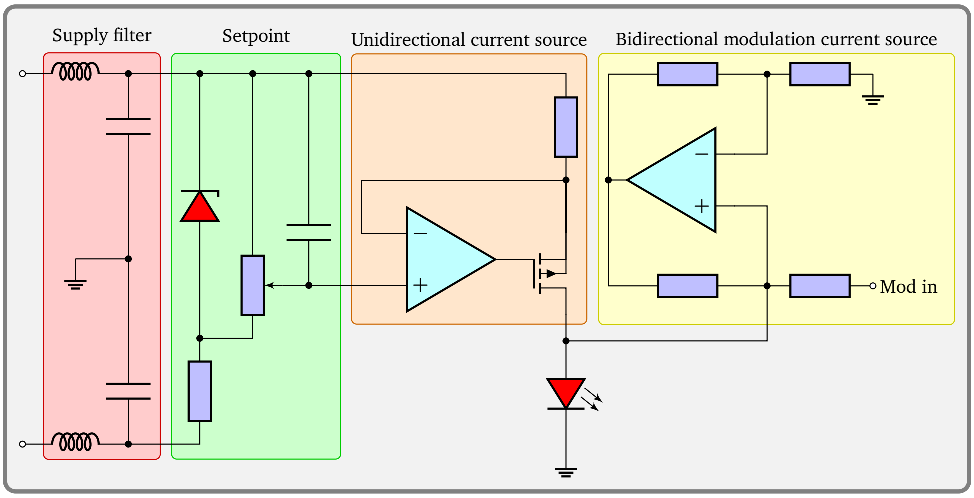


Figure 4.1.: Building blocks of a laser driver based on [120].

The original design is the most straightforward approach and it is possible to reproduce it even on prototype printed circuit boards (PCBs). Erickson et al. [76] replaced the potentiometer with a DAC, but left the other parts untouched. Taubman [210, 211] published some extensive modifications, which not only replaced the reference circuit with a DAC and an `LTZ1000` reference, but also added extensive filtering of the supply. The next sections will discuss these different elements separately and give some in insight into the different versions of the elements found in literature. Those sections also detail problems discoverd and the solution proposed in the design presented here.

4.1.3. Power Supply

The typical power supply scheme in the lab is a dual ±15 V backplane in a 19-inch rack. The subracks are, for example, Fischer Elektronik `BGT 384`. Into these, a backplance PCB [40], currently in development, can be fitted. The backplane has connectors for different modules and power connectors as standard 4 mm connectors. The current controller module uses a DIN41612 Type C, 64-pin connector to mate with the backplane. The pinout of the connector is shown in figure 4.2.

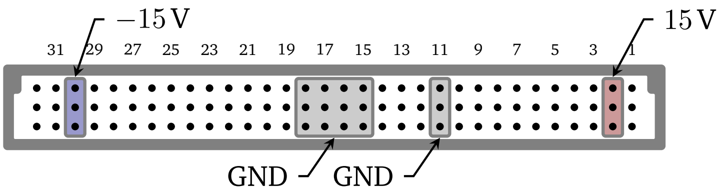


Figure 4.2.: The default male DIN41612 backplane connector pin layout of the digital current controller.

This layout is the default layout and configured on the standard units. All pin not marked, are internally not connected and it does not pose a problem to connect other low voltage signals to those pins. For example, due to legacy compatibility, pins 30–32 are normally tied together to −15 V on the backplane and the same goes for pins 1–3, which are connected together to 15 V.

There is also an alternative layout available, which can be configured using solder jumpers on the PCB. The secondary layout is similar to the layout used by Toptica and allows a convenient upgrade path to replace existing hardware.

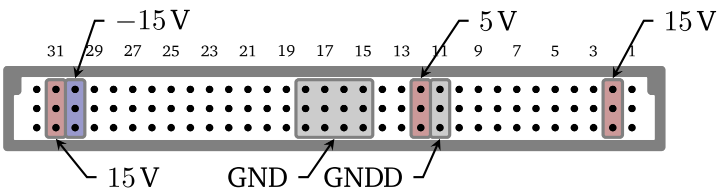


Figure 4.3.: The alternative male DIN41612 backplane connector pin layout of the digital current controller.

In order switch from one layout to another, consult table 4.2.

Jumper	Default Layout	Alternative Layout
JP1 (bottom)	open	closed
JP2	closed	open[1]
JP3	open	closed[1]
JP4	closed	open[1]

Table 4.2.: Jumper positions to enable different layouts. JP2 to JP3 are optional, see text.

[1] optional, see text

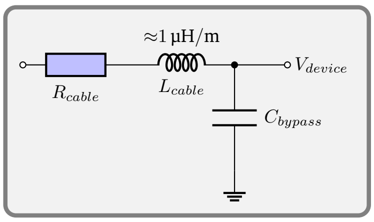
Jumper 1 on the backside of the PCB additionally connects pins 2 to 31. Normally, the digital power rail is fed from the power input as the analog power rail. Jumpers 2–4 are used to disconnect the digital supply from the analog side. The digital part of the laser driver is then galvanically isolated from the analog side and can be supplied by an isolated power supply if desired. Jumper 2 disconencts the digital ground from the analog ground and jumpers 3–4 change the power supply to the 5 V rail in pin 12. The digital circuitry must then be supplied through pins 11–12 as shown in figure 4.3.

The digital current controller is equipped with advanced power supply protection features, that ensure, that the device is not damaged due to incorrect handling. These protection features include over- and undervoltage protection, reverse polarity protection, surge protection and circuitry, that ensures correct power supply sequencing.

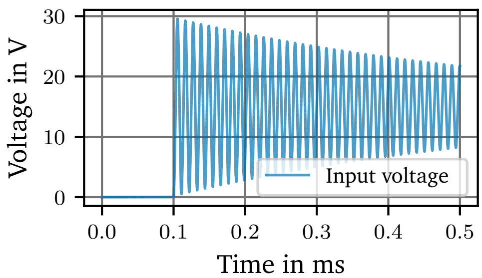
Supply Rail	Working Range	Absolute Maximum
15 V	15 V to 18 V	±40 V
−15 V	−15 V to −18 V	±40 V
5 V	5 V to 18 V	±20 V

Table 4.3.: Voltages above or below the absolute maximum rating may cause permanent damage to the device.

These types of protection are important in the context of the operating environment. Typically, in this lab, the subracks do not have an integrated power supply and the power is supplied using an external lab supply like an HP/Keysight 6632B [3], which is also mounted in the rack. Using this solution, there is no inherent protection against incorrectly setting up the power supply and no means to establish a foolproof procedure. These types of faults are furthered by the need to wire up two of the power supplies mentioned for dual polarity. The most common error encountered by the author, indeed was an incorrect polarity. Another problem is associated with hot swapping.



(a) Power supply parasisitics, forming an underdamped oscillator.



(b) Simulated oscillations of the supply voltage, when connecting to hot a 15 V rail. The parameters were $L = 3\,\mu H$, $R = 10\,m\Omega$ and $C = 1\,\mu F$.

Figure 4.4.: Oscillations of the supply voltage caused by the parasitic cable inductance and hot swapping modules.

The problem with hot swapping is the parasitic inductance of the power supply cable and the input capacitors of the module. Figure 4.4a shows the parasitic elements. The cable resistance is not enough to damp the LC resonator and upon insertion of a module, there will be strong oscillations, easily reaching multiples of the supply line voltage. Figure 4.4b shows a simulation with $3\,\mathrm{m}$ of cable and a very modest input capacitance of $1\,\mu\mathrm{F}$.

Ideally, hot swapping should be addressed by the backplane. Currently, most of the subracks in use are using flying wires, to hook up the connectors. The backplane PCB mentioned above, which is in development, can adress this problem by adding large capacitors (and a damping network) directly on the backplane to bypass the inductance of the power supply cable. Additionally, inrush current limiting per port may be required. This is part of the ongoing development.

Experience has shown, that contrary to advice, hot swapping is practised. It is therefore reasonable to include over- and undervoltage protection for the laser driver, since damage to both the driver and laser diode can be costly. The protection system is designed to safely shut down the laser in case of a fault of any type.

The protection is realized using a `SMBJ40CA` [200] $40\,\mathrm{V}$ bi-directional transient voltage suppressor (TVS) diode, while the overvoltage and reverse polarity protection uses an ADI `LTC4365` [131] controller and a Diodes Inc. `DMN6040SSD` [71] dual $60\,\mathrm{V}$ MOSFET connected back-to-back. One MOSFET is used for reverse polarity protection, the other for overvoltage protection. The `LTC4365` controller will typically shut down the driver within $<5\,\mu\mathrm{s}$ after detecting the fault, eliminating most, if not all, transient events.

Another feature worth mentioning is the builtin inrush current limit. The current driver does have roughly $1.1\,\mathrm{mF}$ of capacitance on each rail, which calls for inrush current limiting, to prevent currents of several Ampere from flowing at startup. The `LTC4365` is also used to limit the inrush current to $<100\,\mathrm{mA}$ in order to play nice with other devices sharing the same power rail.

The full schematic for the input protection can be found in the Github repository at [43].

4.1.4. Supply Filtering

The supply section of the design by Libbrecht et al. as shown in figure 4.1 is simplified. The full input filter consists of a CLC filter or sometimes called π-filter shown in figure 4.5. Do note, that due to the small input capacitance, the filter is basically just an LC-filter.

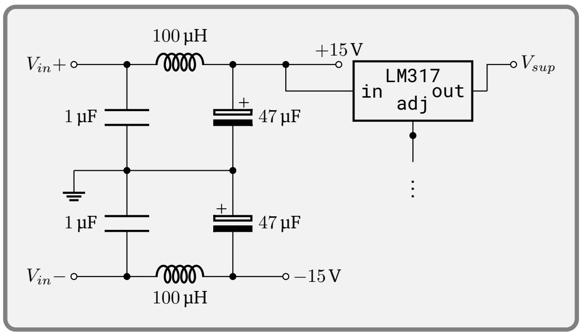


Figure 4.5.: Power supply filter of a laser driver based on [120]. The op-amps are supply by the filtered voltage and the current source is supplied by the `LM317`.

The LC-filter is best suited for a low impedance source like a power supply, because it has a high input impedance. From the transfer function

$$H(s) = \frac{Z_{out}}{Z_{in}} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{1}{s^2LC + 1} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{LC}} = \frac{\frac{1}{LC}}{\left(s + i\frac{1}{\sqrt{LC}}\right)\left(s - i\frac{1}{\sqrt{LC}}\right)}, \tag{4.1}$$

one can deduce, that the passband gain at DC is 1 (obviously) and additionally, there are two complex poles in the imaginary plane at $s = \frac{\pm i}{\sqrt{LC}}$, putting the cutoff frequency of the 2nd order filter at $f_c = 2.3\,\text{kHz}$. Do note, due to the imaginary poles, there is some gain peaking at f_c. Normally, this is damped by the parasitic resistance of the inductor and capacitor. A more detailed analysis follows below, because the cutoff frequency is all that is of interest right now.

In this design, the op-amps are directly driven off the filtered supply rail. Using this information it is possible to estimate the effectiveness of the filter. Using the $30\,\text{nA}_\text{rms}$ in a $100\,\text{kHz}$ bandwidth current noise requirement from specification 3.2, the voltage noise at the sense resistor ($14\,\Omega$, (see section 3.8.7) must be no more than $420\,\text{nV}_\text{rms}$. Now, taking for example a low-noise switch-mode power supply like the Rohde & Schwarz `HMP4040` used at CERN [26], which does have fairly pronouced noise at the switching frequency of around $170\,\text{kHz}$ and harmonics. The author measured these glitches to be about $3\,\text{mV}_\text{pp}$. The noise will have to go through the filter and the supply rejection ratio (PSRR) of the op-amp. The PSRR of the `LT1028` at $170\,\text{kHz}$ is about 10^{-2} under ideal conditions [127] and the filter adds another 10^{-2} when accounting for a $0.5\,\Omega$ series resistance of the output capacitor. The total filtering adds up to about 10^{-4}, which still leaves $21\,\text{nA}_\text{pp}$ of ripple on the drive current in a very small bandwidth.

To have a better rejection of such switch-mode noise, the filter must be improved. The paper presented by Taubman [210] shows a brute-force approach. They applied extremely high values for the capacitor C_{LC} of the LC filter of $10\,\text{mF}$ and then put a second filter based on a so-called capacitance multiplier behind it. This implementation is shown in a simplified form in figure 4.6 and briefly discussed now. For a more detailed schematic and part names see [210].

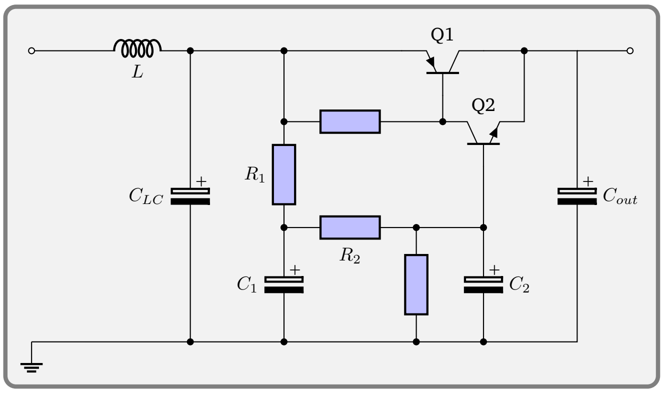


Figure 4.6.: Power supply filter using a capacitance multiplier for a cutoff frequency of 0.5 Hz. This is a simplified schematic based on [210]. Only the positive rail is shown.

Taubman built this filter for a driver with a driving capacity of 2 A, which limits the size of the inductor that can be used, in order to make up for that, he is forced to use giant capacitors. The second stage of the filter comprises a capacitance multiplier, which is formed by wrapping a feedback loop around the 2$^{\text{nd}}$ order filter created by R_1C_1 and R_2C_2. This feedback loop removes the main current from the filter resistors, to allow larger values for R_1 and R_2, while maintaining a fairly low output impedance of the filter. The properties of this construction will be discussed now.

As a first note, the circuit presented by Taubman misses a detail, which should included, when handling such capacitances. The circuit must include a reverse polarity protection, or rather a reverse current protection. If the input is shorted by accident, the 10 mF of capacitance would immediately discharge via the parasitic body diode of the slow start-up transistor, likely vaporizing everything in its path. This could be implemented by adding another transistor to act as a reverse current protection.

The following explanation relies on some basic knowlegde about transistors. A good introduction to transistors is the Transistor Manual [67], some even call the bible of transistors. Armed with some basic knowledge, the capacitance multiplier can be discussed and it must already be said, that the term capacitance multiplier is a bit misleading. It neither multiplies the capacitance, nor does is behave like a real 2$^{\text{nd}}$ order filter. The only thing that is multiplied by the gain of the transitor, is the output capacitance seen by load, making the capacitor look more ideal. Unfortunately, it highly depends on the properties of the transistor(s) and the gain of transistors drops with increasing output current (although it rises with temperature). Make sure to consult the datasheet, which typically gives a plot of the ac current gain h_{fe} vs the collector current I_C to see at which load current, h_{fe} starts dropping. Another issue is the bandwidth of the circuit, because h_{fe} of any transistor rolls off with frequency. The high frequency resonse of this filter then becomes constant. This limits the suppression at around 1 kHz (depending on the output current of course) to around 500–1000. It is usefuly at low frequencies though, as it is shown in the publication of Taubman. Another point is the maximum ripple voltage, that can be filtered. The muliplied capacitance does, of course, not store the same amount of energy as a real capacitor. This means the maximum peak-to-peak input ripple is limited to about one diode drop of 0.68 V from the Collector-Emitter diode. If more ripplle rejection is required an additional resistor from the base of Q_2 to ground like shown in figure 4.6 can be applied. This reduces the output voltage further though. In this design, the current through R_m is sufficient.

As a final remark regarding the capacitance multiplier is the output impedance. The transistor has an output impedance like a diode, so it increases with decreasing current. This means it will also drop about $1.2\,\mathrm{V}$ at $500\,\mathrm{mA}$ and about $0.3\,\mathrm{V}$ at $1\,\mathrm{mA}$. This behaviour must be taken care of by the voltage regulator following the capacitance multiplier. The $2\,\mathrm{V}$ drop is also not a problem in this use case and even comes in handy. If just the supply rail of the laser diode current is fed through the capacitance multiplier, as it is the most sensitive, and the supply rail for the op-amps is not, then those extra $2\,\mathrm{V}$ will do not be a problem. In section 3.8.7 it was already mentioned, that for example the `AD797` op-amp needs a supply that is $3\,\mathrm{V}$ above the diode supply. This means, that less voltage needs to be dropped by the linear regulator that follows the filter. To sum it up, the capacitance multiplier behaves like an ordinary RC filter, but with a lower output impedance and only works at low ripple voltages, is limited in the high frequency domain.

The power supply filters applied in this design use a passive LC filter for the negative and positive rail, then a capacitance multiplier on the diode supply. The negative rail is simply mirrored from the positive rail and pnp instead of npn transistors and vice versa are used. The combined filter is shown in figure 4.7. The diode supply and the analog rail, which is taken before the capacitance multiplier, are fed to low noise post-regulators, the `LT3045` and its negative counterpart, the `LT3094`. Both regulators have excellent power supply ripple rejection (PSRR) out to at least $1\,\mathrm{MHz}$ of more than 10^3. At low frequency the PSRR is even higher and more than 10^5 can be expected. This allows a combined PSRR of better than 10^6 from low to high frequencies, even beyond $1\,\mathrm{MHz}$.

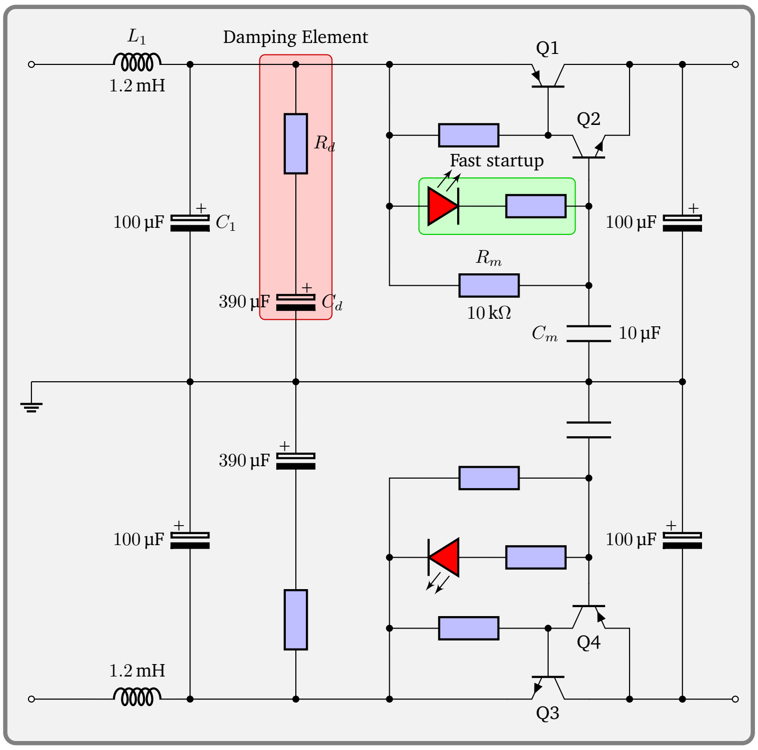


Figure 4.7.: Power supply filter of the digital current driver.

Regarding the filter circuit shown in figure 4.7 a few explaining words on the choice of components are in order before proceeding to the measurement of the PSRR. Going back to equation 4.1, we saw, that the undamped second filter has excessive ringing at the cutoff frequency, because the filter poles are imaginary. To address this, there are several solutions. The most simple one is adding a damping element, either in parallel to the capacitor or in parallel the inductor. In this case a damping element in parallel to the capacitor was chosen, because using a a damping element parallel to the inductor will degrade the filter performance by making the blocking inductor lossy. Using the arangement shown in figure 4.7, the transfer function can be calculated.

$$
\begin{aligned}
H(s) &= \frac{Z_{out}}{Z_{in}} \\
Z_{out} &= (R_d + Z_{C_d})\,||Z_{C_1} = \left(R_d + \frac{1}{sC_d}\right)||\frac{1}{sC_1} = \left(\left(R_d + \frac{1}{sC_d}\right)^{-1} + sC_1\right)^{-1} \\
&= \frac{sC_dR_d + 1}{s^2C_1C_dR_d + s\,(C_1C_d)} \qquad (4.2)\\
Z_{in} &= sL_1 + (R_d + Z_{C_d}) \\
H(s) &= \frac{\frac{sC_dR_d+1}{s^2C_1C_dR_d+s(C_1C_d)}}{sL_1 + \frac{sC_dR_d+1}{s^2C_1C_dR_d+s(C_1+C_d)}} = \frac{sC_dR_d + 1}{s^3L_1C_1C_dR_d + s^2L_1\,(C_1 + C_d) + sC_dR_d + 1} \qquad (4.3)
\end{aligned}
$$

This is the transfer function of a 3rd order filter. This type of filter was discussed by Middlebrook [151] (reprinted in [60] and [61]). Middlebrook derived, that there is an optimal value for the series resitance R_d given a capacitance C_d and the filter components L_1 and C_1. This optimal value has minimal gain peaking, hence a minimal quality factor Q at the resonance freuency. The existance of such an optimal value can be easily understood from the fact, that if $R_d = \infty$ the resonance frequency is $\omega_0 = \frac{1}{\sqrt{L_1C_1}}$ and in case $R_d = 0$ it is $\omega_1 = \frac{1}{\sqrt{L_1(C_1+C_d)}}$. In between ω_0 and ω_1, there is a lossy zone, where R_d due to its lossy nature reduces Q, but at both ends $Q = \infty$, so there must be a minimum in between. By calculating, the minimum value of the transfer function at the point of resonance, Middlebrook found the following results:

$$R_0 := \sqrt{\frac{L_1}{C_1}} \qquad (4.4)$$

$$n := \frac{C_d}{C_1} \Rightarrow C_d = nC_1 \qquad (4.5)$$

$$Q_{optimal} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} \qquad (4.6)$$

$$R_d = R_0 \cdot Q_{optimal} \qquad (4.7)$$

From these equations, it can be seen, that the damping capacitor C_d needs to be fairly large, depending on n. A critically damped system with $Q = 0.5$ would be preferred, but this would require $n \approx 6$ making the C_d prohibitively large. For this filter $n = 4$ was chosen, so making the filter slightly underdamped, so a slight gain peaking at the resonace can be expected. The following componets were chosen. First, a large, low resistance inductor L_1 capable of carrying at least 1 A was chosen. In this case a Coilcraft `MSS1210-125KEB`. High reliability capacitors were chosen to ensure a long lifetime of the device. Choosing capacitors rated with a lifetime of 5000 h at 105 °C gives a expected service life of more than 10 a, when assuming an Arrhenius law with a doubling of the lifetime every 10 K. Apart from the reliability of the capacitors, there are no special requirements for them as there is little ripple current to be expected. The input power supply is supposed to be a filtered low noise supply and not the unfiltered output of a

DC/DC regulator. So it is possible to maximize L_1 and choose a physically smaller C_1 since board space is limited. This results in the following design values, calculated from equations 4.5 and 4.5, given the components values discussed above.

$$
\begin{aligned}
C_1 &= 100\,\mu\mathrm{F} \\
n &= 4 \\
Q_{optimal} &\approx 0.61 \\
R_d &= 0.61 \approx R_0 \approx 2\,\Omega \\
C_d &= 400\,\mu\mathrm{F} \approx 390\,\mu\mathrm{F} \\
f_c &\approx 300\,\mathrm{Hz}
\end{aligned}
$$

Do note, that R_d does include the equivalent series resistance (ESR) of C_d, so the ESR of the capacitor must be subtracted from the final value of the damping resistor placed on the board. This may even absolve one from the need for a discrete resistor if the ESR of the capacitor is high enough. The transistors chose were a combination of a Toshiba `TTA004B/TTC004B` and Onsemi `BC817-40/BC807-40` for the positive/negative rail. The `TTA004B/TTC004B` are good up to about $500\,\mathrm{mA}$. At this point the gains start dropping. A higher power transistor like the Onsemi `D45H8/D44H8` used by Taubman is recommended for Q_1 and Q_3.

Finally, one last part of the capacitance multiplier should be explained. Highlighted in green in figure 4.7 is a fast startup circuit. At startup, the capacitor C_m is discharged and $15\,\mathrm{V}$ will be applied, it will then begin to charge with a current of $1.5\,\mathrm{mA}$ through the $10\,\mathrm{k\Omega}$ resistor. Because the Q_2 is an emitter follower, hence the emitter follows the voltage at the base (minus a diode drop for the base-emitter diode). As a sidenote, when using this kind of circuit, since Q_2 is an emitter follower, all output capacitors, that follow the capacitance multiplier will charge at the same rate as C_m, voltage-wise, this means, that for every $10\,\mu\mathrm{F}$ of output capacitance, a current of $1.5\,\mathrm{mA}$ will flow. While this not significant at this moment it become so, when looking at the fast startup circuit. Applying the input voltage of $15\,\mathrm{V}$ at startup over the LED, a $625\,\mathrm{nm}$ Würth Elektronik `150080RS75000`, it will start conducting, resulting in a 1.8–$2\,\mathrm{V}$ drop. The current flowing into C_m is therefore dependent on the diode series resistor, which was chosen to be $510\,\Omega$, a value not particularly important in this case. So at startup about $25\,\mathrm{mA}$ will flow into C_m, which means $2.5\,\mathrm{mA}/\mu\mathrm{F}$ will flow through Q_1 and Q_2. Assuming roughly $100\,\mu\mathrm{F}$ of distributed bypassing capacitance around the board, this is around $500\,\mathrm{mA}$. All these values are still well below the damage threshold of the transistors ($2.5\,\mathrm{A}$ and $0.5\,\mathrm{A}$) and the LED ($30\,\mathrm{mA}$), but these values must be kept in mind, when adding larger output capacitors. The fast startup circuit ensures an output voltage of $13\,\mathrm{V}$ within $100\,\mathrm{ms}$ instead of around $0.5\,\mathrm{s}$, reducing the time to boot and leaving more time for self-checks without impacting the user experience.

After choosing the values above, the filter was simulated using LTSpice to assert the validity of the parameters chosen. The simulation was conducted with a load current of $500\,\mathrm{mA}$ running through the capacitance multiplier to simulate the worst case. As discussed above, the gain of the transistor h_{fe} drops at higher currents as the transistor saturates. This particularly affects the high frequency behaviour above $10\,\mathrm{kHz}$. The source file can be found at source/spice/input_filter_dgdrive.asc. The simulation additionally includes the series resistance and parasitic parallel capacitance of L_1, the latter will induce some ringing at the self resonance frequency of the inductor at $1\,\mathrm{MHz}$ and limit the useful attenuation beyond that to around 10^3 due to the capacitive coupling of the conductor windings. At the $170\,\mathrm{kHz}$ discussed before, the damping is about the same figure of merit, 10^3.

The suppression is an order of magnitude better than the filter used by Libbrecht et al. and it does not even include the high performance regulators that follow. The transfer function for both the damped LC filter and the LC filter with the capacitance multiplier in series is plotted in figure 4.8. The self resonance peak at 1 MHz can be clearly seen and is not damped, but from the output impedance shows shows that, there is enough capacitance present to compensate for this. The output impedance above 1 MHz, it is dominated by the local bypass capacitors and not accurately represented by the simulation. It can be expected to by lower than the simulated results, which do not include those capacitors.

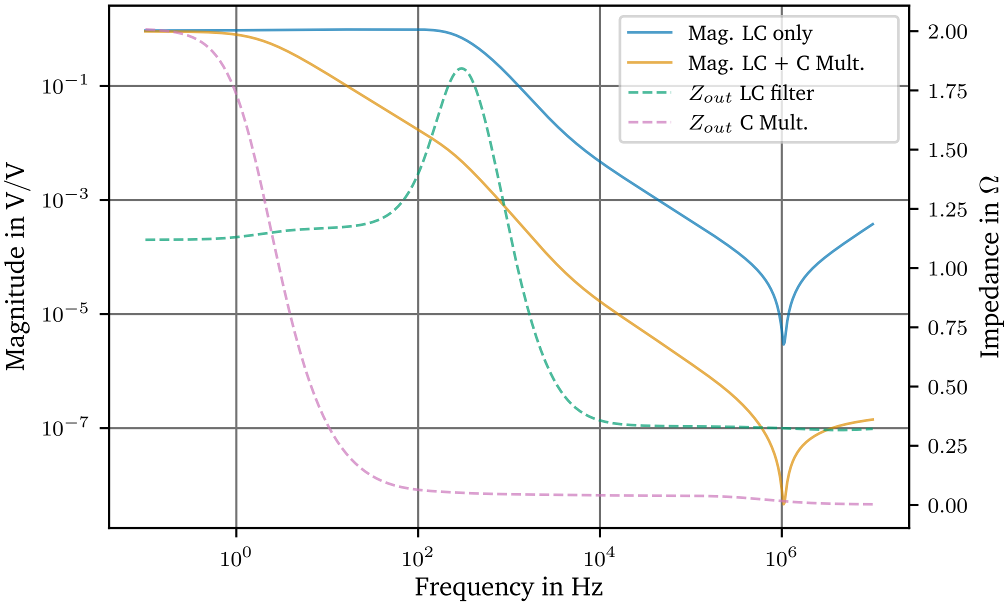


Figure 4.8.: Simulated response of input filter used in the digital current driver. Both magnitude and output impedance of the stages are shown.

At 300 Hz, the LC filter cutoff frequency, the output imedance of the LC filter shows some gain peaking. This is due to the underdamped response chosen and discussed above. This peaking increases the output impeance of 1.2 Ω in the passband, which is mostly the resistance of the inductor, to a total of 1.8 Ω. This can be easily compensated for by the regulators downstream.

The rejection ratio of the LC filter and the capacitance multiplier is better than 10^3 at at 100 kHz an above, delivering the performance estimated above. This is expected to keep switch-mode noise away from the laser driver current.

The high rejection ratio of the filter is expected to make the the experimental validation rather challenging, because there are a number of complications that derive from the active nature of the circuit. The capacitance multiplier must be loaded, preferably at the maximum current to show the worst case and additionally, the ripple voltage must be low enough to not saturate the capacitance multiplier.

This requires a highly sensitive VNA, that has a low frequency range. This setup uses an Omicron Lab `Bode 100`, which can measure from 1 Hz to 50 MHz with an exceptionally low noise floor of about 180 nV/$\sqrt{\text{Hz}}$ [212]. Additionally a Stanford Research `SR560` was used as a pre-amplifier. To apply the ripple voltage to the power supply rails a Picotest `J2123A` negative line injector and a self-designed positive line injector was used. The positive line injector design is available open-source and be found in a Github repository at [38]. This injector is called `PB02`. During the measurement, it was found, that since the expected signal is extremely small, ground currents became an issue. There is an inherent ground loop issue built into the VNA. The outputs and inputs of the `Bode 100` are not isolated. The measurement is a 3-port measurement as shown in figure 4.9. The `Bode 100` is driving the line injectors, measuring the signal going into the line injector and finally sampling the signal accross the output capacitor of the filter. The ground current now has two choices of flowing. One is through the low side of the measurement cables and their resistance, or through the ground plane of the VNA. The latter is the dreaded ground loop. This ground loop becomes more pronouced at higher frequencies, because the return path trough the cable is inductive and its impedance increases with frequency. Typically this problem would be addressed using a common-mode choke inserted into CH2. CH2 is the VNA input measuring the filter output. This common-mode choke prevents any current flowing through CH2, that has not flown through the cable.

Unfortunately, the author did not have a suitable common mode choke at hand, so the only feasable solution to at least suppress the ground loop for low frequencies was to add transformers at the output and the CH2. This isolates the output and the battery powered `SR560` is driving the VNA via the transformer, isolating the input as well. The transformer used for isolating the VNA output, was an injection transformer named `PB01`. It is center tapped to create an anti-symmetrical output for the injection transformers. The center tap reduces the output amplitude by one half. The details regarding this device and its construction can be found in annex A.6. The transformer used at the output of the `SR560` is a Picotest `J2123A`. Both transformers are unfortunately injection transformers and not dedicated isolation transformers as discussed in annex A.6, yet the only transformers available at the time. The consequences of this subtle detail will become imminent in a moment.

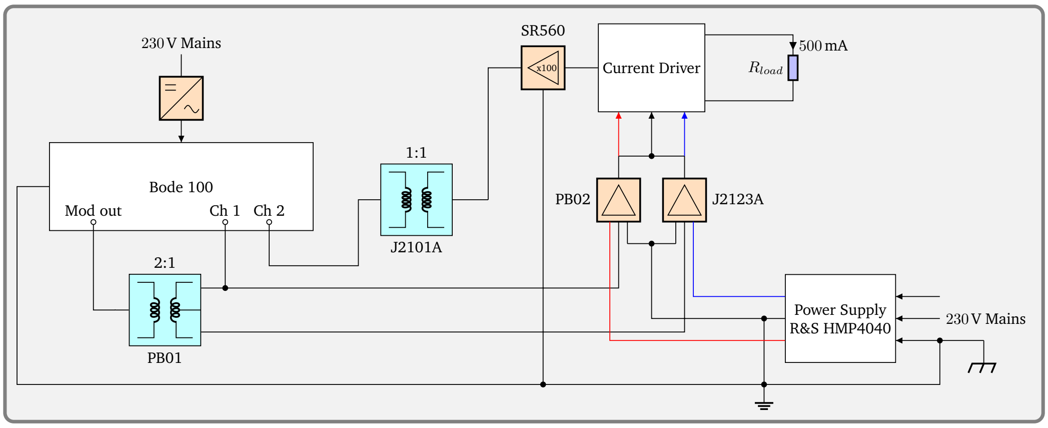


Figure 4.9.: Power and grounding scheme for a low noise measurement of the line filter rejection ratio, minimizing the interfernce of circuit return currents.

The digital current driver is powered by the Rohde & Schwarz `HMP4040` and there is a single point of ground connected to protective earth at the power supply. The power supply feeds into the line injectors, through which the current driver is powered. The output current is set to $500\,\mathrm{mA}$ accross a $10\,\Omega$ dummy resistor. The SR560 measures the ripple voltage after the LC-filter and drives the VNA input via the transformer. The measurement cable used is a short twisted pair to reduce noise pickup.

The ouput of the VNA was set to $-27\,\mathrm{dBm}$ ($10\,\mathrm{mV_{rms}}$), which must by multiplied by about $0.5 \cdot 0.975 = 0.485$ to give the ripple voltage on the positive supply, the latter term comes from the line injector [38]. Putting this into perspective, given a $-60\,\mathrm{dB}$ (10^{-3}) suppression, results in a ripple voltage of only $4.9\,\mu V_{rms}$.

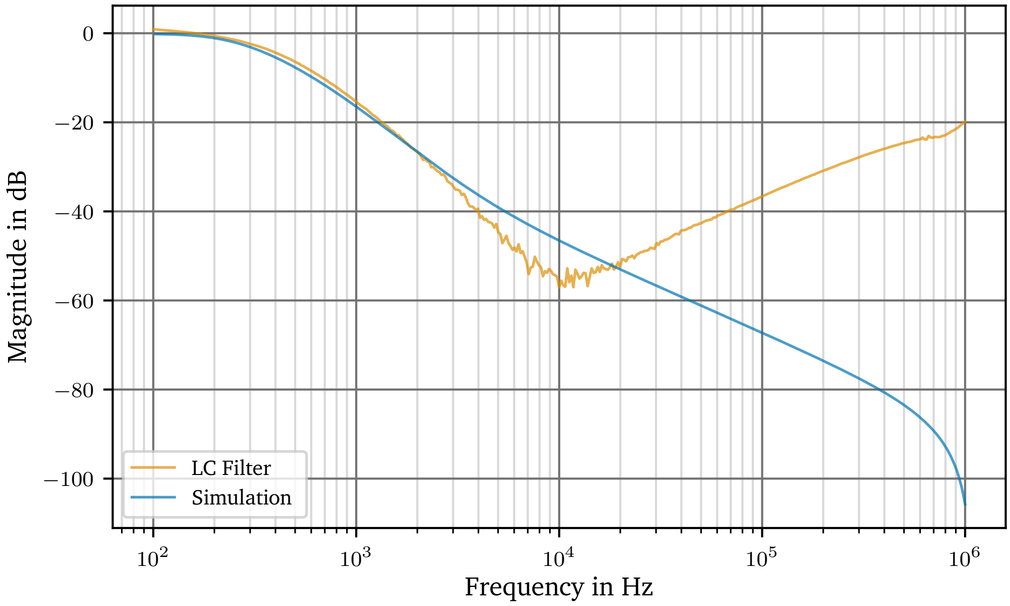


Figure 4.10.: Measured response of input filter used in the digital current driver. Above $10\,\mathrm{kHz}$ capacitive coupling through the transformer can be seen.

Figure 4.10 shows the measurement of the LC-filter output. At low frequencies, there is good agreement with the simulation and the filter rolls off with $-40\,\mathrm{dB/decade}$. At around $10\,\mathrm{kHz}$ the noise floor of the measurement is reached at $-55\,\mathrm{dB}$. Then the ground loop shows itself again coupling through the transformers. The magnitude rising with $20\,\mathrm{dB/decade}$ is an artifact, which can be significantly influence by changing the type of probing and the location of probing, so at this point it is clear that for any usable data above $10\,\mathrm{kHz}$ a common-mode choke is required. Additionally the noise floor of the measurement is reached at around the same frequency, requiring a lower noise amplifier. Due to the lack of another amplifier and the choke, the author left the measurement as is. It is still a good example to show the pitfalls of a 2- or 3- measurement. This topic will be revisted later in section 4.1.10, when measuring the current noise of the driver.

To conclude, the measurements show that the LC supply filter is correctly damped with the expected corner frequency of $300\,\mathrm{Hz}$ and it will likely perform as intended, but above $10\,\mathrm{kHz}$ the filter performance cannot be accurately measured due to the limited setup.

4.1.5. Voltage Reference and Setpoing Adjustment

The voltage reference used in this Design is the ADI `LM399` and alternatively the ADI `ADR1399`, but the latter was not available in sufficient quantities for production. So only preliminary tests were conducted on this reference. The `LM399` is used by most current drivers tested for the laser system of highly charged ions, because, as discussed in section 3.8.7, it is the most economical solution, although this statement will by qualified again in section 4.1.13 with regard to the Zener diode selection process.

Nonetheless, the `LM399` is also used in this design and there are two approaches to integrate it as a voltage reference. The first concept is the one used by Libbrecht et al. [120] and shown in figure 4.1. It is a simple arangement of a resistor in series with the Zener diode. The resistor is used to regulate the current through the diode, according to the data sheet around $1\,\mathrm{mA}$ [125] should be used. The circuit presented by [120] and carried over unmodified by Seck et al. [185] uses $2\,\mathrm{mA}$ at the maximum diode supply voltage of $12\,\mathrm{V}$. The maximum diode supply voltage is defined by the minimum dropout voltage of the `LM317`, which is $3\,\mathrm{V}$ [126] and the input supply voltage of $15\,\mathrm{V}$. The diode supply voltage can also be regulated down to limit the compliance voltage. This in turn reduces the Zener current accordingly.

As a sidenote, the diode supply voltage adjustment circuitry is rather questionable, because it allows output adjustment of the `LM317` up to a maximum $24\,\mathrm{V}$, although there is only a $15\,\mathrm{V}$ input. Maybe the resistance value ($5\,\mathrm{k\Omega}$) of the trimmer was intended for a higher input voltage or was a left-over from a previous design phase. Using [126]

$$V_{out} = 1.25\,\mathrm{V}\left(1 + \frac{R_2}{R_1}\right) = 1.25\,\mathrm{V}\left(1 + \frac{R_{trim}}{275\,\Omega}\right),$$

half the value of R_{trim} would have sufficed to provide the $12\,\mathrm{V}$ minimum output.

This voltage adjustend option is furthermore advertised as a current/voltage limit, but its use is particularly hideous, because as discussed in section 3.8.5, the MOSFET needs a certain voltage to remain in the saturation region for correct operation. So adjusting the supply voltage to be within the safe operating range of the laser diode will force a limit on the output current. Assuming a minmum saturation voltage of $2\,\mathrm{V}$ for the `VP0106` MOSFET [65] and the $50\,\Omega$ sense resistor used by Libbrecht et al. [120] results in an output current reduction of $\frac{2\,\mathrm{V}}{50\,\Omega} = 40\,\mathrm{mA}$. This limit is only a soft limit though, not limiting the output current, but rather defining the limit at which the current source will work as intended. When bringing the driver above this current, the output impedance will be seriously reduced as shown in figure 3.46 and noise starts entering the system as detailed by [185]. A somewhat undesirerable side-effect, especially, as there is no way to monitor the current in this design.

Going back the Zener diode, the adjustable supply voltage has another drawback, because it influences the Zener voltage of the reference. The `LM399` has a dynamic impedance of $1.5\,\Omega$ [125]. This means that the Zener voltage changes $1.5\,\mathrm{mV/mA}$. Using the value of the Zener diode bias resistor $R_{bias} = 7.5\,\mathrm{k\Omega}$ given in [120] gives a suppression of

$$\frac{1.5}{7500 + 1.5} \approx \frac{1.5}{7500} = \frac{1}{5000},$$

totally obliterating the stability of the diode. So it is best adjusted once, then left alone.

Fortunately, this has little bearing on the high frequency performance due to the filter that follows. Coming to the filter, there is another issue in the original design due to the filter impedance. As shown in equation 3.103, the output impedance of an RC filter at low frequency is R_{filt}. For the filter circuit shown in figure 4.1 taken from [120] $R_{filt} =$

$6\,k\Omega + (0 \text{ to } 5)\,k\Omega$, depending on the potentiometer setting, with the maximum of $5\,k\Omega$ in the center position. Considering the current noise of the LT1028 results in a whooping $47\,nV/\sqrt{Hz}$ at $10\,Hz$ compared to the voltage noise of only $1\,nV/\sqrt{Hz}$. So the current noise is clearly dominating the noise figure of the op-amp. This number is, thankfully, still a lot less than the $170\,nV/\sqrt{Hz}$ of the LM399, but when considering the ADR1399, which has a noise density of $65\,nV/\sqrt{Hz}$, it is no longer negligible.

All of these issues were addressed in the design detailed now. An abridged version of the setpoint generation circuit is shown in figure 4.11, which is reduced to the most impotant components. The full schematic can be found in the Git repositories [33, 43].

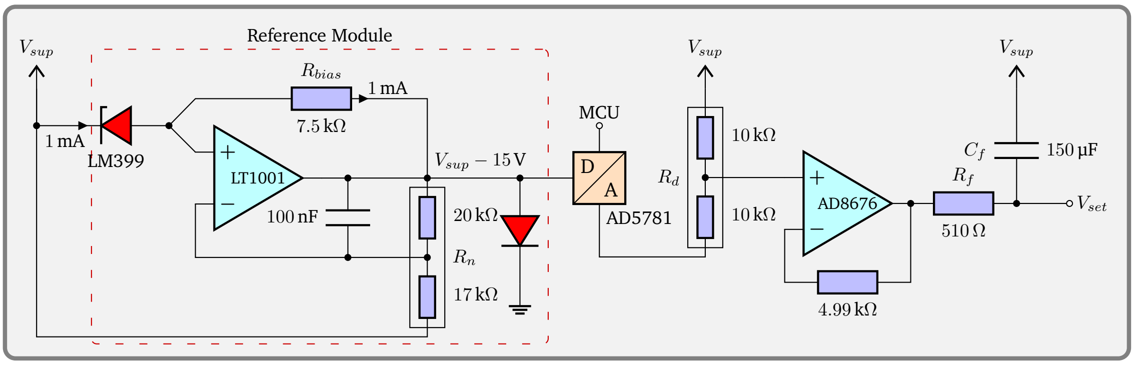


Figure 4.11.: Setpoint generation using the LM399, an AD5781 DAC followed by a 2:1 divider and an RC filter.

To better follow the discussion of figure 4.11 it is essential to remember, that all voltages are referenced to the diode voltage supply V_{sup} as opposed of ground like in a normal circuit. For example the $-7\,V$ Zener voltage becomes $V_{sup} + (-7\,V)$ and so on. This is done, because as it was shown in figure 3.45 when the precision current source was introduced, the setpoint to adjust the current is referenced to V_{sup}. For reasons of simplicity, the author only refers to these values as $-7\,V$.

To understand the circuit shown in figure 4.11, it is best to split it into two pars, the voltage reference, which is a separate module, shown in a red dashed box and the setpoint generation using a DAC. The DAC will be discussed first, because some of the design decisions are rooted there.

While there are many DACs available on the market, fine grained adjustment of the laser current is desired limiting the choice of DACs. Erickson et al. [76] used an AD5541 $16\,$bit DAC in their design, which provides up to $70\,mA$, having a resolution of about $25\,\mu A$. Taubman et al. [211] presented a higher power $1\,A$ controller with an $18\,$bit Texas Instruments (TI) DAC9881SB DAC granting a resultion of $4\,\mu A$. The resolution required by this design is mostly defined by the blue laser and as it was mentioned in section 3.2.1, the laser is stable over the range of a few tens of µA and requires up to $145\,mA$. A $16\,$bit DAC would offer a resolution of $2\,\mu A$, which would only allow a few DAC codes of stable operation. Therefore a DAC for $18\,$bit or more is desirable. The choice between an $18\,$bit and a $20\,$bit DAC is predetermined by the choice of reference. The noise of the LM399 of around $0.5\,\mu V/V$ at low frequency and its drift performance [125] makes an $18\,$bit DAC the more reasonable choice. The choice of the current source circuit limits the choice of DACs further. Using a current source, which is referenced to V_{sup} requires the DAC to either accecpt V_{sup} as the positive reference voltage directly or one must float the DAC like Erickson et al. did. Additionally both Erickson et al. and Taubman et al. divided down the reference voltage from $-7\,V$ to $-5\,V$ to match the maximum input voltage of the DAC. Floating the DAC brings about a number of problems, like having to level-shift all control signals as Erickson et al. [76] did. Correct power suppply sequencing also becomes vital.

A simpler approach can be taken if the DAC can accept both V_{sup} as the postive reference and $V_{sup} + V_{ref}$ as the negative reference without being floated to $V_{sup} + V_{ref}$. A typical bipolar output DAC almost matches those requirements. The only problem is a constraint imposed by the negative reference input, which must typically be at a potential less than ground. This problem can be solved by amplifying the reference voltage to ensure $|V_{ref}| \geq V_{sup}$. This also has the added benefit of reducing the noise contribution and offset drifts of the added circuitry, like thermal electromotive force (EMF). The suppression is inversely proportional to the gain. This approach is persued here and its implementation is explained below.

The only bipolar DACs meeting the requirements are the TI `DAC91001` [63] and the ADI `AD5781` [9], both 18 bit devices with a pin compatibe upgrade path to a 20 bit should the need arise. Both devices are fairly similar in regard to this application. The device chosen was based on the availability, the `DAC91001` is only available through TI, while the `AD5781` is available through multiple distributors. Additionally, the author has worked on multiple DACs from ADI before and the implementation details are similar, which cut down the development time.

The DAC reference inputs and the output is buffered using an ADI `ADA4077-4` using a remote sensing arrangement as per the data sheet recommendation [9]. V_{sup} is taking directly from the four-wire sense resistor, a Vishay `VPR221Z` (see section 4.1.6 for details). The negative reference voltage $V_{sup} + V_{ref}$ is the amplified Zener voltage taken from the reference module output. Typically, $V_{ref} = -15\,\mathrm{V}$ is used, because this ensures that $|V_{ref}| \geq V_{sup}$ at all times. The typical supply voltage of the current driver is $\pm 15\,\mathrm{V}$ (maximum $\pm 18\,\mathrm{V}$) and therefore $V_{sup} \leq 15\,\mathrm{V}$ due to required headroom of $3\,\mathrm{V}$ for the op-amps used as discussed in section 3.8.7.

The $-15\,\mathrm{V}$ reference must then be divided down after the DAC, because the setpoint, which is the voltage dropped across the sense resistor must naturally be smaller than V_{sup}. The resistor network forming a 2:1 divider is a Vishay `DSMZ` $10\,\mathrm{k\Omega}/10\,\mathrm{k\Omega}$ network. The factor of $\frac{1}{2}$ is chosen because it is the most stable ratio, as the resistor values are of the same magnitude. An ADI `AD8676` [12] dual op-amp buffers both V_{sup} to supply one arm of the divider and also the output of the divider using the the second op-amp. This output buffer uses a $5\,\mathrm{k\Omega}$ compensation resistor in the feedback path to match the input resistance. This compensates for an input offset drift, due to changes in the input bias current to cover the worst-case specifications of the op-amp ($\pm 4.5\,\mathrm{nA}$). The drift would result from the difference in the input bias current, because one input sees $5\,\mathrm{k\Omega}$, while the other input sees zero resistance in a unity-buffer configuration. The divider is then followed by single pole filter in contrast to the second-order filter used by [76, 120, 185] shown in figure 4.12.

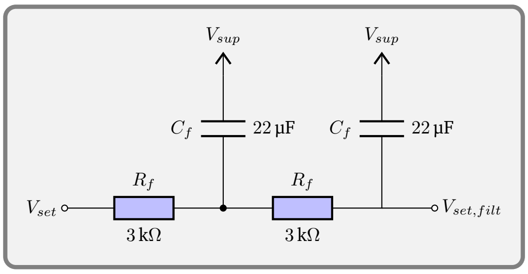


Figure 4.12.: Setpoint filter used in [76, 120, 185].

The second-order filter is overdamped ($\zeta = 1.5$) with a cutoff frequency of $2.4\,\mathrm{Hz}$. The likely reason this filter is applied, is because the LT1028, used in the design by Libbrecht et al., does have a very low flicker noise corner of $3.5\,\mathrm{Hz}$ [127].

There are two problems with the approach though. First, the current noise of the LT1028 is dominating at low frequencies with more than $5\,\mathrm{k\Omega}$ of input resistance and second the bias current cancelling circuit inside the op-amp, which requires a matched source resistance. The bias current bias current cancelling circuit is injecting a small amount of correlated current noise into both inputs. Using a large input resistance, this noise needs to be considered as well. With unmatched source resistances it is not suppressed by the common mode rejection as explained in section 3.8.6. Therefore, contrary to intuition, using an additional resistor at the inverting input reduces the total noise.

A filter designed for the AD797 needs to be less aggressive, because the flicker noise corner frequency is about an order of magnitude higher and around $30\,\mathrm{Hz}$ [10]. Therefore, a first order filter shown in figure 4.14 can be used. According to the discussion regarding the current noise above, the capacitor size should be maximized so that the resistive part of the RC filter can be reduced. Ideally R_f is kept well below $1\,\mathrm{k\Omega}$ to make sure that the total noise

$$\begin{aligned} e_{total} &= \left(e_n^2 + 4k_BTR_{total} + i_nR_{total}\right)^{\frac{1}{2}} \\ &= \left(e_n^2 + 4k_BT\left(R_f + R_S\right) + i_n\left(R_f + R_S\right)\right)^{\frac{1}{2}} \end{aligned} \tag{4.8}$$

is minimized. e_n is the voltage noise of the op-amp, i_n the current noise, R_f the filter resistor and R_S the sense resistor used for the current source as shown in figure 3.45. In order to determine to optimal filter components several options were simulated and also verified experimentally. The simulation results are shown in figure 4.13.

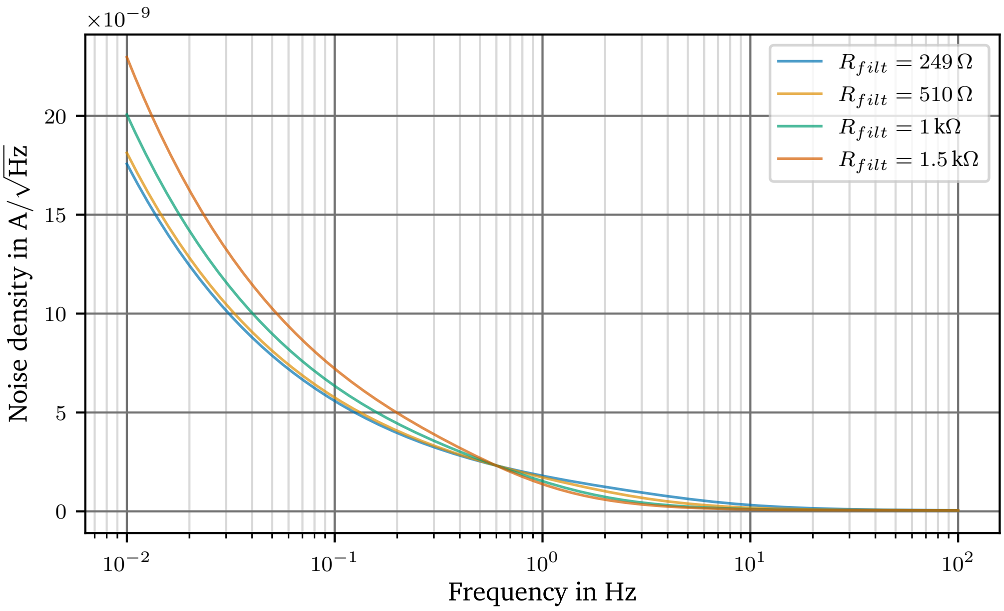


Figure 4.13.: Simulated noise density of a $250\,\mathrm{mA}$ current source running at $50\,\mathrm{mA}$ built according to figure 4.11. Compared are different values of the filter resistor.

The simulation is a slightly simplified simulation of the circuit shown in figure 4.11. The reference module, including the DAC is replaced by a noise model, which includes the flicker noise and the white noise of the Zener reference and the DAC. The noise of the buffers is neglected here, because the DAC noise quoted in the datasheet [9] already includes the noise and the noise of the Zener buffer is an order of magnitude less than the Zener noise. The `AD8676` buffer following the resistive divider is included and so is the `AD797` current source. The sense resistor value is $30\,\Omega$. The output current was set to $50\,\mathrm{mA}$. The full LTSpice simulation can be found among the supplemental material at source/current_regulator_AD797_noise.asc.

The simulation was conducted for several different values of R_{filt}. A range of values between $249\,\Omega$ and $1.5\,\mathrm{k\Omega}$ were simulated. The values reflect resistor values, that are commonly available. I can be seen that for higher values of R_{filt}, the current noise starts dominating the low frequency noise of the `AD797` amplifier as discussed above. The inflection point is around $0.6\,\mathrm{Hz}$ and the current source becomes increasing noisy below this frequency with increasing filter resistance. The choice of the capacitor ($150\,\mathrm{\mu F}$) is explained below.

As expected, the difference between a $249\,\Omega$ and a $510\,\Omega$ resistor is pretty small and the noise contribution only becomes noticable for $R_{filt} \geq 1\,\mathrm{k\Omega}$ underpinning the statement above. Using a larger resistor R_{filt}, does improve the noise contribution above $0.6\,\mathrm{Hz}$ though. This advantage is fairly small however. Table 4.4 shows the integrated noise of the plot shown in figure 4.13 and it can be seen that the numbers are close together, yet there is a slight improvement with a value of $510\,\Omega$. In order to derive these values from the simulation, it is important to remember, that LTSpice can only deal with voltage noise, so a shunt resistor is used for the simulation. Its noise must subtracted from the result.

R_{filt}	Integrated noise, $10^{-2}\,\mathrm{Hz}$ to $10^{2}\,\mathrm{Hz}$
$249\,\Omega$	$4.58\,\mathrm{nA_{rms}}$
$510\,\Omega$	$4.34\,\mathrm{nA_{rms}}$
$1\,\mathrm{k\Omega}$	$4.43\,\mathrm{nA_{rms}}$
$1.5\,\mathrm{k\Omega}$	$4.81\,\mathrm{nA_{rms}}$

Table 4.4.: Integrated noise of the current source for different values of R_{filt}. The integration range is $10^{-2}\,\mathrm{Hz}$–$10^{2}\,\mathrm{Hz}$.

Using the simulation as a guideline, several values of R_{filt} were tested and $510\,\Omega$ was found to be optimal in the final circuit regarding the noise and the temperature coefficient of the filter. The latter is discussed in more detail below. The resistor is a high quality Susumu `RG3216P-5100-B-T1` $0.1\,\%$ resistor in a large 3216 package, because Seifert [188] has shown, that larger resistors are of consistenly higher quality and exhibiting less noise. Having settled on the optimal resistor value, the filter capacitor must be considered.

The capacitor value can be estimated from the required suppression. The `AD797` reaches $1\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ at $100\,\mathrm{Hz}$ and the `LM399` has a noise density of about $90\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ at $100\,\mathrm{Hz}$. This means, that a cutoff frequency of $1.1\,\mathrm{Hz}$ is desired, requiring a filter capacitor of $280\,\mathrm{\mu F}$. Taking into account the limited board space, it is clear, that the $280\,\mathrm{\mu F}$ marks an upper bound when considering capacitors with low dolumetric efficiency. This warants the examination of different types of capacitors and their properties. Libbrecht et al. [120] for example used special tantalum capacitors, but there are also other options available. Modern low-leakage electrolytic capacitors and film capacitors were also investigated. Ceramic capacitors were not considered for this filter, because they are either piezoelectic (X5R, X7R, etc.) or too big in physical size (C0G). The properties studied are the leakage current and its temperature stability, while optimizing volumetric efficiency. Table 4.5 shows an overview of different dielectric materials and their suitability according to those properties. More information on other types of capacitors can also be found in [240] or [239].

Capacitor type	Volumetric efficiency	Leakage	Temperature stability
Electrolytic	$++$	$--$	$--$
Tantalum	$+$	$-$	$-$
Film	$-$	$+$	$+$
Ceramic C0G	$--$	$++$	$++$

Table 4.5.: Capacitor properties of different dielectrics.

The leakage current of a capacitor is interesting, because it affects the filter performance at low frequency. Considering figure 4.14, which shows the first order filter of the digital current driver, two major sources of error currents are present. The leakage of the capacitor and the input bias current of the op-amp.

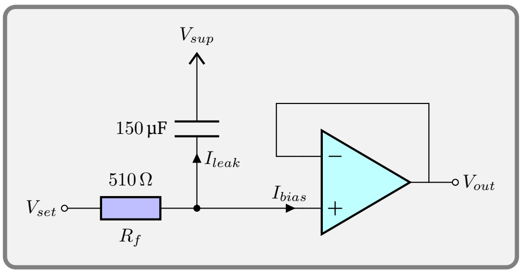


Figure 4.14.: Setpoint filter and error currents.

According to the datasheet, the AD797 [10] has temperature dependent input bias current of about $2\,\mathrm{nA/K}$, which results in a drift of about $-1\,\mu\mathrm{V/K}$ using the $510\,\Omega$ resistor. Do note, that an increase in the leakage and bias current causes a negative temperature coefficient. All in all, the drift caused by the leakage and bias current should not exceed the reference drift of about $0.3\,\mu\mathrm{V}/(\mathrm{V\,K})$ ($2.25\,\mu\mathrm{V/K}$ for a $7.5\,\mathrm{V}$ reference voltage), which results in $4.4\,\mathrm{nA/K}$. Using a larger filter resistor like $R_f = 6\,\mathrm{k\Omega}$ in the second-order filter used by Libbrecht et al. [120], a current of only $1\,\mathrm{nA}$ would cause an offset of $6\,\mu\mathrm{V}$, almost $1\,\mu\mathrm{V/V}$ considering the maximum setpoint voltage of $-7\,\mathrm{V}$ in their design. The leakage current of a capacitor in this context is comprised of two efffects, the first is the bulk resistance of the capacitor and the second is dielectric absorption. Dielectric absorption is an effect that describes how the molecules of the dielectric material slowly align to the external electric field applied. It can be modelled as a parallel RC circuit with a very large resistance. Depending on the type of capacitor, this effect can take several days to subside.

Electrolytic capacitors do have the highest capacitance per volume, but suffer from dielectric absorption and large changes in capacitance over temperature. Tantalum capacitors also have large leakage currents [82], which also heavily depend on temperature, but they are even more compact than electrolytic capacitors. Film capacitors are very stable capacitors along with paraelectric NP0 ceramic capacitors. NP0 capacitors, being the most stable unfortunately have the worst volumetric efficiency. PET Film capacitors are a compromise between stabilty and size.

In order to asses the magnitude of the problem, the leakage current over temperature was measured for several capacitors.

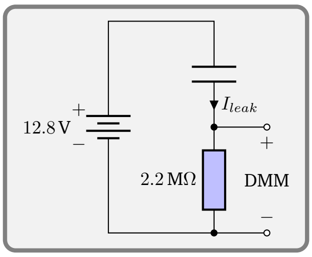


Figure 4.15.: Setup for measuring the leakage current of a capacitor.

The simple measurement setup is shown in figure 4.15. A Vishay `MRS25` 2.2 MΩ resistor was placed in series with the capacitor and a 12.8 V test voltage from a low noise DC supply – in this case a stack of 8 alkaline batteries was applied. Batteries were used to ensure, that no AC line noise is mistakenly recorded as leakage. The voltage drop accross the resistor was measured using an HP `3458A` DMM. The input bias current of the `3458A` is on the order of a few pA [119, 173], which is small compared to the leakage current measured in this setup and henceforth neglected. The capacitor was first allowed to settle so that dielectric absorption can subside. The initial temperature was set to 24 °C and then stepped to 45 °C and left until the reading had settled again, then stepped down to 24 °C. This was done to observe the settling behaviour which would cause a low frequency random walk in a filter arangement as outlined in section 3.6.1. This setup forms a low- pass filter due to the large value resistor. This means, that any fast settling dielectric absorption cannot be observed.

Three samples each from the same batch of the following capacitors where compared:

- Nichicon `UKL1V331KHD` low leakage 330 µF, 35 V electrolytic capacitor
- Kemet `T491X157K020AT` 150 µF, 20 V tantalum capacitor
- WIMA `MKS4B061507G00JSSD` 150 µF, 50 V polyester (PET) film capacitor

The capacitors were chosen to have at least 150 µF and a voltage rating of ≥16 V, because the maximum setpoint voltage applied is −7.5 V and a derating of at least 50 % is applied to improve reliability.

This test was conducted with all capacitors and the results are summarised in table 4.6.

Capacitor	Capacitance	DC leakage at 12.8 V, 24 °C	Temperature stability
Nichicon `UKL1V331KHD`	330 µF	1.1 nA	390 pA/K
Kemet `T491X157K020AT`	150 µF	19 µA	16 nA/K
WIMA `MKS4B061507G00JSSD`	150 µF	2.2 nA	150 pA/K

Table 4.6.: Capacitor leakage current and temperature stability of the leakage current for different types of capacitors.

The electrolytic capacitor showed strong dielectric absorption and it took about 24 h to settle from about 500 nA to 1 nA. All measurements were therefore taken after 24 h. After settling, stability was suprisingly good, altough long relaxation constants were observed when changing the temperature. A measurement of a typical sample is shown in figure 4.16.

The tantalum capacitor was a standard industrial grade capacitor and not a special low leakage version. The reason was that low leakage wet slug tantalum capacitors are very costly and similar in size to the PET film capacitor. The `T491` capacitor was tested to get an idea about the performance of standard tantalum capacitors. The leakage current observed for the tantalum capacitor was the highest of the components tested. Especially the temperature stability was unacceptable, as it was almost an order of magnitude larger than was considered acceptable above.

The type of film capacitor was chosen, so that its size was still possible to fit it onto the PCB. The $150\,\mu F$ WIMA `MKS4B061507G00JSSD` is the $50\,V$ version. There is also a $63\,V$ available, but these were not available in small quantities for testing. If available the $63\,V$ version should be preferred, because higher voltage capacitors have better leakage specifications, when biased at the same voltage. Dielectric absorption was not observed during testing, but could haven been masked by the large $RC = 330\,s$ time constant, as the initial current takes about $40\,min$ to decay to $3\,nA$.

Based on the results summarised in table 4.6 the $150\,\mu F$ WIMA `MKS4B061507G00JSSD` capacitor and a $510\,\Omega$ Susumu `RG3216P-5100-B-T1` resistor were chosen for the filter.

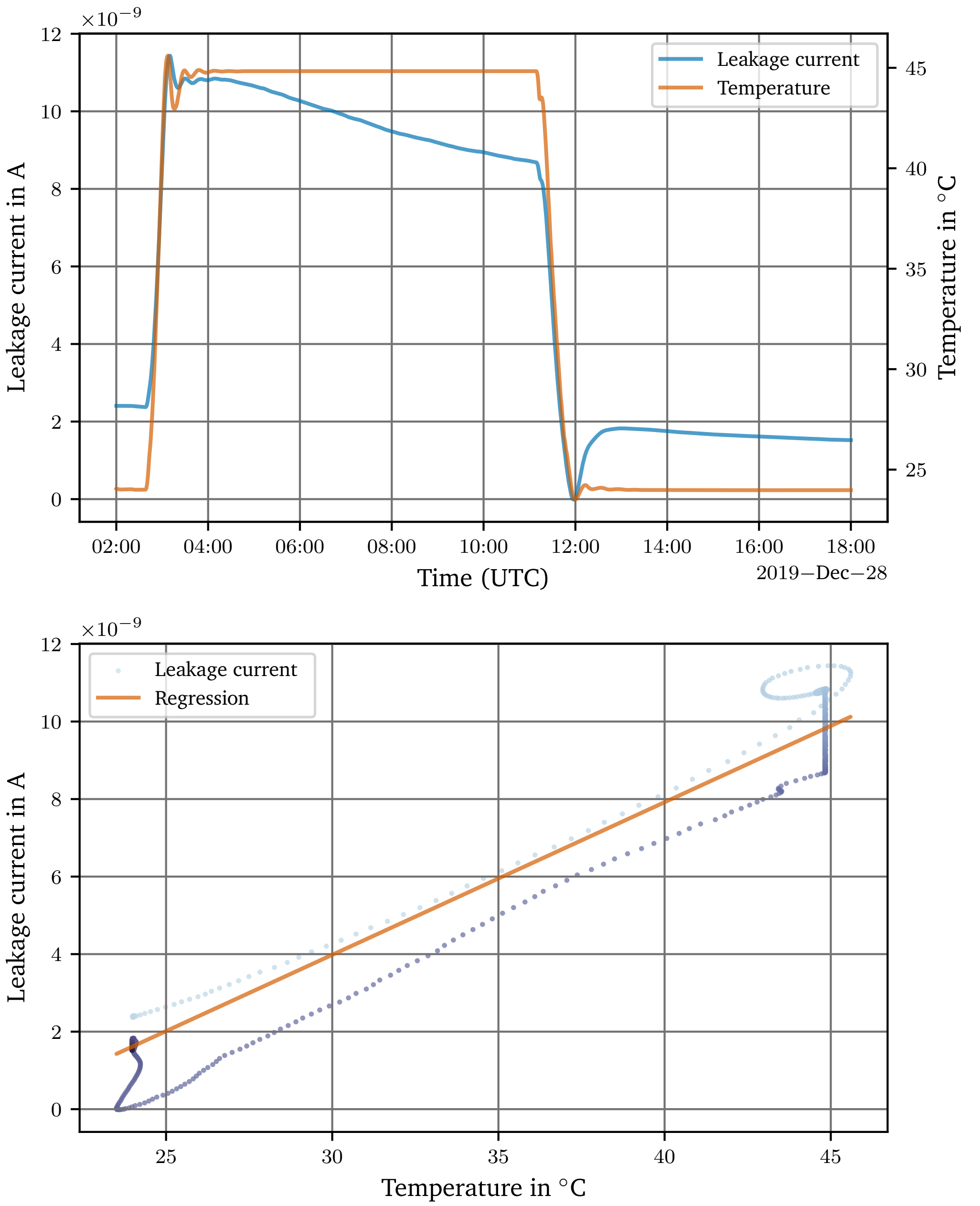
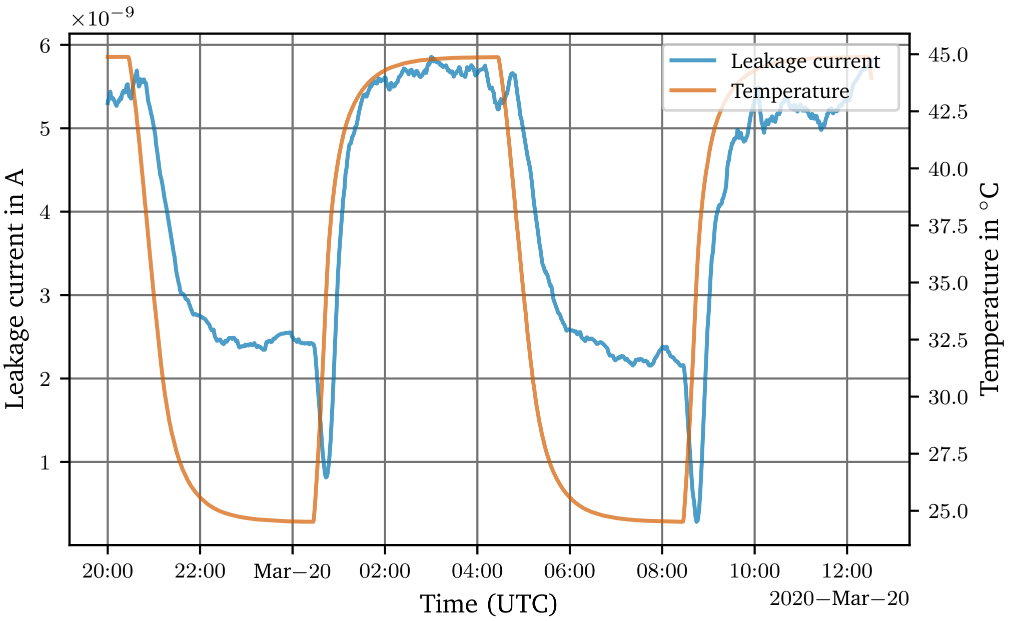


Figure 4.16.: Leakage current over temperature of a Nichicon UKL1V331KHD $330\,\mu F$ electrolytic capacitor biased at $12.8\,V$. The capacitor was allowed to soak for $24\,h$ prior to the measurement.



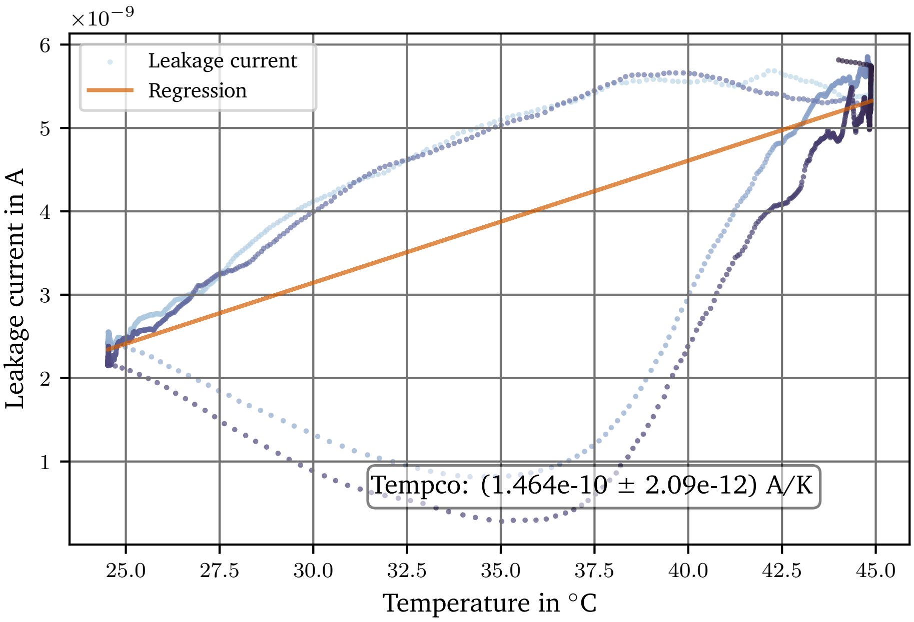


Figure 4.17.: Leakage current over temperature of a WIMA `MKS4B061507G00JSSD` $150\,\mu F$ PET capacitor biased at $12.8\,V$.

Having discussed the DAC it is clear, that the Zener voltage must be amplified to meet the requirements of the negative reference input of the DAC. An optimal value of $-15\,\mathrm{V}$ was also found above.

The voltage reference can now be realized as an additional reference module, that is connected to the main PCB via a pin header. This allows additional testing of the much simpler reference module in a special test bench. These tests are detailed in section 4.1.13, but the circuit is discussed here first.

Amplifying the Zener voltage has the advantage that it is possible to bootstrap the Zener diode to produce its own reference current. As it was discussed above, the `LM399` only has a limited rejection of $\frac{1}{5000}$ against changes of the supply voltage. Bootstrapping the Zener improved the rejection ratio to changes of the diode supply voltage and noise, because any changes have to additionally get past the amplifier CMRR as discussed in 3.8.6.

The bootstrapped circuit ist shown in figure 4.11 on page 109 and can be divided into an upper part with R_{bias} and a lower part with a feedback network. The feedback network, based on a Vishay `300144Z` [1] resistor network, is used to amplify the $-7\,\mathrm{V}$ to

$$-7\,\mathrm{V}\cdot\left(1+\frac{Rn_1}{Rn_2}\right)=-7\,\mathrm{V}\cdot\left(1+\frac{20\,\mathrm{k\Omega}}{17\,\mathrm{k\Omega}}\right)=-15.2\,\mathrm{V}\,,$$

again referenced to V_{sup}. With $-15.2\,\mathrm{V}$ at the output and $-7\,\mathrm{V}$ at the Zener diode, R_{bias} will then provide

$$I_{bias}=\frac{-15.2\,\mathrm{V}-(-7\,\mathrm{V})}{7.5\,\mathrm{k\Omega}}\approx 1.1\,\mathrm{mA}\,,$$

which is independent of V_{sup}, just as desired. R_{bias} is a $0.1\,\%$, $10\,\mathrm{\mu\Omega/(\Omega\,K)}$ Panasonic `ERA-6ARB752V` resistor, but in principal any $100\,\mathrm{\mu\Omega/(\Omega\,K)}$ resistor can be used, because the sensitivity of R_{bias} is roughly

$$\frac{-1\,\mathrm{mA}\cdot 1.5\,\Omega}{7\,\mathrm{V}}\frac{\mathrm{d}R_{bias}}{R_{bias}}\approx 215\times 10^{-6}\cdot\frac{\mathrm{d}R_{bias}}{R_{bias}}\,. \tag{4.9}$$

Using a $100\,\mathrm{\mu\Omega/(\Omega\,K)}$ resistor instead, would cause a temperature coefficient of $-20\,\mathrm{nV/(V\,K)}$, which is small compared to the $0.3\,\mathrm{\mu V/(V\,K)}$ of the `LM399`. The higher quality resistor was chosen to limit long termn drift. The feedback network is far more critical in this regard, hence the use of the Vishay `300144Z` high-precision network.

There is one last issue to address. Unfortunately, the reference circuit is not guaranteed to start up correctly. The self-biased circuit has two stable points of operation. The expected one is at $V_{sup}-V_z$ and the other is $V_{sup}+V_f$, when the Zener diode is forward biased. The cause of this is the positive feedback to the `LT1001`. At startup, the Zener diode has a very high impedance (see the typical Zener diode I-V curve, e.g. [187]) can be considered open, while R_{bias} is small compared to the feedback network. The op-amp inputs are slighly capacitive and the non-inverting input then pulls high faster, delivering positive feedback. This unwanted operating point can be reached if the op-amp supply comes up before V_{sup}. The Zener diode will then be forward biased by the op-amp. Of course, this can only happen if the op-amp supply voltage is higher, than V_{sup}, which in this case is true. This situation is critical, because the output would then go above $0\,\mathrm{V}$. This output is connected to the negative reference input of the DAC, which must not be positively biased. To prevent this case and damage to the DAC, the second diode at the output of the op-amp is used. It ensures that the output cannot go above ground and the gain of the positive feedback is drastically reduced and the negative feedback takes over quickly.

The setpoint circuitry can now be summarised as follows. The reference module outputs an amplied reference voltage of $-15\,\mathrm{V}$, which is fed to the negative reference input of an ADI `AD5781` $18\,\mathrm{bit}$ DAC, which is used to create a setpoint voltage between V_{sup} and $V_{sup} - 15\,\mathrm{V}$. This voltage is divided down by a factor of two (with regard to V_{sup}) using a Vishay `DSMZ` resistor network. The resulting voltage between V_{sup} and $V_{sup} - 7.5\,\mathrm{V}$ is then buffered and low-pass filtered using a $510\,\Omega$ and $150\,\mathrm{\mu F}$ RC low pass. The filter capacitor, a WIMA `MKS4B061507G00JSSD`, was tested for low leakage to ensure stability of the filter. The filtered voltage is at last fed to the precision current source discussed in the next section.

The reference module, mounted on a standoff, secured with a screw is shown in figure 4.18.

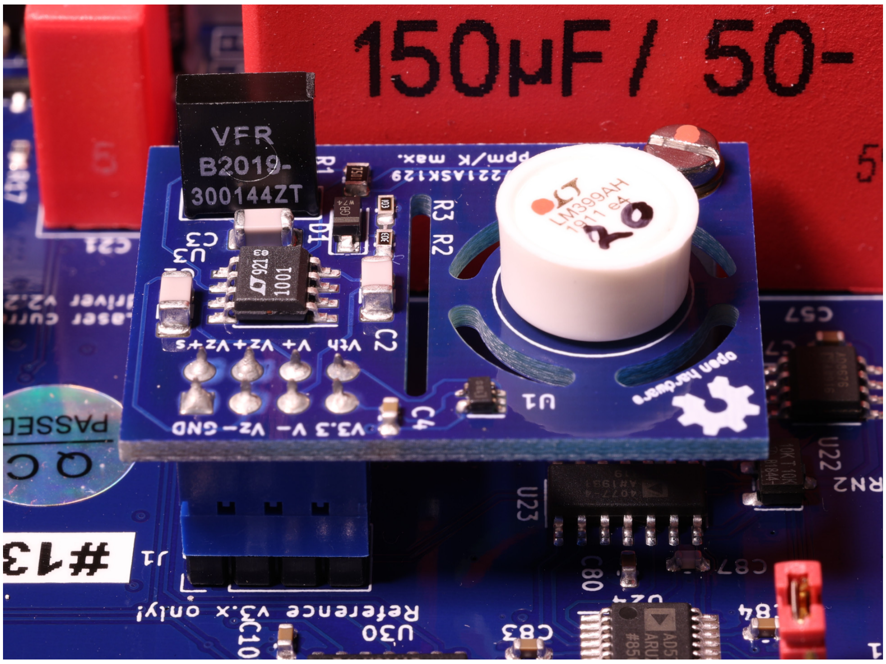


Figure 4.18.: The voltage reference module mounted on its socket on the main current driver PCB. `LM399` no. 20 is marked with a red dot because it passed quality control. In the background, the large filter capacitor can be seen.

4.1.6. Precision Current Source

The theory behind the precision current source was laid out in section 3.8.4, but some interesting details were not discussed yet. One of the main drawbacks of the current source from section 3.8.4 is that the setpoint has a direct impact on the compliance voltage as discussed in section 3.8.5. Given an input supply voltage of $15\,\mathrm{V}$ it is immediately evident that with a reference voltage of $7.5\,\mathrm{V}$ a compliance voltage of $8\,\mathrm{V}$ as defined in specification 3.2 is impossible to reach.

One solution would be to increase the division ratio after the DAC to, for example, one to three, reducing the reference voltage to $5\,\mathrm{V}$. Unfortunately, this solution increases the current noise by a factor of $\sqrt{\frac{7.5}{5}} \approx 1.2$ according to equation 3.99, because the sense resistor has to be scaled by the factor of $\frac{5}{7.5}$ for the same current output.

Another option is to use the negative $-15\,\mathrm{V}$ rail instead of ground to connect the cathode of the laser diode. This would directly increase the compliance voltage by $15\,\mathrm{V}$ and include enough headroom for other components of the current source, like the MOSFET. This solution, however, brings along the problem that was discussed in section 3.8.1. Directly connecting the laser diode to a voltage rail can have catastrophic consequences for the diode in case the other side is accidentally connected to ground.to a voltage rail can have catastrophic consequences for the diode in case the other leg is accidentally connected to ground.

This work therefore presents a novel third option which separates the current source from the compliance voltage requirement. This circuit is shown in figure 4.19.

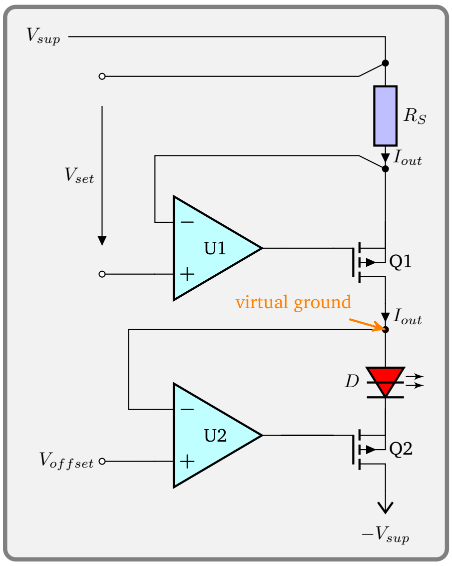


Figure 4.19.: Simplified current source implemented in the digital current driver. The voltage across R_S is Kelvin sensed. The laser diode is wrapped in a feedback loop and the current source is shielded from the load. U1 is an ADI `AD797B` and U2 is an ADI `ADA4625-1`.

This circuit consists of the current source introduced in section 3.8.4 and a current sink, that wraps the laser diode in a feedback loop to shield the current source from the load. In addition to the circuit shown in section 3.8.4 in figure 3.45 on page 74, the sense resistor is connected differently. The sense resistor used is a Vishay `VPR221Z` [223], a four-terminal device. This allows Kelvin sensing of the voltage across R_S. The positive voltage is buffered using several precision ADI `ADA4077-4` [13] op-amps. This voltage is then supplied to the votage reference circuit, the DAC and the divider that follows. It is important that no current is drawn from this net. The negative voltage is sensed by the AD797 to close the feedback loop around R_S.

To understand the current sink, it is easiest, to assume, for now, $V_{offset} = 0\,\mathrm{V}$ and the non-inverting input is grounded. Applying a setpoint voltage V_{set} to U1, an `AD797`, will cause the upper current source to source a current into the virtual ground and the laser diode. This will cause the virtual ground potential to rise and the op-amp U2, an ADI `ADA4625-1`, will then see a positive voltage at its inverting input. U2 will then pull its output low, pulling the gate of the p-channel MOSFET Q2 low as well, causing the current sink to drain the appropriate amount of current. U2 will always steer the Q2 in such a way that the virtual ground is maintained. Do note, that using a p-channel MOSFET for Q2, will put an extra burden on the op-amp U2, because it has to follow the load voltage with its output. Fortunately, remembering the Shockley equation 3.2 it is clear that the voltage of a diode changes little with the current and this issue can therefore be neglected. On the other hand, using a p-channel MOSFET does not require the op-amp output to go down all the way to the negative diode supply to turn off the MOSFET. The p-channel MOSFET turns at $V_{offset} - V_{th}$, which is close to $0\,\mathrm{V}$. This arangement is similar to the concept of a transimpedance amplifier, except that instead of a resistive element in the feeback path, a (laser) diode is used. Putting the laser diode into a feedback loop has a number of advantages.

The most important one is that the current does no longer see the load and only has to source the current into the virtual ground. The compliance voltage required from the upper current source is reduced to V_{offset}, because this is the voltage at the virtual ground. The compliance voltage, that can be provided by the combined source is increased to more than $10\,\mathrm{V}$ with this design, more than adequate to meet the design specifications of $8\,\mathrm{V}$ given in specification 3.2. Using a higher voltage negative supply is also possible to further increase the compliance voltage. The compliance voltage is essentially independent of the upper current source.

The circuitry supplying the voltage for the current source is essentially a voltage source, so additional care must be taken to protect the laser diode. This is where the offset voltage comes into play. Choosing a positive offset is interesting, because if the virtual ground is accidentally shorted to the system ground, the inverting input of U2 will be pulled low and the output of U2 will go high, closing the MOSFET Q2 and protecting the laser diode. If V_{offset} were negative, U2 would open Q2, likely destroying the laser diode. V_{offset} must therefore always be $\geq 0\,\mathrm{V}$. In this design $V_{offset} = 500\,\mathrm{mV}$ has proven successful, but in future, it may be reduced to around $100\,\mathrm{mV}$ for improved headroom of the MOSFET Q1 in higher current designs. The reduced load voltage seen by the current source is also important for the modulation current source discussed in section 3.8.9.

In addition to solving compliance voltage related problems, there is another benefit from this configuration. Since the current source does not see the load and the voltage seen is constant, the output impedance is considerably improved. In essence, any change of the virtual ground potential is suppressed by the gain of the op-amp U2, which means the output impedance is multiplied by the open-loop gain of U2, typically a gain 10^7 at low frequency can be expected from the `ADA4625-1` [14]. The output impedance at this level is then longer limited by the current source, but rather other circuit parasitics. This helps the rather poor output impedance of the Howland current source to meet the requirements stated in specification 3.2.

There one downside of this solution though. As the laser diode is wrapped in a feedback loop, that loop needs to close around the diode. This means, that the physical size of the loop is (regarding op-amps) enormous, because the laser head is not located on the PCB, but rather seprated by several meters of cable, located on an optical table next to the rack. This introduduces a considerable phase shift into the control loop at high frequencies. The details of the cabling are discussed in section 4.1.8 in more detail For now only the propagation speed of light is of interest. The DVI cables used have a velocity factor of 0.8 and a typical $3\,\mathrm{m}$ cable will therefore cause a delay of about $\tau = 12.5\,\mathrm{ns}$. The signal has to go through the cable twice, so at $1\,\mathrm{MHz}$, this will introduce a phase delay of

$$\theta = 2\pi \cdot 2\tau \cdot 1\,\mathrm{MHz} \approx \frac{\pi}{20} = 9^\circ\,.$$

While at $1\,\mathrm{MHz}$, this is not yet critical, a phase delay of $\frac{\pi}{2}$ at $10\,\mathrm{MHz}$, would send the control loop into oscillations. The bandwdith of U2 is therefore limited to $1\,\mathrm{MHz}$, which works well with cables of up to $3\,\mathrm{m}$. Longer cables are not compatible with this device and should not be used. More details on circuit surrounding U2 can be found in the schematics [43].

The last issue that needs to be discussed is the power dissipation in the sense resistor. This is less problematic for low currents, but at currents of several hundred mA it becomes more and more of a problem. Using a constant maximum setpoint voltage (in this case $7.5\,\mathrm{V}$) results in a power loss in the resistor, that is proportional to the output current. An output of $500\,\mathrm{mA}$ for example requires the sense resistor to dissipate $3.75\,\mathrm{W}$. The resistor used is rated for $8\,\mathrm{W}$ when mounted to a heat sink, but only at $25\,^\circ\mathrm{C}$ ambient, which is illusory, when mounted inside a box without active cooling. The chassis will be warmer.

The $8\,\mathrm{W}$ maximum is also detrimental to the long-term stability target. The datasheet of the sense resistor gives a maximum load life stability of $150\,\mathrm{\mu\Omega/\Omega}$ for $2000\,\mathrm{h}$ [223] at $8\,\mathrm{W}$ when using a proper heat sink. A technical note from Vishay [208] gives some hints regarding the long-term drift of the older C-Foil resistors. Assuming the Z-foil `VPR221Z` resistors behave in a similar way, the load life drift for $30\,\mathrm{d}$ ($720\,\mathrm{h}$) can be estimated as

$$\frac{\Delta R}{R}(30\,\mathrm{d}, 3\sigma) = 150\,\mathrm{\mu\Omega/\Omega} \cdot \sqrt[3]{\frac{720}{2009}} = 107\,\mathrm{\mu\Omega/\Omega}\,.$$

Using the power derating curve in the datasheet [223] to estimate a thermal resistance of $14\,\mathrm{K/W}$ from the metal foil to ambient gives a temperature of $112\,\mathrm{K}$ above ambient for an $8\,\mathrm{W}$ load. Consulting [208], this temperture sees in increase of a factor 15 over ambient temperature. Reducing the load to $2\,\mathrm{W}$ will decrease the internal temperature of the metal foil considerably to $28\,\mathrm{K}$ above ambient and therefore reduce the drift to less than a factor of 4. The drift estimated above would then reduce to about $29\,\mathrm{\mu\Omega/\Omega}$, far less than the required $240\,\mathrm{\mu\Omega/\Omega}$ as given by specification 3.1, leaving a safe headroom for higher ambient temperatures.

In order to reduce the power lost in the resistor to the desired $2\,\mathrm{W}$, R_S can be split into multiple resistors to distribute the current. While this idea neither novel nor interesting, it does become so, when looking at it in a different context. Splitting the sense resistor only becomes neccessary at currents above a few hundred mA. At these the sense resistor is of a fairly low value, typically $<50\,\Omega$, and the current noise of the current source is no longer dominated by the sense resistor. As a reminder, the voltage noise of the `AD797` op-amp is $0.9\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$. This is the equivalent to a $50\,\Omega$ resistor at room temperarture. With $R_S < 50\,\Omega$, the op-amp becomes the dominant noise source of the current source. The voltage noise of the op-amp, in contrast to the thermal noise of the resistor, does see an improvement when averaged. Using a n op-amps improves the high frequency noise by a factor of $\sqrt{n}$. This assumes, that the wideband noise is uncorrelated, which should be the case, as any power supply noise is effectively filtered out and the noise is only produced in the op-amp. At low frequencies, the reference noise still dominates the noise and using a single reference and setpoint DAC, which is fed into both current sources, the noise is correlated and therefore adds normally, so it does not see any improvement using this technique.

The simulation found at source/current_regulator_AD797_noise.asc can be used to estimate the potential improvement. The simulation gives a more complete picture, because it included the noise caused by the resistors and the feedback network around U1. Table 4.7 gives the simulated output noise for two solutions of a $500\,\mathrm{mA}$ laser driver, one with a single current source and the other with two current sources. The sense resistors in this case are smaller than the $50\,\Omega$ discussed above. The single source uses a $15\,\Omega$ resistor, while the dual current source solution employs two $30\,\Omega$ resistors.

	LF Noise $10^{-2}\,\mathrm{Hz}$ to $10^{2}\,\mathrm{Hz}$	HF noise, $10^{2}\,\mathrm{Hz}$ to $10^{5}\,\mathrm{Hz}$
Single	$27.2\,\mathrm{nA_{rms}}$	$21.7\,\mathrm{nA_{rms}}$
Dual	$27.2\,\mathrm{nA_{rms}}$	$17.0\,\mathrm{nA_{rms}}$

Table 4.7.: Noise comparison of a single current source with $R_S = 15\,\Omega$ and dual current source with $2 \times 30\,\Omega$, both at $500\,\mathrm{mA}$.

From the simulation it can be easily gathered that the high frequency noise of a dual current source design is about $\frac{1}{\sqrt{2}}$ that of a single source design, just as predicted above. The low frequency noise is also not affected as discussed above. In order to improve this, a lower noise reference like the `ADR1399` must be used. Judging from the datasheet of the `LM399` and `ADR1399` [16, 125], the `ADR1399` should only have about two-thirds of the noise. While the dual source design is already very close to the desired $30\,\mathrm{nA_{rms}}$ in $100\,\mathrm{kHz}$ bandwidth, it is still slightly above the desired target with a total noise of $31.9\,\mathrm{nA_{rms}}$. A low noise reference is therefore key and given the loose datasheet specifications of those references, selecting references for low noise is important. This is discussed in section 4.1.13.

Using a second current source also helps with reducing the statistical spread of both the drift and the temperature coefficient of the sense resistor and the `AD797` op-amp U1. While the datasheet gives a *typical* value of $\pm 0.05\,\mathrm{\mu\Omega/(\Omega\,K)}$, the 3σ range is more like $\pm 2\,\mathrm{\mu\Omega/(\Omega\,K)}$ [236]. Using two current sources results an inherent statistical averagingging regarding the temperature coefficient, making the $1\,\mathrm{\mu A/(A\,K)}$ target of specification 3.1 easier to reach.

Configuration	Sense resistor value	Maximum current
`DgDrive-150`	$50\,\Omega$	$150\,\mathrm{mA}$
`DgDrive-250`	$30\,\Omega$	$250\,\mathrm{mA}$
`DgDrive-300-LN`	$2 \times 50\,\Omega$	$300\,\mathrm{mA}$
`DgDrive-500-LN`	$2 \times 30\,\Omega$	$500\,\mathrm{mA}$

Table 4.8.: Different configurations of the digital current driver current tested and built.

Combining more than two current sources unfortunately yields diminishing returns as the noise scales as $\frac{1}{\sqrt{n}}$, whereas cost and complexity goes with n. Therefore a maximum of two current sources are used in this design. Table 4.8 lists the current source configures, that were built for this project. The `DgDrive-300-LN`, which is using a $50\,\Omega$ resistors is at the limit where adding a second current source has any benefits regarding the wideband noise. The difference in wideband noise is about $15\,\%$ ($13.8\,\mathrm{nA_{rms}}$ versus $11.6\,\mathrm{nA_{rms}}$) when comparing the simulation against a $25\,\Omega$ configuration. Only two of these units were built to test the extra stability added by the statistical averaging scheme.

A final note regarding the second current source and the Kelvin sensing mentioned above. The the positve voltage as shown in figure 4.19 is not connected. Only the input voltage of the first current sense resistors is used. One could, in theory, connect the sense pins of both resistors via another resistor to the op-amp and average the voltage, but since both resistors are the same, the offset error is unaffected. It is therefore better to leave the second sense pin unconnected, before introducing other errors.

Noise measurements of the digital current driver in comparison to other current drivers and the simulation results shown above are presented in section 4.1.10.

To summurize the results, a novel current source configuration was presented that addresses the limited compliance voltage by removing the load seen by the current source. The compliance voltage in this design is more than $10\,\mathrm{V}$, limited by the power supply rails of $\pm 15\,\mathrm{V}$. In addition, a solution was given to limit the increasing current noise contribution of the op-amp in high ouput current configurations, that use small sense resistors. These results were underpinned by simualtions and a simulation model was provided to estimate the expected improvements.

4.1.7. Modulation Current Source

Several options for the modulation current source are presented in literature. Some designs use a simple AC coupled input [120, 227], which drastically reduces the high-frequency output impedance of the current source. Others use a JFET in parallel to the laser diode to divert some of the current [25, 214, 233], which causes poor DC performance due to the missing feedback loop. Libbrecht et al. [76, 120] in addition to the AC coupled input also presented a more rugged approach similiar to a Howland current source, which delivers a reasonable DC performance and a claimed bandwidth of more than 10 MHz. The modulation circuit shown by Libbrecht et al. is likely an update to ealier version of the same laser driver printed in the paper of Wieman et al. [227]. This would explain the rather peculiar arangement. This circuit is shown in figure 4.20 and will be discussed in more detail, because it also used in the legacy laser drivers found in this group.

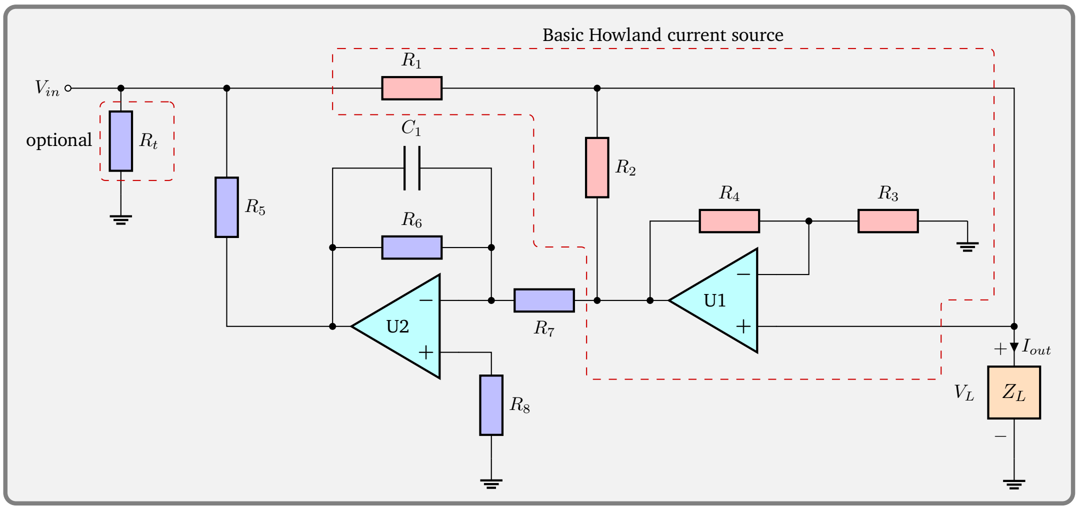


Figure 4.20.: The modulation current source used by [76, 120]. $R_1 = R_2 = R_3 = R_4 = 1\,\mathrm{k\Omega}$ are matched resistors. $\frac{R_5}{2} = R_6 = R_7 = R_8 = 1\,\mathrm{k\Omega}$. $C_1 = 1\,\mathrm{\mu F}$ $R_t = 54\,\Omega$ is optional.

Looking at figure 4.20, the current source can be identified as the classic HCS. The Howland current source does require a very low impedance input V_{in}, because otherwise the matching of the four resistors $R_1 = R_2 = R_3 = R_4$ is imbalanced. This obviously brings about the problem of having to either short the input or keep it connected to low impedance source at all times. This imbalance is created by the load voltage, which forces to op-amp output to a non-zero voltage to drive the inverting input to the same voltage as the non-inverting input, which is at V_L. Libbrecht et al. addressed this problem by using an inverting amplifier U2 to invert the ouput of U1. Since the output current of U1 is equally divided between the arm with R_3 and R_4 and the other arm with R_1 and R_2, the current U2 injects into the input must be divided by half, hence $R_5 = 2 \cdot R_x$. At this point things need to be mentioned. The design by Libbrecht et al. uses the `OP07` [157] precision op-amp for both U1 and U2. This op-amp does have an internal bias current compensation scheme[2] so its input bias current is very low and on the order of a few nA. This makes one wonder about the purpose of the resistor R_8, which is not

[2]This detail can either be read from the simplified schematic as there is a current source feeding into both inputs or from the input offset current and input bias current specification. The offset current is about the same as the bias current and does not have a defined polarity indicating the possibility of over- and undercompensation.

only the wrong value, as it should be $R_8 = R_6||R_7 = 500\,\Omega$, but also pretty much unneeded. The matching of the resistors fortunately $\frac{R_5}{2} = R_6 = R_7 = R_x$ does not need to be as tight as the ones of the HCS as discussed in section 3.8.9, because U2 forms an inverting amplifier with gain $A_2 = \frac{R_6}{R_7} \approx -1$ and feeds back a current of $\frac{A_2 \cdot V_{o,U1}}{R_5}$ into the input node. The error current is then distrubuted between the input source and the HCS, so only the fraction $R_1||R_{in}$ of the error current flows into R_14. Assuming an output impedance of the modulation source of $50\,\Omega$, the error curret flowing into R_1 is attenuated by a factor of $\frac{1}{47}$, relaxing the requirements of the matched resistor network by the same factor.

Another issue, that can be identified with the above circuit is the capacitor C_1. It forms a low-pass filter with the cutoff frequency

$$f_c = \frac{1}{2\pi R_6 C_1} \approx 159\,\mathrm{Hz}$$

for $R = 1\,\mathrm{k}\Omega$ and $C_1 = 1\,\mu\mathrm{F}$ as given in [76, 120]. This has two detrimental effects, as it offsets the careful balanced created by the feedback of U2. First, it will dramatically decrease the output impedance of the current source. It was already shown in appendix A.7, that the Howland current source is very sensitive to an imbalance of the resistor ratios and if the feedback of U2 is reduced, the output impedance of the source starts playing a role. This effect can be seen in figure 4.21, which shows an LTSpice simulation of the circuit shown in figure 4.20 and with a $50\,\Omega$ source at the modulation output. The fast modulation input found in [120] was omitted. This would additionally limit the output impedance above $1.59\,\mathrm{kHz}$ to $10\,\mathrm{k}\Omega$ ($CR = 10\,\mathrm{nF} \cdot 10\,\mathrm{k}\Omega$). The simulation file can be found at source/spice/modulation_input_LibrechtHall.asc.

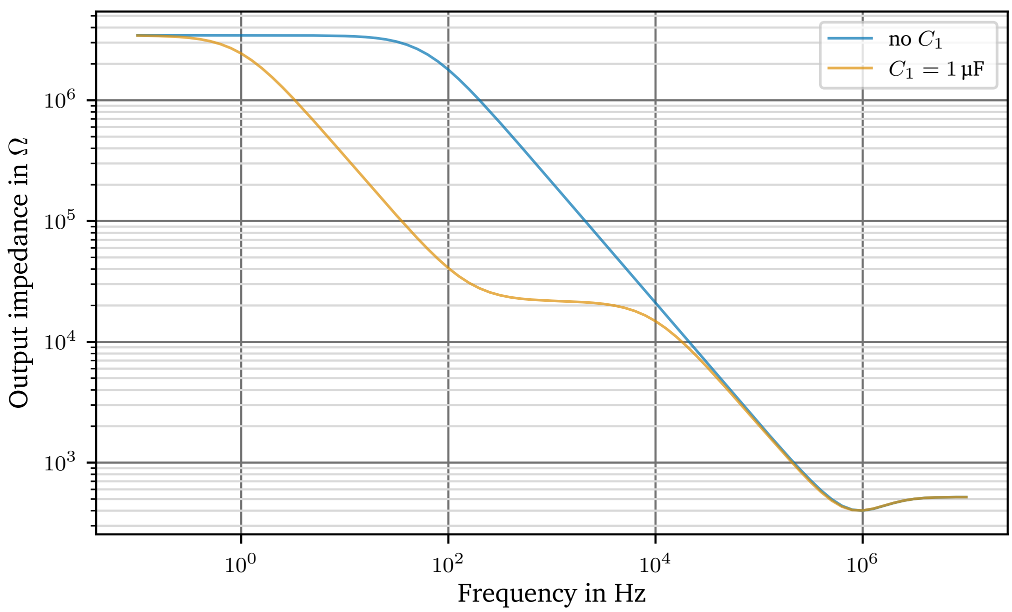


Figure 4.21.: The output impedance of the HCS from figure 4.20 with $0.01\,\%$ worst-case resistor matching and a $50\,\Omega$ source. The simulation was repeated with and without the capacitor C_1.

Figure 4.21 shows the output impedance for two configurations. With and without the capacitor C_1. The output impedance at low frequencies is around $3.5\,\mathrm{M}\Omega$ for both configurations, which is the limit found This changes rapidly when inserting the capacitor, in which case the output impedance drops to around $20\,\mathrm{k\Omega}$ at $1\,\mathrm{kHz}$, created by the $50\,\Omega$ mismatch added to R_1. At even higher freuqencies, the results converge, because the op-gain is the more limiting factor.

The second effect of the capacitor C_1 is more subtle, but poses a serios problem for a high bandwidth servo. This has to do with the input impedance presented by the circuit. By inspection of figure 4.20, one finds the input impedance to be $R_6||R_1 \approx 667\,\Omega$. The required termination resistor R_t would be $54\,\Omega$, or two $27\,\Omega$ resistors in series, which can be bought off-the-shelf. Unfortunately as soon as the current source balance is disturbed by the gain of U2 rolling off, its input impedance changes. This makes proper input matching impossible and causes unpredictable high frequency behaviour depending on the load. This was also mentioned by [76] and investigated closer by Preuschoff [166] and therefore not covered here. Preuschoff showed that this type modulation current source shows a highly load dependent behaviour above $1\,\mathrm{MHz}$.

This work proposes a different approch to the problem. Instead of using the more complicated approach above, a simple buffer as part of a dual op-amp (Texas Instruments `OPA2140` [158]) is used. Earlier versions also used an ADI `AD8672` [11], which were replaced because the `OPA2140` is similarly specced, but has a lower bias current and a rail-to-rail output. A buffer allows proper termination and has no bandwidth dependent issues. The design focusses on a maximally flat responce up to $1\,\mathrm{MHz}$ and then a well defined roll-off follows. The full circuit is shown in figure 4.22

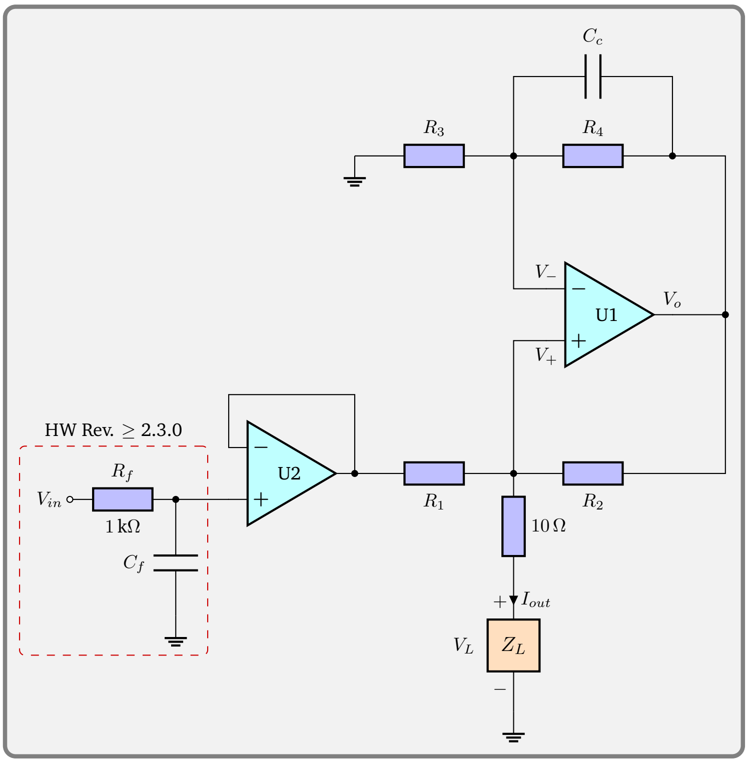


Figure 4.22.: The modulation current source used by digital laser driver DgDrive. $R_1 = R_2 = R_3 = R_4 = 1\,\mathrm{k\Omega}$ and $C_c = 10\,\mathrm{pF}$. The op-amp is a Texas Instruments `OPA2140`.

The idea behind this design is simple. A current source is, by definition, a high impedance source. It can not be matched to the typical $50\,\Omega$ transmission line. Reflections, cable and load dependent behaviour are an inevitable consequence. By deliberately limiting the bandwidth to around $1\,\mathrm{MHz}$ these problems can be avoided. If a faster solution is desired, it is best to move the current source directly into the laser like it was done in [167].

As discussed in section 3.8.9, the four resistors R_1, R_2, R_3 and R_4 need to be closely matched. This is achieved using a Vishay `MORNTA1001AT5` [155] array. The ratio matching is $0.05\,\%$ and according to table 3.11 on page 90 one can expect a mediocre worst case output impedance of $500\,\mathrm{k\Omega}$, which is in parallel to the high output impedance precision current source, discussed in section 3.8.4, with an output impedance of several $\mathrm{G\Omega}$. The latter can be neglected in the parallel circuit, because it is orders of magnitude higher and the combined output impedance can be approximated by the impedance of the Howland current source.

Normally, this would be disastrous and a better array would be required, but in this implementation it is not a problem, because as shown in the last section 4.1.6, the modulation output is sinked into a virtual ground node created by a current sink. The final output impedance is therefore multiplied by the gain of the `ADA4625-1`, which has plenty of bandwidth and open-loop gain with a unity-gain-bandwidth of $45\,\mathrm{MHz}$ and a gain of 10^7. This brings up the next subject to be discussed, the bandwdith of the current source.

The bandwidth of the modulation source is limited by the op-amp bandwidth and circuit parasitics. Erickson et al. [76] used a $50\,\Omega$ dummy load (and likely a very short cable) to test the modulation current source. To give a more realistic pciture of the performance, a laser diode was used and the modulation was recorded using a photodiode.

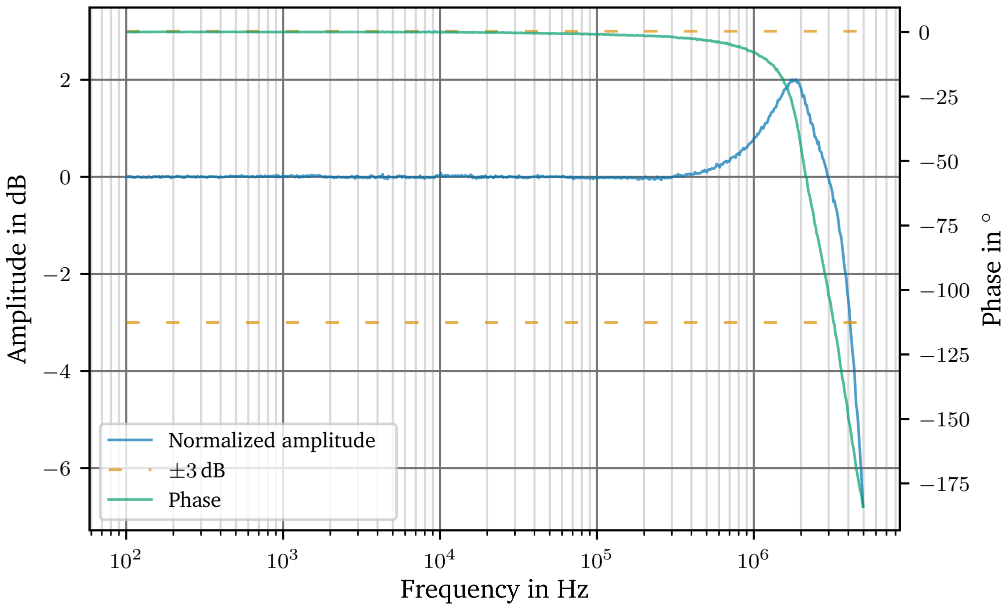


Figure 4.23.: Modulated output current over frequency, measured using a Thorlabs `LD785-H1` laser diode as the load. Source: [166].

Figure 4.23 shows a frequency sweep of the modulation input using a Keysight `DSOX1102G` oscilloscope, which can has a signal generator output to create simple Bode plots. The DgDrive current source was driving an ECDL with a Thorlabs `LD785-H1` laser diode via a $2\,\mathrm{m}$ cable. The laser diode was impedance matched using a matching network in the laser head as presented in [167]. The amplitude was recorded using a Hamamatsu `S9055-01` [178] photodiode with a transimpedance amplifier as discussed in [166] and a $1\,\mathrm{V_{rms}} \equiv 1\,\mathrm{mA_{rms}}$ modulation. The laser power is proportional to the current modulation [227] and can therefore be used to test the modulation capability of the driver. The graph was finally normalized to the average of the flat part below $100\,\mathrm{kHz}$ to calibrate out the transimpedance amplifier gain of the photodiode. The op-amp used for the DgDrive current source was an ADI `AD8672` as a hardware revision 2.2.1 laser driver was used.

The frequency response of the modulation shown in figure 4.23 demonstrates the excellent capability of the integrated modulation current source for the purpose of frequency stabilisation or frequency steering. It has a very flat and predicatable amplitude and phase response up to $1\,\mathrm{MHz}$. A slight gain peaking of $2\,\mathrm{dB}$ ($26\,\%$) is seen at $1.8\,\mathrm{MHz}$, due to the op-amp input capacitance and other parasitic capacitance of the PCB. At $1\,\mathrm{MHz}$ the gain peaking is $0.78\,\mathrm{dB}$ ($9\,\%$) with a phase shift of $-8°$. The $3\,\mathrm{dB}$ bandwidth is $4\,\mathrm{MHz}$. Due to the gain peaking, the phase shift is $-150°$ – unsuitable for a control loop, but it may still be used for modulation purposes. These values provide excellent behaviour in a control loop up to $1\,\mathrm{MHz}$ as desired in specification 3.2. Future revisions > 2.3.0 include an additional input resistor R_f, which serves two puposes, first it makes the design more robust against electrostatic discharge (ESD) and second, it can be configured as a low pass using C_f. This can help mitigate gain peaking if a faster op-amp is used. Additionally, the optional capacitor C_c was added in case a faster op-amp needs some compensation to prevent oscillations. This makes the design more flexible.

To summurize the modulation current source properties, the following list is given.

Device Properties 4.1: DgDrive Modulation input

- Transconductance $1\,\mathrm{mA/V}$
- $3\,\mathrm{dB}$-bandwidth $4\,\mathrm{MHz}$
- Recommened maximum control loop bandwidth $1\,\mathrm{MHz}$ with $8°$ phase shift

4.1.8. Cables and Connectors

This section covers the cables used for the digital current driver. These include the external connection to the laser head and the internal connection of the analog board with the frontend user interface board.

Several cabling options for connecting the current driver with the laser head were investigated while testing different current drivers. Both twisted pair cables and coaxial solutions are employed in the field by different solututions. Vescent, for example, uses an SMA connector and a coaxial cable. This solution is a universal approach as an abundance of SMA cables can be found in any lab. On the other hand, the typical RG-316 cable used with SMA connectors is geared towards high frequency applications. This is reflected in the rather poor capacitance of around $100\,\mathrm{pF/m}$, which can be improved to $75\,\mathrm{pF/m}$ by using larger RG-58X cables with a polyethylene (PE) foam dielectric [4]. A coaxial cable works well, when only a single conductor is required.

During the design of the new digital laser driver, it was clear, that the number of conductors would increase. The past has shown that additional features like protection circuitry and a fast current modulation input should be moved into the laser head. The laser head design was presented in [167] and shall not be discussed here, but this design requires are dual voltage supply and a signal line to enable the protection relay. On top of that, the voltage of the laser diode is remote sensed directly at the laser diode, adding another pair of conductors.

In addition to the need for more conductors, experience has shown, that self-made cables are a common source of problems in the lab. The legacy laser driver design, employed in this group, used a LEMO `ERA.0S.303.CLL` socket at the laser and the driver. For these sockets, cables were typically self-made using a LEMO `FFA.0S.303.CLAC44` plug on both ends. While the quality of the connectors is excellent, their assembly requires skilled hands, which resulted in fluctuating quality. Switching to professionally made cables considerably improved the situation in the past. Using custom specialty cables has the disadvantage of limited availability, high cost and likely longer lead times, so an off-the-shelf solution was preferred.

The laser head connector must be mechanically secure, be able to handle a current of $500\,\mathrm{mA}$ or more and idealy be widely available. Moglabs and Troxel et al. proposed a Digital Visual Interface (DVI) connector [216] and the same approach was also adopted in this work. The DVI cable, which nowadays is a DVI-D cable, only contains twisted pairs. There are also DVI-I cables, wich carry legacy analog signals through caoxial conductors, but these are obsolete. This brings up the question of the difference between a twisted pair cable and the coaxial cable, which needs to be addressed.

The most mundane difference is that twisted pairs are cheaper to manufacture than coaxial cables and they require less space, allowing more conductor in a cable. On the other hand, at high frequencies, the impedance of a coaxial cable is more uniform than that of the twisted pair. Fortunately, the latter property has greatly improved in recent years and, for example, the DVI cable is rated for clock rates of $165\,\mathrm{MHz}$, far more than is needed for this application. While the price difference and space savings are interesting for large installations it is less critical for this application, where signal integrity is premium.

Regarding noise immunity there is a profound difference between the two types of cables and elelectric and magnetic field coupling must be distinguished. A coaxial cable grounded at one end with a floating load offers fairly good protection against low frequency electric and magnetic field coupling, while at freuqencies above about $100\,\mathrm{kHz}$, the shield forms an antenna. For the floating shield to work, the laser head construction must ensure, that it does not have a ground connection, because as soon as both ends are grounded, a ground loop is formed.

At low frequencies, magnetically induced noise current can then flow in the shield which also serves as the return conductor. This introduces a noise voltage via the shield resistance, which must be minized with a thick braid to reduce the resistance. On the other hand, high frequency noise above around $100\,\mathrm{kHz}$ is kept out and the skin effect helps to confine the current loop closer to the inner conductor, thereby reducing magnetic pickup. With a coaxial cable one must choose between good low frequency protection or high frequency noise attenuation.

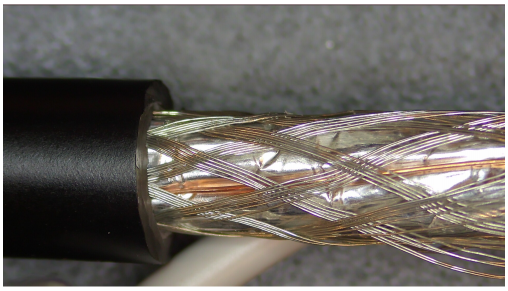
Looking at the twisted pair cable, it has very good magnetic shielding due to the twisted wires as long as there are enough turns per unit length [58, 159]. The mutual capacitance between the two conductors of a twisted pair is also lower, although the conductors are closer together, but the different geometry helps. In case of a shielded twisted pair, in addition to the mutual capacitance, there is also the capacitance between the shield and the two conductors. Through this capacitance, electric field interference can still couple into the cable. In order to reduce the succeptibility, it is important to keept the cable capacitance as low as possible, which can be achieved using insulators with low relative permittivity ϵ_r for the twisted pairs. To meet their high frequency specification, DVI cables typically use a low ϵ_r dielectric, namely air injected PE foam. Another option would be expanded PTFE (ePTFE), which is a microporous structure that contains air and is held together by strands of PTFE. Finally, having a dedicated shield allows to use a hybrid grounding scheme, by only capacitively coupling the shield at one end. This breaks the ground loop, yet keeps the shield effectively grounded at high frequency. A good overview of different grounding schemes can be found in [159, p. 72]. In order to find a suitable cable several options were explored.

The cables tested for the laser driver are a DVIGear `SHR` [194], a SUPRA Cables `DVI-DVI Single-Link` [207], an unbranded DVI cable for comparison and a Gore `RCN9034-24` Category 6a Ethernet cable [88]. The former three cables have a PE foam dielectric and 17 conductors while the latter is an ePTFE cable with 8 conductors. The DVIGear `SHR` cable does have the largest conductors with $0.33\,\mathrm{mm}^2$ (AWG22) followed by the SUPRA cable with $0.26\,\mathrm{mm}^2$ (AWG23), then the Gore `RCN9034-24` with a conductor diameter of $0.20\,\mathrm{mm}^2$ (AWG24) and finally the unbranded cable with just $0.05\,\mathrm{mm}^2$ (AWG30).

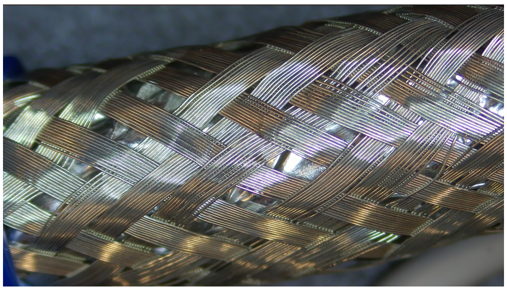
The twisted pairs of all DVI cables are tripple shielded, using a foil around the pairs, then another foil around all conductors and finally a braid over the outer foil. The Gore Category 6a cable has a foil shield around the 4 pairs and a braid around all conductors. The braid helps with low frequency shielding and the more coverage it has, the better. Since the braid can never fully shield a cable, the foil is added to help with high frequency shielding. Figure 4.24 shows the samples of the shielding of different cables tested. The Supra Cables `DVI-DVI Single-Link` is not shown, because the braid coverage is similar to the DVIGear cable and a photo can be found in the datasheet [207].

The braid of the unbranded cable has the least coverage with about $60\,\%$ calculated according to ANSI/SCTE 51 2018 [149]. In addition the braid is made of aluminium instead of copper. This increases the impedance and typically reduces the effectiveness by about $20\,\mathrm{dB}$ [230]. The other cables have an $80\,\%$ and a $95\,\%$ copper braid coverage. The braid and foil shield combination is fairly effective up to several dozen MHz [159, p. 84] covering the most important frequency range for this device if a decent copper braid is used.

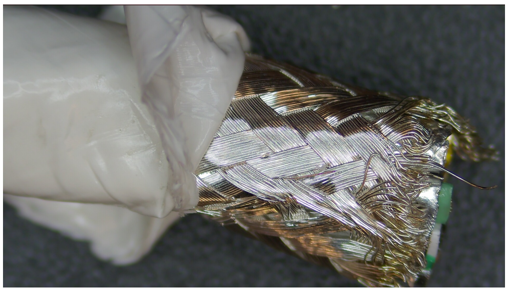
As it was mentioned above, using a shielded cable brings more capacitance to the table. A factor that needs to be considered when shielding a current source as it reduces the output impedance. It is therefore important to have a more detailed look at the cable capacitance in this application. Neglecting the parasitic capacitances to earth ground, the two capacitances most interesting are the mutual capacitance between the conductors and the capacitance between each conductor and the shield, for details see [108]. The cable capacitances of all



(a) Unbranded DVI cable. $60\,\%$ braid coverage.



(b) DVIGear SHR. $80\,\%$ braid coverage.



(c) Gore `RCN9034-24`. $95\,\%$ braid coverage.

Figure 4.24.: Braid coverage of several DVI cables and a Category 6a cable.

cables were measured using an LCR Research `LCR Pro 1 Plus` and the results are given in table 4.9. To measure the capacitance between the conductor and the shield, one of the twisted pairs was shorted to the shield using a DVI connector with a solder joint connecting both the shield and the conductor. The Category 6a cable was soldered together without a connector. The capacitance was then measured between the shorted pins and the remaining conductor. The measured total capacitance C_{tot} is the paralleled capacitance between the shield and the conductor C_{ws} and the mutual capacitance C_m. C_{ws} can then be calculated as

$$C_{ws} = C_{tot} - C_m \,.$$

DVIGear SHR	C_m	C_{ws}	Conductor Size
DVIGear SHR	(49 ± 1) pF/m	(46.0 ± 1.5) pF/m	$0.33\,\text{mm}^2$
`SUPRA DVI Single-Link`	(42 ± 1) pF/m	(36.0 ± 1.5) pF/m	$0.26\,\text{mm}^2$
Gore `RCN9034`	(43 ± 1) pF/m	(36.0 ± 1.5) pF/m	$0.20\,\text{mm}^2$
Unbranded DVI dual link	(42 ± 1) pF/m	(41.0 ± 1.5) pF/m	$0.05\,\text{mm}^2$

Table 4.9.: Measured cable capacitance for two DVI cables using a PE foam dielectric and an ePTFE Category 6a cable.

All DVI cables tested fared well regarding the capacitance and gave a similar performance when compared to the ePTFE Category 6a Gore `RCN9034`. Given the measurement uncertainties of the `LCR Pro 1 Plus` no distinction can be made.

To give a figure for the capacitance seen by the current source, it must be determined, whether the circuit is balanced or unbalanced. Remembering section 4.1.6 with figure 4.19

on page 120 where the current source schematic was shown, it is clear, that the circuit is unbalanced. The virtual ground has very little impedance, while the other conductor presents a high impedance current source. The virtual ground is the most sensitive node. Any noise current injected into it cannot be distinguished from the drive current. Injecting current into the return path on the other hand has very little effect. Noise current is injected via capacitive coupling, so the capacitance seen by the virtual ground node is the most important. This capacitance seen is C_{tot} measured above. Looking at the number it can be seen that a DVI cable is indeed a good choice for this use case as it has less capacitance than the typical coaxial cable, which has about $100\,\mathrm{pF/m}$.

The final decision was made in favor of the SUPRA `DVI Single-Link` cable, because of its decent shielding and very low capacitance, while being readily available in constrast to the also excellent Gore cable, which would need further assembly. A word of caution regarding the unbranded DVI cable needs be said. In addition to the meager shielding, the $0.05\,\mathrm{mm}^2$ wire is not recommended for carrying $500\,\mathrm{mA}$ because it has a resistance of about $330\,\mathrm{m\Omega/m}$, Using a cable of $3\,\mathrm{m}$ length would already drop $1\,\mathrm{V}$ (or $500\,\mathrm{mV}$ when using a dual link cable) at $500\,\mathrm{mA}$. The $0.26\,\mathrm{mm}^2$ cable chosen, for comparison, only drops $82\,\mathrm{mV}$ using a single link.

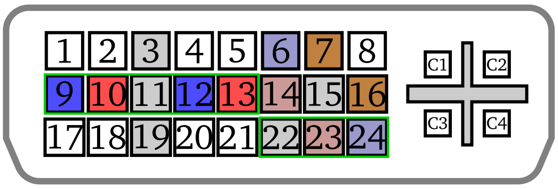


Figure 4.25.: Pin layout of the DgDrive DVI connector. Ground is gray, positive voltages red, negative voltages blue, digital i/o pins brown. White pins are not connected.

Pin	Function	Pin	Function
1–2	NC	14	$+12\,\mathrm{V}$
3	Shield, GND	15	GND
4–5	NC	16	Open-collector, enable laser
6	$-12\,\mathrm{V}$	17–18	NC
7	EEPROM	19	Shield, GND
8	NC	20–21	NC
9	LD cathode	22	Shield, GND
10	LD anode	23	LD voltage sense postive
11	Shield, GND	24	LD voltage sense negative
12	LD cathode	C5	GND
13	LD anode		

Table 4.10.: DVI connector pin layout. See figure 4.25 for the pin labels.

With the cable chosen, the connector layout needs to be discussed. DVI-D cables come in two flavors, single link and dual link cables. Dual link cables have twice the number of data lines. These data lines called called TMDS data lines. Those and the clock line are shielded twisted pairs. Depending on the type of cable, it has 17 or 23 pins, of which 4 are used to connect the shields of the 4 or 7 twisted pairs. So, apart from the twisted pairs, there 5 conductors available for additional functions. The connector layout of the DVI port of the digital current driver is shown in figure 4.25. A number of conductors are left unconnected (NC) for future applications. Pin 8 and pins C1 to C5 are can only be used with a DVI-I digital and analog cable. As mentioned above, these cables have became rare as analog displays are mostly extinct.

Pin 9 and 10 are used to deliver the laser diode drive current, while the shielded clock line on pins 23 and 24 are used to sense the diode voltage. When a dual link cable is used, pin 12 and 13 are also used to carry current. Using a single link cables reduces the capacitance in compararison to a dual link cable.

The other conductors are used to supply the laser head with a $\pm 12\,\mathrm{V}$ rail and to enable the protection relay in the laser head. Additionally, there is an electrically erasable programmable read-only memory (EEPROM) chip inside the laser to indentify it. This chip can be read using a one-wire protocol via pin 7.

As discussed above, the shielding and grounding plays an important role to supress noise. The digital current driver uses a floating DC power supply to supply the sub rack. the subrack is connected to chassis ground and so is the front panel of the current driver. The DVI cable shield is then connected to the front panel of the current driver and from there connected to the system ground. This chassis forms the path of least impedance for any noise current present and diverts it around the PCB ground plane. On the laser side the shield is connected to the laser head, which is grounded as well to protect the laser from electrostatic discharge (ESD). The PCB inside the laser head is only connected to the return conductor, effectively staying inside the shield.

Finally the cable between connecting the display board with the analog board is presented. The cable is shielded as well and is grounded on the analog board as well as capacitively coupled via a $10\,\mathrm{nF}$ capacitor on the digital front panel board to keep the digital signals from interfering with the analog board. It is a 5 conductor cable as shown in figure 4.26. For example, a Belden 9535 cable can be used, but any other foil shielded cable can be used. Use of a foil is recommended to shield the high frequency signals from leaking. The connectors are from the JST PHR series.

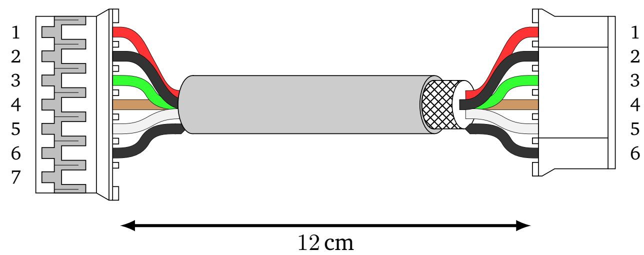


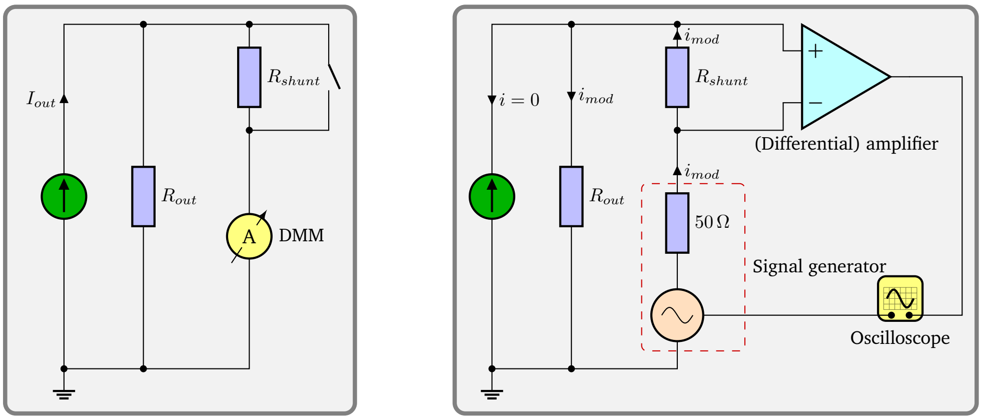
Figure 4.26.: Display cable used to internally connect the display board with the analog board. The 7-pin header is deprecated and will be replaced with a 6-pin header.

Pin	Function	Colour	Pin	Function	Colour
1	$+3.3\,\mathrm{V}$	Red	4	I²C SCL	Green
2	GND	Black	5	Interrupt	White
3	I²C SDA	Brown	6	Shield	–

Table 4.11.: Display cable pin layout. See figure 4.26 for the cable layout.

4.1.9. Test Results: Output Impedance

There are several ways of measuring the output impedance of a current source. Two such methods were used to test the output impedance of the current source and are shown in figure 4.27.



(a) DC scheme.

(b) AC scheme.

Figure 4.27.: Two methods for measuring the output impedance of a current source.

Figure 4.27a shows the more simple scheme. It can be used to determine the output impedance at very low freuqencies or rather DC and only requires three component an ammeter or multimeter, a resistor and a switch. The resistor value should be scaled such that $R_{shunt} \cdot I_{out}$ is just below the compliance voltage of the current source to maximize the resolution. To calculate the output impedance, the following steps are required. The current flowing through the shunt is measured using the ammeter and then switch is closed and the current is measured again. Assuming the resistance of the switch and the internal shunt of the ammeter is very small in comparison to R_{out}, essentially shorting the current source and it can be seen in figure 4.27a that all current I_{out} is flowing through the switch and the ammeter when the switch is closed. When opened, the current is split between R_{shunt} and R_{out}. This allows calculating R_{out} as

$$R_{out} = \frac{R_{shunt} \cdot I_{shunt}}{I_{out} - I_{shunt}} = \frac{V_{shunt}}{\Delta I} \,. \tag{4.10}$$

Both the shunt resistance R_{shunt} and the current flowing through it I_{shunt} can usually be determined with sufficient accuracy. The difference ΔI between to current with and without shorted ΔI is an entirely different matter though. Given a high impedance source, ΔI is naturally small. For educational purposes this problem can be illustrated by measuring the output impedance of the digital laser driver.

The measurement shown in 4.28 was conducted according to figure 4.27a. The ammeter was a Keysight `34470A` and the shunt resistor value was $R_{shunt} = (3.298 \pm 0.002)\,\mathrm{M\Omega}$, which was measured using a Keysight `3458A`. The output current was chosen as low as reasonably possible to allow for a larger shunt resistor to improve the sensitivity. $30\,\mathrm{s}$ into the measurement, the switch was disengaged and the current source had to drive the shunt resistor. The DMM settings were $10\,\mathrm{PLC}$ with autozeroing enabled. Since the output impedance of the current source is so high and even though the noise of the current source is extremely low with only $1.5\,\mathrm{nA_{rms}}$ ($\equiv 6\,\mathrm{nA/A}$ refered to full scale output) over $30\,\mathrm{s}$, the difference is hardly recognizable. Figure 4.28 also shows, in orange, the mean value of both measurements before and after switching in the shunt resistor. Longer measurement or integration times to supress more noise are ineffective due to the presence of flicker noise in the current source, rendering longer integration times useless.

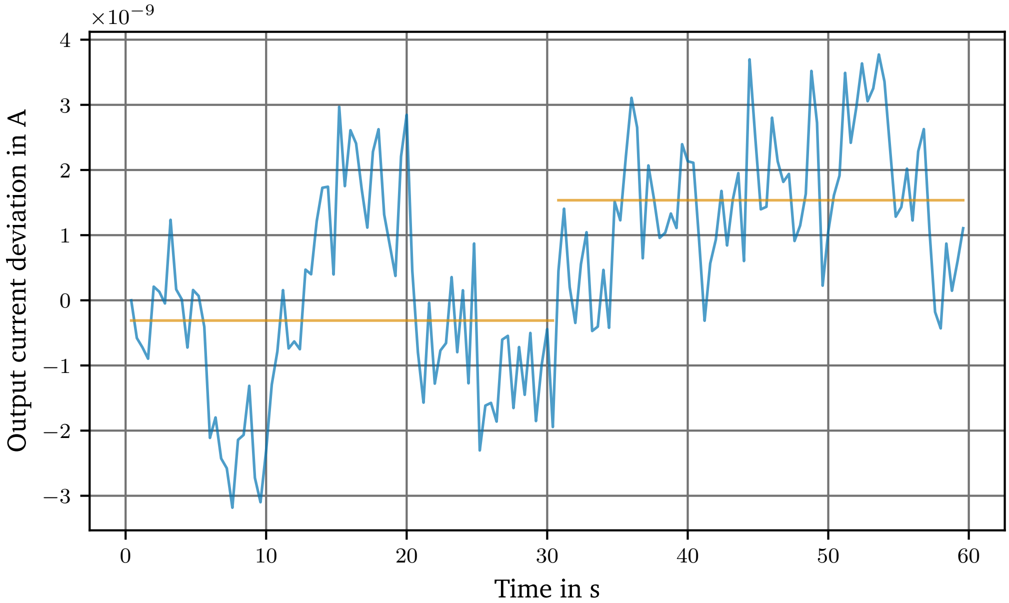


Figure 4.28.: Measurement of the output current using the technique illustrated in figure 4.27a. $R_{shunt} = (3.298 \pm 0.002)\,\mathrm{M\Omega}$, $I_{out} = (2.7200 \pm 0.0015)\,\mathrm{\mu A}$. The switch position was changed at $T = 30\,\mathrm{s}$. The DMM was nulled before the measurement.

The results nulled to $2.72\,\mathrm{\mu A}$ and extracted from figure 4.28 are

$$I_{out} = (-0.31 \pm 1.48)\,\mathrm{nA} \qquad I_{shunt} = (1.54 \pm 1.10)\,\mathrm{nA}\,. \tag{4.11}$$

As it can be seen the statistical uncertainty is in excess of $50\,\%$ of the desired quantity $\Delta I = 1.85\,\mathrm{nA}$. This means that applying the conventional approach of using the mean value and applying the simplified formula for the propagation of uncertainty

$$\sigma_f(x, y, ...) \approx \sqrt{\left(\frac{\partial f}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial f}{\partial x}\sigma_y\right)^2 + \ldots}$$

will yield improper results, since it requires the uncertainty to be limited to a close neighbourhood around the mean, because it is only an approximation. The same advice goes for using the mean of $I_{\Delta I}$ to calculate the expected mean of the result R_{out}. More information on working with asymmetric uncertainties can be found in [30].

In order to overcome this problem, a Monte Carlo simulation in Python was prepared was prepared. The source file can be found at data/simulations/sim_output_impedance_mc.py as part of the online suplemental material [42]. For the simulation, the uncertainties of R_{shunt} and I_{out} were neglected, because they are small in comparison to the uncertainty of ΔI. The same goes for a systematic uncertainty resulting from the DMM calibration error of R_{shunt} and I_{out}. The simulation uses 1×10^8 samples, which are drawn from a normal distribution of $(1.85 \pm 1.56)\,\mathrm{nA}$ to calculate the output impedance applying equation 4.10. Using those samples, 81.6×10^6 results were found the range $\left[0, 2 \times 10^{10}\right]$ and used to create the histogram shown in figure 4.29.

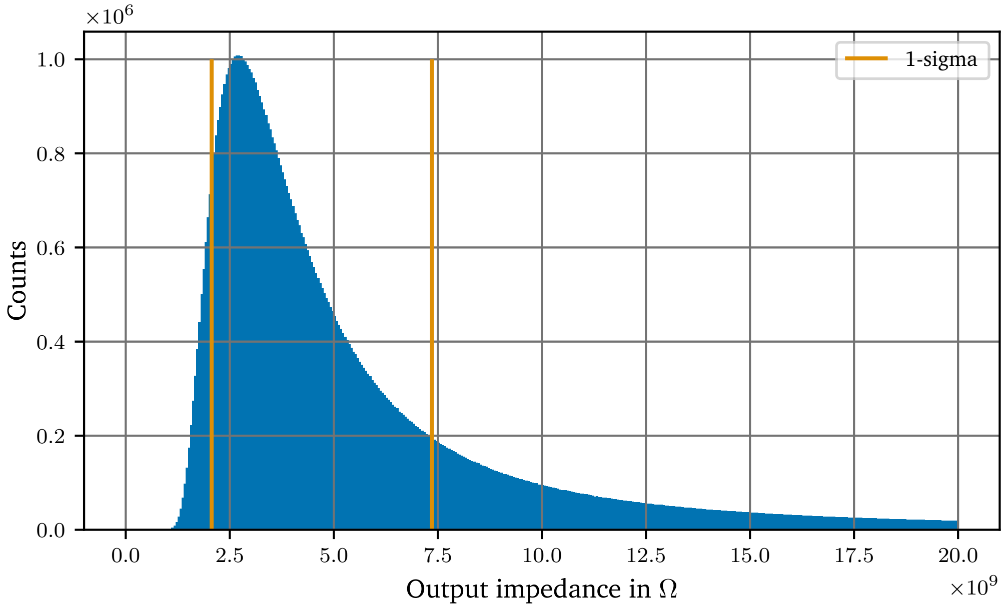


Figure 4.29.: Monte Carlo simulation to derive the output impedance of the DgDrive from the parameters given in 4.11.

This leads the following result for the low frequency output impedance

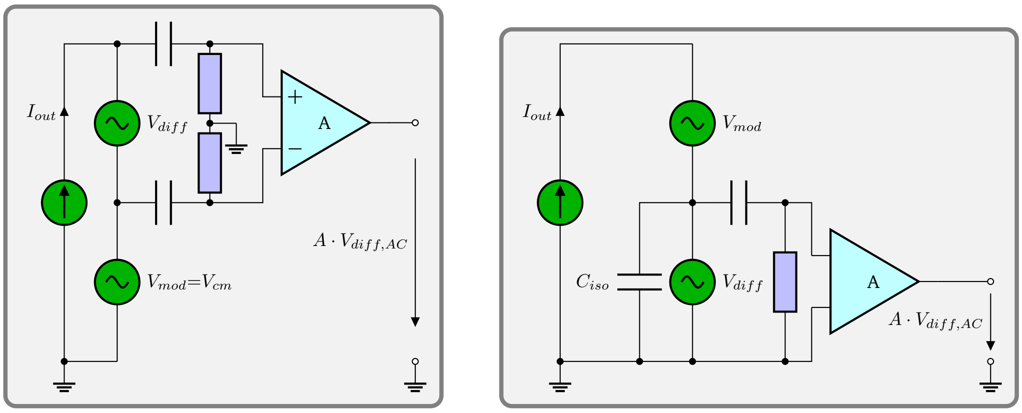
$$R_{out} = 2.7^{+4.70}_{-0.64}\,\mathrm{G\Omega}\,.$$

Overall it can be said that the DC output impedance is likely a lot higher than $2\,\mathrm{G\Omega}$, but more conclusive results need a different approach. Using the method shown in figure 4.27b, a signal signal generator in combination with an oscilloscope or a network analyser can be used to determine the output impedance over a wide frequency range. Since the laser driver current source relies on op-amps and their gain, going to higher frequencies will see a dropping output impedance. This way, the limited resolution of the previous measurement can be evaded. Additionally having the output impedance spectrum reveals a lot of details about the current source as will be discussed next.

The setup shown in figure 4.27b puts a few requirements on the signal generator, the amplifier and the oscilloscope that need to be addressed first. This setup can com in two version shown in figure 4.30. One requires a floating signal generator, because the resistor is ground referenced and the other requires a floating or differential amplifier. Due to this nature, the whole measurement becomes more of a test of CMRR or ground isolation than anything else. This will be briefly elucidated now to help understand the final solution implemented.

Starting with a ground referenced amplifier as shown in figure 4.30a, it is clear that the amplifier sees the common-mode voltage produced by the signal generator on poth inputs. An ideal amplifier is unaffected by V_{cm} and has a common-mode gain $A_{cm} = 0$, but any physical amplifier has both a common-mode and a differential mode gain A_d with the CMRR given as [215, p. 328]

$$CMRR = \frac{A_d}{A_{cm}}\,.$$



(a) Grounded signal generator.

(b) Floating signal generator.

Figure 4.30.: Different configurations of the amplifier have a profound effect on the measurement. Placing the sigal generator on top gives significant coupling to ground, yet allows a single-ended amplifier.

The common-mode rejection quickly becomes a problem, because a larger output impedance implies a smaller differential signal albeit the same common-mode signal. Therefore the CMRR of the amplifier presents an upper limit to the sensitivity in this configuration. The most common amplifier for this situation is a so-called instrumentation amplifier, which is a difference amplifier with buffered high impedance inputs [215]. It is a design based on three op-amps and the CMRR is mostly limited by the matching of four internal resistors. To give some numbers, a $1\,\mathrm{k\Omega}$ shunt resistor in combination with a modern high precision instrumentation amlifier like the Texas instruments `INA821` [96] will give a CMRR of around $120\,\mathrm{dB}$ limiting the maximum output impedance that can be measured to $R_{out} \ll 1\,\mathrm{G\Omega}$. Increasing R_{shunt} is impractical, because of the limited compliance voltage of most laser current drivers. AC coupling the instrumentation amplifier as shown in figure 4.30a aggrandizes the CMRR problem discussed before. In addition to the common-mode rejection of the amplifier, the transfer function of both high-pass filters has to be closely matched as well, because a gain mismatch translates into a decreased CMRR. Matching those filters to 10^{-6} is a tedious venture.

Alternatively, the signal generator can be floated and placed above the shunt resistor as shown in figure 4.30b. This allows to use a singled ended amplifier, which can be easily AC coupled to remove the DC offset. While this takes care of the CMRR issues, it creates another one. The signal generator may have a isolated outputs, but there is still a capacitive coupling C_{iso} to earth and if either the amplifier the current source, or the oscilloscope is connected to protective earth an AC leakage current can be observed. Using a high value shunt resistor of several $\mathrm{M\Omega}$ to improve the signal-to-noise ratio (SNR) of the measurement exaggerates the problem, especially around the line frequency, and multiples thereof. This foiled any attempts to measure the output resistance at low freqencies. Using a low capacitance isolation transformer could solve this issue, but there is a solution, that promises fewer sources of error.

Instead of floating the signal generator or using a differential amplifier, the amplifier can be floated. Battery driven amplifiers, like the Stanford Research `SR560` or the Signal Recovery `5113` are available off-the-shelf and include additional filters and a variable gain. Using a floating amplifier either requires an isolated oscilloscope or a differential input like a differential probe typically used for high voltage applications. The common-mode rejection specification of the differential probe can be a lot more relaxed when compared to a differential amplifier, since the signal is already amplified. Both approaches were used to measure the output impedance of several commercial laser drivers and the digital current driver presented in this work.

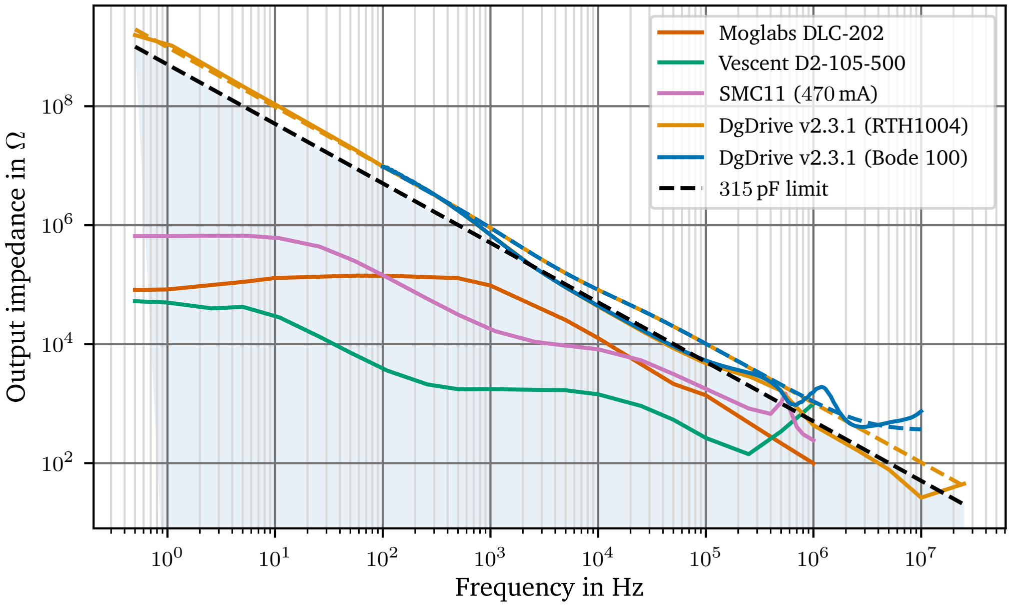


Figure 4.31.: Measured output impedance of several laser diode drivers. The shaded region is the limit imposed by a 3 m RG-213 coaxial cable with a capacitance of 315 pF between conductor and shield. The coloured dashed lines show the results of an LTSpice simulation.

The instruments used for the dynamic output impedance measurement were a Keysight 33522B signal generator, a Stanford Research SR560 [202], a Sapphire HVP70 differential probe and a Keysight MSO9254A oscilloscope. Additionally, a Rohde & Schwarz RTH1004 [170] isolated battery powered oscilloscope and an Omircon Bode 100 [212] vector network analyser (VNA) was used for some of the measurements. The devices tested were the Vescent D2-105-500, the Moglabs DLC-102, the Sisyph SMC11 and the DgDrive-250, a 250 mA version of the new current source. The cable length used for testing was about 1 m in length to reduce the effect of the cable capacitance. The commercial drivers were tested using a 10 Ω shunt resistor, the DgDrive-250 required several different shunt resistors to meet the signal-to-noise rquirements. The measurements are shown in figure 4.31.

Both the Sisyph SMC11 and the Vescent D2-105-500 show a familiar characteristic. The output impedance is reminiscent of the simulation shown in section 4.1.7 as figure 4.21 on page 4.21. This is not suprising as Vescent even points to their heritage, the design by Libbrecht et al. [120], in the datasheet [62]. Regarding the SMC11 it is unclear, but likely that a similar circuit is used. Moglabs on the other hands employes a different scheme, which does not have this limitation. The output impedance of those drivers shows the expected behaviour of being limited by the resitstor matching in the modulation current source and then rolling off when going to higher frequencies as the amplifier gain drops. The sudden rise seen in the output impedance of the Vescent D2-105-500 starting at 250 kHz is a resoance likely produced by the test setup parasitics. These parasitics will be discussed below in more detail, when discussing the output impedance of the DgDrive.

All commercial current driver share the same problem though, the limited output impedance of the modulation current source confines the combined system to below $1\,\mathrm{M\Omega}$. None of these drivers manage to get close to the physical limit imposed by a hypothetical $3\,\mathrm{m}$ RG-213 coaxial cable used as the connection the laser. Those cables commonly used in this group have a measured capacitance of $105\,\mathrm{pF/m}$ and are found throughout the labs. Since both the `SMC11` and the `D2-105-500` have an SMA output for the laser current, such a connection would likely be adopted in a real situation. The limit is shown as a dashed black line with a light blue fill below in figure 4.31. The values within the blue shaded region markes the output impedance physically possible when a $315\,\mathrm{pF}$ capacitor is connected in parallel.

This means that those drivers do not utilize the full potential granted by physics to suppress external noise sources. This will be discussed in the next section 4.1.10. The wiggles and bumps in the output impedance at high frequencies stem from the test fixtue and parasitic effects, including the impedance mismatch from connecting a current source to a low impedance sink. Some of the parasitic effects are shown in figure 4.32 and will discussed along with the output imput impedance of the `DgDrive-250`.

The `DgDrive-250` (serial number: #13) required special attention when measuring its output impedance. First of all, the current source has a differential output, so a floating output of the signal generator is mandatory. This means, that the oscilloscope input measuring the signal generator voltage needs be isolated as well. To facilitate this, the `MSO9254A` was replaced with the fully isolated `RTH1004`. In addition to those changed, the drivers output impedance is so high, that at low frequencies up to $110\,\mathrm{Hz}$ a $1\,\mathrm{M\Omega}$ resistor had to be used to reach the required signal-to-noise ratio. The test current was set to $5\,\mathrm{\mu A}$ for the $1\,\mathrm{M\Omega}$ ressitor, although this had no bearing at the result, which was independent of the output current or the compliance voltage within the specified limits. Between $260\,\mathrm{Hz}$ and $250\,\mathrm{kHz}$ a $1\,\mathrm{k\Omega}$ resistor was used and beyond that a $50\,\Omega$ resistor. Along with the change to a $50\,\Omega$ shunt resistor the amplifier was removed and the signal was directly fed into the `RTH1004`, because the `SR560` only has a bandwidth of $1\,\mathrm{MHz}$ [202]. The $10\,\mathrm{bit}$ ADC of the `RTH1004` [170] for compensated the loss in gain. In order to reduce parasitic effects, the shunt resistor was a small 0805 SMD resistor, that was soldered between the pins of an isolated BNC connector. Using this scheme an output impedance of $1\,\mathrm{G\Omega}$ at $1\,\mathrm{Hz}$, was measured. Towards lower freuqencies, the measurement is limited by noise and the cutoff limit of the bandpoass filter in the `SR560` of $0.03\,\mathrm{Hz}$ [202]. The low freuency results nonetheless confirm the measurement using the DC measurement shown above. Towards higher frequencies, the output impedance drops by an order of magnitude per decade of frequency – the typical behaviour of an RC filter.

The parasitic capacitance of the test fixture and the $1\,\mathrm{m}$ cable used to connect the driver to the fixture, was found to be responsible. The cable used was constructed by cutting apart a DVI cable and soldering an RG-213 cable to the current source outputs. The test fixture including the cable was measured as having a capacitance of $155\,\mathrm{pF}$. Including the $12\,\mathrm{pF}$ input capacitance of the `RTH1004` [170], a simulation in LTSpice was done and it is shown as a dashed line of the same colour as the corresponding trace in figure 4.31. From the agreement of the simulation results with the impendance measurement, it can be inferred that the measurement delivers only a lower bound, constrained by the test setup. Nonetheless, the $155\,\mathrm{pF}$ of the test setup give a reasonable approximation for the real-life situation. As measured in section 4.1.8 and presented in table 4.9 132, the DVI cable has a mutual capacitance of $42\,\mathrm{pF/m}$–$49\,\mathrm{pF/m}$, which is about the same number for a typical $3\,\mathrm{m}$ cable. The results in figure 4.31 can therefore be be anticipated in real-life situations.

To confirm these results obtained using a signal generator and the oscilloscope, the measurement was repeated using a different setup. Instead of using a generator and an oscilloscope, a VNA, the Omicron `Bode 100`, was used. The VNA does not have isolated inputs and outputs. To inject the modulation, a Picotest `J2101A` injection transformer was used to isolate the output. In addition, the signal behind the transformer was sampled using a Sapphire `HVP70` differential probe. The remaining input was then connected in parallel to the shunt resistor. This secondary measurement was only conducted with a $1\,\mathrm{k\Omega}$ shunt and without the `SR560` pre-amplifier. Therefore is was only possible to measure from $100\,\mathrm{Hz}$ to $10\,\mathrm{MHz}$. The upper limit is given by the injection transformer bandwidth of about $10\,\mathrm{MHz}$ [64]. As can be seen in figure 4.31, both measurements agree very well. The long tail at above $1\,\mathrm{MHz}$ is caused by the input capacitance of the VNA. To understand this, figure 4.32 can be consulted.

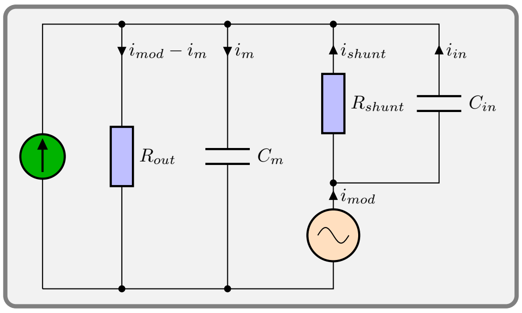


Figure 4.32.: Simplified setup for measuring the output impedance including the mutual capacitance of the cable C_m and the parasitic input capacitance C_{in} of the instrument.

Figure 4.32 shows a model of the test setup aloing the most prominent parasitic effects. The mutual capacitance between the two conductors C_m of the current driver output lead is in parallel to the output R_{out} resistance. In addition, the input capacitance C_{in}, including the lead cpaacitance, of the measurement instrument is in parallel with the shunt resistor R_{shunt}. The measurement instrument is either the `Bode 100` VNA, the `SR560` amplifier, or the `RTH1004` oscilloscope along with a $25\,\mathrm{cm}$ BNC cable.

The output impedance is then calculated from the ideal asumption without parasisitcs as

$$R_{out} = \frac{v_{mod}}{i_{mod}} - R_{shunt} = \frac{v_{mod}}{i_{shunt}} - R_{shunt}\,. \tag{4.12}$$

With C_{in} in parallel to R_{shunt}, not all the current is flowing through R_{shunt} and $i_{shunt} < i_{mod}$ and equation 4.12 no longer holds. With $i_{shunt} < i_{mod}$ R_{out} looks bigger and with parts of i_{mod} flowing into C_m, R_{out} looks smaller. Since C_m is in parallel with R_{out} its effect is immediately visible, causing R_{out} to roll off at very low frequency. C_{in} can only be seen as high frequencies, because R_{shunt} is a lot smaller and therefore the impedance of C_{in} needs to be a lot lower than the impedance of C_m to cause a noticable effect. In addition, C_m is typically larger than C_{in}. For this reason, R_{shunt} must be reduced with increasing frequency. This was done by going from a $1\,\mathrm{M\Omega}$ resistor to a $1\,\mathrm{k\Omega}$ to a $50\,\Omega$ resistor. The effect can therefore only be seen in the measurement conducted with the VNA. Using a fixed $1\,\mathrm{k\Omega}$ resistor in combination with the larger input capacitance of the VNA leads to this aparant increase in the output impedance, which is entirely due a measurement error. This can also be reproduced in the simulation by inserting the capacitor $C_{in} = 55\,\mathrm{pF}$ as shown in figure 4.32 into the model. The simulation results were also added to figure 4.31 and are shown as a dashed blue line.

The drop in output impedance of a factor of 2 at $1\,\mathrm{kHz}$ is currently not well understood. It could be part of the test fixture, which was already the limiting element of the measurement.

There are plans to revise the test fixture with a dedicated PCB and amplifiers directly on the board to reduce the capacitance seen by the current source. This would give a clearer picture on the source of this phenomenon.

The result shown in this section can be summurized as follows. A high impedance is desirable to give a good noise immunity of the current source as shown in figure 3.38b. The output impedance of all commercial laser drivers tested were limited by the output impedance of the modualtion current source and were below $1\,\mathrm{M\Omega}$ at low freuqencies. This limit was shown to be result of the resistor matching in the modulation current source. It can be expected to vary widely between individual samples of the devices as shown in figure 3.53 on page 89. The DgDrive was demonstrated to have a very high output impedance of at least $1\,\mathrm{G\Omega}$ at $1\,\mathrm{Hz}$, which was the limit of the test setup. At higher freuqencies, the field of tested drivers moves closer together, because here the output impedance is limited by the bandwidth of the op-amps used. All drivers are within a range of $1\,\Omega$ to $1\,\mathrm{k\Omega}$.

None of the commercial current drivers tested reached the limits imposed by a typical $3\,\mathrm{m}$ RG-213 coaxial cable, chosen as an example for connecting the laser driver. Of those drivers, the Moglabs `DLC-102` has shown the best performance between $100\,\mathrm{Hz}$ and $100\,\mathrm{kHz}$, a range containing a lot of noise sources like overhead lights and switch mode supplies. The Vescent `D2-105` and Sisyph `SMC11` demonstrated the bahaviour seen in the original paper presented by Libbrecht et al. [120] and could do lot better by improving on their modulation current source. The Vescent `D2-105` driver has gererally shown the worst performance in this test, so a poorer noise immunity is expected in the next test.

The DgDrive in turn managed to push the limits of a low capacitance DVI cable and delivered a performance only limited by the cable capacitance, which translates in a superior noise rejection capability compared to the commercial devices. The noise performance will be discussed next.

4.1.10. Test Results: Current Noise

The spectral current noise density is a quantity, that is both seemingly trivial to measure and also easy to understand and graphically compare, therefore it is a figure widely and many devices are emblazoned by such graphs. The upside is that these numbers can used for reference although comparing devices is not trivial, because the noise depends on the maximum output current as discussed in section 3.8.6.

Defining the bandwidth of such a measurement is a matter of debate and depends on the future use-case of the current driver. In this work an upper frequency of $1\,\mathrm{MHz}$ was chosen for two reasons, first, to limit the number of amplifiers required. As the noise power rises with the bandwidth (in the best case as $\sqrt{\Delta f}$ for white noise) and impedance matching comes into play, a higher power amplifier is required, a trait that does not bode well with low noise, low frequency frontends. So for frequencies above a few MHz more than one amplifier is called for. More amplifiers make the whole measurement more intricate, because the amplifiers are the most critical parts in the whole chain.

The second reason does not root in the lazyness of the researcher, but has a physical origin. Cables used in the lab like RG-58 or RG-223 have a capacitance of about $100\,\mathrm{pF/m}$. With a cable length of around $3\,\mathrm{m}$ resulting in $300\,\mathrm{pF}$, one finds, that at $10\,\mathrm{MHz}$ the impedance seen by the laser diode approaches $50\,\Omega$, not unsurprising, given that the cable design impedance is $50\,\Omega$ and a signal at $10\,\mathrm{MHz}$ has a wavelength of $\lambda \approx 2\,\mathrm{m}$. At this point the performance of a current source is naturally limited and the laser should be stabilized to an exteranal refrence using a current source closer to the laser diode like in the laser head as demonstrated by [167] [167].

The measurement technique used in this section is a simple shunt resistor through which the current is sourced, then an AC coupled amplifier is used to amplify the resulting voltage noise. This measurement setup is owed to the fact, that most publications [76, 120, 185, 210] and commercial drivers [85, 133] use this method and it therefore allows intercomparison. A better signal-to-noise ratio could be achieved using a transimpedance amplifier configuration, which would provide more gain and less noise. Nonetheless, sufficient SNR can be achieved using a $10\,\Omega$ resistor and a low noise pre-amplifier (LNA). A $10\,\Omega$ resistor was chosen to keep the load on the current driver low to exclude any compliance voltage related effect. These will be discussed separately at the end of this section. Using a test current of $50\,\mathrm{mA}$ gives a voltage drop of only $500\,\mathrm{mV}$, well below the typical voltage of a diode, so compliance voltage related effect should not be visible. In addition, using $250\,\mathrm{mA}$ and $2.5\,\mathrm{V}$ is close to the Thorlabs `L785H1` used in this group, so both scenarios can be easily test using a single resistor.

A $10\,\Omega$ resistor creates a thermal noise density of $400\,\mathrm{pV}/\sqrt{\mathrm{Hz}}$. This exlcudes the `SR560` as a pre-amplifier, because it would severly limit the measurement by its own noise figure of $4\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ [202], the equivalent of a $1\,\mathrm{k}\Omega$ resisor. While the `SR560` uses a FET input amplifier with a very high input impedance and low current noise, the $10\,\Omega$ input impedance allows the use of bipolar transistors at the input of the amplifier. Bipolar transistors are more sensitive to the source impedance since they do not present a high impedance input like a JFET, resulting in a higher current noise, but achieving a lower voltage noise in return. A design based on the amplifier presented in [147] with a noise floor of $460\,\mathrm{pV}/\sqrt{\mathrm{Hz}}$, a $3\,\mathrm{dB}$-bandwidth of $10\,\mathrm{Hz}$ to $1.5\,\mathrm{MHz}$ and a fixed gain of $80\,\mathrm{dB}$ (10^4). The amplifier is powered by alkaline batteries to revent mains hum from entering the amplifier through the power supply. The input impedance is only $500\,\Omega$, so this has to be taken into account for a source impedance of $10\,\Omega$ as proposed above. The voltage divider formed will reduces the gain to about 9800.

To measure the spectral density, multiple devices were used since no low frequency FFT

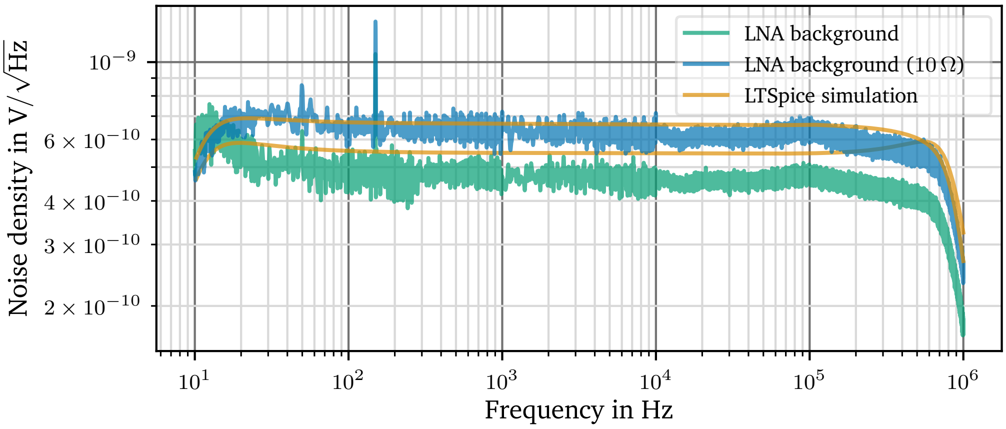


Figure 4.33.: Noise floor of the low noise amplifier based on [147].

analyser or the like was available. At low freuqency the digitizing function of the Keysight `34470A` [70] with a sampling rate of up to 10 kHz was was configured for the range between 0.1 Hz–260 Hz. A Keysight `MSO9254A` oscilloscope was used between 260 Hz and 40 kHz. Above that, a Tektronix `RSA306` USB spectrum analyser (SA) served to measure up to 1 MHz. Each range was band-filtered using the `SR560`, which is also responsible for the gain (and noise) dropping off towards 1 MHz. The time-domain data collected by the DMM and the oscilloscope, was then converted into its frequency representation using the algorithm developed by Welch [225].

Figure 4.33 shows the noise of the amplifier in two configurations. One with the input shorted and the other with a 10 Ω resistor between the inputs. The latter was corrected for the reduced gain and presents the instrinsic noise of the current noise measurement setup. The white noise floor including the 10 Ω resistor is at 615 pV/$\sqrt{\text{Hz}}$, which agrees well with theory, which predicts

$$\sqrt{\left(400\,\text{pV}/\sqrt{\text{Hz}}\right)^2 + \left(450\,\text{pV}/\sqrt{\text{Hz}}\right)^2} = 610\,\text{pV}/\sqrt{\text{Hz}}\,.$$

The noise floor is mostly white with two exceptions. Although the amplifier is battery powered, the 50 Hz mains frequency and its third harmonic shows a problem, that such low noise amplifiers are susceptible to magnetic pickup and must be shielded (preferably in mild steel) and kept away from linear power supplies with transformers. In practice, a distance of 3 m from the linear power supply was found to be sufficient.

An LTSpice simulation of the circuit was also conducted, but it slightly overestimates the noise figure of the `THAT300` NPN transistor array yet still gives reasonable results. The SPICE model used was the noise model provided by the manufacturer [57]. The simuated data is included in figure 4.33 as solid orange lines for reference.

The noise floor of 615 pV/$\sqrt{\text{Hz}}$ or 61.5 pA/$\sqrt{\text{Hz}}$ limits the resolution of the test setup to laser drivers with a current noise of about 100 pA/$\sqrt{\text{Hz}}$. In this case an error of 17 % is incurred, which must be kept in mind. Lower noise drivers require a larger resistor though. In this case a 1 kΩ resistor was chosen and the test current was reduced to 5 mA. Since the LNA does not work well with a source impedance of 1 kΩ and the thermal noise of the resistor is 4 nV/$\sqrt{\text{Hz}}$, on par with the `SR560`, the latter can be used without the low noise amplifier in front.

The test setup requires utmost care in order to not introduce ground loops, that erroneously show up as noise from the current source. To ensure this, all measurement instruments were floating or battery powered without a direct connection to earth. The Keysight 34470A DMM has floating inputs, the Keysight MSO9254A oscilloscope was used with an insolated differential probe and the Tektronix RSA306 SA was powered via an ALLDAQ ADQ-USB 3.0-ISO-W USB 3.0 isolator to remove the high frequency coming noise from the host computer. The RSA306 was then supplied through an Agilent E3631A linear power supply on the isolated side. The amplifiers were powered from battery. The full setup including grounding is shown in figure 4.34. All laser drivers except the DgDrive-250 used the power supply that came with the unit to give an example of the full system performance that can be expected out of the box. The DgDrive-250, which does not come with power supply of its own, was connected to both an Agilent E3631A linear supply and a R&S HMP4040 switch-mode supply to demonstrate the suppression of the input filter, which was already examined in section 4.1.4. As mentioned above, both the custom low noise amplifier and the SR560, are very sensitive to magnetic coupling. Since shielding at this level either requires an expensive mu-metal box or a thick steel box, the amplifier was instead moved away from any large transformers, namely the power supplies of the drivers. The shunt resistor was connected via a 3 m cable, also resulting in a more realistic scenario, when compared to a typical laser setup. The effect of different cables and lengths on the current source was discussed in the last section 4.1.9.

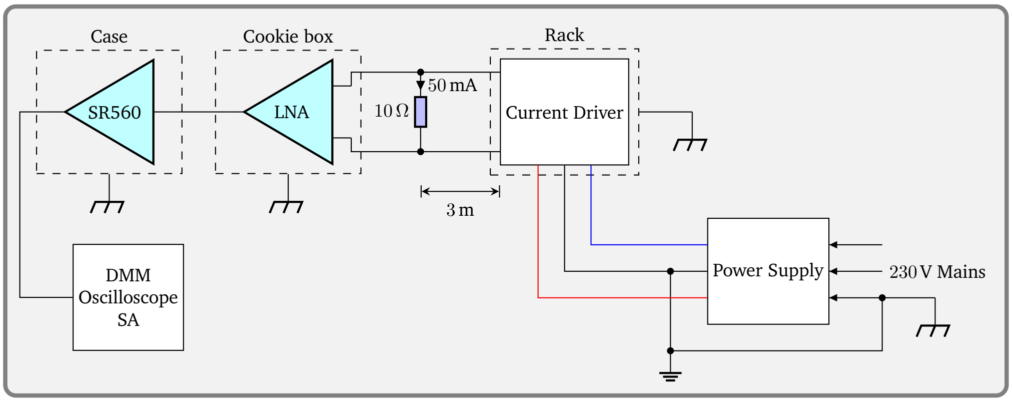


Figure 4.34.: Power and grounding scheme for the current noise measurement. The shunt and amplifier are located 3 m away from the power supply to prevent magnetic coupling.

The results of the measurements are shown in figure 4.35 and each device will now be briefly discussed, starting with the Toptica DCC 110. While the DCC 110 is legacy device no longer sold by Toptica, it was widely sold and is still used labs. Unfortunately, it has shown the worst performance of all devices tested. A whole whole range of mains harmonics can be seen. Additionally, there is some interference around 15 kHz and its harmonics. On top of that, the baseline noise density is around $10\,\mathrm{nA}/\sqrt{\mathrm{Hz}}$ dropping off at 10 kHz probably due filtering of the reference, because it levels off at $300\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$, likely the baseline noise of the current source.

The next device evaluated was the LQO LQprO-140 developed at the neighbouring group *Laser und Quantenoptik* [84]. The author would like to thank the group for providing a sample for testing. The unit came in small rack with an integrad power supply. It must be noted that there is only little mains hum or harmonics present from this supply. The noise floor was better than the one presented by Toptica, but unfortunately the $<300\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ claimed in the

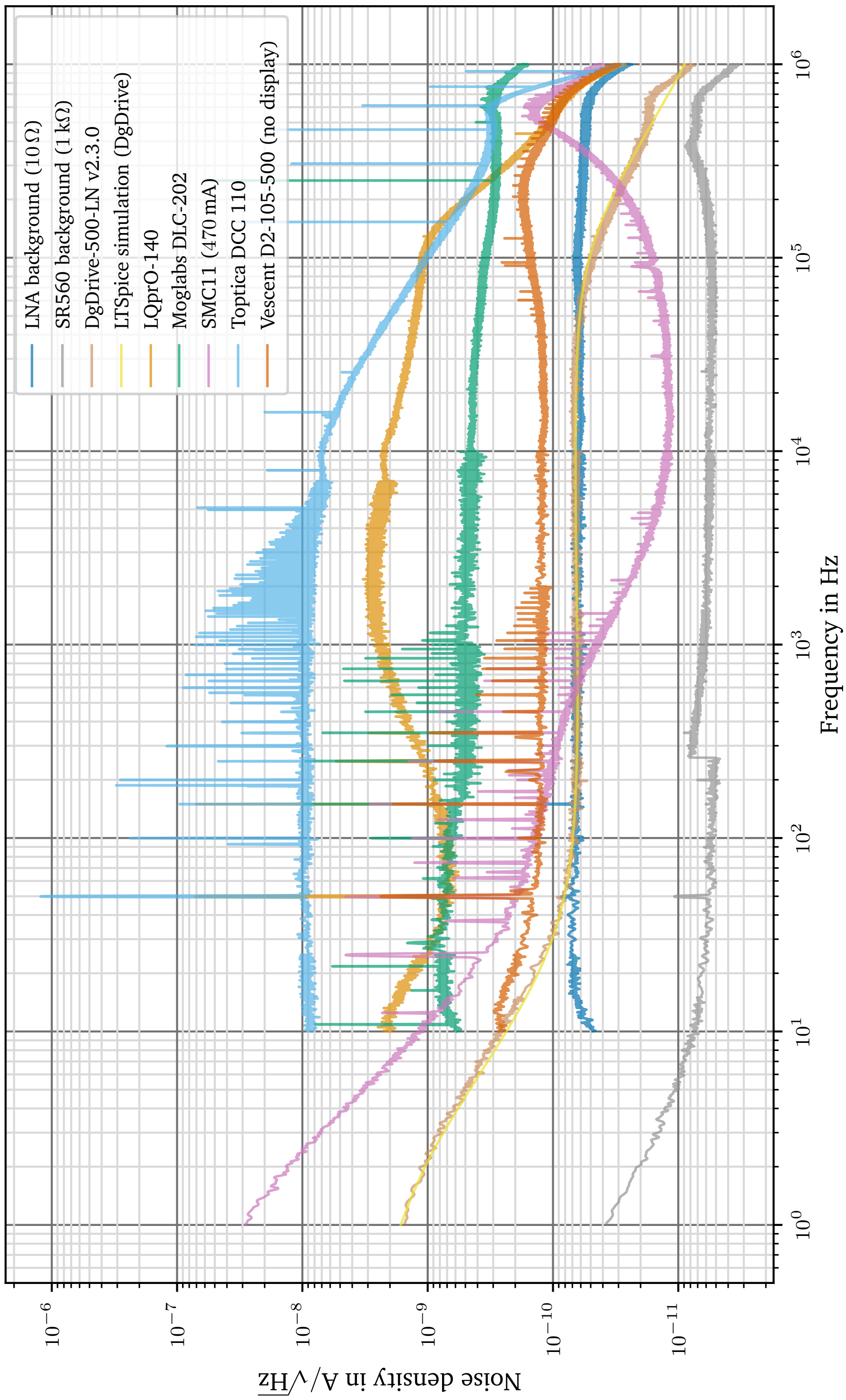


Figure 4.35.: Noise spectra of several current drivers tested. The `DgDrive-500-LN` and the `SMC11` were measured to $1\,\mathrm{Hz}$ using the `SR560` and a $1\,\mathrm{k\Omega}$ shunt resistor improve the SNR. The amplifier noise floor was not subtracted form the measurements.

datasheet cannot be confirmed. A feature of this driver might be a boon and a bane at the same time and be responsible for the high noise floor. The driver has a fullscale modulation input with a bandwidth of $200\,\mathrm{kHz}$ [85], so aggressive filtering of the current source setpoint with a low cut-off is not possible and therefore quite some noise is present up to $200\,\mathrm{kHz}$. Some of this noise though is introduced through the supply line though, and during evaluation, some improvements were made by John [100], bringing the driver closer to the performance of the Moglabs `DLC-102`.

The Moglabs `DLC-102` has a fairly flat noise floor of $500\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$, which is more than the $250\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ claimed by Moglabs. This is owed to the fact that the `DLC-102` has a fairly high compliance voltage of $6\,\mathrm{V}$. Using an architecture similar to the precision current source discussed in 3.8.4, a low value sense resistor must be employed. This in turn results in less transconductance gain, wich increases the current noise as discussed in section 3.8.6 and shown in table 3.6 on page 83. The $50\,\mathrm{Hz}$ noise and harmonics are likely due to the unshielded transformer integrated into the unit for the power supply. The large peak at $250\,\mathrm{kHz}$ is due to the local oscillator of the integrated lock-in amplifier. The oscillator can be disabled completely by adjusting internal trimpots, but this was not done for the noise measurement, because the origin of the peak is known, deliberately placed there and had no bearing on the measurement.

The Vescent `D2-105-500` delivered a performance close the amplifier noise limit discussed above. With a measured noise of $120\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$, the actual noise floor is closer to $100\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$, which is well below the conservatively specified $<200\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ [62]. The driver is succeptible to mains hum though, most likely coupled magnetically. The power supply that came with driver, a Vescent `D2-005`, had a fairly short lead and used an E-core transformer inside, resoponsible for some magnetic leakage. Apart from the $50\,\mathrm{Hz}$ noise, the driver is capable of meeting the required $30\,\mathrm{nA_{rms}}$ noise figure (see table 3.2). At frequencies below $10\,\mathrm{Hz}$ no measurement was done, because the the compliance voltage of the driver is limited to around $2.5\,\mathrm{V}$ and going above that, severe noise peaking was seen. Nonetheless, the noise corner frequency of the reference filter can be surmised to around $10\,\mathrm{Hz}$ below which the noise should rise in a similar fashion as the noise of the `DgDrive-500-LN` at $20\,\mathrm{dB/decade}$. As a final note, all noise tests were conducted with the display board disconnected, as there was strong radiated noise coming from this board. The issue was reported to Vescent and reported fixed as of 2018-08 [195], but no retesting was done.

The Sisypth `SMC11` claims even lower noise and the noise floor reached $12\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ at $15\,\mathrm{kHz}$, but was not flat at all. The low noise figure seems to come from some very aggressive filtering of the current source using a very large inductor found on the PCB. This in turn likely results in the gain peaking due to the cable capacitance at higher frequencies. It must also be noted that this driver was tested with an increased shunt voltage of $5\,\mathrm{V}$ with a $1\,\mathrm{k\Omega}$ shunt as discussed above. A issue with the power supply, that came with the unit can also be seen, as there is a significant amount of sub-harmonics of the $50\,\mathrm{Hz}$ mains present, which point towards inductive saturration. Another problem can be seen below a few Hz. The increase of noise is well above the $20\,\mathrm{dB/decade}$ expected from a reference filter. It is closer to $30\,\mathrm{dB/decade}$ pointing towards a mix of random walk and drift as discussed in section 3.6. This will be picked up in the next when discussing the stability of the driver.

The last device to be discussed is the `DgDrive-500-LN` (serial number: #14), which is the low-noise $500\,\mathrm{mA}$ version. It does have a flat noise floor of $65\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ up to around $100\,\mathrm{kHz}$ where it appears to drop off. This is due to the reduced output impedance getting closer to the shunt resistor value of $1\,\mathrm{k\Omega}$ as can be seen in figure 4.31 on page 139. Towards the lower frequencies the noise rises with the expected $20\,\mathrm{dB/decade}$ due to the reference filter. There is no noise coming from the mains as can be seen at any frequency. Electrical coupling from mains is suppressed by the power supply filter and magnetic coupling into the cable is effeciently suppressed using a twisted pair (DVI) cable. The very high output impedance also effeciently eliminates other radiated noise entering the cable. The performance of the the input filter can also be seen in figure 4.36, which shows the `DgDrive-500-LN` in comparison to another `DgDrive-500-LN`, the latter being powered by a R&S `HMP4040` switch-mode supply, which does have substantial noise at the switching frequency of 130 kHz and its harmonics harmonics [26], which was confirmed by the author in a separate measurement.

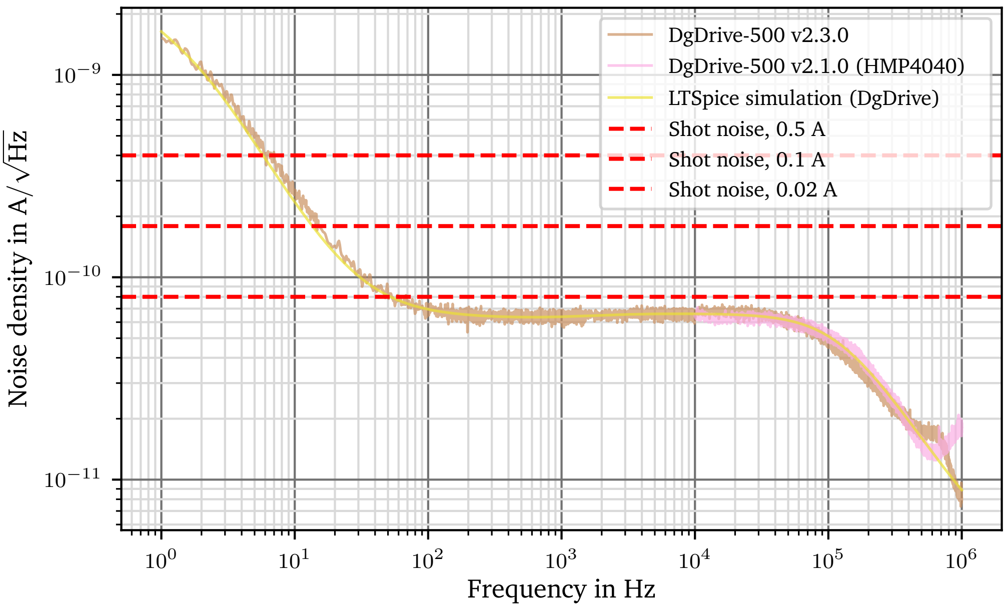


Figure 4.36.: Current noise of two `DgDrive-500-LN`. One is powered by a R&S `HMP4040` switch-mode supply, the other by a linear supply. There is no noise measureable at the switching frequency of 130 kHz.

The noise floor of the two drivers is identical and only the high freuqency behaviour is slightly different, which is the gain peaking of the amplifier and can also be seen in the background measurement shown in figure 4.35. The simulation results which contain an accurate noise model of the Zener diode (and the DAC, but this is negligible), shows very good agreement with the measurements. The noise model was built by the author from noise measurements of the `LM399`. At low currents, most of the noise comes from the reference filter and the input current noise of the `AD797`. At currents above 10 % of the maximum output, the `LM399` starts contributing the majority of the noise below 1 Hz, so using a lower noise refrence in future revisions would improve the noise performance.

Figure 4.36 also includes the shot noise limit

$$S_{shot} = \sqrt{2eI} \tag{4.13}$$

for different laser diode injection currents. The shot noise limit translates into an amplituide noise limit at the quantum level for, e.g. optically pumped lasers. The drive current noise of a current source as described in this work can be well below the shot noise limit though. Given a high impedance current source such as the `DgDrive`, the laser light can be amplitude squeezed below the shot noise limit as demonstrated by Machida et al. [136]. This was not measured in this work, but given the performance of the DgDrive amplitude squeezing can be assumed and it can also be seen in figure 4.36, than even the $500\,\mathrm{mA}$ driver already surpasses the shot noise limit at $20\,\mathrm{mA}$ of drive current, well below below typical laser diode operating currents. Ampltiude squeezed light can considerably increase the detection sensitivity beyond the standard quantum limit in applications where amplitude changes are observed, like michelson interometers [146] or small amplitude modulations [234]. The loss is phase/frequency noise due to the squeezing is negligible, because an ECDL is typically not quantum limited in its linewidth.

A final aspect that must be mentioned is the behaviour of the current sources when increasing the load voltage. As discussed in section 3.8.5, the MOSFET must be biased into saturation for optimal performance. Failing to do so, will create strong noise peaking due to the reduction in loop-gain of the precision current source. Since the design of the digital current driver is different and less pronounced, the Vescent D2-105-500 is shown as an example. For this measurement, the $10\,\Omega$ shunt resistor was used and the current was gradually increased, thus increasing the load voltage and decreasing the drain-source voltage drain-to-source voltage V_{DS} of the MOSFET. This behaviour effectively defines a maximum usable compliance voltage, were the device is still within its noise specifications. This is unfortunately only a soft limit, as the driver will still function beyond this limit and give no indication of the reduced performance. There is also no possibility to sense the load voltage on the driver side in any of the commercial drivers tested. The DgDrive senses the load voltage both digitally and has an integrated analog output to check the voltage.

In addition to a noise peak appearing at higher frequencies, more from the voltage reference is introduced at low frequencies, because the voltage reference divider ratio is increased to increase the setpoint. Figure 4.37 shows several values of the drain-to-source voltage between

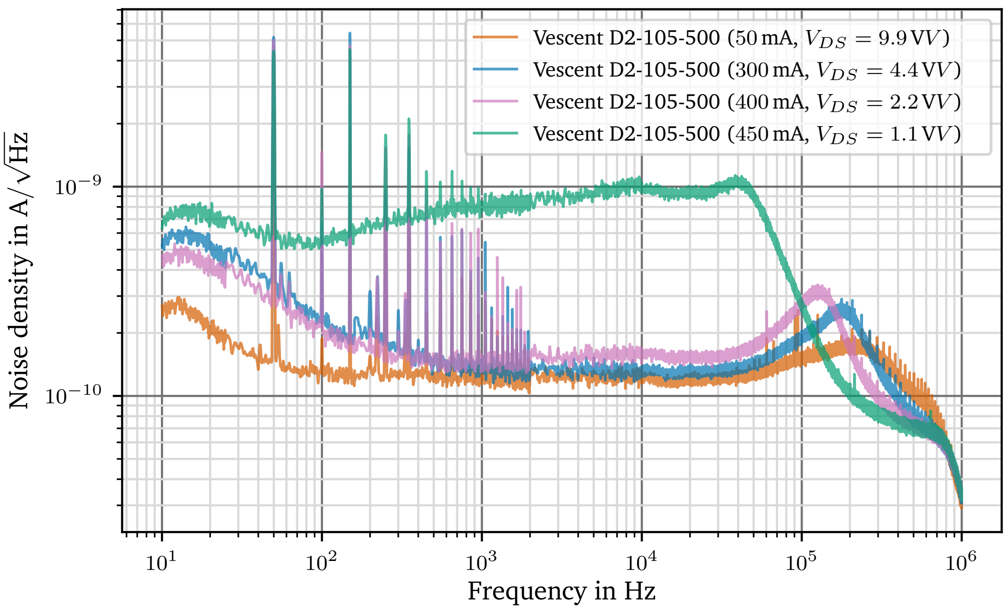


Figure 4.37.: Increasing current noise as the load voltage is increased up to a total failure of regulation at a load of $4.5\,\mathrm{V}$. Measured using a $10\,\Omega$ shunt resistor.

$9.9\,\mathrm{V}$ and $1.1\,\mathrm{V}$. Starting a load voltage of $3\,\mathrm{V}$ and $300\,\mathrm{mA}$, resulting in $V_{DS} = 4.4\,\mathrm{V}$, a noise peak starts building up degrading the performance of the current source. Taking $V_{DS} = 2.2\,\mathrm{V}$

as the minimum V_{DS} acceptable and a supply voltage of $11\,\mathrm{V}$ one can see that at full output dropping $7\,\mathrm{V}$ across the internal sense resistor of the current source, the maximum compliance voltage is limited to $1.8\,\mathrm{V}$, way too low for typical laser diodes. It must therefore be assumed that under full load, the `D2-105-500` will inject significant noise into the laser or even drop out out regulation as at $V_{DS} = 1.1\,\mathrm{V}$ the current source will effectively cease to function correctly. This must always be kept in mind when using these commercial current drivers and as discussed by [185], the only way around this problem is staying well below the maximum output current, which effectively increases the current noise of the laser driver as one has to use a larger, noisier driver than necessary.

To summarize the results of this section, it was shown that the unique architecture of the `DgDrive` combined with the ultra low noise performance ensures a predictable low noise performance of the current driver even under full load and high compliance voltage requirements. Other commercial current drivers tested had a noise floor magntitudes higher, stability problems and severe compliance voltage issues at the higher end output. Additionally due to the excellent filtering and very high output impedance, noise in the current source output is effectively suppressed and was not measurable and well below the noise floor of the device. This was demonstrated for both $50\,\mathrm{Hz}$ mains hum and noise from a switch-mode power supply used to power the driver. Finally, the driver fully complies to the requirements set out in table 3.2, which none of the commercial alternatives is able to meet in terms of noise and at high load voltages. The diode driver noise is so low, that even the `DgDrive-500` is below the shot-noise limit at currents above $20\,\mathrm{mA}$ and combined with the high output impedance demonstrated in section 4.1.9 allows for sub-shot-noise amplitude squeezing of the laser output.

4.1.11. Test Results: Temperature Stability

As it was shown in section 3.2.1, the ambient temperature can vary by as much as $2\,\mathrm{K}$ in a so-called temperature stabilized environment. The laser driver must therefore be tested in a representative temperature range. The racks typically have a temperature of around $26\,°\mathrm{C}$ when actively cooled. The laser driver was tested in a custom thermal chamber and the ambient temperature was swept several time from 20 to $30\,°\mathrm{C}$. At the same time, the voltage over a $10\,\Omega$ or $100\,\Omega$ Vishay `VPR221Z` shunt was measured using a Keysight `3458A`, depending on the output current of $50\,\mathrm{mA}$ and $510\,\mathrm{mA}$. The shunt was mounted in an aluminium housing and its temperature was monitored using a Keithley `DMM6500` and a PT100 glued to the shunt.

Repeating the test for a low current output and additionally for the maximum output allows to discriminate between the temperature coefficient (tempco) resulting from offset drifts and voltage reference drifts. At low outputs the effect of the reference voltage is scaled down. In case of the `DgDrive-500-LN` at $50\,\mathrm{mA}$ by factor of $50/510$ making it almost negligible. In this case op-amp offsets and other offset drifts from effects like thermal EMF dominate the temperature coefficient, but effects that are proportional to the ouput are suppressed. At full output the voltage reference drift is added unscaled and also the drift of the sense resistor can then be seen.

Another effect that can only be seen at full output is burst noise coming from the Zener diode used as a reference. This manifest a sudden jumps in the other continuous plot. The `DgDrive-500-LN` #14 tested is one of the earlier versions and contains a faulty reference, which was discovered during those tests. This issue is discussed in more detail in the next section.

The results shown in figures 4.38 and 4.38 are typical results. As it was discussed in section 4.1.5, great care taken to reduce the offset related drift, which can be seen in figure 4.38. The efforts were rewarded with a very low temperature coefficient of $97\,\mathrm{nA/K} \equiv 0.19\,\mathrm{\mu A/(A\,K)}$ when running at $10\,\%$ output current. At a maximum current of $510\,\mathrm{mA}$ the drift of the sense resistor and the Zener diode add to the result and the temperature coefficient increases to $348\,\mathrm{nA/K} \equiv 0.68\,\mathrm{\mu A/(A\,K)}$. These results are typical for this batch. The reason is that all resistors are from the same batch, order directly from Vishay. In addition, the summing and averaging of the two current sources reduces the spread even further.

Other laser driver were tested in a similar fashion when it was possible to fit them into the thermal chamber. Not all drivers were tested, because as can be seen on the timescale of figure 4.38, these tests took a very long time to suppress hysteresis effects. Larger devices like the Moglabs `DLC-102` or the Vescent `D2-105` were placed in a cooling cabinet and once the current had stabilised, the door was opened to apply a step change of about $10\,\mathrm{K}$. The results are summarised in table 4.12

Device	Tempco at $50\,\mathrm{mA}$	Tempco at full current
`DgDrive-500-LN`	$0.19\,\mathrm{\mu A/(A\,K)}$	$0.68\,\mathrm{\mu A/(A\,K)}$
`D2-105`	$1.3\,\mathrm{\mu A/(A\,K)}$	–
`DLC-102`	$3.5\,\mathrm{\mu A/(A\,K)}$	–
`LQprO`	$8.6\,\mathrm{\mu A/(A\,K)}$	–

Table 4.12.: Tempco of several laser drivers tested. Scaled to full scale output.

The `SMC11` was tested, but found to not stable enough over time to make out a temperature coefficient from the drift already present.

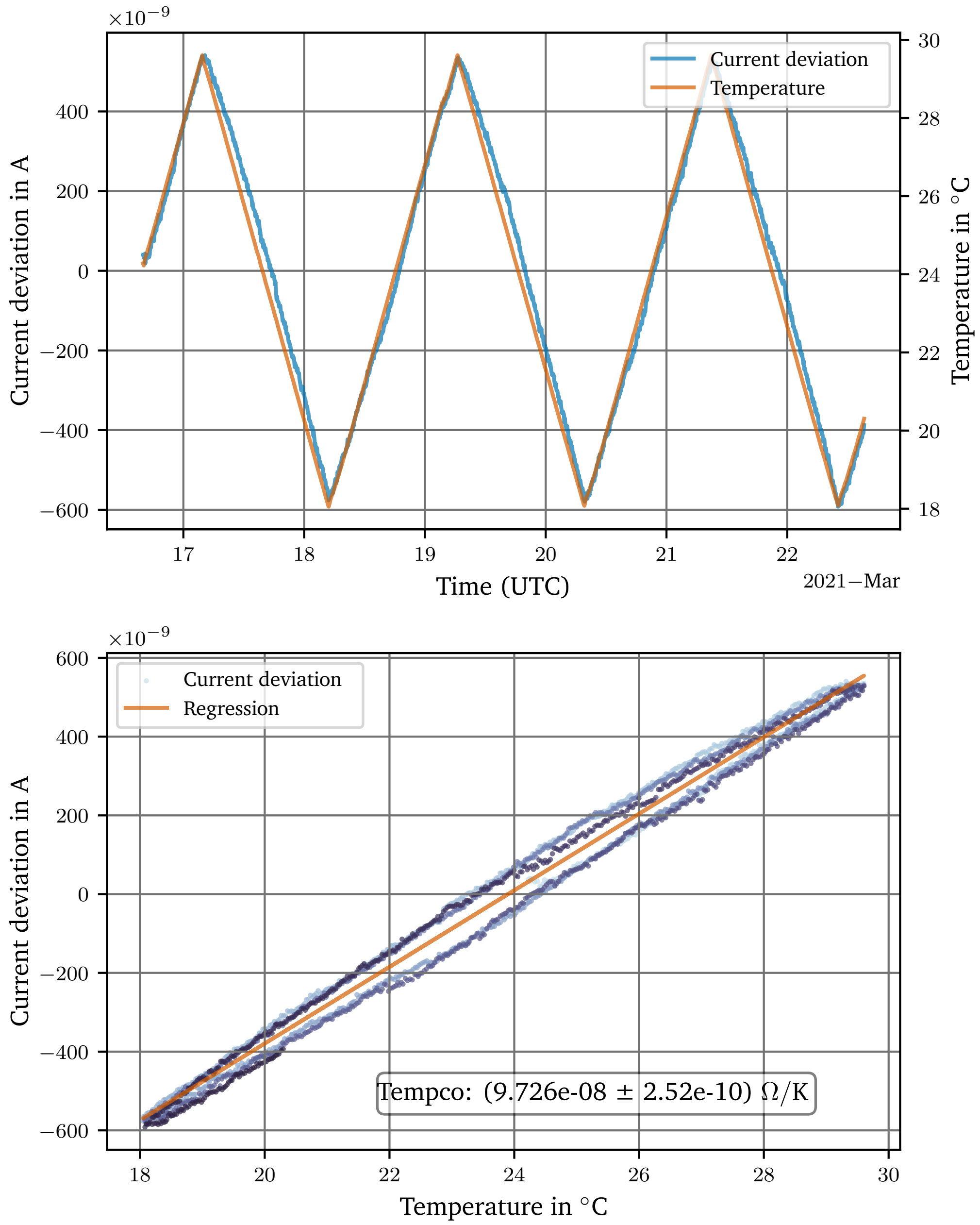


Figure 4.38.: Output current drift of a `DgDrive-500-LN` running at $50\,\mathrm{mA}$ over an ambient temperature range of 20 to $30\,°\mathrm{C}$.

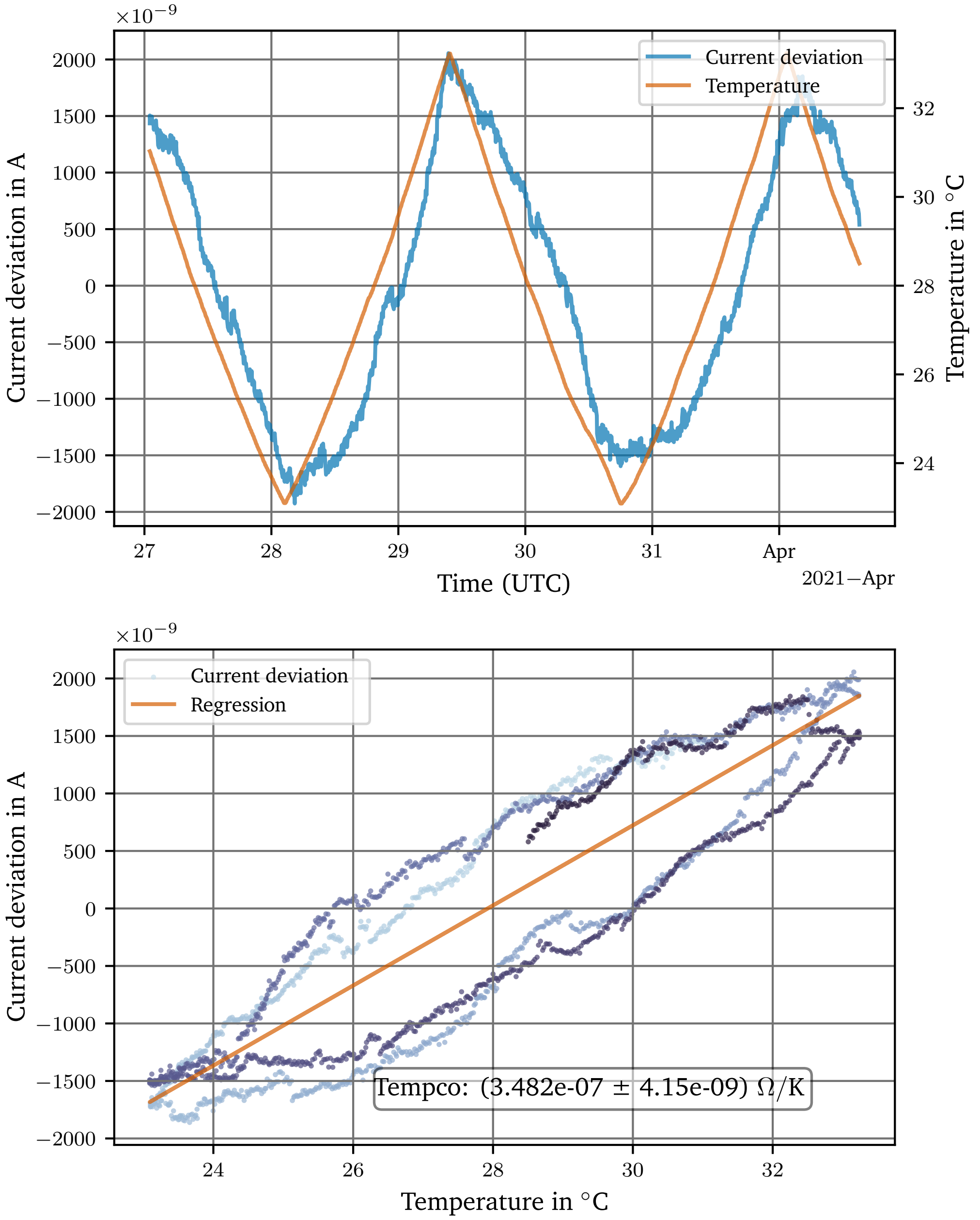


Figure 4.39.: Output current drift of a `DgDrive-500-LN` running at $510\,\mathrm{mA}$ over an ambient temperature range of 20 to $30\,°\mathrm{C}$.

To conclude, the temperature coefficient of the DgDrive-500-LN was tested to be better than

$$0.1\,\mu\mathrm{A/K} + 0.5\,\mu\mathrm{A/(A\,K)} \cdot I_{out}\,. \tag{4.14}$$

This is well below the required target of $\leq 1\,\mu\mathrm{A/(A\,K)}$ set in table 3.1. Since the tests typically take a week or more, only the most promising commercial drivers were tested. Unfortunately, no other driver was able to meet the target, although the Vescent `D2-105` came close with $1.3\,\mu\mathrm{A/(A\,K)}$. The SISYPH `SMC11` was not even stable enough to be tested at all. This topic will be discussed in the next section.

4.1.12. Stability

When remotely controlling a laser system, stability of the laser driver is of immediate concern, because uninterrupted operation of the system is a key requirement and if the laser cannot be locked again remotely, it is time consuming, if possible at all, to go to the remote laser lab and readjust the laser current driver. The development of the past years have also shown a greater demand of remote working rendering readjustment unfeasable.

4.1.13. Zener Diode Selection

As hinted in section 4.1.11, early temperature stability tests with the LM399 Zener diode as a reference confirmed, what the data sheet [125] already suggest in the 'Low Frequency Noise Voltage' plot. There are random bi-stable voltage step changes. This phenomenon is called burst noise or popcorn noise.

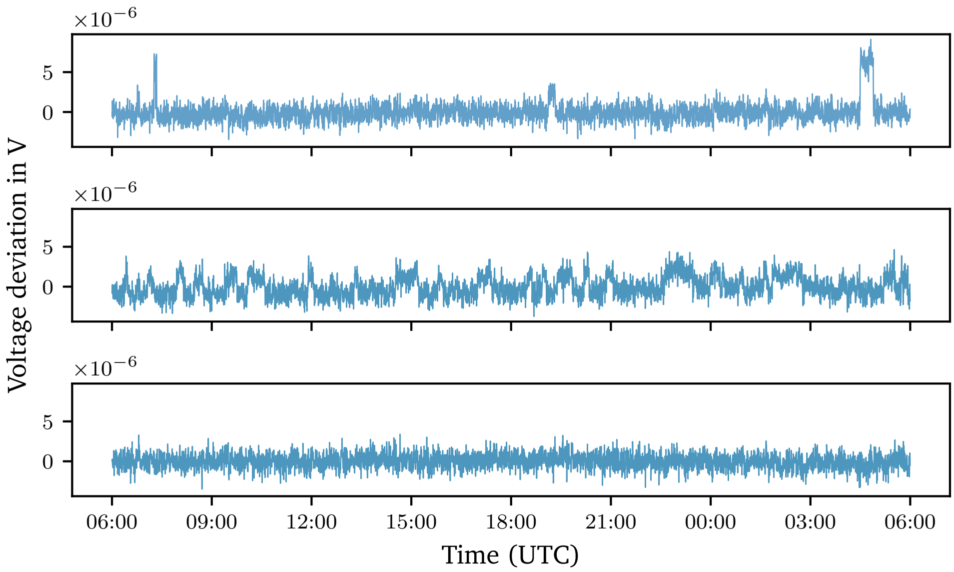


Figure 4.40.: Popcorn noise in different samples of the LM399 over a $24\,\mathrm{h}$ period as recorded by the scanner system discussed in this section.

Figure 4.40 shows two samples of the LM399, that exhibit popcorn noise, while the last one does not.

The sources of popcorn noise in semiconductor devices are not yet fully understood, but some sources have been identified. Defects in the semiconductor crystal lattice and contamination of the semiconductor material have been linked to popcorn noise [104]. This problem has improved over the years as manufacturing processes and wafer quality has evolved. Unfortunately the LM399 is built around a process from 1991, as can be seen etched into the die [103].

The popcorn noise caused by defects and contamination can be reduced by lowering the strain on the lattice and removing surface contaminants on the die. This can be can be achieved using a high-temperature burn-in process. Manufacturers like Fluke and Keysight use similar techniques in their products. Fluke, for example, uses a period of $60\,\mathrm{d}$ burn-in for their references [171].

Fortunately, the LM399 is a heated reference, which regulates its die to $90\,°\mathrm{C}$ when turned on, so it is only required put the diodes in a simple test circuit and wait. The use of a separate test setup instead of the final circuit has both advantages and disadvantages. The disadvantage is that the Zener diode will subjected to mechanical stress again, when soldered in the final PCB, this stress will, of course, not be removed by the burn-in process as it is applied after testing, but this mainly affects the voltage drift properties of the Zener diode and not the popcorn noise. The drift of the diode is only of secondary concern for the laser current driver, as the output drift is mainly caused by the reference resistors used and, which are typically at least an order of magnitude worse than the the drift of the diode judging by the data sheet [125, 223].

The advantages of testing the Zener diodes separately, on the other hand, are that more diodes can be tested at the same time, as a special compact test fixture can be used. It is also simpler to remove the diodes from the test fixture, because they be socketed. Therefore for this application a separate test board was used. Building this test setup is detailed in the next sections.

4.1.14. Building a Test Setup for Zener Diodes

There are several ways to measure the popcorn noise of semiconductor devices. The most trivial one is to directly monitor the device in the time-domain. In this case, the Zener voltage can be monitored with a long-scale multimeter. This requires a low noise DMM, that can reliably distinguish between both voltage levels, which are about $4\,\mu\mathrm{V}$ apart. A related option is to use a second reference, whose voltage is similar to the device under test (DUT). Measuring the the voltage difference between the two references, less resolution is required. Directly comparing the difference of two references using a millivolt meter is commonly done when intercomparing primary voltage references. This method, however, increases the measurement noise by a factor of $\sqrt{2}$, if both references produce the same level of uncorrelated noise. The noise of the LM399 with a $100\,\mathrm{PLC}$ integration time ($2\,\mathrm{s}$) is about $1.5\,\mu\mathrm{V}_{\mathrm{pp}}$ as can be determined from the data in figure 4.40.

Measuring two references against each other would then result in around $2.1\,\mu\mathrm{V}$ of noise. This makes distinguishing the jumps possible, but challenging.

A third option is to use a high-pass filter and an amplifier. Additionally, the signal can be low-pass filtered to remove any excess high frequency noise. This approach also requires less resolution than directly measuring the voltage, because the signal-to-noise-ratio is improved du the amplifier. It is therefore possible to use an off-the-shelf analog-to-digital converter (ADC). One such circuit, along with some examples, is demonstrated in [104, 105]. It must be noted, that due to the high-pass filtering, it not possible to measure slow voltage drifts using this method.

The forth and final option presented here, is approaching the problem in the frequency domain and requires a low-noise amplifier with a low frequency cutoff. As it was already discussed in section 3.6.1, popcorn noise is found to have a frequency dependence of $1/f^2$. This can be used to distinguish it from other random noise processes that show a frequency dependence of $1/f$. A good example of an op-amp, that has excessive burst in comparison to a good sample is given in *The Art of Electronics* [94, p. 478]. Going to frequencies below $10\,\mathrm{Hz}$, one can sort the references by their noise spectrum.

In this work only options one and two were were tested and as it was said above, with options three and four there is a chicken and egg problem. One needs a number on known good devices to compare other DUTs to. At the start of the evaluation, most of the data available about the

LM399 was from the data sheet. Compiling a dataset of the performance of dozens of LM399 is expensive and time consuming and companies typically treat such data as a closely guarded secret.

The next section deals with the choice of multimeter to satisfy the requirements test the Zener diodes according to options one and two, so either directly measuring the output voltage or difference of a known good sample against the DUT.

4.1.15. Choosing a Multimeter for Testing Zener Diodes

The DMM used plays an important role for the test setup. In this section, some of the challenges, that can be encountered will be discussed. The expected amplitude of the popcorn noise is around $0.5\,\mu V/V$ or $3.5\,\mu V$ of the output voltage, when considering the $7\,V$ Zener voltage of the LM399 diode.

The $7\,V$ will typically be measured on a $10\,V$ range. This is not a trivial task, because a signal-to-noise-ratio of $0.35\,\mu V/V$ or more than $130\,dB$ is required. This calls for a device, that not only has the required resolution, but also the stability over time and temperature to ensure the measurement will not be distorted by the DMM.

Therefore, a voltmeter with lower noise and a more stable reference, than the DUT is mandatory. This only leaves the class of very low noise 7.5 or 8.5 digit multimeters. These multimeters feature a different type of voltage reference, because the LM399 is not suitable due to its noise. The only Zener diodes that meet those requirements are the ADI LTZ1000 [134], the Motorola SZA263 (out of production) and the Linear Technology (LT) LTFLU-1, a proprietary design by Fluke and LT. The LTZ1000, for example, is specified for a typical noise of $1.2\,\mu V_{pp}$ in a frequency range of $0.1\,Hz$–$10\,Hz$ [134]. Additionally, in comparison to the LM399, those Zener diodes do not suffer from the popcorn noise issue to this extend.

Comparing only 7.5 and 8.5 digit voltmeters, narrows down the choice of multimeters considerably. The market for high-end 8.5 digit DMMs is limited and therefore every device on the market caters for a certain niche. It is therefore prudent to look at their specifications to choose the correct device for this purpose. In table 4.13 a list of popular 8.5 DMMs can be found. Several models included in the table, are already discontinued, but these DMMs can still be acquired on the second-hand market.

Manufacturer	Model	Remarks
Advantest	`R6581`	Discontinued. Scanner cards available.
Datron/Wavetek	`1812`	Discontinued. Wavetek was bought by Fluke.
Fluke	`8508A`	Discontinued. $20\,V$ range.
Fluke	`8588A`	In production.
Keithley/Tektronix	`2002`	In production. Scanner card available. $20\,V$ range.
Keysight	`3458A`	In Production.
Solartron	`7081`	Discontinued. Slow.
Transmille	`8104`	In Production. External scanner available. Slow.

Table 4.13.: Overview of 8.5 digit multimeters.

While the author has not tested every multimeter in table 4.13, it is possible to judge some of them apriori by their specifications. The `Solartron 7081` (also sold as `Guildline 9578`) is a less optimal choice, because a conversion takes $52\,s$ for 8.5 digits. The discontinued `Fluke 8508A` and the `Wavetek 1812` multimeter are very similar devices, because Fluke bought

Wavetek in 2000 and as a result, the Fluke 8508A is more of an update to the Wavetek 1812 than a new device.Again they are both fairly slow, taking $25\,\mathrm{s}$ for a conversion at 8.5 digits. The other multimeters are still in production and similar in price, but their field of use is slightly is different. The Fluke 8588A excels at stability and features a modern user interface, whereas the Keysight 3458A is unbeaten in linearity and noise. A detailed comparison of those two meters can be found in the work of Lapuh, Kucera, Kovac, and Voljc [115]. The Keithley Model 2002 focuses on its scanning capability and the Transmille 8104 does have electrometer functions. Unfortunately, the 8104 is also fairly slow at 8.5 digit with conversions taking $4\,\mathrm{s}$ at its fastest setting [5], so it will not be considered.

To narrow it down even further, several 7.5 and 8.5 digit multimeters were tested. The results of those tests will be discussed here to give an impression of the performance of these devices. The tested multimeters are the Keysight 3458A, the Keithley Model 2002, the Keysight 34470A and a Keithley DMM6500. The 3458A was chosen, because of its low noise. The Model 2002 was chosen for its internal scanning unit. The 34470A was chosen as a lower-end and cheaper alternative and because it is a fairly low noise device. Finally the DMM6500 is on the list to compare a DMM with an LM399 reference.

A test was conducted with this selection of devices. A Fluke 5440B calibrator supplied $10\,\mathrm{V}$ to all mulimeters and readings were taken over the course of a week. This data was used to estimate the noise and the stability of the multimeters, including burst noise. The noise of the DMM at $10\,\mathrm{V}$ is typically not found in the datasheet, because the noise performance is usually quoted for shorted inputs, which does not include the internal reference noise. This test also allows to check for popcorn noise of the internal reference. The calibrator has a specified output noise of $<1.5\,\mu\mathrm{V}$ within a bandwidth of $0.1\,\mathrm{Hz}$–$10\,\mathrm{Hz}$ at $1\,\mathrm{V}$ and is stable to within $5\,\mu\mathrm{V}_{\mathrm{rms}}$ over $30\,\mathrm{d}$, a specification far superior to the LM399. Based on this tess, a multimeter was chosen for an automated test setup to bin the LM399s.

The test was conducted in a stable and monitored lab environment, with a temperature deviation of at most $\Delta T = \pm 0.2\,\mathrm{K}$. All multimeters were connected to the same DUT. Although this might potentially cause interference between the multimeters due to the pump out current spikes caused by the switching interals, no ill effects, like voltage offsets or increased noise, were observed during the setup of the tests. A more detailed discussion of the pump out current of the 3458A can be found in [173].

The two 8.5, one 7.5 and one 6.5 digit multimeters were connected using shielded cables, either Pomona 1167-60 or self-made cables. The GUARD terminal of the calibrator was connected to chassis GROUND at the calibrator and then connected to the cable shield. On the 3458A, the shield was connected to the GUARD terminal and the GUARD switch was set to open according to the manual [107]. For the other multimeters, that do not have a GUARD terminal, the shield was left floating at the DMM side. Additionally the Fluke 5440B, the HP 3458A and the Keysight 34470A have an autocalibration routine, which was run once prior to the measurement. The detailed settings used for the DMMs can be found in the appendix A.1 on page 205, a summary ist given in table 4.14 to show the important differences.

All DMMs were configured to have a similar conversion time. This lead to different integration times, which are given in power line cycles at $50\,\mathrm{Hz}$. The Model 2002 takes considerable longer for a measurement than the Keysight multimeters. The reason is the auto-zero function, which is shown in figure 3.27 on page 56. The Model 2002 does three steps when doing auto-zeroing, it measures the signal, the zero point for an offset compensation and also the reference voltage for a gain correction. In comparison, the 3458A only corrects for the offset drift. The gain is adjusted when using the ACAL function. The former auto-zero routine, therefore takes longer by one half, but results in more stable measurements. The results of the comparison are shown

DMM	Integration time in NPLC
`HP 3458A`	100
`Keithley Model 2002`	40
`Keysight 34470A`	100
`Keithley DMM6500`	90

Table 4.14.: Concise list of differences in the settings used for comparing the DMMs. The total integration time of each DMM is about $2\,\mathrm{s}$.

in figure 4.41.

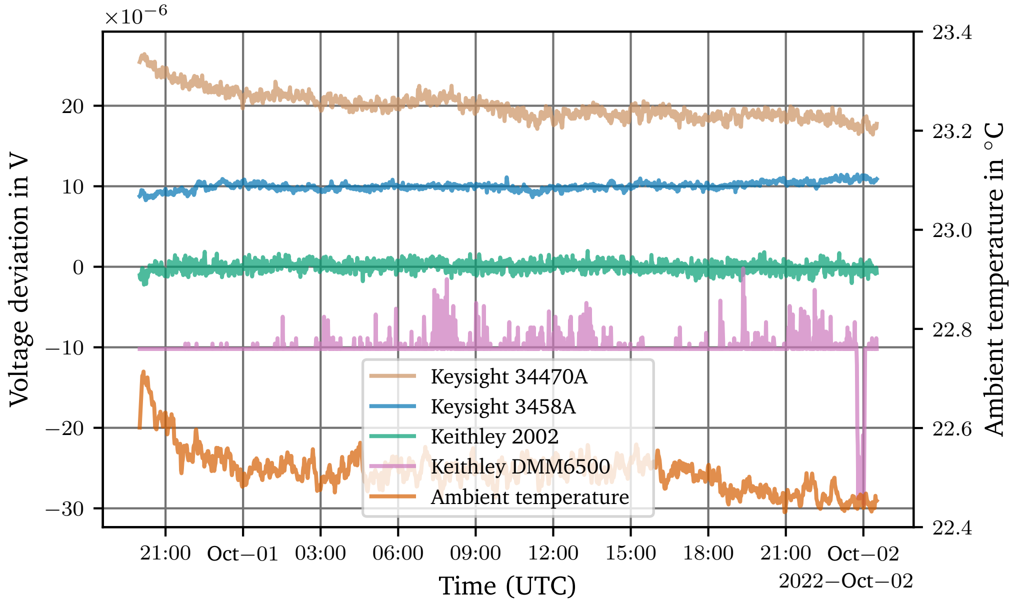


Figure 4.41.: Comparison of several DMMs measuring the $10\,\mathrm{V}$ output of a Fluke `5400B` for $24\,\mathrm{h}$. The traces are spaced $10\,\mu\mathrm{V}$ apart.

From these measurements it is easily discernable that the `DMM6500` 6.5 digit multimeter is not suitable, because it suffers from popcorn noise itself. The `34470A` fared better, but had a larger temperature coefficient and about the same noise as the Keithley `Model 2002`. The two 8.5 digit multimeters performed best and the choice was limited to either model.

In order to test a large amount of Zener diodes, and considering the duration of the burn-in process, which can take anything between $100\,\mathrm{h}$–$1000\,\mathrm{h}$, it is necessary to have an automated setup that can multiple Zener diodes at once. Such a setup consists of a digital multimeter (DMM), a scanner and test board, that holds the Zener diodes and provides the necessary infrastructure for the diodes. The latter two will be discussed in the next section.

4.1.16. A Scanner System for Testing Zener Diodes

As discussed before the diodes need to be tested for up to $1000\,\mathrm{h}$ and it is not be feasible to test them individually. So a minimum of at least 10 diodes should be tested at the same time.

Several commercial options currently available were considered for this project and are shown in table 4.15.

	Keysight		Keithley			Fluke	Rigol
	DAQ973A	34980A	DAQ6510	2750	3706	2680	M300
DMM	6.5	6.5	6.5	6.5	7.5	18 bit	6.5
Channels	3x20	8x40	2x10	5x20	6x60	6x20	5x32
FET	✓	✓	✓	✓	✓	✗	✗
Voltage	120 V	80 V	60 V	60 V	200 V	75 V	300 V
Card	DAQM900A	34925A	7710	7710	3724	2680A-PAI	MC3132
USB	✓	✓	✓	✗	✓	✗	✓
Ethernet	✓	✓	✓	✗	✓	✓	✓
GPIB	✓	✓	✓	✓	✓	✗	✓

Table 4.15.: Overview of scanner mainframes

A recent trend to more compact devices has led major manufacturers to include multimeters in the scanner mainframe creating so called data acquisition units. Legacy devices, that only have switching capabilities are no longer available. For example Keithley replaced the small desktop switch mainframe `Model 7001` with the `DAQ6510` and Keysight is offering the `DAQ973A`, a scanning 6.5 digit DMM, that accepts extension cards. Unfortunately, for this project, as discussed above, an integrated 6.5 digit multimeter does not add any value.

The simplest option is to go with an 8.5 digit multimeter that already includes a scanner option or buy a used `Keithley 7001` from a second-hand dealer to use with the `3458A`. The author tested both options and the simplicity and space saving of only having a single device to connect and program makes the integrated scanner card of the `Model 2002` very attractive.

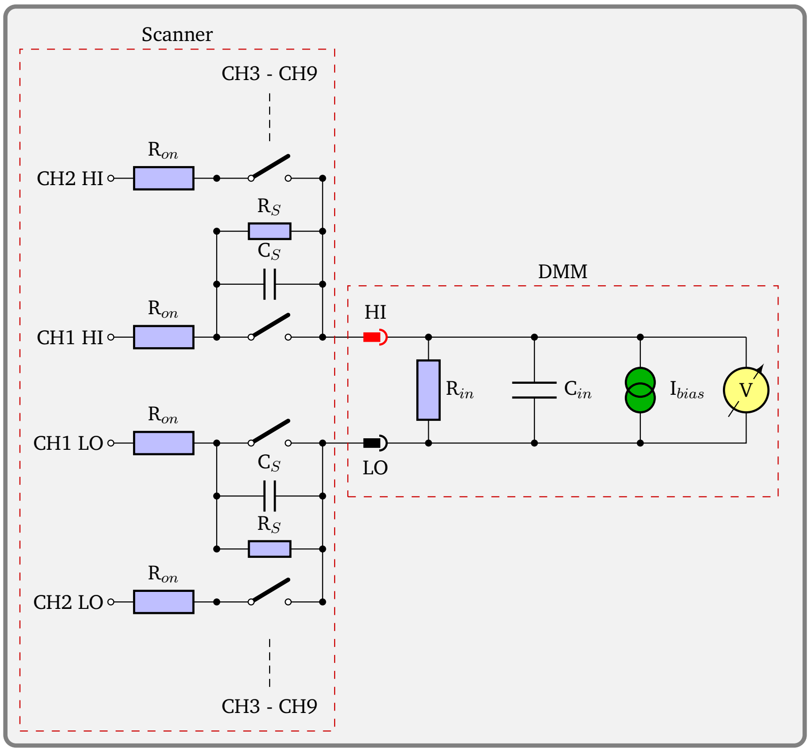


Figure 4.42.: Simplified schematic of the scanner front-end with parasitic elements

The scanner card used to multiplex the DMM does have to meet several specifications. The most important aspects are the number of channels and the lifetime of the relays. Other factors, such as channel to channel isolation, the contact potential, resistance and maximum voltage are not the limiting factors for this setup, because the voltage is low, there is no ac component involved and the the typical input impedance of high-end multimeters is far more than $100\,\mathrm{G}\Omega$ [6, 119, 154, 173].

In this work the Keithley (now Tektronix) `Model 2002` was chosen for three reasons. It is a very compact system requiring only a half-sized 2U rack in comparison to the other DMMs, that are typically full-sized 2U rack devices. The other two advantages are the integrated scanner card slot that allows to to fit a 10 channel scanner card and finally the $20\,\mathrm{V}$ range. The latter is interesting for testing the final voltage reference boards, as these have a $15\,\mathrm{V}$ output.

The standard scanner card that comes with the `Model 2002` is the `Model 2000-SCAN` card. These cards have a number of 2-pole relays mounted to them, which have an expected lifetime of 10^5 to 10^8 actuations. Considering 3 actuations per minute and a measurement inverval $1000\,\mathrm{h}$ per batch and 10 batches for a total of 100 Zener diodes. Roughly 10^6 actuations are required, which could lead to premature failure of the test setup. To solve this issue, a solid state scanner card was designed. This card fits into the standard slot and uses the same connector shown in figure 4.43.

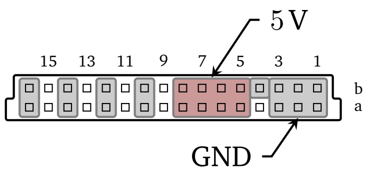


Figure 4.43.: The extension connector used in several Keithley multimeters

Pin	Function	Cable Colour	Pin	Function	Cable Colour
a1, b1	$+6\,\mathrm{V}$–$+20\,\mathrm{V}$	brown	6	GND	green/white
a2, b2	PD cathode	red	7	LD Cathode	blue/white
a3, b3	LD case (GND)	red/white	8	LD Anode	blue
a4	PD anode (GND)	red/white	9	LD current	green
a5	$-6\,\mathrm{V}$–$-20\,\mathrm{V}$	brown/white			

The scanner card is based on an `ICE40HX1K` FPGA and its design files are available opensoruce at [35]. This scanner card can either be equipped with 10 channels or 20 channels. The `Model 2002` only supports 10 channels, while newer Tektronix mulimeters like the `DMM6500` support up to 20 channels. The connector is still the same, yet the communication protocol has changed. Both protocols were implemented on the FPGA are are selectable via a solder bridge during production. This soldder bridge can also be changed later to convert a 20 channel card into a 10 channel card if the need arises. This is one of the reasons for using the FPGA, instead of a simple shift register like it was done for the `Model 2000-SCAN`. An attempt was also made using a microcontroller, but the legacy protocol used by the `Model 2002` does not bode well with an $8\,\mathrm{bit}$ aligned microcontroller. Although some mircrontrollers allow adjustable packet sizes, there is more than the scanner controller talking on the bus and both controllers have different packet sizes (and clock rates). This could not be implemented using the hardware controllers of the MCU. In addition, the high clock rate of the scanner controller of $2\,\mathrm{MHz}$

made directly sampling the signal in software impossible. A picture of the finished board is shown in figure 4.44.

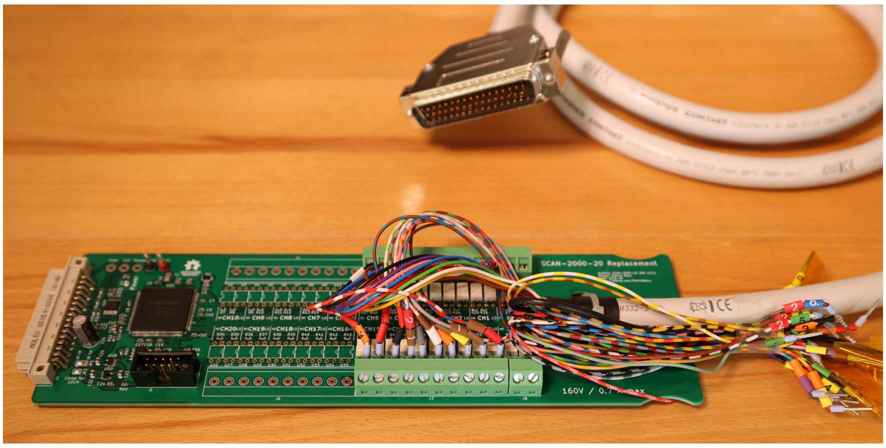


Figure 4.44.: A 10 channel solid state scanner module for the Keithley `Model 2002` including the connector for the reference PCB.

With a scanner and multimeter selected, the focus now lies on the test fixture for the Zener diode. This test setup consists of a mounting PCB, that holds up to 20 Zener diode. It provides power regulation and a minimal circuit required to support each diode. This circuit is given in figure 4.45.

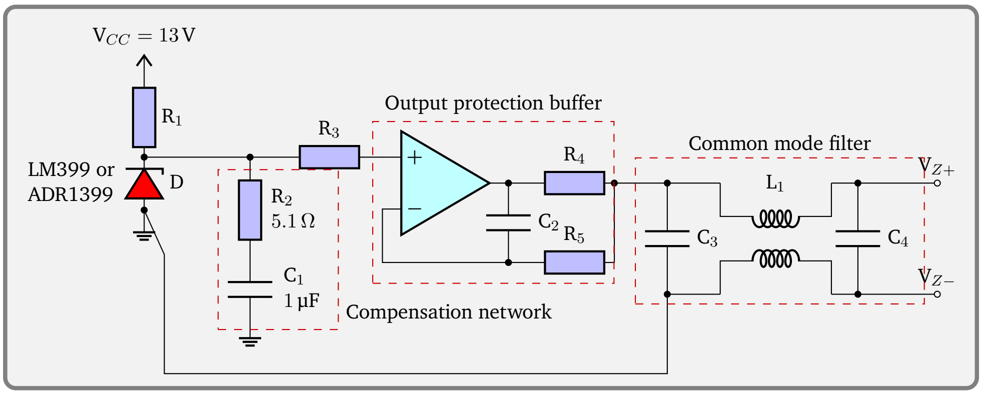


Figure 4.45.: Circuit used for burning in the Zener diodes.

The compensation network is required when using the ADR1399, because of its very low dynamic impedance as recommended in the data sheet [16]. It is not strictly required for the LM399, but fitted nonetheless, because there are no downsides to it. This makes the board compatible with both types of references. Each Zener output is protected using an output buffer, which provides isolation and short circuit protection. Finally there is a common mode filter at the output to suppress high frequency noise via ground loops.

Using this test setup, 102 `LM399` Zener diodes were tested up to this date (2023-04). Of those 39 were found suitable for the `DgDrive`. This low yield of only 40 % is a matter of concern and has significantly delayed the development and production of the drivers. Using the automated test setup, the latter issue is manageable, but still slows down production. Tests are currently conducted to replace the `LM399` with the newer `ADR1399` Zener diode, but unfortunately, it is still not available in quantities surpassing individual samples. Two of those samples were thoroughly tested and showed significantly better performance in terms of both white noise and burst noise. Although these results are promising more samples need to be tested to give a final verdict. Using those screened Zener diodes a batch of laser drivers was built and the system performance with regard to stabilty was tested. This shown in the next section.

4.1.17. Test Results: Stability

The stability measurements were conducted for $24\,\mathrm{h}$ and the output current was recorded using a Keysight `34470A` and a Vishay `VPR221Z` $10\,\Omega$ shunt resistor, which proved more stable than the internal shunt. All measurements were done at $50\,\mathrm{mA}$, because at that time the author was not aware of the the flicker noise issues of the `LM399` used in those drivers. The flicker noise of the commercial drivers can therefore not be judged from these measrements.

The only device found unsuitable for the use in high precision spectroscopy was the `SMC11` for showed large drifts, which were in part due to the trimmer resistor used to set output current. This was already seen in section 4.1.10 and the full effect can be seen in figure 4.46.

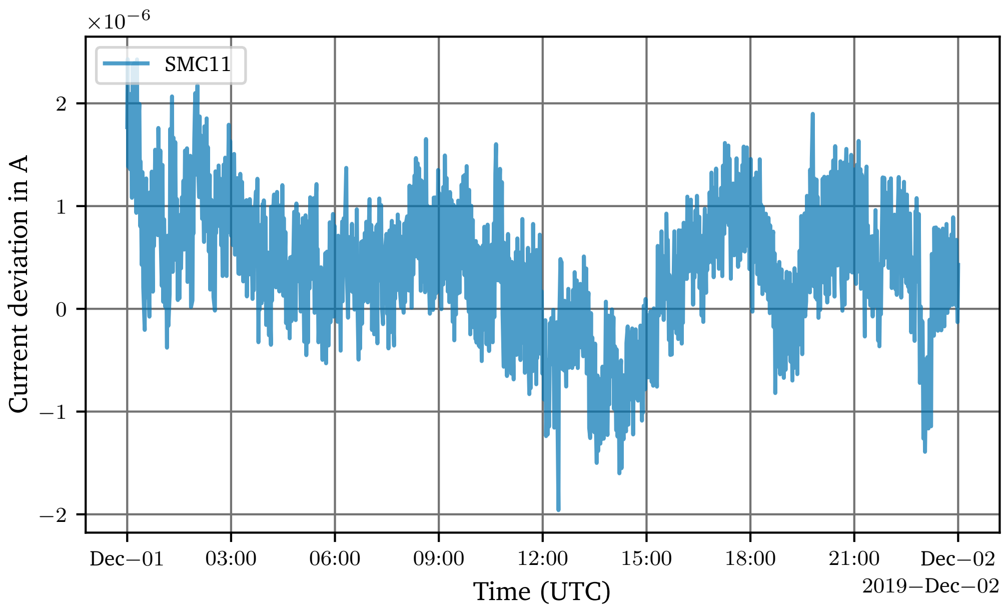


Figure 4.46.: Output current stability of the `SMC11` over the course of $24\,\mathrm{h}$.

The `SMC11` showed both short-term drift issues and also instability over the long term. This makes adjusting the driver difficult, especially if a lock to an atomic reference or a high finesse resonator is desired.

On the other hand the `DgDrive-500-LN` tested shows no discernable drift over the $24\,\mathrm{h}$ period as can be seen in figure 4.47.

The drift is entirely masked by the measurement noise of the `34470A` and the test should be repeated with future revisions to evaluate the actual drift. Given that more capable devices, including the Fluke `5440B` calibrator with a very low specified drift [2] and several characterised shunt resistors are available by now, far superior results can be expected. Nonetheless, an upper bound of less than $10\,\mathrm{nA/d}$ can be given. Evaluating the calibration of the laser drivers currently in use after one year showed a drift of $<(1 \pm 2)\,\mathrm{\mu A/A/a}$ against a freshly calibrated `3458A`. This surpasses the requiremets put up by the specification of $240\,\mathrm{\mu A/A/30d}$ given in table 3.1 on page 17 by a far margin.

The other drivers are shown here for brevity, but appart from the higher temperature coefficient, there no drift issues found. Long-term data is not available though, because the devices were on loan.

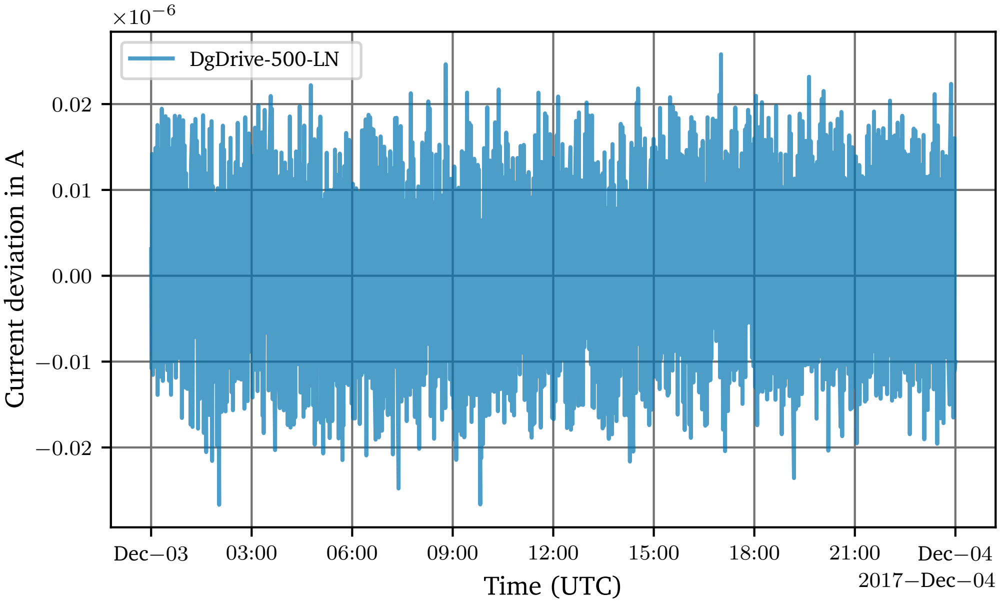


Figure 4.47.: Output current stability of the `DgDrive-500-LN` over the course of $24\,\mathrm{h}$.

To get a better idea on the long-term more data needs to be collected on those drivers currently in use. This is expected to happen at yearly calibration intervals, when the drivers will be compared to the calibrated `3458A`.

4.1.18. Summary

This section evaluated several commercial current driver currently representing the state of the art. These drivers were all found to be unsuitable to drive the Osram `PL 450B` [164] blue laser diodes used in the ARTEMIS experiment to generate the $441\,\mathrm{nm}$ probe laser. The most common problem identified was the limited compliance voltage, especially with low-noise drivers like the Vescent `D2-105`, which can barely drive modern high power laser diodes like the Thorlabs `L785H1` [110], which require more than $2.5\,\mathrm{V}$ at high currents. To achieve this a higher output current driver needs to be used, which increases the overall output noise. Additionally, hardly any drivers have a digital interface to allow remote control.

To address these problems, a new laser driver design is presented, that not only surpasses all commercial alternatives, both in performance and price, but also delivers the much needed digital interface to integration into modern experimental setups.

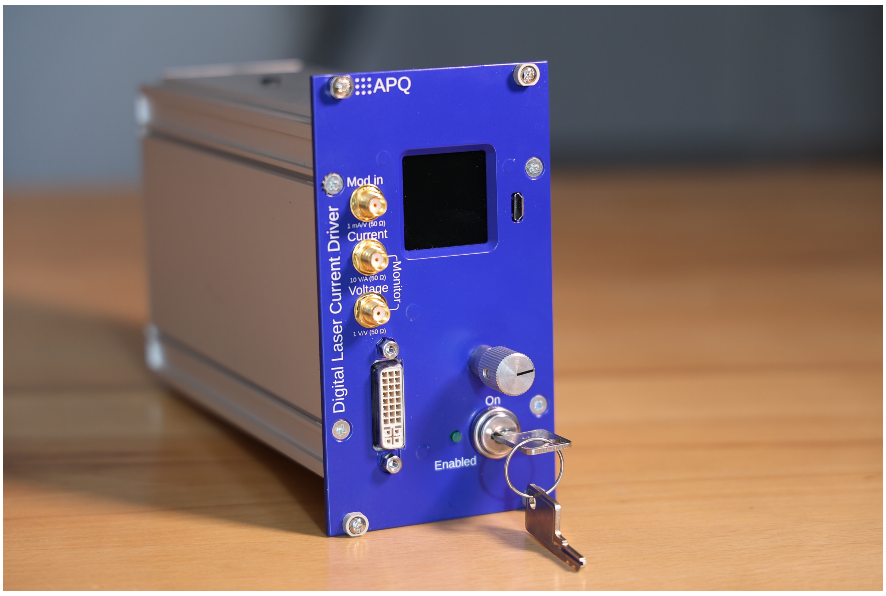


Figure 4.48.: Picture of the digital current driver `DgDrive`.

The driver was demonstrated to have the following performance:

- Maximum output current: $500\,\mathrm{mA}$ (p. 123)
- Temperature coefficient: $0.1\,\mu\mathrm{A/K} + 0.5\,\mu\mathrm{A/(A\,K)} \cdot I_{out}$ (p. 152)
- Stability: $<(1 \pm 2)\,\mu\mathrm{A/A/a}$ (p. 165)
- Noise: $<70\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ above $100\,\mathrm{Hz}$ (146)
- Below shot noise above $20\,\mathrm{mA}$ for the $500\,\mathrm{mA}$ version. (p. 148)
- Modulation bandwidth: $1\,\mathrm{MHz}$ (p. 128)
- Ouput impedance: $>1\,\mathrm{G\Omega}$ resulting in excellet noise immunity (p. 139)
- Digital control
- Python API for remote control

4.2. Labkraken

Labkraken allows real-time monitoring of experiments or environmental parameters. It was extensively used through this work to collect for example the measurement data produced by the Zener diode test setup described in section 4.1.16 to give real-time feedback on the diode voltages. Other applications, among many other, are the monitoring of the laser output power for experiments or logging the ambient temperature, humidity and air pressure.

As outlined in section 3.4, Labkraken was built using Python 3.7 or higher and the Asyncio framework running inside Docker containers. The full source code can be found at [41]. It is programmed using a functional programming style and based on datastreams to aggregate data from different sensors, which currently include

- Tinkerforge sensors
- GPIB devices via GPIB Ethernet adaptors
- LabNodes (see section 4.3)
- Ethernet capable SCPI devices

The collected data is sent an MQTT server, which decouples the data acquisition from the data storage backend. It also allows the aggregate the data of multiple Kraken instances running in parallel. A simple flow diagram is show in figure 4.49 to give an idea about the flow of the data.

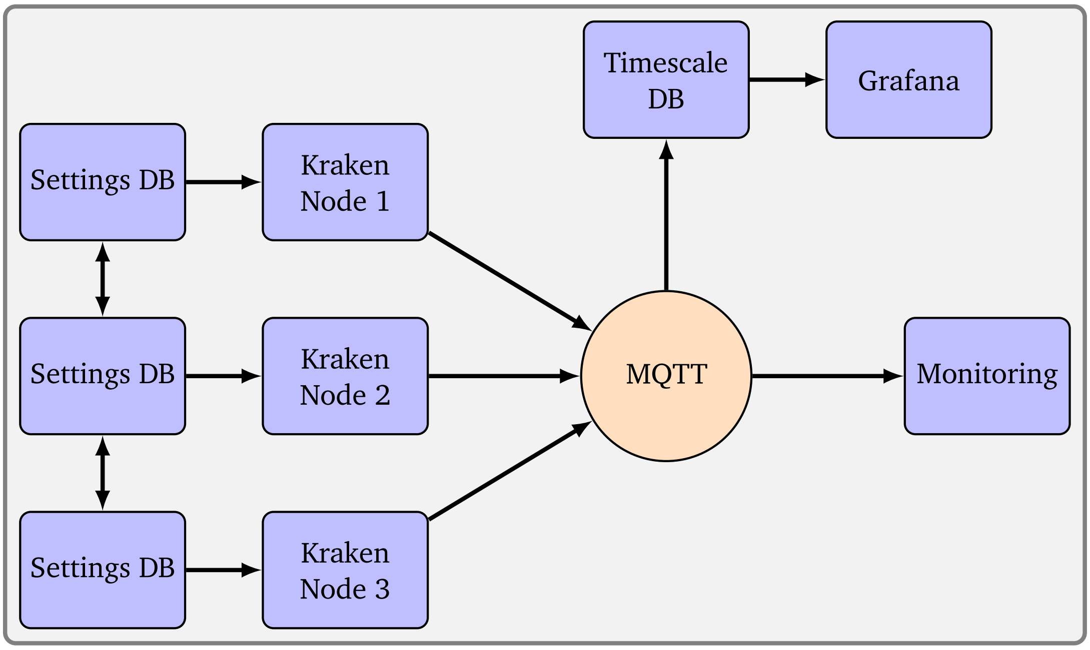


Figure 4.49.: Flow diagram showing the lab monitor system.

Figure 4.49 shows multiple Kraken nodes connected to the MQTT server. These nodes are independent data collectors, but also serve backup for the other nodes. Sensors can be dynamically moved between nodes for example to restart a server without data loss. Each Labkraken node is connected to a settings database with currently is a MongoDB cluster that keeps itself synchronized.

The MQTT server add a layer of resilience to the system as it makes sure that the data get delivered to the clients using a quality-of-service protocol. This means, that messages will be buffered and retained if a client disconnects. It also makes sure no duplicates are produced in an attempt to resend a message.

The data distrubuted by the MQTT can be monitored in real-time for debugging or monitoring purposes. It also stored in a Timescale database with based on a PostgreSQL database.

The data stored in the database is finally visualized using a Grafana backend and can be accesed by the end-user to keep tabs on the experiment and the lab.

4.2.1. Perfomance

To give an idea of the workload that is currently being processed a few number can be given. As of 2023 more than 250 sensors across several labs and offices are being moitored. The typical monitoring interval is currently between 1 s and 60 s. The database currently handles about 2×10^6 inserts/d and the MQTT server processes around 5×10^6 datagrams/d. Figure 4.50 gives an idea on the increasing data load created by adding new experiments and therefore more sensors.

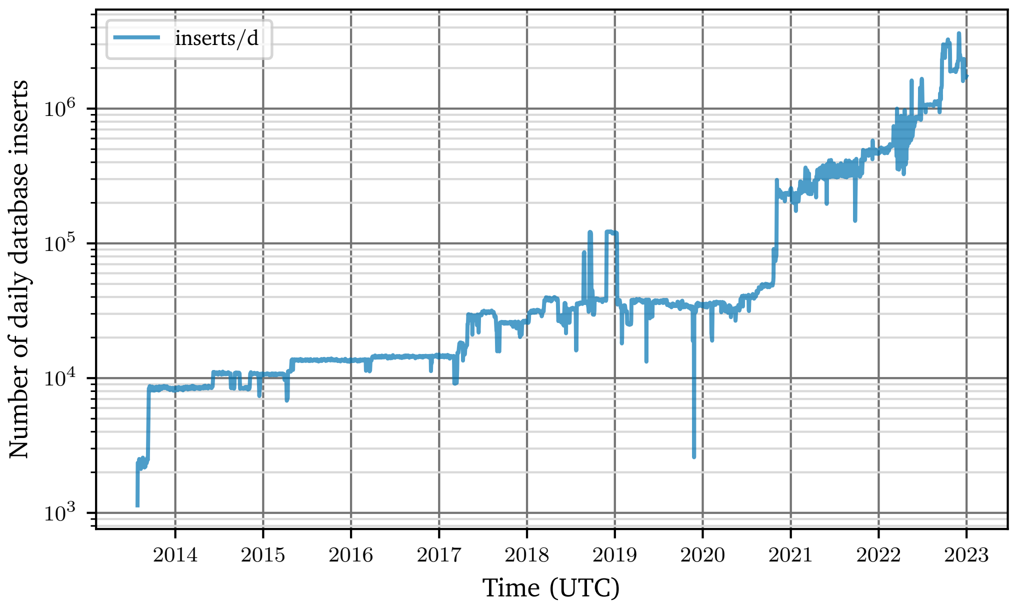


Figure 4.50.: Daily number of datasets collected by the Labkraken system over recent years.

4.2.2. Reliability

As mentioned above, Labkraken is designed to be resilient and fault tolerant. This includes the ability to share the data acquisition between multiple instances of Labkraken. Theses instances are hosted in Docker containers on different servers. The use of the Docker container system allows to easily build test Labkraken and in case of problems roll back to a previous version.

Using multiple instances of Labkraken gives it the ability to move sensors from one instance to another to for example reboot a server for updates. Even a full operating system migration was sucessfully conducted twice without service interruption. As of 2023 the uptime of the system is being monitored. To test the full chain a sensor that sends in data in 1 s intervals is used. This timestamped data is binned into bins of 1 min intervals and the number of bins is then counted and compared to the number of minuted passed in this year. So far the results is

Year	Availability
2023	99.96 %

Table 4.16.: Availability of the Labkraken system.

This number is mostly due to operator errors, which are due the missing configurion frontend, which could prevent malconfiguration. Future software revision ill include a web frontend for configuring Labkraken instead directly manipulating the database. Also a move away from MongoDB is planned, because the clustering features of the MongoDB have been found unreliable whith the database spradically not coming back up after updates without manual intervention. The mid-term goal, for now, is a four 9s availability, but this might also need further improvements on the network layer, which currently is not redundant, but could be achieved by moving a backup node into a different lab.

4.3. Lab Temperature Controller

4.3.1. Controller Hardware

The lab temperature controller also called LabNode is an Ethernet connected PID controller which is shown in figure 4.51.

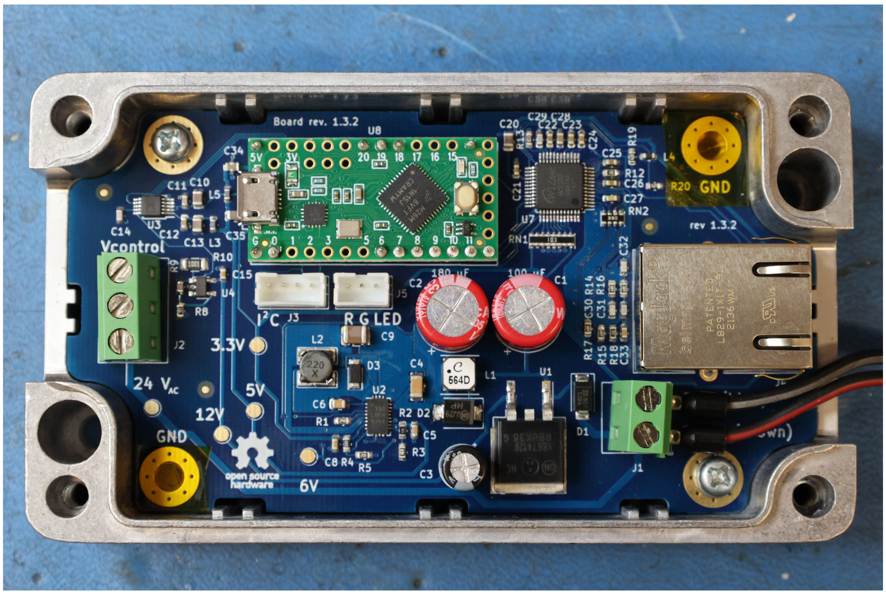


Figure 4.51.: Picture of a LabNode, an ethernet connected PID controler using a `Teensy LC` microcontroller.

It was developed during the Bachelor's thesis of Liebmann [121] and is mainly used to connect to themostats like the Heimeier `EMO T` which used a 24 V_{AC} supply and accepts a 10 V input for the setpoint. It is far more versatile though and can be used for many other tasks as well.

The controller is based on the `Teensy LC` microcontroller, which has a 32 bit Cortex M0+ 48 MHz MCU. It can be controlled and configure via Ethernet using CBOR [46] encoded commands. The CBOR encoder is similar to a binary form of the poplar JSON [47] text format for data transfer. The CBOR encoder is available for many programming languages and Python implementation for the LabNode API is provided by the author [37], which can be install via the PyPI repository. These controllers feature a highly optimized PID library [39], written in C++ and assembly by the author. It was optimized for the Cortex M4 and performs even better on that platform. It used the so called saturated math instructions, which is available as part of the DSP (Digital Signal Processing) extension of the M4. For details see [39]. On the `Teensy LC`, a pid update rate of more than 100 kHz was achieved, allowing the LabNode to be customized for more demanding tasks that require PID control. An external connection to the I^2C bus of the MCU can be used to integrate custom sensors.

The controller is capable to work with a broad range of actuators as is has switchable gain block at the out to accomodate the most common input ranges of linear actuators of 5 V and 10 V. The 12 bit DAC is an ADI `AD5681R` [7], but for more demanding tasks a 16 bit version `AD5683R` can be installed as well with minimal software changes required. The full design files can be found at [36].

4.3.2. Controller Implementation

The labtemperature swings were already introduced in figure 3.3 on page 15 and it became obvious that given a 2 K temperature swing precision measurement requiring stability below 1 ppm are not possible. An example is shown in figure 4.52.

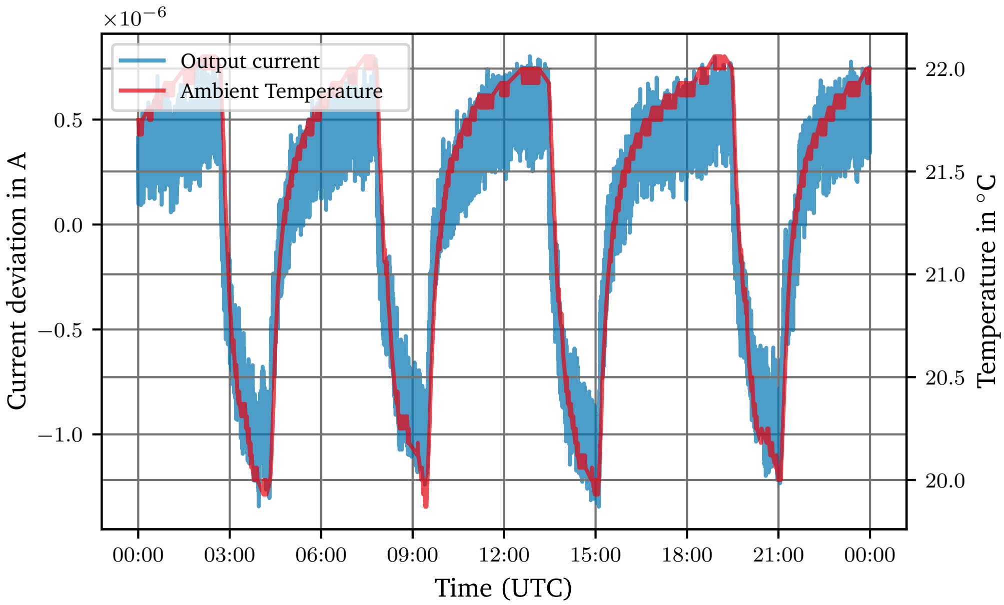


Figure 4.52.: Measurering a 50 mA current using a Keysight 34470A with changing ambient temperature. The current source is based on the design of Erickson et al. [76].

The device under test (DUT) for the measurement shown above was sitting in a temperature controlled environment and the Keysight 34470A[3] was exposed to the lab enviroment. Admittedly, the example in figure 4.52 does not reflect the correct way to measure current with high stability, but serves as an excellent example to highlight the problem.

In order to resolve the situation, a LabNode was used to replace the commercial temperature controller of the airconditioner box. These boxes use an active radiator and are connected to the cooling water system of the building. The water flow can be regulated via an actuator to control the air temperature blown into the room. The lab which is shown as an example below has two such boxes and both are controlled by individual temperature controllers. To reduce the influence one box has on another it is desirable to keep the sensors close to the airconditioner unit. This also descreases the response time of the sensors and thereby increases the regulation bandwidth. On the other hand, only regulating the output temperature of the airconditioner unit will result in a temperature offset measured on the optical tables depending on the local heat load. This problem can be addressed using multiple sensors. One sensor that is close to output of the airconditioner and the other one at the location, where the temperature is to be stabilized. This idea is hardly new and commonly used in industry [199] another intriguingly simple solution was also presented by Znaimer et al. [238].

The bandwidth of the PID control loop required is not not very high and it was found that a controller update rate of 1 Hz was more than sufficient. This allows to connect the sensors via

[3] 100 mA range, 10 PLC, AZERO ON, $f_s = 0.5$ Hz

Ethernet instead of a direct connection to the PID controller. Using Ethernet greatly simplifies the wiring, because the infrastructure is already there. Additionally, when routing these sensor signals through a small computer, more elaborate control schemes can be implemented on the fly in software without modifying the LabNode controller firmware. This also simplifies testing and open up more option regarding the sensor choice.

Such a control scheme is currently implemented and several sensors made by Tinkerforge are distributed throughout the lab to give the desired feedback. A Docker container running on a server is processing these sensors and then hand the computed value to the LabNode controller. Should the server fail to update the data, a fail-safe enganges and the controller will use a backup sensor to control the room temperature, albeit with a reduced bandwidth, because the sensor is located inside the controller box and not directed attached to the airconditioner.

To adjust the PID controller a model of the room was used as described in section 3.5.2 on page 30. This is shown next.

4.3.3. Controller Tuning

The FOPDT model was found to sufficiently describe the room under most circumstances and gave usable results for a PI controller. An exeption to the rule is the case were the sensor is very close to the airconditioner. As an example this procedure is explained using the airconditioner in the back of a lab that contains two such units. This is also the lab were the measurement shown in figure 4.52 was conducted. To fit the model to the data later, timestamped values of the controller output and the room temperarture are required. The Labkraken software shown in section 4.2 greatly simplifies this task as these values are routinely logged by Labkraken, so there is no special setup required. To extract the data later, either an SQL query can be used to have the Timescale database prepare a usable dataset or the Grafana web interface can be used to select the appropriate timeframe. One my consult appendix A.2 for details on the SQL queries required.

The procedure to extract the PID parameters is as follows

1. Place the sensors and log their output with a timestamp
2. Make sure the room temperature is stable
3. Set the controller to manual mode, which leaves the output at the previous output value
4. Wait a few minutes to collect some data. This gives the fit a better starting point.
5. Apply a step pulse by increasing the output by 500 counts. Do make this gives a reasonable response. Some rooms may require a smaller step like 100 counts.
6. Wait again until the temperature has settled to ensure a good fit result.
7. Return the controller to automatic mode

With the timestamped temperature and output values in hand, a fit to the model can be attempted. The type of model depends on the location of the sensor. Sensors located a few meters away from the airconditioner can be modeled, to a good degree, using a first-order plus dead-time model. The result will then be a set of parameters for a PI controller as discussed in sections 3.5.2 and 3.5.3. Sensors located very close to the airconditioner or placed directly in the airflow require a different approach and can best be simulated with a second-order model. The result will then be a set of PID controller parameters.

Figure 4.53 shows the fit to the backup sensor of the LabNode controller in the back of the lab. The controller is located about 1.5 m from the airconditioner and not directly located in the airflow. Both the time delay until the cold air reaches the controller and the long decay time can be seen. The latter is due to the placement of the sensor inside the controller and is also the reason for the seemingly high temperature of more than $35\,°$C. The test typically takes about 3–4 h and the lab should best be undisturbed during this time. Fortunately, the controller is remote-controlable and can be programmed using the Python API introduced above. A Python example to set the controller output can be found with the example source of the library [37].

From the fit the following values for the FOPDT parameters were found:

$$K = (2.137 \pm 0.005)\,\mathrm{mK/bit} \qquad \tau = (2264 \pm 17)\,\mathrm{s} \qquad \theta = (113 \pm 8)\,\mathrm{s}$$

K is the normalized gain, τ the decay time, and θ the dead time or lag.

As it was shown section 3.5.4, the tuning rules that give, in this situation, the smoothest response with very little overshoot are the SIMC and the AMIGO rules. The results for those two are given below and were calculated according to table 3.4 on page 39.

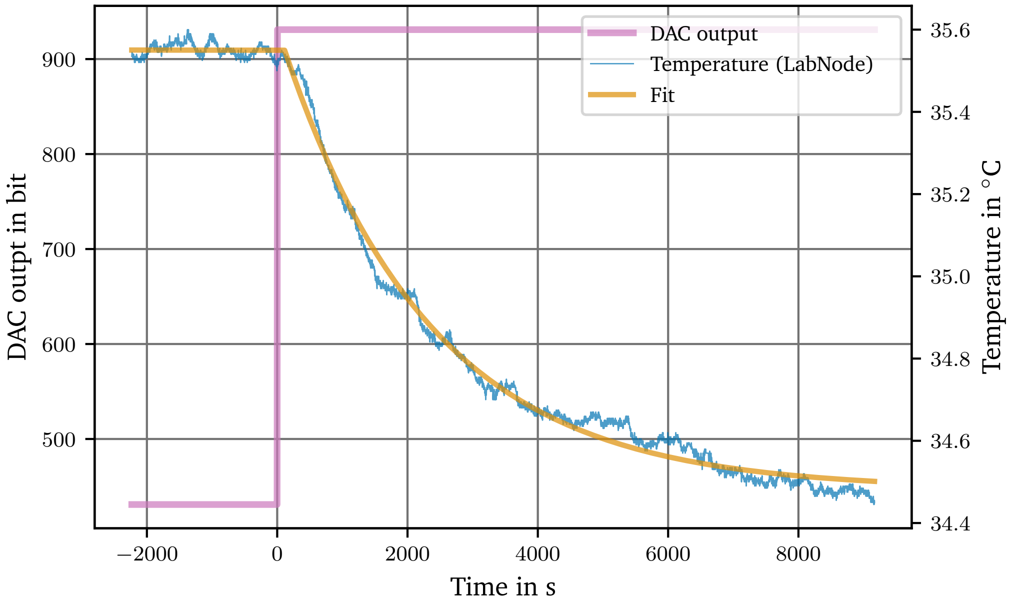


Figure 4.53.: Fitting a FOPDT model to the response of an output step of 500 bit.

Tuning Rule	k_p	k_i
SIMC PI	4688.57 bit/K	5.186 bit s/K
AMIGO PI	2927.71 bit/K	3.087 bit s/K

Table 4.17.: PI tuning rules for lab 011 extracted from a fit to a FOPDT model.

For the backup sensor, the more conservative result is chosen as a starting point for tuning the controller. The reason is that the backup mode is supposed to 'just work' even under adverse conditions. The results obtained will like need some more tuning to achieve optimal results. Retuning might be neccessary if larger changes were made, either to the heat load in the room or the airconditioning system. A well tuned system is shown in the next section.

4.3.4. Test Results: Temperature Stability

Having tuned the PID controllers the system was found stable even for fast changing loads like high power devices being turned. To demonstrate the performance a typical week was chosen and the temperature is shown in figure 4.54.

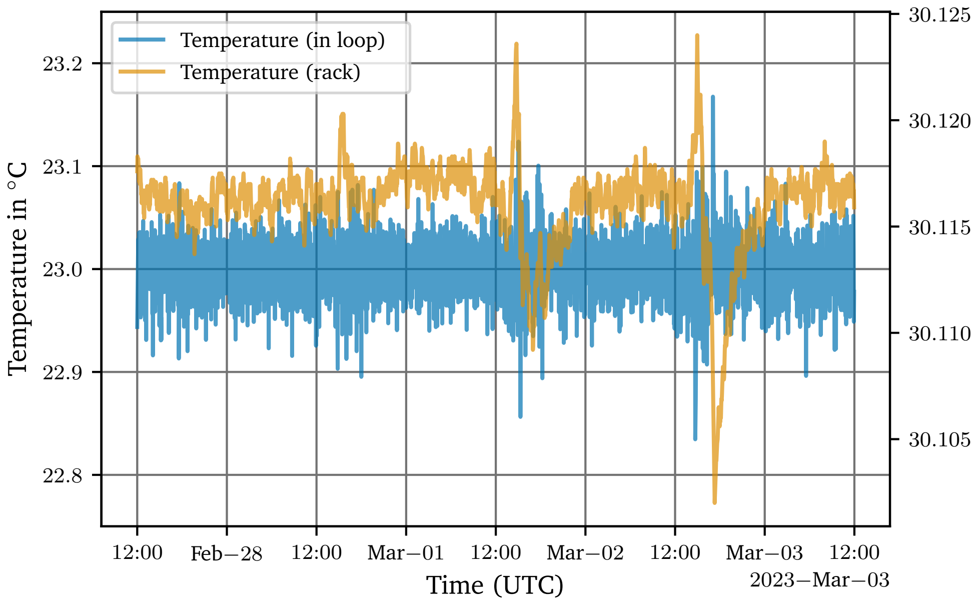


Figure 4.54.: Temperature measured by two sensors in the lab. The in loop sensor and another sensor located inside a rack nearby.

Figure 4.54 shows the temperature as measured by sensor attached to the airconditioner and a Fluke `5615-12-P` PT100 sensor which is stuck into the rack that monitors the Zener diodes as described in section 4.1.16. The only deviations seen are caused by people working on the table right next to this rack. Typically the temperature is stable within $\pm 0.5\,\mathrm{K}$ over weeks, a significant improvement over the system previously installed.

It must also be mentioned that over the course of the last $6\,\mathrm{a}$ none of the 14 units currently in use has failed or shown issues. This highlights the build quality and realiability of the system. One of the key parameters for such an important aspect of the infrastructure.

4.4. Digital Temperature Controller

A diode laser as implemented by this group requires two temperature controllers. One is used to stability the internal resonator and the other is used to control the temperature of the laser diode. For this purpose, a two-channel controller was concived in this work and a prototype has been tested. This section is split into two parts, just like temperature controller. There is the analog frontend, that does the data acquisition and there is a power driver board that is responsible for driving two thermoelectric coolers (TEC). The analog frontend was developed during the Master's thesis of Sattelmaier [181]. An in-depth characterisation can be found there. The most imported findings will also be reproduced here.

The second part deals with driver board. The driver board features two 12 V, 5 A output drivers and also contains the microcontroller.

4.4.1. Analog Board

The analog front end is isolated on a separate board to reduce noise coming from the power power which drives the TECs running several A. The frontend consists of two channel, that are identical. Each channel has a current source, a multiplexer (MUX) and an ADC, including a buffer. A simplified overview is given in figure 4.55.

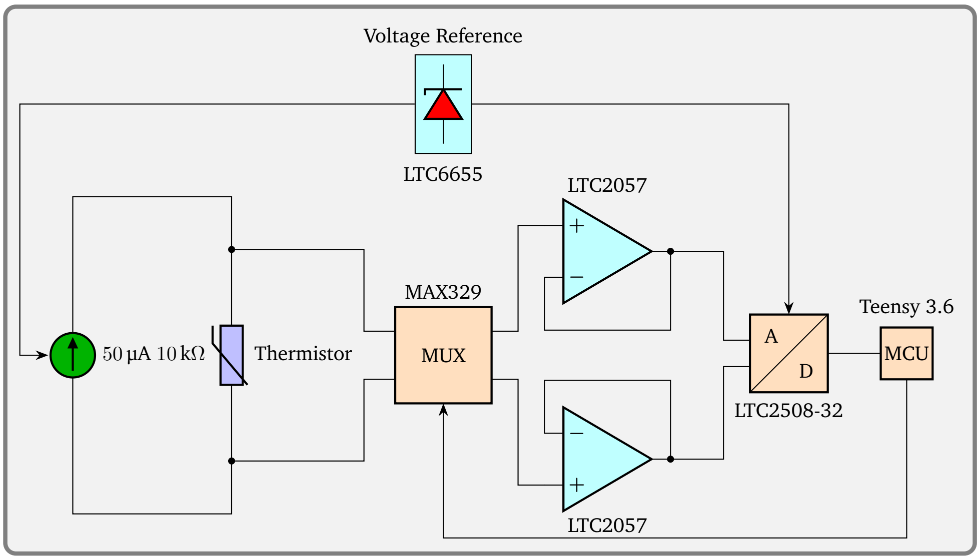


Figure 4.55.: Simplified overview of the digtal temperature controller. Shown is a single channel. Not included is the internal reference resistor that is accessible through the MUX.

As can be seen in 4.55 the same reference voltage is used to create the current and fed into the ADC. This has the advantage of cancelling out low frequency noise and drift of the reference, at least the linear effects. Therefore the reference is a far simpler design than the one shown in section 4.11. The reference is an ADI `LTC6655-4.096` [132], a bandgap reference as opposed to the Zener diode used in the laser current driver design. These voltage references can operate with a lower supply voltage, but are less stable. The latter is of little concern as drift mostly cancels out in this arrangement.

The voltage reference is buffered and split into two paths, one feeds into the ADC, a 32 bit ADI `LTC2508-32` [130] successive approximation (SAR) ADC. This ADC runs at 1 MHz, but does apply and internal filtering and decimation algorithm. Decimating the output is necessary, because the MCU, a `Teensy 3.6` with a 180 MHz Cortex M4 32-bit processor is not capable of transfering the 32 bit data value within the 650 ns as required by the `LTC2508-32`. Deimating the input allows to spread the readout over multiple acquisitions as detailed by Sattelmaier [181]. This has some drawbacks as discussed later though.

The other path is going to the current source. The reference voltage is again buffered and followed by an ADI `AD5760` [8] 16 bit DAC with a very low temperature coefficient of $<0.05\,\mu V/V$. The DAC is used to adjust the output current, which is useful when using other temperature transducers than a $10\,k\Omega$ thermistor. The design of the current source is very similar to the design discussed in section 4.1.6 on page 120. This main current source (acutally a sink) is a buffered howland current source as shown in figure 3.54 on page 91 instead of the MOSFET based current source. The Howland current source is based on an ADI `LT1997-3` [128] difference amplifier as proposed in section 3.8.9. The reference resistor is a $100\,k\Omega$ Vishay `S102` [177] non-hermetic moulded precision resistor. The current is then fed into a CR-filter (do note the inverted order, because it is a current filter). The low output impedance of the filter requires a cascode with a an InterFET `SMP4338` JFET to bring the output impedance back up. After the cascode, the current is sourced from a a virtual ground as shown in figure 4.19 on page 120. The current flow is in the opposite direction as in the laser current driver in section 4.1.6 because the voltage across the thermistor must be of positve polarity for the ADC.

The extremely high output impedance is required mostly because it reduces the settling time of the current source. This is important when considering the autozeroing routine. The acquisition will be discussed next in more detail.

4.4.2. Data Acquisition

The thermistor to measure the temperature of the laser resonator and diode mount is an Amphenol `RL0503-5820-97-MS` [175] large bead thermistor. The large size of the thermistor results in a lower noise as compared to smaller thermistors like the TDK `B57550G103+` used in earlier designs [176]. To reduce self-heating effects due to the sense current, a current of $50\,\mu A$ was chosen as opposed to the industry standard of $100\,\mu A$. This will produce a voltage of 500 mV across the the thermistor at room temperature. Using the equation derived by Steinhart et al. [205]. The temperature sensitivity can be estimated as is found to be $1.91\,\mu V/100\mu K$ at $23\,°C$ and $450\,nV/100\mu K$ at $60\,°C$. To reach the design goal of $100\,\mu K$ small voltages have to be measured. This brings up several problems like low frequency noise from the amplifier and thermal EMF which can introduce an offset voltage between two dissimilar metals of different temperature – an issue that arises at every connector and solder joint between the sensor and the analog frontend. Copper oxide buildup on connectors can produce peltier voltages as high as 1 mV/K, orders of magnitude higher than the measurement quantity. Both the low frequency flicker noise and the random walk seen from thermal EMF can be treated as the same problem. The details and their nature were discussed in section 3.6.

To mitigate this low frequency noise autozeroing is required. As explained in section 3.7 autozeroing requires a second measurement of the zero value to subtract it from the measurement. This obviously slows down the measurement frequency by half. An alternative is to invert the polarity of the measurement, then average both as

$$\frac{V_{th} + V_{off} - (-V_{th} + V_{off})}{2} = V_{th}\,. \tag{4.15}$$

V_{th} is the voltage across the thermistor and V_{off} the offset voltage introduced by noise. This way the result is the desired thermistor voltage, but with the offset removed. This approach still bears the cost on bandwidth, but has no dead time. This is a considerable advantage as discussed in section 3.7.1 because it allows to increase the switching frequency until one ends up in the white noise region of the spectral density. The limits will be discussed down below, but first the multiplexer is shown. The design uses a low leakage ADI `MAX329` [144] dual four-channel multiplexer. This multiplexer is used to switch the current source between four configurations.

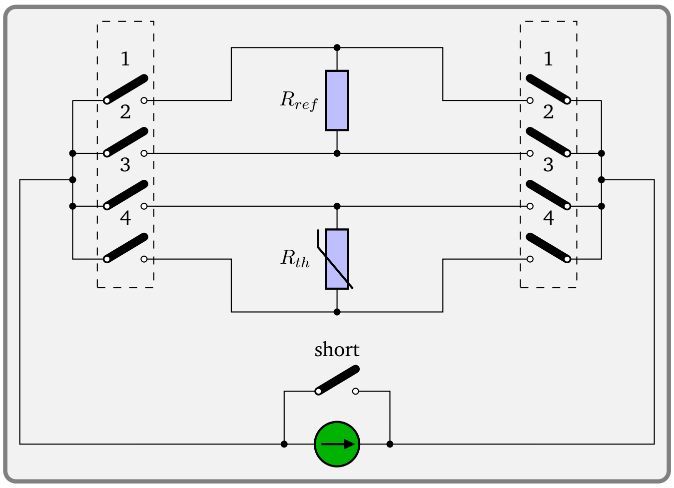


Figure 4.56.: Multiplexer of the analog frontend to invert the current source or switch to a reference resistor.

The multiplexer shown in figure 4.56 allows to invert the current source, but also switch it to an internal reference resistor. A hermetic $10\,\mathrm{k}\Omega$ Vishay `VHP101` [220]. This resistor gives the analog frontend a number of unique capabilities. The instrument can not only autocalibrate its frontend by checking against the known value of the reference resistor. It can also be used as a resistance bridge because both channels have access to this reference resistor. So by measuring a known resistor on channel 1 and an unknown resistor on channel 2, its ratio can be computed by comparing both channels with the reference resistor. This feature is not yet implemented in firmware, but can prove interesting for high precision resitance measurement. The shorting switch shown above the current source is enganged before opening any of the multiplexer switches to ensure the current source does not raise the voltage to the supply rails in an attempt to keep the current flowing.

Switching the polarity of the current source brings a number of challenges. First, the load is switched between a short and the $10\,\mathrm{k}\Omega$ within $1\,\mu\mathrm{s}$ [144] as this is the transistion time of the `MAX329`. The current source must follow these load changes as fast as possible, because in between the transistion, the ADC cannot collect data. This requires a very high dynamic output impedance as mentioned above. The second issue has to do with the cable. The has a capacitace and additionally the dielectric used as insulator displays dielectric absorption. This topic was already section 4.1.5 on page 113 in the context of capacitor leakage. The impedance involed in this situation is typically an order magnitude higher than the $510\,\Omega$ discussed in section 4.1.5. Sattelmaier showed that using a $2\,\mathrm{m}$ twisted pair cable with a PVC insulator a decay time of $(0.1876 \pm 0.0794)\,\mathrm{s}$ could be ovbsered. Given that the dielectric loss tangent of PCV is 2 orders of magnitude higher than that of PTFE [148] and the relative permittivity of

PTFE ϵ_r is about $60\,\%$ that of PVC. PTFE cables were chosen to minimize the dead time. The cables used are 4-wire MIL-DTL-27500 `M27500A 18WJ4T24` cables. These cables did not cause an degradation of the measurement.

The autozeroing also affects the ADC. As mentioned above, the ADC has an internal filter and then decimates the result to reduce the amount of data that needs to processed by the microcontroller. The interal filter is, unfortunately, not aware of the switching and cannot apply equation 4.15. The filter has therefore always be reset and it is required to wait for the settling of the filter before processing new data. The internal filter needs 7 (decimated) samples to settle. The smallest downsampling factor the $180\,\text{MHz}$ `Teensy 3.6` could cope with was 4096 which results in $244\,\text{samples/s}$ with a bandwith of $30\,\text{Hz}$. The 7 sample wait time introduces a dead time of $29\,\text{ms}$. Figure 4.57 can be used to judge the impact of the slow data processing.

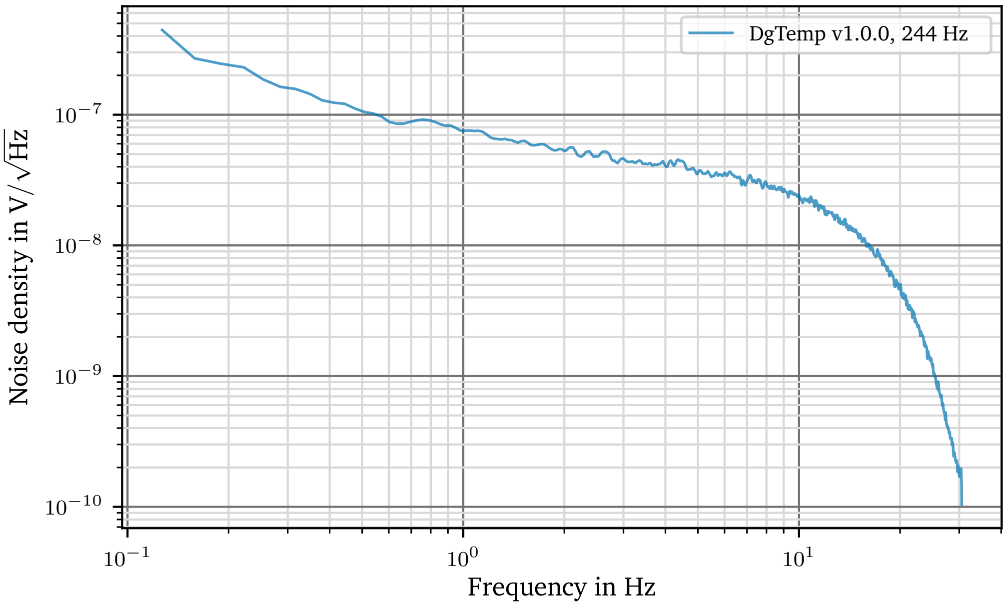


Figure 4.57.: Noise spectrum of the analog frontend using an `LTC2508-32` with shorted inputs.

The most prominent feature of figure 4.57 is the bandwidth of $30\,\text{Hz}$ which can be clearly identified. Although is mostly shaped by the digital filter of the `LTC2508-32`, it can be seen that towards low frequencies, the noise floor takes the form of flicker noise. The `LTC2057` used a buffer has a noise floor of $10\,\text{nV}/\sqrt{\text{Hz}}$. From the data it can be expected, that towards higher frequencies, the noise should drop to that value. Unfortunately, this frequency range is out reach. The inversion was therefore chosen to be done every 512 samples, that is, with a frequency of about $2\,\text{Hz}$, which is about a decade away from the filter cutoff.

4.4.3. Test Results: Linearity

Another property of the analog frontend is the linearity, not for the resistance bridge function mentioned above, but also an accurate determination of a calibrated temperature. This test was done by sweeping a Fluke `5440B` calibrator over the input range from 0 to $4.096\,\mathrm{V}$. A Keysight `3458A` with a specified linearity of $<0.1\,\mathrm{\mu V/V}$ was used to confirm the voltage. This lead to a barabolic calibration curve that was programmed into the microcontroller and was then corrected for in real-time. The integral non-linearity (INL) measurement is shown in figure 4.58.

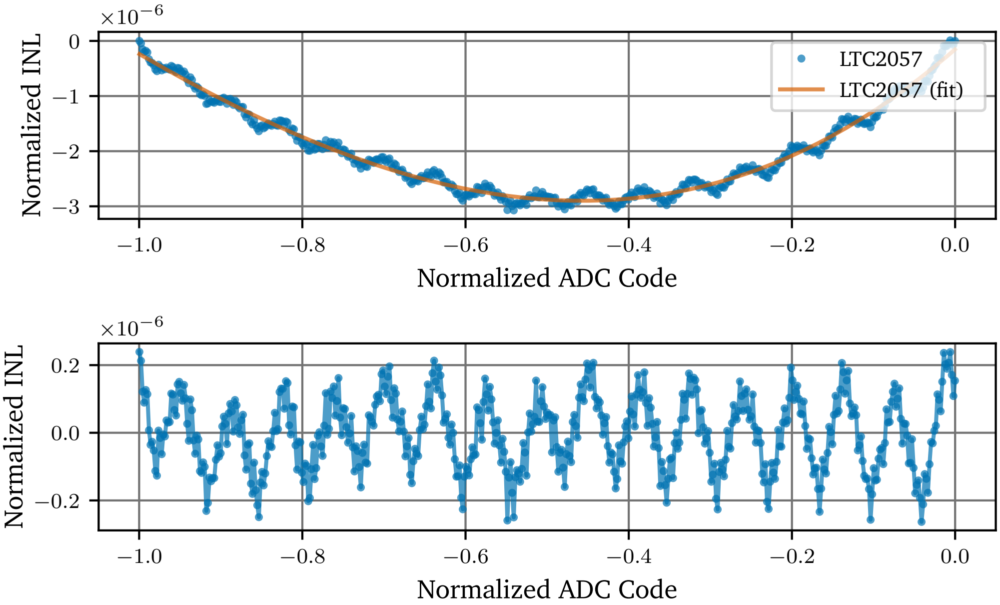


Figure 4.58.: INL of the analog frontend with an `LTC2057` buffer. Top: uncorrected INL. Bottom: With calibrated correction applied.

Several other op-amps like the Texas Instruments `OPA827`, another chopper op-amp or the ADI `ADA4625-1`, a JFET op-amp were also tested. While the `LTC2057` did not have the lowest INL of all device tested, it had the most predicatable and the only one having a parabolic shape [181]. The sinusoidal shape superimposed on the parabola is inherent to the architecture of the ADC and also shown in the datasheet [130]. The parabola was stable over several weeks and tested multiple times, so it can be calibrated out. To rule out a drift with temperature, the INL was also tested at both $20\,^{\circ}\mathrm{C}$ and $30\,^{\circ}\mathrm{C}$, which also showed negligible difference.

4.4.4. Test Results: Temperature Stability

The temperature stability of the analog frontend was also investigated. To give a realistic impression, a reference resistor $10\,\mathrm{k\Omega}$ (ab-precision `RS2-10k`) was used to simulate a thermistor. The temperature controller was then placed inside a thermal chamber and while measuring the resistance of the reference resistor located outside of the chamber. The temperature was then stepped from $23\,°\mathrm{C}$ ambient to $45\,°\mathrm{C}$. The results are shown in figure 4.59.

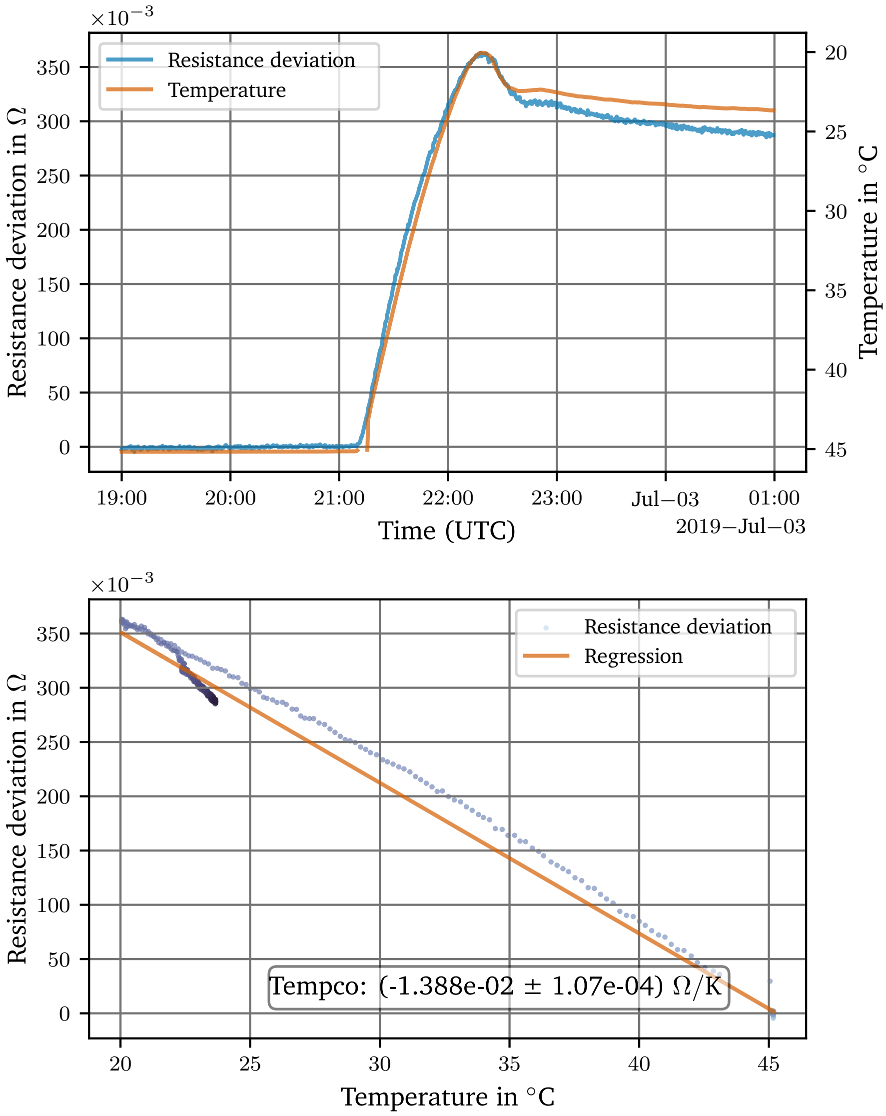


Figure 4.59.: Temperature coeficient of the temperature controller frontend. Measured by stepping the ambient temperature by $22\,\mathrm{K}$ while measuring a $10\,\mathrm{k\Omega}$ reference resistor located outside the thermal chamber.

With the autozero function enabled a result of $(-13.9 \pm 0.1)\,\mathrm{m\Omega/K}$ was measured, the equivalent of $-40\,\mathrm{\mu K/K}$. Related to full scale input that is $-0.17\,\mathrm{ppm/K}$, which can be attributed to the drift of the current source. It can therefore be assumed, that the ambient temperature of the lab has little to no influence on the setpoint of the controller.

4.4.5. Test Results: Longterm stability

The final aspect was the longterm stability. In order to do so, the controller was left in the thermal chamber while measuring the $10\,\mathrm{k\Omega}$ `RS2-10k` reference resistor and the internal reference resistor as well. The thermal chamber did not contain a dry pouch, so the humidity was uncontrolled. This revealed a small dependence on said ambient humidity which is most likely the $100\,\mathrm{k\Omega}$ Vishay `S102` molded epoxy resistor because epoxy resin swells when exposed to humidity. This puts pressure onto the resistive element which then changes its resistance.

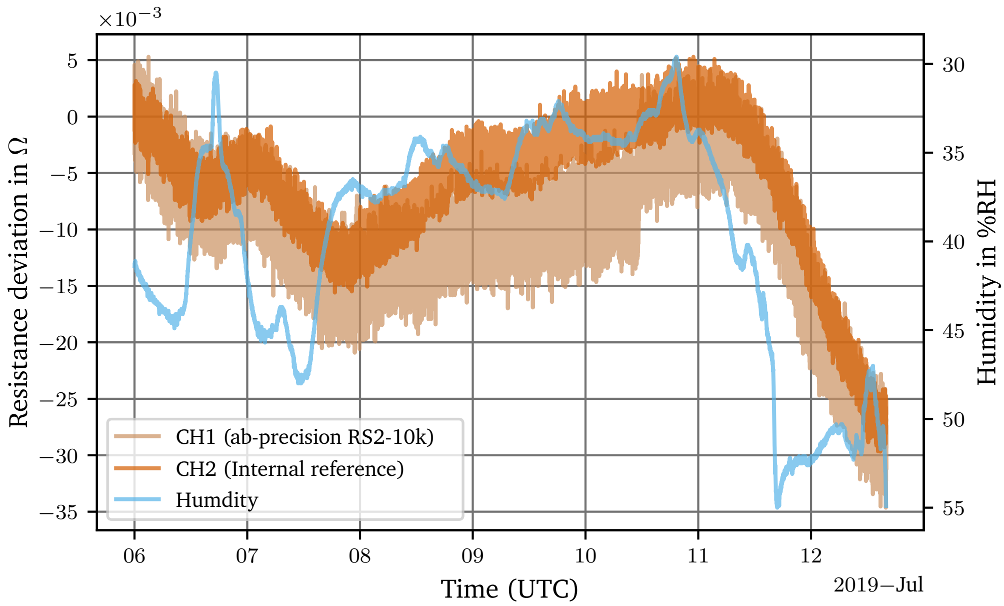


Figure 4.60.: Longterm comparison of the internal reference resistor against an external $10\,\mathrm{k\Omega}$ reference.

This sensitivity can either be addressed in hardware by replacing the resitor for a hermetic version, or software solution can be applied activating the autocalibration routine in regular invervals. The device can then recalibrate itself using the internal hermetic reference resistor.

Nonetheless, the sensitivity to humdity is on the order of $80\,\mathrm{\mu K}$ between $30\,\%\mathrm{RH}$ and $60\,\%\mathrm{RH}$, which is close to the full swing of the lab. The noise of the individual channel is $1.65\,\mathrm{m\Omega_{rms}}$ for the external channel and $0.89\,\mathrm{m\Omega_{rms}}$, which is $4.1\,\mathrm{\mu K_{rms}}$ and $2.2\,\mathrm{\mu K_{rms}}$ respectively. This is all well below the requirements listed in table 3.4.

4.4.6. Output Driver

The output driver of the temperature controller is based on an ADI `LT8714` bipolar DC/DC regulator that is designed for an output of $\pm 12\,\mathrm{V}$ and $\pm 5\,\mathrm{A}$. Two of these controllers were integrated on a PCB. The switching frequency of $300\,\mathrm{kHz}$ can be filter better than the typical switching frequency on the order of a few kHz typically found in H-Bridge designs. Additionally, the DC/DC regulator can be supplied with a higher voltage of up to $20\,\mathrm{V}$ which reduces the required current accordingly. This allows to integrate more devices into a subrack powered by the same power supply. The output driver board also houses the microcontroller which ist connected to the analog board via a digital isolator to reduce the noise coupled into the analog side. Both board were tested together and laser resonator was temperature controlled for a test. This revealed the first problem. The laser resonators used in this group are housed in an aluminium case that is not airtight. This allowed air draft to considerably affect the resonator tempeature. Even walking past the laser caused temperature swings of $\pm 1.5\,\mathrm{mK}$ or more.

To address this issue, the laser is additionally shielded using a silicone sleeve that is put on the external laser resontor housing. The silicone sleeve was molded in house using a 3D printed replica of the laser resonator. The type of silicone used is and RTV2 platinum silicone with a shore hardness of $32\,\mathrm{ShoreA}$. This extra protection reduced the disturbances by half and the result is shown in figure 4.61

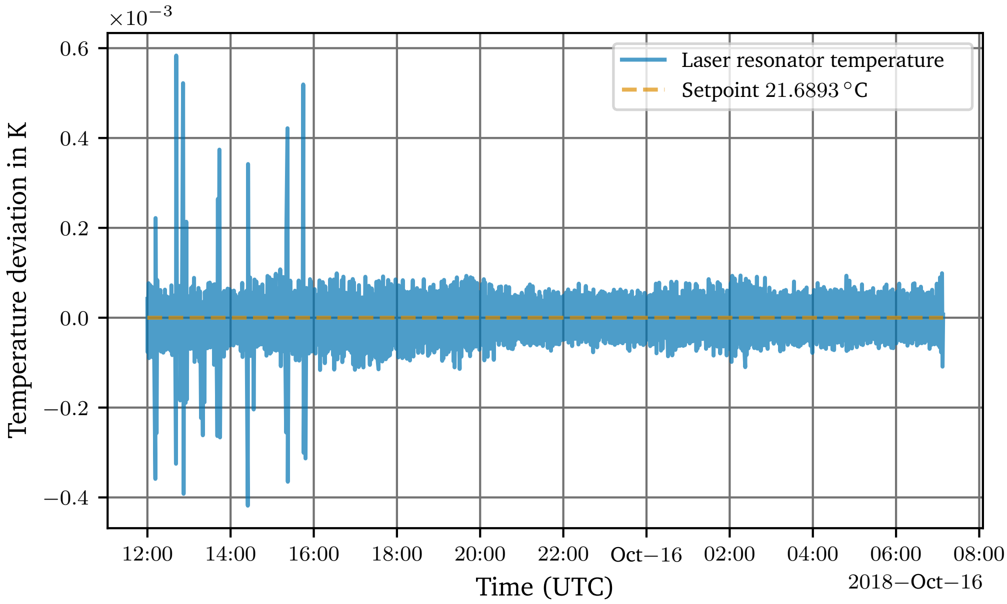


Figure 4.61.: Stability of a temperature controlled laser resonator. The disturbances are caused by air drafts, that rapidly change the temperature of the aluminium housing.

The temperature controller typically keeps the resonator within $\pm 150\,\mu\mathrm{K}$ and the only problem are disturbances causes by airdrafts due to people walking past or working in the vicinity. To rule out the controller as the source of the problem, an aluminium block as a testmass was placed in a thermal chamber and a Keysight `3458A` was used to monitor the voltage across the $10\,\mathrm{k\Omega}$ thermistor, used by the PID controller. The thermistor used was a calibrated Fluke `5611T-P` to rule out a problem like burst noise coming from the sensor. The temperature in the chamber was then changed from $20\,°\mathrm{C}$ to $15\,°\mathrm{C}$.

The first observation made was that the disturbances were gone for as long as the internal fan was turned off and there was only natural convection present. This gives a strong indication, that turbulant air is a major factor for the temperature noise seen in the lab.

Addiationally, using the thermal chamber the gain of the controller can be estimated. To do so, a $5\,\mathrm{K}$ temperature step was applied and the controller tried to keep the temperature stable. As can be seen an aparant error of about $100\,\mu\mathrm{K}$ remains, so the gain can be estimated to be about $50\,000\,\mathrm{K/K}$. Do note at this point that there is an enormous uncertainty on this value, because the voltage difference is only $2.5\,\mu\mathrm{V}$. The measurement of the `3458A` does not compensate for thermal EMF and $500\,\mathrm{nV/K}$ can be introduced by almost any junction. See [94] for a list materials.

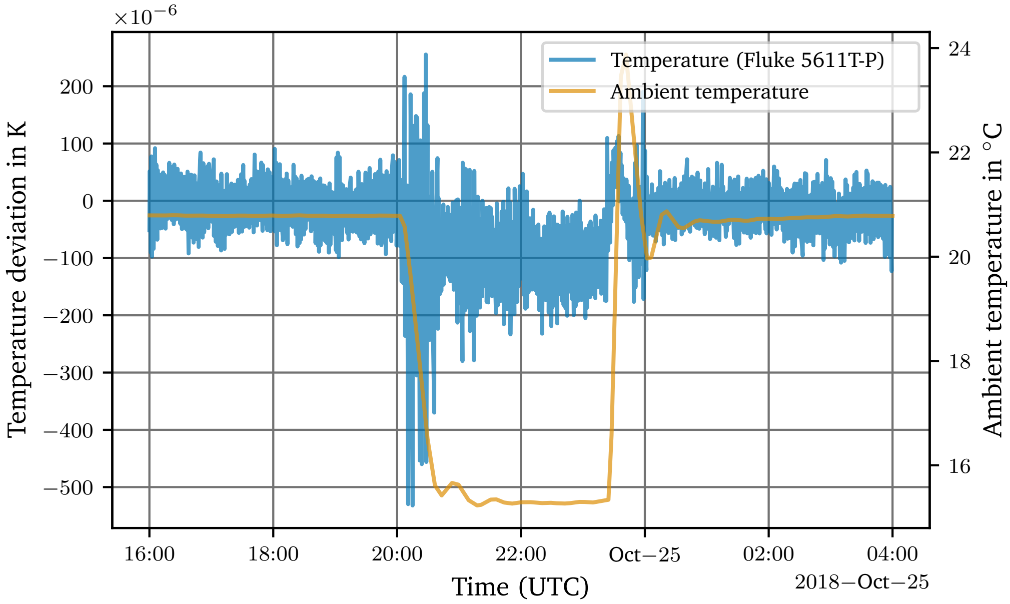


Figure 4.62.: Measurement of the gain of the PID controller. The voltage across the sense thermistor of the PID controller is measured by a Keysight `3458A` and the temperature is stepped $5\,\mathrm{K}$.

To improve the stability of the laser resonators, this problem should be addressed in the future with an updated laser resonator design that is at least to some degree air tight. An experimental version of such a laser was alread designed for the ARTEMIS experiment [142] and is currently part of the laser system.

4.4.7. Summary

A high precision temperature controller with two $12\,\mathrm{V}$, $5\,\mathrm{A}$ channels was presented. With a non-linearity of $<1\,\mu\mathrm{V/V}$ and an internal reference, calibration is as simple as having a known resistor. It was shown that the system brings the current mechanical laser resonator design to its limits. The performance of the laser system is now defined by the mechanical stability of the resonator. To improve the situation with commonly found non-hermetic laser resonators the necessity of using an partially air tight silicone sleeve around the laser resonators was stressed and the benefits of such a construction are shown.

The temperaturee controller was also tested in a variety of environmental conditions and a supression of ambient temperature fluctuations of 2×10^{-5} was estimated. This number is hard to confirm though and might easily be a lot higher due to the small voltages and large temperature gradients to be measured. Along with a sensitivity of $4.1\,\mu\mathrm{K_{rms}}$ and a humidity drift without autocalibration of $2.8\,\mu\mathrm{K/\%RH}$ puts the temperature controller at the same level as a high stability thermistor that are known to drift about $100\,\mu\mathrm{K/a}$ [72].

5. Outlook

This work has pushed the boundaries of diode lasers to a new level. Diode lasers built the technology presented in this work will see their limits in the mechanical design rather than the control electronics. The design present allow precision laser spectroscopy even in the presence of adverse environment conditions, that would otherwise pose serious challenges to the experimenter.

The development of such high precision devices, especially for small series as they are typically required for university labs, has become more challenging over the recent years due to the decline in availabilty of many electronic parts. These shortages mostly affect microcontrollers and power electronics. It is the authors desire to make the instruments presented in this work available to public via an open-source contribution of the design files and also to keep them available. This necessitates some updates to the designs to make them resilient to supply chain issues. The author proposes the following changes to design presented in this work.

The DgDrive laser current driver should be modified with the careful addition of an FPGA on the analog side to simplify the digital isolation between the front panel and the analog electronics. Recent developments have demonstrated that reducing the number critical components like the digital isolators creates more independence. The use of an FPGA can also simplify many of the logic elements on the board by bringing them into the FPGA core. Being able to change this logic sofware also improves the compatibility with future part changes. The FGPA also allows for smaller and simpler microcontroller, giving more options when adapting to supply changes. Another interesting new development that is currently persued is the integration of the new ADI `ADR1399` reference diode which has shown promising results regarding the burst noise issue as well as deliver a lower floor. Given those early results future batches of the current driver may not even need the additional screening process of the Zener diodes discussed in section 4.1.13. This would greatly simplify the build process in terms of hardware and time required.

Much of the same as said above can be said about the temperature controller. The supply situation is even more dire in this case, because many of the many specialised parts with tight specifications. For example the `LTC2508-32` ADC used in the analog frontend is tailored to the microcontroller. Adding an FPGA frontend for the ADCs can greatly simplify and even improve the performance of the system as it is right now. With the advent of new high performance ADCs in recent years an FPGA also gives the needed flexibility in choosing a suitable substition if the need arises. The work on these changes was already started, but is not yet finalised.

Regarding the driver board of the temperature controller. The availabilty of the new ADI `LT8722` TEC driver is much anticipated because it integrates many of the components used on the driver board into a single chip. Appart from the filtering the board can be greatly simplified, reducing the production cost and complexity. A key feature to further the public adoption of the design.

These developments are independent of the benefits than can already be reaped from the designs currently available. The new laser system built for the ARTEMIS experiment is expected to deliver the much desired stability for the spectroscopy of Ar^{13+}. Clearing the commisioning of the Penning trap is major step forward and and also opens up the road towards the spectroscopy of Bismuth ($^{209}Bi^{82+}$) as the new electronics can provide the needed stability for the seed laser at 976 nm. Parts of this system integrating the new electronic are currently being performance tested. Some of this work was already presented in [166].

A final aspect that has come up with the new electronics are the limits of the mechanical design of the laser resonators. While not suprising, the unsealed laser makes for a good barometer as shown in figure 5.1.

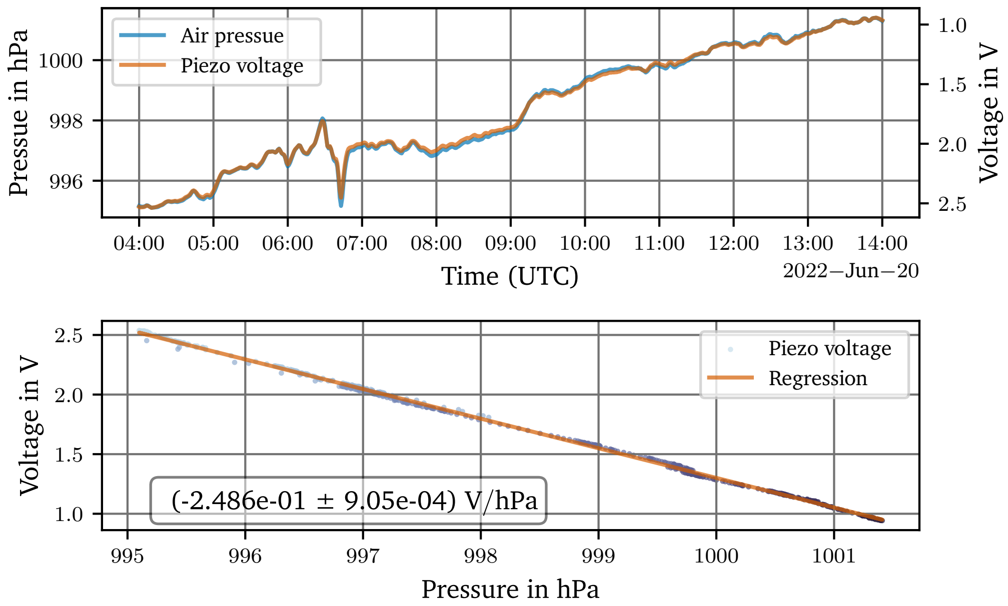


Figure 5.1.: A 780 nm laser locked to the rubidium ^{85}Rb D_2 line. The piezo mounted inside the resonater moves the back reflecting mirror to follow the air pressure and keep the laser frequency locked to the rubidium transistion.

An o-ring seal, like demonstrated by [54], can improve the stabilty of the diode lasers and ensure the performance of the lock becomes insensitive to the weather. In addition the ample use of epoxy resin inside the resonator causes a sensitivity to humidity as the epoxy resin swells when exposed to water [99]. Both the back reflecting mirror and the lenses forming the *cat-eye* [29] are fastened using epoxy resin. The currently employed technique of using silica gel to maintain a constant humidity should be superseded by a more permanent solution. A sealed resonator can therefore add an import part to the day-to-day stability of the experiments.

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A. Appendix

A.1. Multimeter Settings for the Comparison Test

The Multimeters were configured for maximum stability and similar conversion times using the following settings via SCPI. For better readability, all commands are shown unabridged.

HP 3458A

```
PRESET NORM;  # reset the device
TARM HOLD;  # stop readings
BEEP;
OFORMAT ASCII;  # return text
TRIG AUTO;  # trigger when ready
NRDGS 1,AUTO; # take 1 reading
DCV 10;
AZERO ON;  # enable autozero
NDIG 9;
NPLC 100;
FIXEDZ OFF;  # High input impedance
TARM AUTO;  # enable readings
```

Keithley Model 2002

```
*CLS;  # clear events and errors
*RST;  # reset all settings
*OPC?;  # wait until device is reset
:INITiate:CONTinuous OFF;  # disable continuous initiation
:ABORt;  # place K2002 in idle
:SYSTem:AZERo:STATe ON;  # enable autozero
:SYSTem:AZERo:TYPE SYNChronous;  # azero for every reading
:SYSTem:LSYNc:STATe ON;  # line sync
:SENSe:VOLTage:DC:RANGe:UPPer 20;
:SENSe:VOLTage:DC:DIGits 9;
:SENSe:VOLTage:DC:NPLCycles 10;
:SENSe:VOLTage:DC:AVERage:COUNt 4;  # the averaging length
:SENSe:VOLTage:DC:AVERage:TCONtrol REPeat;  # filter type
:SENSe:VOLTage:DC:AVERage:ADVanced:STATe OFF;
:SENSe:VOLTage:DC:AVERage:STATe ON;  # Enable averaging
:FORMat:DATA REAL,64;  # read data as doubles
:FORMat:ELEMents READing;  # only return the reading
:FORMat:EXPonent HPRecision;  # Scientific notation
:INITiate:CONTinuous ON;  # Enable continuous triggering
```

Keysight 34470A

```
:SYSTem:BEEP;
:ABORt;
*RST;
*CLS;
:CONFigure:VOLTage:DC;
:SENSe:VOLTage:RANGe 10;
:SENSe:VOLTage:ZERO:AUTO ON;  # enable autozero
:SENSe:VOLTage:NPLCycles 100;
:SENSe:VOLTage:IMPEdance:AUTO ON;  # High input impendance
:FORMat:DATA ASCii,9;  # return 9 digits ASCII
```

Keithley DMM6500

```
SYSTem:BEEPer 500, 0.2;
ABORt;
*RST;
*CLS;
:SENSe:FUNCtion:ON "VOLTage:DC";
:SENSe:VOLTage:DC:RANGe:UPPer 10;
:SENSe:VOLTage:DC:LINE:SYNC ON;
:SENSe:VOLTage:DC:AVERage:COUNt 9;  # the averaging length
:SENSe:VOLTage:DC:AVERage:TCONtrol REPeat;  # filter type
:SENSe:VOLTage:AZERo:STATe ON;  # enable autozero
:SENSe:VOLTage:DC:NPLCycles 10;
:SENSe:VOLTage:INPutimpedance AUTO;  # High input impedance
:SENSe:VOLTage:DC:AVERage:STATe ON;  # Enable averaging
:FORMat:DATA ASCii;  # read data as double instead of text
:FORMat:ASCii:PRECision 16;  # return 16 digits ASCII
:DISPlay:VOLTage:DC:DIGits 6;  # set the screen to 6 digits
```

A.2. Querying the TimescaleDB via SQL

SQL query to extract binned data from the Timescale DB of two sensors in lab 011: 011_humidity and 011_temperature. The data returned is of the form date,humidity,temperature. The database groups the asynchronous data into bins of 6 h and averages the data inside those bins. The timeframe is from 2022-01-01 until 2023-01-01. In addition, the Tinkerforge sensors will only send a new value, if it has changed, the last observation must be carried forward. This is done using the *locf()* function call in SQL query.

SQL query

```sql
SELECT
  time
  ,data_values [1] humidity  --1st value in the array
  ,data_values [2] temperature --2nd value
FROM (
  SELECT
    bucket as "time"
    ,array_agg("data") as data_values
  FROM (
    SELECT
      time_bucket('6h',"time") AS "bucket"
      ,sensor_id
      ,locf(avg(value)) AS "data"
    FROM sensor_data
      WHERE
      time BETWEEN
        '2022-01-01T00:00:00.00Z' AND '2023-01-01T00:00:00Z'
      AND sensor_id IN (
        SELECT id
        FROM sensors
        WHERE
          label = '011_humidity' OR
          label = '011_temperature' AND
          enabled
      )
    GROUP BY bucket, sensor_id
    ORDER BY bucket
  ) t1
  GROUP BY "bucket"
) t2
```

When attempting to derive PID parameter for the lab temperature controller, controller output and the sensor output is needed. This query will compile the data in buckets of 2 s. Missing data from the Tinkerforge sensors, which will only update their output on a change, is interpolated by filling the gap with the previous value. The order of the array depends on the values of the sensor ids and needs to be adjusted accordingling for each query.

SQL query

```sql
SELECT
  timestamp as time,
  arr[2] as output,
  arr[1] as "temperature room",
  arr[3] as "temperature labnode"
FROM (
  SELECT
    timestamp,
    array_agg(value) arr
  FROM (
    SELECT
      time_bucket_gapfill('2.000s',"time") AS "timestamp",
      sensor_id,
      locf(avg(value)) as value
    FROM sensor_data
    WHERE
      "time" BETWEEN
        '2022-09-22T04:10:00Z' AND '2022-09-22T10:30:00Z'
      AND
      sensor_id IN (
        SELECT id FROM sensors
        WHERE
          label = '011_temperature_back' OR
          label = '011_temperature_labnode_back' OR
          label ='011_output_aircon_back'
      )
    GROUP BY timestamp,sensor_id
    ORDER BY timestamp,sensor_id
  ) t1
  GROUP BY timestamp
) t2
```

A.3. The Transconductance Amplifier with a MOSFET

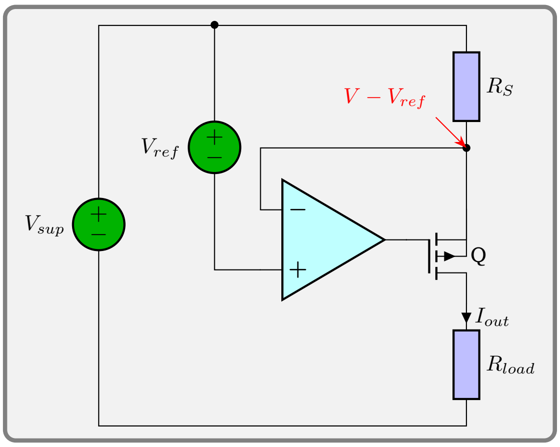


Figure A.1.: Transconductance amplifier with a p-channel MOSFET.

The amplifier shown in figure A.1 is a feedback transconductance ampplifier as discussed in [187]. Its transfer function can be derived using the techniques presented in section 3.5.1. As a reminder, the general transfer function is definded as:

$$P(s) = \frac{I_{out}}{V_{ref}} \equiv A_f \,. \tag{A.1}$$

The closed-loop transfer function is sometimes also called gain-with-feedback A_f [187] or noise-gain.

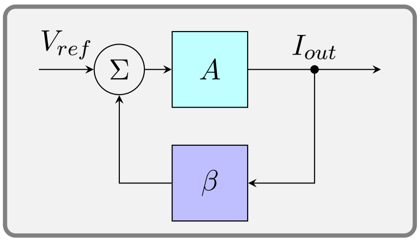


Figure A.2.: Block diagram of an amplifier with feedback β and gain A.

For the system shown in figure A.2, the closed-loop gain A_f can be written as

$$A_f = \frac{A}{1 + A\beta} \overset{A \to \infty}{=} \frac{1}{\beta} \,. \tag{A.2}$$

For the ideal transconductance amplifier with infinite open-loop gain A it follows, that the gain is simply reduced by the feedback factor β. For the MOSFET source voltage shown in figure A.1, β can be easily determined by inspection. The ideal op-amp with infinite open-loop gain A_{ol} has the same voltage at the inverting and non-inverting input. This means that below R_S at the source node of the MOSFET denoted in red, the voltage must be $V - V_{ref}$. This implies, that the voltage V_{ref} is dropped across R_S, defining I_{out}. Using equation A.2, β can be calculated

$$A_f = \frac{I_{out}}{V_{ref}} = \frac{\frac{V_{ref}}{R_S}}{V_{ref}} = \frac{1}{R_S} \approx \frac{1}{\beta} \,. \tag{A.3}$$

Calculating the transconductance amplifier gain A requires a little more work and it is useful to switch to the small-signal model of the circuit. To build the small-signal model, a number of simplifications can be applied. In the same same way as it was done for the MOSFET with a source resistor in figure 3.44 on page 73, the AC component of V_{ref} can be set to zero, because it is considered constant and so can the supply voltage V. The load is also considered constant and hence shorted to ground. In order to ground V_{ref}, the non-inverting input of the MOSFET must be disconnected, because there still is the voltage v_id connected to it. The model includes the differential input resistance R_{id} between the inverting and non-inverting input of the op-amp, because for bipolar input op-amps, the differential input resistance can be as low as a few kΩ and must be considered. The common-mode input resistance of the op-amp inputs is typically serveral dozens of MΩ or higher and can be safely neglected. This leads to the small signal model shown in figure A.3. The MOSFET model is the Thévenin model introduced in figure 3.43b on page 72. Do note, that this model is for low frequencies only, as it neglects capacitive effects of the op-amp and mosfet. Capacitors are treated as having infinite impedance in this model.

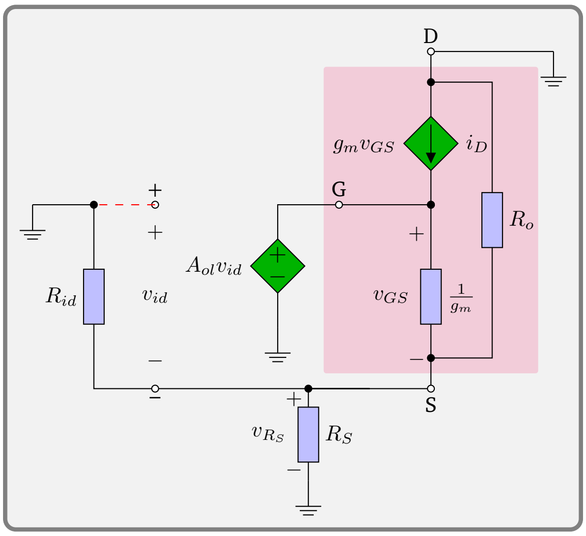


Figure A.3.: Small signal model for a transconductance amplifier with a MOSFET as shown in figure A.1

From the model in figure A.3, the following equations can be extracted in a similar fashion as it was done for the common-gate amplifier and equation 3.90 on page 73.

$$v_{GS} = A_{ol} v_{id} - V_{R_S} \tag{A.4}$$

$$V_{R_S} = i_D \left(R_o || R_S || R_{id}\right) = g_m v_{GS} \left(R_o || R_S || R_{id}\right) \tag{A.5}$$

$$\begin{aligned} A\beta = \frac{V_{R_S}}{V_{id}} &= \frac{g_m v_{GS} \left(R_o || R_S || R_{id}\right)}{\frac{1}{A_{ol}} \left(1 + g_m \left(R_o || R_S || R_{id}\right)\right) v_{GS}} \\ &= A_{ol} \frac{g_m \left(R_o || R_S || R_{id}\right)}{1 + g_m \left(R_o || R_S || R_{id}\right)} \end{aligned} \tag{A.6}$$

Dividing by R_S yields the open-loop gain of the transconductance amplifier, a quantity, that is interesting for calculating the MOSFET noise contribution:

$$A = \frac{A_{ol}}{R_S} \frac{g_m \left(R_o || R_S || R_{id}\right)}{1 + g_m \left(R_o || R_S || R_{id}\right)} \tag{A.7}$$

This leads to the closed-loop transfer function

$$A_f = \frac{A_{ol}}{R_S} \frac{g_m \left(R_o||R_S||R_{id}\right)}{(A_{ol}+1) g_m \left(R_o||R_S||R_{id}\right) + 1}\,, \tag{A.8}$$

and finally the output impedance of the transconductance amplifier can be calculated using the output impedance of the common-gate amplifier 3.91, calculated on page 73.

$$\begin{aligned} R_{out} &= \left(1 + A\beta\right) R_{out,cg} \\ &= \left(1 + A_{ol} \frac{g_m \left(R_o||R_S||R_{id}\right)}{1 + g_m \left(R_o||R_S||R_{id}\right)}\right) \left(g_m R_S R_o + R_o + R_S\right) \\ &\overset{A_{ol} \gg 1}{\approx} A_{ol} \frac{g_m \left(R_o||R_S||R_{id}\right)}{1 + g_m \left(R_o||R_S||R_{id}\right)} \left(g_m R_S R_o + R_o + R_S\right)\,. \end{aligned} \tag{A.9}$$

Equation A.9 can be simplified for typical applications by approximation of $g_m \left(R_o||R_S||R_{id}\right)$. Using the example parameters for the `IRF9610` in saturation used previously on page 71 and additionally the ADI `AD797` [10] op-amp $g_m \left(R_o||R_S||R_{id}\right)$ with the following parameters

$$\begin{aligned} &I_D = 250\,\mathrm{mA}\,, \lambda = 4\,\mathrm{mV}^{-1}\,, V_{DS} = 3.5\,\mathrm{V}\,, R_S = 30\,\Omega\,, \\ &R_{id} = 7.5\,\mathrm{k\Omega}\,, \kappa = 0.813\,\mathrm{A\,V}^{-2}\,, A_{ol} = 20\,\mathrm{V\,\mu V}^{-1} \end{aligned}$$

one finds

$$\begin{aligned} R_o &= \frac{I_D}{\frac{1}{\lambda} + V_{DS}} = 1014\,\Omega \\ g_m &= \sqrt{2\kappa I_D \left(1 + \lambda V_{DS}\right)} = 0.642\,\mathrm{S} \\ g_m \left(R_o||R_S||R_{id}\right) &\approx g_m R_S \approx 18.63 \\ \frac{g_m \left(R_o||R_S||R_{id}\right)}{1 + g_m \left(R_o||R_S||R_{id}\right)} &\approx 0.95 \end{aligned}$$

Using typical parameters, it can be seen, that dropping the $\frac{g_m(R_o||R_S||R_{id})}{1+g_m(R_o||R_S||R_{id})}$ term will only lead to error of about $5\,\%$. Given the datasheet uncertainties for the MOSFET related parameters on the order of $50\,\%$–$100\,\%$, it can be safely neglected, leading to the following approximations

$$R_{out} \approx A_{ol} \left(g_m R_o R_S + R_o + R_S\right) \tag{A.10}$$

$$A_f \approx \frac{1}{R_S}\,.$$

The approximation for the output impedance holds true when $g_m R_S \gg 1$, which typically is the case. While R_S might become small, this is compensated by an increase in g_m in this application, because a smaller source resistor implies a higher output current, demanding a MOSFET with a higher transconductance, so the product of $g_m R_S$ remains constant.

It can therefore be said, that the op-amp is simply amplifying the output impedance of the MOSFET along with the source resistor and the closed-loop gain is defined entirely by R_S, a very convenient property.

If the model is to be considered at frequencies $\omega > 0$, A_{ol} can be replaced by the first order approximation of the op-amp gain as

$$A_1(\omega) = \frac{A_{ol}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}\,, \tag{A.11}$$

which is valid for most compensated op-amps, which have a dominant pole at $\omega_c \approx 1\,\mathrm{Hz}$.

A.4. Simulating Current Source Properties in LTSpice

This section explains some more advanced concepts of LTSpice [152] to simulate device properties and circuit properties used when working with the current source presented in section 3.8.4. This section does not aim at explaining the basic functions of LTSpice, but rather some special functions. It is left to the interested reader to acquire those basic skills. The example presented here, allows to generate the MOSFET *Typical Output Characteristics* plot found in datasheets, the transconductance of a MOSFET and the (dynamic) output impedance of a current source. The typical output characteristics can be used to compare the model with the datasheet or with measurements taken. Comparing these model parameters with the datasheet can establish confidence, that the simulation results can be transferred to a real circuit.

A.4.1. MOSFET Typical Output Characteristics

The output characterisic is a graph found in all MOSFET datasheets and is shown below in figure A.4.

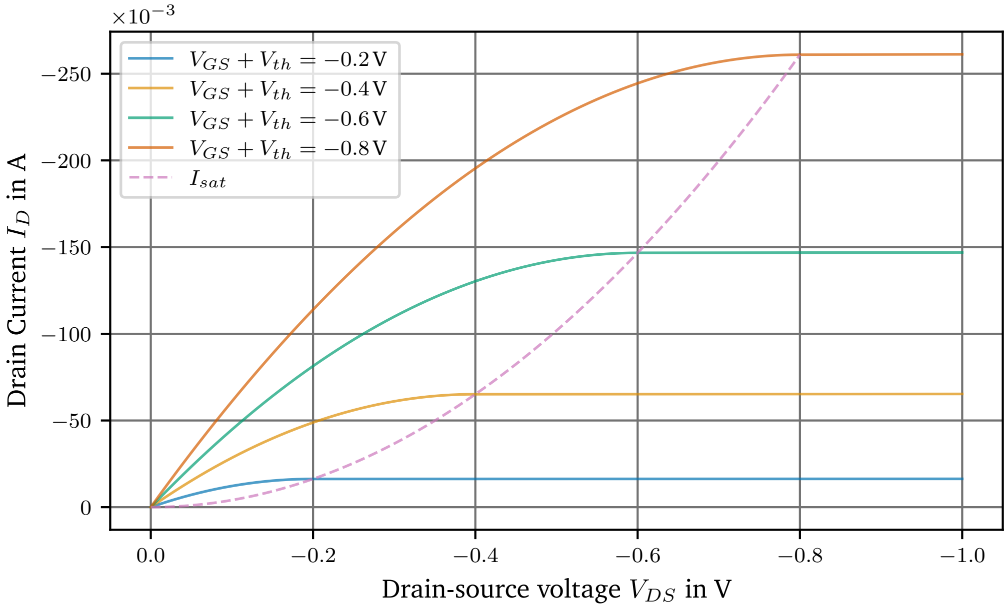
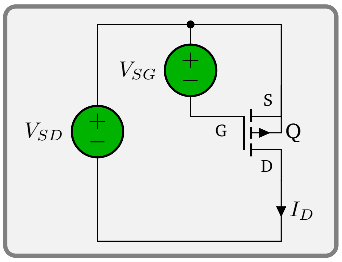


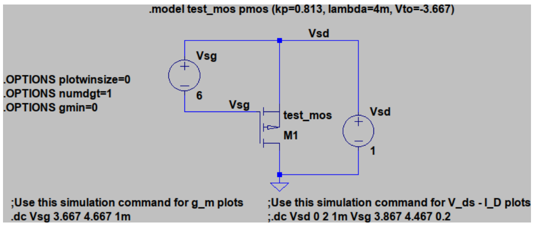
Figure A.4.: Simulated drain current over the drain-source voltage, also called output characteristics of a MOSFET.

Plotting this graph allows to compare the model to the datasheet or the measured values in order to tweak the model. To create this graph, the simulation file found in the folder source/spice/mosfet_gm-id.asc as part of this document can be used. The SPICE simulation for the output characteristics of the MOSFET simulates the following circuit shown in figure A.5a.

Do note, that V_{DS} and V_{GS} are inverted and given as V_{SD} and V_{SG}. The reason is, that the plotter in LTSpice works better with positive numbers to guess the correct scaling of the axis.



(a) P-channel MOSFET under test.



(b) LTSpice model.

Figure A.5.: P-channel MOSFET circuit and its LTSpice model.

Figure A.5b shows the same circuit drawn in LTSpice. The MOSFET parameters are entered using the **.model** syntax

.model test_mos pmos (kp=0.813, lambda=4m, Vto=−3.667)

with the parameters $\kappa = 0.813\,\mathrm{A\,V^{-2}}$, $\lambda = 4\,\mathrm{mV^{-1}}$ and $V_{th} = -3.667\,\mathrm{V}$. The options **plotwinsize** and **numdgt**, shown in figure A.5b, make sure, that LTSpice does not compress the output data and increases the floating point precision. This is important, because I_D spans a large range of values. Setting **gmin** to 0 prevents LTSpice from adding a small transconductance to every pn-junction, thus changing the MOSFET model. Finally, the most important command is the **.dc** command, which instructs LTSpice to step the voltage sources V_{SD} and V_{SG} to evaluate I_D over V_{SD}. The command

.dc Vsd 0 2 1m Vsg 3.867 4.467 0.2

steps the voltage source V_{SD} from 0–$2\,\mathrm{V}$ in steps of $10\,\mathrm{mV}$ and for each step of V_{SD}, steps V_{SG} from 0.2–$0.8\,\mathrm{V} - \mathrm{V_{th}}$ in steps of $200\,\mathrm{mV}$. Plotting

Id(M1)

results in the plot shown in figure A.4, which can be found in datasheets as the *Typical Output Characteristics* plot. To draw a line in the graph showing the point where the MOSFET enters the saturation region, denoted I_{sat} in figure A.4, as given by equation 3.79, add the following plot command to the graphing window and resecale the axis.

0.5*0.813*1A/1V**2*V(vsd)**2

This command must be adjusted for the value of κ and do note, that κ is entered with units of $\mathrm{A/V^2}$ to correctly display the output in A.

A.4.2. MOSFET Transconductance

Another interesting property to plot is the transconductance g_m of the MOSFET. Again, using the same model used previously in figure A.5b and from equation 3.80 we known that g_m is defined as

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{DS}=const}.$$

To derive g_m, we need to generate values of $I_D(V_{GS})$. This can again be done by stepping V_{GS}

.dc Vsg 3.667 4.667 1m

To produce a smooth plot, the the steps size of V_{SG} was decreased to 1 mV. V_{DS} is constant in this plot and can be set using the voltage source V_{SD}. The MOSFET is intentionally biased into the saturation region at $V_{DS} = -1\,\mathrm{V}$ as can be seen in figure A.4.

LTSpice is now able to numerically differentiate the data, which can be invoked by by plotting

```
−d(Id(M1))
```

The minus sign comes from the inverted $V_{SG} = -V_{GS}$. To plot g_m over I_D, the formula for g_m given above needs to entered manually into the *Expression Editor* by right clicking the expression label on top of the graph. Finally, the x-axis must be changed to $Id(M1)$, leading to the plot in figure A.6.

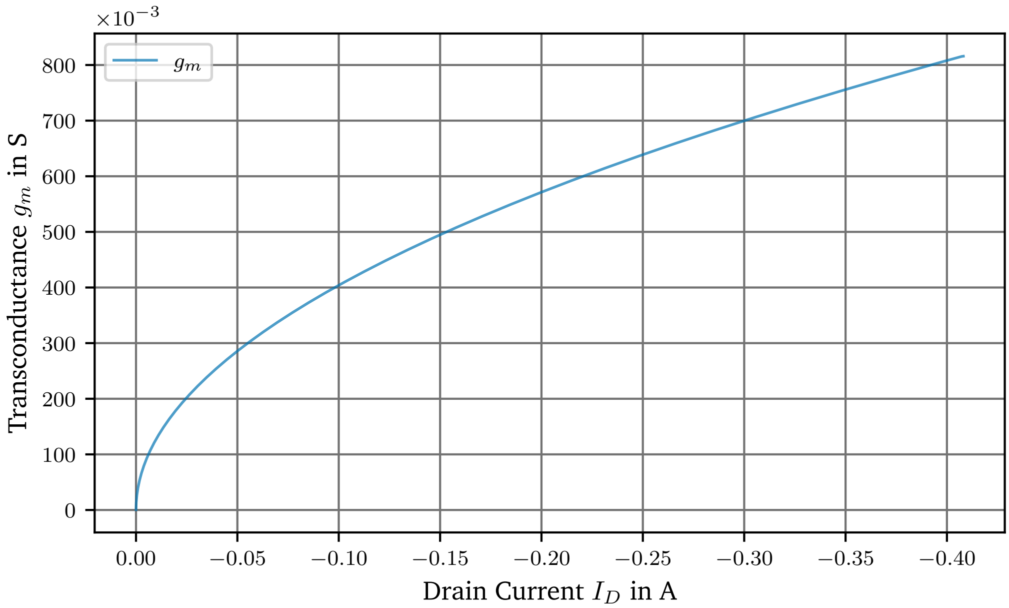


Figure A.6.: Simulated transconductance in saturation at $V_{DS} = -1\,\mathrm{V}$.

As expected from equation 3.80, g_m is proportional to the square root of I_D when the MOSFET is in saturation.

As a sidenote, if the MOSFET model includes gate leakage, this leakage current may influce the calculation of g_m, especially at very low currents. In this case, it it better to plot the positive derivative of the source current $Is(M1)$, which does not include the leakage current.

```
d(Is(M1))
```

A.4.3. Output Impedance

This sections will explain how to calculate the dyncamic output impedance using LTSpice. The example circuit used, is the precision current source from section 3.8.4. The dynamic output impedance was defined in equation 3.83 as the inverse of the conductance leading to

$$R_{out} = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} .$$

Using the technique presented in the previous section, the obvious solution would be to again use the **.dc** sweep command and then numerically differentiate the result. Unfortunately this will lead to disappointing results, because the output impedance in question is very large and the limits of the numerical precision will be reached, nicely demonstrating the boundries of numercial methods. LTSpice allows to increase the numeric precision to double floating point values using the option **numdgt**

```
.options numdgt=15
```

Unfortunately, this only forces LTSpice to interally use the double floating point number format, which does have a precision of 53 bit which means $\log_{10}\left(2^{53}\right) = 15.95$ decimals. So instead of using the large-signal model of the MOSFET, it becomes more convenient to evaluate the small-signal model

$$R_{out} = \frac{v_{load}}{i_D} = \frac{v_{DS}}{i_D}$$

at several different points of V_{DS}, thereby reconstructing the large-signal model from rasterized versions of the small-signal model. For the small-signal model, $v_{DS} = v_{load}$, because the supply voltage and the voltage across the sense resistor can be considered constant, so any change in the voltage across the load must cause the opposite change in the source-drain voltage $v_{SD} = -v_{DS}$.

To run this simulation the small-signal simulation must be used and additionally some commands not available through the graphical user interface need to be entered by hand.

The LTSpice simulation is shown in figure A.7 and will now be explored.

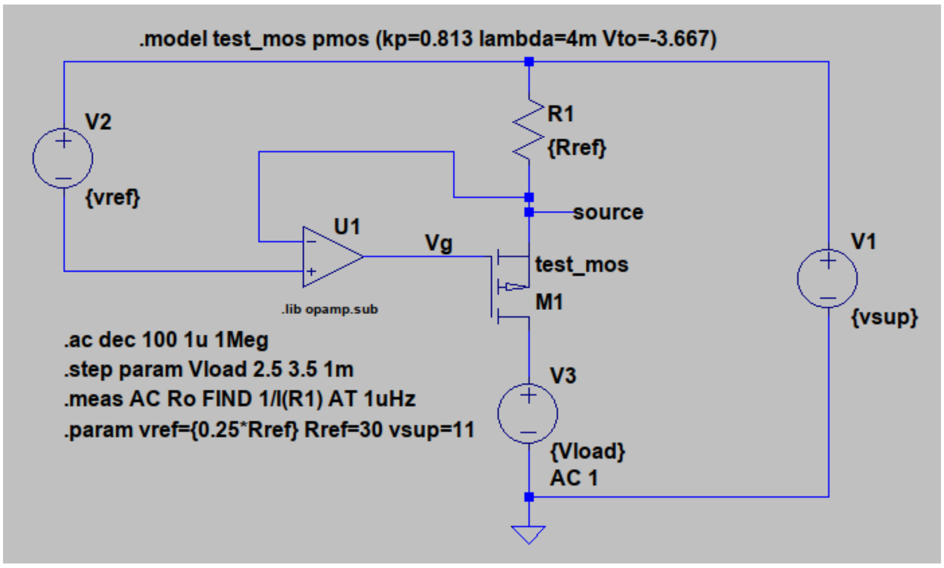


Figure A.7.: LTSpice model.

The simulation uses the same MOSFET model as above and adds an ideal op-amp to control the loop. The op-amp model has a open-loop gain of 2×10^6 and a gain-bandwidth product of 10 MHz as can be approximated from the the datasheet of the AD797 [10] and is also given in table 3.10. This leads to a 3 dB corner frequency of 5 Hz, which will be interesting later.

To access the small-signal model the **.ac** command is used, because LTSpice uses the small-signal model to calculate the ac response of a circuit at a given working point. The command

```
.ac dec 100 1u 1Meg
```

calculates the ac response from $1\,\mu$Hz to $1\,$MHz with 100 points/decade. Additionally, as discussed, the load will be stepped, by stepping voltage source in the source leg of the MOSFET. We use a voltage source in this case instead of a resistor, because the AC impedance of a laser diode is typically very small. For the working point, it does not matter whether V_{load} is resistive or not. To step the voltage source, the command

```
.step param Vload 2.5 3.5 1m
```

is used to change V_{load} from 2.5–$3.5\,$V in steps of $1\,$mV, which is exactly the maximum V_{DS}, which is $V_{sup} - V_{ref} = 3.5\,$V. This is done to show the effect of the complete loss of regulation. The last thing to do, is to extract the desired output impedance from the many stepped small-signal simulations. This can be done using the **.meas** command telling LTSpice to save a single value at a certain frequency from each step.

```
.meas AC Ro FIND 1/I(R1) AT 1uHz
```

The **.meas** command shown will save the value of $\frac{1}{i_D} = \frac{1}{I(R1)}$ at $1\,\mu$Hz to the (error) log file whenever the **.ac** command is run. The value of v_{DS} was already set to $1\,\mathrm{V_{rms}}$ in the LTSpice simulation as shown in figure A.7, thus $\frac{1\,\mathrm{V}}{I(R1)} = R_{out}$. The current through sense resistor instead of i_D was chosen because it is numerically more stable and since there is no gate current it is the same as i_D. The frequency were the R_{out} is measured was chosen to be well below the corner frequency of the op-gain, which was calculated above to be $5\,$Hz. This gives the near DC output impedance of the current source.

To plot the values are stored in the log file, click on *View* in the top menu, then *SPICE Error Log*. Now right-click on the error log and select *Plot stepp'ed .meas data*. This will open a new plot window showing the output impedance curve.

Those results are discussed in more detail in section 3.8.5.

A.5. MOSFET Noise Sources

This section gives the reader a quick overlook of the noise sources found in MOSFETs. A good overview of different types of noise in MOSFETs can also be found in [135] and goes beyond the scope presented here.

The MOSFET wideband noise can be attributed to thermal noise in the channel [68]. Der Ziel developed a model for the thermal noise in the saturation region of the MOSFET, while the classic Johnson–Nyquist noise [101] can be used for the ohmic region as it behaves like a voltage controlled resistor. This results in the noise density of

$$i_{n,thermal} = \begin{cases} \sqrt{4k_B T \frac{2}{3} g_m} & \text{saturation} \\ \sqrt{4k_B T g_{DS}} & \text{ohmic} \end{cases} \tag{A.12}$$

Using the example parameters from table 3.10, one finds

$$\begin{aligned} g_m &= \sqrt{2\kappa I_D \left(1 + \lambda V_{DS}\right)} = 0.642\,\mathrm{S} \\ T &= 25\,°\mathrm{C} \\ i_{n,thermal} &\approx 83.9\,\mathrm{pA}/\sqrt{\mathrm{Hz}}\,, \end{aligned} \tag{A.13}$$

the equivalent noise of a resistor $R_D = \frac{3}{2g_m} = 2.3\,\Omega$.

A more detailed analysis, which also points out the limits of the model above can be found in [213].

Additionally the MOSFET also suffers from shot noise due to leakage through the gate, but this can be neglected because this leakage current is very small and even a relatively large current of $1\,\mathrm{mA}$ only produces

$$i^2_{n,shot} = \sqrt{2eI_D} \tag{A.14}$$

$$\approx 1.8\,\mathrm{pA}/\sqrt{\mathrm{Hz}}\,. \tag{A.15}$$

Shot noise becomes interesting, when the MOSFET is used well below threshold or at higher frequencies, because, then the parasitic gate-drain capacitor C_{GD} will leak from the input to the ouptut as can be seen in figure A.8. Figure A.8 shows the different parasitic capacitances of a MOSFET.

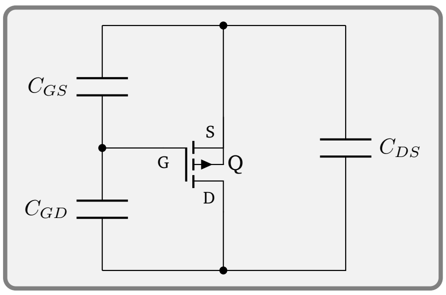


Figure A.8.: Parasitic capacitances of a MOSFET.

These capacitances can also be found in datasheets, although not directly, because they are defined as

$$C_{iss} = C_{GD} + C_{GS} \quad \text{input capacitance} \tag{A.16}$$
$$C_{oss} = C_{DS} + C_{GD} \quad \text{output capacitance} \tag{A.17}$$
$$C_{rss} = C_{GD} \quad \text{reverse transfer capacitance}\,. \tag{A.18}$$

Regarding low frequencies, MOSFETs also show strong flicker noise. It is known from section 3.6.1, that the sources of flicker noise are not clearly understood, so there are several theories regarding flicker noise models for MOSFETs.

An an empirical model given by [135, 172] can be used to describe the flicker noise as

$$i_{n,flicker} = \sqrt{\frac{K_f I_D}{C_{ox} L^2} \frac{1}{f}}\,. \tag{A.19}$$

This model is presented here, because it is both supported and easy to implement in LTSpice. While the parameter K_f is approximately $2 \times 10^{-10}\,\mathrm{fC}^2/\mu\mathrm{m}^2$ [135] for p-channel MOSFETs, the gate width and length W, L are device specific und unfortunately not given by the manufacturers. The typical corner frequency for MOSFETs, though, is between a few hundred kHz and a few dozen MHz depending on the size of the transistor. Larger transistors tend to show lower noise. Hence older processes are prefered in this regard. Given that the noise is uncorrelated, the total noise of the MOSFET in saturation can be written as

$$i_n = \sqrt{4 k_B T \frac{2}{3} g_m + \frac{K_f I_D}{C_{ox} L^2} \frac{1}{f}} \tag{A.20}$$

As a reminder, the MOSFET is a (transconductance) amplifier, that takes a voltage at the input and outputs a current. To make the noise figures comparable, the noise is divided by the gain g_m. This called the input referred noise. The input referred (voltage) noise e_n is given by:

$$e_{n,thermal} = \sqrt{4 k_B T \frac{2}{3 g_m}} \tag{A.21}$$
$$e_{n,flicker} \overset{3.81}{\approx} \sqrt{\frac{K_f}{2 \kappa C_{ox} L^2} \frac{1}{f}} \tag{A.22}$$

We can see, that flicker noise is fully determined by process parameters in this model.

A.6. Building an Injection Transformer

Typically devices in the lab at APQ are supplied with a positive and a negative voltage – usually $\pm 15\,\mathrm{V}$. This is readily achieved using two floating outputs of a power supply and connecting them in series, then tapping off the center as the common voltage around which the $\pm 15\,\mathrm{V}$ is centered.

When testing new devices like the current driver or temperature controller developed for this work, it is sometimes necessary to inject a disturbance into the power rails. This setup requires a positive and a negative line injector like the positive injector `PB02` presented in [38] and the negative Picotest `J2123A` line injector. When driving these injectors it is desirable to drive them both from a single output of a VNA. The Picotest `Bode 100` used for many low frequency applications does not have galvanically isolated inputs and outputs. Galvanic isolation can be achieved using a transformer to drive the injectors. Additionally using a transformer, it is easy to create two outputs, that are π rad out phase. Building one such transformer is explained in this section.

Before proceeding to the build instructions it is useful to have look at amodel of the transformer with some parasitics to better understand the design decisions. A simple model is shown in figure A.9.

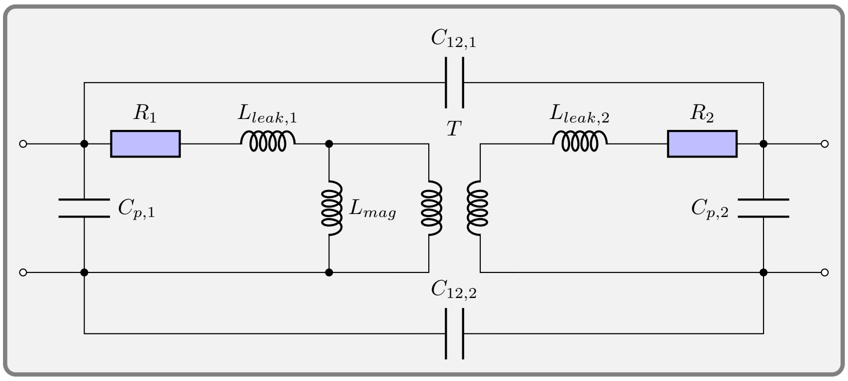


Figure A.9.: A simple model of a transformer, neglecting core losses and frequency and loading dependent effects.

The model only includes the major parasitic effects and their importance will now be discussed briefly. Starting with the resistance of the coil R_x, which should be well below $1\,\Omega$ plays a rather small role, but will introduce some losses and dampen any resonances. The magnetizing inductance L_{mag} represents the energy, that is stored in the core. In this model it is the flux, that travels inside the core. Using a material with higher permeability increases L_{mag}, which is better, but one must look out not to saturate the core. The leakage inductance L_{leak} is the part of the magnetic field, that is lost, where the field lines do not pass through the secondary winding. This should idealy be low and can be lowered by tightly winding the transformer. Tightly winding the transformer has the downside of increasing the isolation capacitance $C_{12} = C_{12,1} + C_{12,2}$. Having less leakage inductance improves the high frequency behaviour of the transformer though, therefore a tighly wound bifilar winding scheme is chosen. Finally, there is a coupling capacitance C_p between the two input (and output) nodes, which becomes problematic at higher frequencies, when the impedance of the transformer goes up, while the impedance of the C_p goes down.

To summarize, for a good injection transformer, that has a flat transfer function out to high frequencies, it is important to keep L_{mag} high, by using a high permeability material like a nanocrystalline core and to keep L_{leak} low by tightly coupling the windinds. These choices unfortunately make a bad isolation transformer as will be shown later based on the electrical parameters of the finished transformer.

It was already said, that a flat transfer function is desired, so the frequency range of interest must be definded. The `Bode 100` covers a frequency range from 1 Hz to 50 MHz. The whole range is a bit too much to ask for, because the low frequency end requires a large core to cope with the increased flux. The many windings required, will then cause problems at the high end, due to L_{leak}, which then limits the high frequency response. This transformer aims for a good compromise to cover most of the range, while accepting a limited performance at the corners.

This concludes the discussion of the design choices as the intricate details of the parasitic effects of different types of transformers, their geometry and materials are not discussed here for simplicity. The interested reader may look up [186] for more details. This section is only intended to be a simple instruction manual to allow the reader to build an affordable alternative to fairly expensive commercial solutions with similar performance.

The materials required are:

- A box like the the Hammond `1590B`.
- A nanocrystalline ferrite core is preferred for example, a Vacuumschmelze `T60006-L2040-W452` or `T60006-L2040-W424`.
- 3 m of Cat5e Ethernet cable. Preferably FEP insulated like Belden `7928A`, but any other will also do.
- 2–3 isolated BNC connectors like the Amphenol `031-10-RFXG1`. You will need 3 connectors for the center tapped version and 2 for a 1:1 transformer.
- 1 Cinch Connectivity Solutions `111-2223-001` earthing connector.
- Drills in sizes 6 mm and 9.7 mm.
- Kapton tape

The author used a Vacuumschmelze `T60006-L2040-W452`, because it was available at the time, but the `T60006-L2040-W424` might be a better choice, because of its higher inductance per turn (101 µH at 10 kHz vs. 12.2 µH at 10 kHz). The `T60006-L2040-W452` has a slightly smaller inner diameter (25 mm vs 32 mm), so less windings will fit onto the core, this may offset some of the higher inductance coefficient of the core, but fewer windings also reduce the inter-winding capacitance due to the shorter cable length.

The target is 46 turns of the twisted pair cable around the core for a `T60006-L2040-W452`. This should give a tight fit. When center tapping the transformer do make sure to accurately count and then exactly cut one wire in the center. Do not cut the wire in advance, because you will need to leave some overhead at the beginning to leave plenty of room to solder the cable to the BNC connectors.

When done winding the transformer, wrap it with Kapton tape to secure the windings. It is recommended to test it before final assembly. Carefully solder the BNC connectors to the ends and test it with a VNA,. These connectors will later be removed again. Make sure to calibrate the VNA beforehand and when the transformer matches the requirements, it is time to mount it in the box.

The box requires one 6 mm hole for the earthing connector and 3 9.7 mm holes for the BNC connectors. The finished device is shown in figure A.10.

Figure A.10.: Photo of the finished injection transformer in its box. The black rubber is used to secure it in the box.

After final assembly, the injection transformer was tested using a Picotest `Bode 100` VNA and also compared against a commercial Picotest `J2101A` 1:1 transformer.

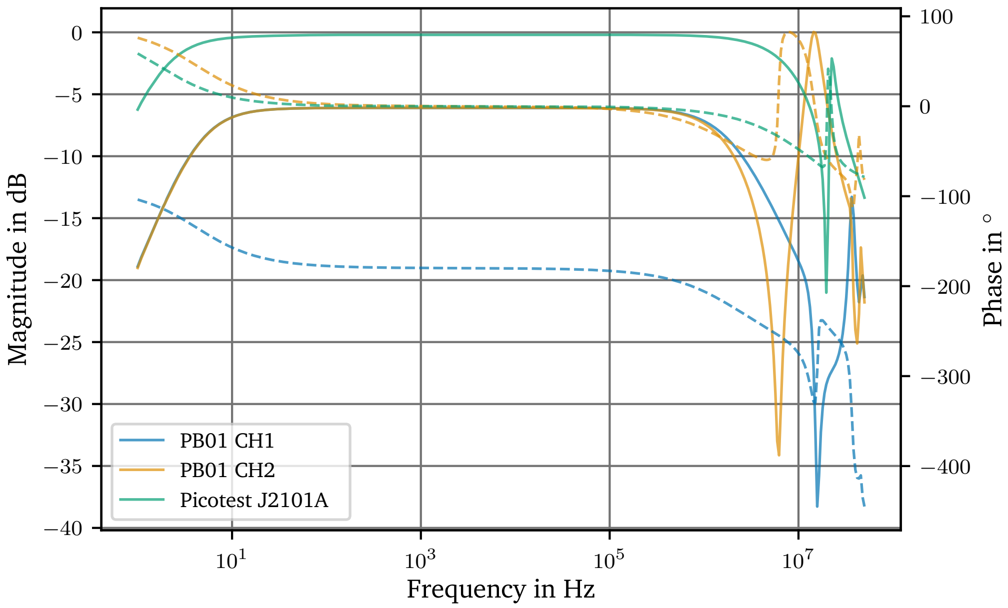


Figure A.11.: Bode plot of both channels of the injection transformer `PB01` and the Picotest `J2101A`. The solid lines are the magnitude, the dashed lines is the phase.

The VNA settings for the Bode plot were chosen to make sure, the core does not saturate, So the exitation is very moderate. CH1 is used to monitor the primary side and CH2 monitors the secondary side. The most important VNA parameters can be summarized as

- Output $50\,\Omega$ $223.6\,\mathrm{mV_{rms}}$ ($0\,\mathrm{dBm}$)
- CH1 (measuring VNA output) set to $1\,\mathrm{M}\Omega$
- CH2 set to $50\,\Omega$
- Receiver bandwidth $1\,\mathrm{kHz}$

The choice of terminating the transformer output into $50\,\Omega$ is fairly arbitrary, but does have a bearing on the frequency response. It seems Picotest is terminating into $5\,\Omega$ and $50\,\Omega$ [64] in their setup. Using a high impedance termination on the secondary side leads to a self-resonance peak around $6\,\mathrm{MHz}$. The self resonance is caused by the inductance of the transformer together with the parallel capacitance of the winding. See below for the electrical parameters.

From the Bode plot shown in figure A.11 It can be seen, that the transfer gain of the two outputs is identical within the limits of the measurement and as expected at $-6\,\mathrm{dB}$. Do remember, that the `PB01` transformer has a 1:0.5 ratio, because it is bifilar wound and center-tapped. Additionally, CH1 is π rad out out phase. The reason is the center tapping, both output are referenced to the same ground in the middle, so one output must be out of phase compared to the other.

The lower $-3\,\mathrm{dB}$ point is at around $4.5\,\mathrm{Hz}$ and the upper at $2\,\mathrm{MHz}$ and $1.7\,\mathrm{MHz}$ for CH1 and CH2 respectively. The Picotest `J2101A` has a $-3\,\mathrm{dB}$-bandwidth of $2\,\mathrm{Hz}$ to $8.4\,\mathrm{MHz}$, which is quite a bit better at the low end a fair bit worse at the high end. The claimed *usable Bandwidth* (whatever that is) is $10\,\mathrm{Hz}$ to $45\,\mathrm{MHz}$.

Lastly, some electrical properties of the injection transformers as measured using the `Bode 100` and confirmed using an LCR Research `LCR Pro1 Plus`.

Device	`PB01`	Picotest `J2101A`
Inductance @ $1\,\mathrm{kHz}$	$20.3\,\mathrm{mH}$	$68.8\,\mathrm{mH}$
Series resistance R_1	$540\,\mathrm{m}\Omega$	$470\,\mathrm{m}\Omega$
Isolation capacitance @ $10\,\mathrm{kHz}$ C_{12}	$95\,\mathrm{pF}$	$80\,\mathrm{pF}$
$-3\,\mathrm{dB}$-bandwidth	$4.5\,\mathrm{Hz}$ to $1.7\,\mathrm{MHz}$	$2\,\mathrm{Hz}$ to $8.4\,\mathrm{MHz}$

A final word regarding the isolation capacitance of the transformers. These two transformers are by no means isolation transformers, the isolation capacitance is far too high for this use-case. The reason for such a high capacitance is the type of wiring and winding chosen. For better high frequency performance a twisted pair was chosen. Here, the wires are in very close contact to each other and there is no shield in between. The twisted pair was measured to have about $25\,\mathrm{pF/m}$ @ $10\,\mathrm{kHz}$ after removing the jacket, which resulted in much looser twists so the $95\,\mathrm{pF}$ seems to be a reasonable deviation from the expected $150\,\mathrm{pF}$ (Cat5e is supposed to have around $50\,\mathrm{pF/m}$).

A.7. The Howland Current Source

This section discusses the Howland current source and derives an equation for the output impedance with regard to several imperfections found in the non-ideal circuit. The discussion includes both the classic Howland current source (HCS) [190] and the *improved* Howland current source.

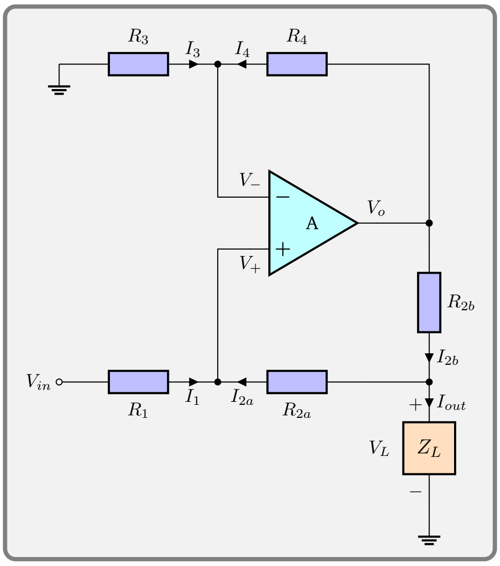


Figure A.12.: The Howland current source. Using $R_{2a} = 0\,\Omega$ is the classic version, while $R_{2a} \neq 0\,\Omega$ is the *improved* version.

First, an ideal circuit is discussed with perfectly matched resistors and an ideal op-amp, then the effects of an imperfect resistor matching and an non-ideal op-amp with finite gain are discussed, finally an equation including both effects is given. This model can then be used to create a list of requirements for the components. Other parasitic effects like stray capacitance or the input capacitance of the op-amp is neglected. This is valid for the low frequency range of interest. The op-amp is also assumed to be ideal with regard to the input bias current and voltage offset. While the input bias current depends on the type of op-amp, it is typically less than a few nA or even pA if a JFET op-amp is used. This is far less than the currents required for the applications in this work. The same argument applies to the offset voltage. The interested reader may look up some of those details in [140]. A discussion of the effect of the parasitic capacitance and its compensation can be found in [217], which demonstrates a Howland current source with a -3 dB bandwidth of 450 kHz with an output impedance of more than 1 MΩ.

The calculations to derive the model were done using SymPy [52], a Python framework for symbolic calculation. The full source code can be found in data/simulations/howland_current_source.ipynb as part of the online suplemental material [42].

In order to calculate the output impedance, the voltage at the load is required, because the output impedance of the current source is

$$R_{out} = -\frac{\partial V_L}{\partial I_{out}} \tag{A.23}$$

The negative sign is due to the direction of the current I_{out}, which flows out of the output node as shown in figure A.12, but the passive sign convention is that the current must flow into the device, hence a minus is applied, see for example [109]. V_L can be found using Kirchhoff's current law applied to the inverting input node, non-inverting input node and output node. The inverting node is the most simple one, so it is best to start there. Assuming, that no current is flowing into the op-amp pin it can be seen that

$$\begin{aligned} I_3 + I_4 &= 0 \\ \frac{-V_-}{R_3} + \frac{V_o - V_-}{R_4} &= 0 \\ \Rightarrow V_o &= V_- \left(1 + \frac{R_4}{R_3}\right) . \end{aligned} \tag{A.24}$$

The non-inverting node can be calculated as follows.

$$\begin{aligned} I_1 + I_{2a} &= 0 \\ \frac{V_{in} - V_+}{R_1} + \frac{V_L - V_+}{R_{2a}} &= 0 \\ \Rightarrow V_+ &= \frac{R_1 V_L - R_{2a} V_{in}}{R_1 - R_{2a}} . \end{aligned} \tag{A.25}$$

Finally, the output node is given as

$$\begin{aligned} I_{2b} - I_{2a} - I_{out} &= 0 \\ \frac{V_o - V_L}{R_{2b}} - \frac{V_+ - V_L}{R_{2a}} - I_{out} &= 0 . \end{aligned} \tag{A.26}$$

The missing piece of the puzzle is a relationship between V_+ and V_-. Since the feedback loop is closed the relationship below exists. It can be simplified by neglecting the offset voltage and assuming an infinitely high open-loop gain A_{ol}. The latter assumption will be treated separately later.

$$\begin{aligned} V_{out} &= A_{ol}(V_+ - V_-) \\ V_- &= V_+ - \frac{V_{out}}{A_{ol}} \approx V_+ \end{aligned} \tag{A.27}$$

Using equations A.24, A.25, A.26, and A.27, the load voltage V_L can now be calculated.

$$V_L = \frac{(R_1 R_{2b} R_3 + R_{2a} R_{2b} R_3)\, I_{out} - ((R_{2a} + R_{2b})\, R_3 - R_{2a} R_4)\, V_{in}}{R_1 R_4 - (R_{2a} + R_{2b})\, R_3} \tag{A.28}$$

Having the load voltage, the dynamic output impedance is

$$\begin{aligned} R_{out} = -\frac{\partial V_L}{\partial I_{out}} &= \frac{R_1 R_{2b} R_3 + R_{2a} R_{2b} R_3}{(R_{2a} + R_{2b})\, R_3 - R_1 R_4} \\ &= \frac{R_{2b} + \frac{R_{2a} R_{2b}}{R_1}}{\frac{R_{2a} + R_{2b}}{R_1} - \frac{R_4}{R_3}} \end{aligned} \tag{A.29}$$

Looking at the denominator from equation A.29 it is clear, that the output impedance goes to infinity if

$$\frac{R_4}{R_3} = \frac{R_{2a} + R_{2b}}{R_1} . \tag{A.30}$$

It is also obvious that any deviation from the equality given in equation A.30 leads to a finite output impedance. This output impedance shall now be estimated. Similar to [81] an imbalance factor ϵ is introduced to describe the matching of the resistors.

$$\frac{R_4}{R_3} = \frac{R_{2a} + R_{2b}}{R_1} (1 - \epsilon) \tag{A.31}$$

Substituting equation A.31 into equation A.29 leads to the output impedance due to resistor mismatch

$$R_{out,m} = \frac{R_1 R_{2b} + (R_{2a} R_{2b})}{\epsilon (R_{2a} + R_{2b})} = \frac{(R_1 + R_{2a}) R_{2b}}{R_{2a} + R_{2b}} \frac{1}{\epsilon} \tag{A.32}$$

To give a value for the mismatch factor ϵ the resistor tolerances must be considered. Typically when building a Howland current source, a resistor array is used, to ensure tight matching of the resistors to satisfy equation A.30, so it is safe to assume the tolerance T for all four or five resistors is the same. This tolerance is typically between $5\,\%$ and $0.01\,\%$ for the highest quality resistors. Further assuming $R_{2a} + R_{2b} = R_2$, $R = R_1 = R_2 = R_3 = R_4$ and a maximum mismatch due to the tolerance equation ϵ can be calculated from equation A.31.

$$\begin{aligned} \frac{R(1+T)}{R(1-T)} &= \frac{R(1-T)}{R(1+T)} (1 - \epsilon) \\ \Rightarrow \epsilon &= \frac{4T}{(1-T)^2} \end{aligned} \tag{A.33}$$

For equal resistors values R, the output impedance due matching errors of those resistors degrades to

$$R_{out,m} = \left(\frac{R^2 - R_{2a}^2}{R} \right) \frac{(1-T)^2}{4T} \tag{A.34}$$

From equation A.34 the output impedance for the classic Howland current source with $R_{2a} = 0$ is easily found to be

$$R_{out,m,HCS} = R \frac{(1-T)^2}{4T} \approx \frac{R}{4T} \tag{A.35}$$

using the the Taylor expansion

$$\epsilon = \frac{4T}{(1-T)^2} = 4T + 4T^3 + \mathcal{O}(T^5)\,.$$

The improved Howland curret source is better treated with respect to R_{2b}, because R_{2b} defines the output current sensitivity with respect to V_{in}. Since R_{2b} defines the output current, the other resistor values can be chosen to be very large. The output impedance in case $R \gg R_{2b}$ can be calculated as

$$\begin{aligned} R_{out,m} &= \frac{R_{2b} (2R - R_{2b})}{R} \frac{(1-T)^2}{4T} \\ R_{out,m,iHCS} &= \lim_{R \to \infty} R_{out,m} = 2R_{2b} \frac{(1-T)^2}{4T} \approx \frac{R_{2b}}{2T} \end{aligned} \tag{A.36}$$

The result is that for $R \gg R_{2b}$, the output impedance of the improved Howland Current source is about twice as high as the basic Howland current source. The size of the resistors R

are only limited by the desired bandwidth, because circuit parasitics like the input capacitance of the op-amp must then be considered.

Resistor mismatch is not the only element that negatively affects the output impedance. Another limiting factor is the finite op-amp gain A, which, on top of that, also decreases with frequency. Not applying the approximation in equation A.27, yields a rather lengthy term for V_L

$$V_L = \frac{\begin{array}{r} AI_{out}R_1R_{2b}R_3 + AI_{out}R_{2a}R_{2b}R_3 - AR_{2a}R_3V_{in} - AR_{2a}R_4V_{in} \\ - AR_{2b}R_3V_{in} + I_{out}R_1R_{2b}R_3 + I_{out}R_1R_{2b}R_4 + I_{out}R_{2a}R_{2b}R_3 \\ + I_{out}R_{2a}R_{2b}R_4 - R_{2b}R_3V_{in} - R_{2b}R_4V_{in} \end{array}}{(A-1)R_1R_4 - (A+1)(R_{2a}+R_{2b})R_3 - R_1R_3 - (R_{2a}+R_{2b})R_4} \tag{A.37}$$

Again, differentiating to find the output impedance yields

$$R_{out} = \frac{\begin{array}{r} A_vR_1R_{2b}R_3 + A_vR_{2a}R_{2b}R_3 + R_1R_{2b}R_3 \\ + R_1R_{2b}R_4 + R_{2a}R_{2b}R_3 + R_{2a}R_{2b}R_4 \end{array}}{(A+1)(R_{2a}+R_{2b})R_3 - (A-1)R_1R_4 + R_1R_3 + (R_{2a}+R_{2b})R_4} \tag{A.38}$$

This time, assuming perfect matching of the resistors with $R_{2a} + R_{2b} = R_2$, $R = R_1 = R_2 = R_3 = R_4$, R_{out} can be further simplified, yielding a term similar to equation A.34.

$$R_{out,A} = \left(\frac{R^2 + R_{2a}^2}{R}\right)\frac{A+2}{4} \tag{A.39}$$

For a typical compensated op-amp, the frequency dependent gain was already introduced in equation A.11 as

$$A(\omega) = \frac{A_{ol}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}},$$

with the open-loop gain A_{ol} and corner frequency ω_c of the dominant pole at which the gain starts rolling off with an order of magnitude per order of magnitude in frequency ($20\,\mathrm{dB}$ per decade).

Comparing equations A.34 and A.39 it is clear, that the sensitivity of the output impedance to the resistor tolerances and the op-amp gain are of the same magnitude since $\frac{(1-T)^2}{4T} \approx \frac{1}{4T}$ for small T and $\frac{A+2}{4} \approx \frac{A}{4}$ for typical values of A. With regard to the resistor tolerances and the gain of op-amps, it is clear that at low frequencies, the contribution of precision op-amp with a gain $A \geq 10^6$ is insignificant, even when $0.01\,\%$ resistors are used. This makes trimming or selection of components inevitable if a high output impedance is required. Only at freuqencies above $1\,\mathrm{kHz}$, when the op-amp gain has dropped to values comparable to $\frac{1}{\epsilon}$, the op-amp needs to be considered.

Finally, the same calculations can be done including both the finite gain and the resistor matching. These calculations are omitted here for brevity, but can be found in the Jupyter notebook mentioned above. The result is

$$R_{o,m,A} = \left(\frac{R^2 - R_{2a}^2}{R}\right)\frac{(AR + R - \epsilon + 1)}{A\left(R + \epsilon - 1\right) + 2R - 2\epsilon + 2}\,. \tag{A.40}$$

Another representation is also given by Mahnam et al. [140]. They decompose the output impedance into several components to build an equivalent circuit. This allows to treat the gain dependent part as a capacitance, hence the term output capacitance is sometimes used. The formula given here is more suited for an analytical approach or for Monte Carlo simulations though.

Finally, the compliance voltage must be discussed. The output voltage of the op-amp can again be calculated using Kirchhoff's current law and the details are found in the Python notebook data/simulations/howland_current_source.ipynb as part of the online suplemental material [42]. The result is

$$V_o = \frac{2\left(RV_L + R_{2a}V_{in}\right)}{R + R_{2a}} \tag{A.41}$$

For the classic Howland current source ($R_a = 0$) one finds

$$V_{o,HCS} = 2V_L\,, \tag{A.42}$$

which is independant of the input voltage. It is largely independant of the resistors as well in case of a laser diode, because V_L is fairly constant with the output current. The improved Howland current source behaves differently and the op-amp output voltage for $R \gg R_{2b}$ becomes

$$V_{o,iHCS} = \lim_{R\to\infty} V_o = V_L + V_{in}\,. \tag{A.43}$$

In this case part of the load dependence is traded for an input voltage dependence. Whether that is an advantage depends on the application.

List of publications

Kanika, P.Baus, G.Birkl, Z.Guo, A.Khodaparast, J.Klimes, W.Quint, M. Shaaban, M.Vogel, *The ARTEMIS experiment for precision measurements of the electron gfactor in highly charged ions*, In HYPERFINE 2019 proceedings (2019).

A. Martin, P. Baus, and G. Birkl, *External cavity diode laser setup with two interference filters*. Applied Physics B **122.12** (Dec. 2019).

T. Preuschoff, P. Baus, M. Schlosser and G. Birkl, *Wideband current modulation of diode lasers for frequency stabilization*, Review of Scientific Instruments **93**, 063002 (2022).

Kanika, A. Krishnan, J. W. Klimes, B. Reich, K. Anjum, P. Baus, G. Birkl, W. Quint and M. Vogel, *Production of highly charged ions inside a cryogenic Penning trap by electron-impact ionisation*, submitted to Journal of Physics B (2023).

J. W. Klimes, Kanika, A. Krishnan, B. Reich, K. K. Anjum, P. Baus, G. Birkl, W. Quint, W. Schott, and M. Vogel, *Cryogenic vacuum valve with sub-second operation times*, submitted to Review of Scientific Instruments.

P. Baus and G. Birkl, *An Open-Source, Low Noise, High Stability Digital Laser Driver for the next Generation of Laser Diodes*, in preparation.

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Die Arbeit hat bisher noch nicht zu Prüfungszwecken gedient.

Darmstadt, 15. Mai 2023

Patrick Baus