Model and Analysis of Charybdis' Whirlpool in Homer's *The Odyssey*

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Abstract

The pupose of this endeavor was to apply the laws of fluid mechanics to the fictional whirlpool described in *The Odyssey* in order to test the validity of the author's description and hopefully gain a greater appreciation for the scene in question. This was done through the use of a computer model which took into account several situational details gathered from the book and applies the solutions of several known fluid dynamics equations and relations. A relationship was found which mapped the approximate size and shape of such a whirlpool. Initially, it was found to be much smaller and more docile than the description in the text had indicated. Some paramaters were evaluated and reset to create a whirlpool that was slightly more worthy of such a description. Ultimately, it is still doubtful that such a phenomenon could happen.

1 Introduction

Homer's ancient Greek epic *The Odyssey* tells the story of a hero's travel from war through all kinds of trials and disasters to return to his family and home. This fictional tale contains many fantastical devices that create intriguing perils, such as witchcraft, gods and goddesses, and mythical monsters. In one of the iconic scenes of the story, the hero Odysseus and his crew must guide their boat through a strait in the ocean between two great monsters, Scilla and Charybdis. The former reaches out of a cave and picks off crew members who get too close. The latter lives underwater and is said to cause whirlpools by sucking in large volumes of water and then spewing them out. The whirlpool is said to draw nearby ships inward with a force from which they cannot escape, eventually tearing them apart in the violent water.

The story of Charybdis is an intriguing one from a physics perspective, because although it is caused by a clearly mythical creature, the effect itself is a very real scientific phenomenon. Whirlpools can arise naturally from a number of different scenarios, one of which being a strong suction force inside a body of water. Other factors come into play, but in the end it is clear that such a whirlpool can be modelled analytically. Still, it is a very complex system and I previously had very little understanding of how it behaves.

My whirlpool model essentially places a vacuum source inside a body of water with an inherent circulation, and evaluates how the velocity of the water changes at different locations with respect to this source. I am then able to use these velocities to map out the whirlpool's surface gradient. This model can show us just what a whirlpool in the given setting would look like, and how different parameters affect its size and shape.

2 Theoretical Background

2.1 Setting the Parameters

Before modelling a whirlpool, it is necessary to know as much as we can about the body of water in which we will envision it. A number of inferences can be made from Homer's poem, specifically on pages 273-275 of the Rober Fagles translation.[1] The area is described as a narrow pass, about an "arrow's shot" accross. This is estimated to be about 200 meters. The nearby land is described as sheer cliffs that are impossible to climb, so we will assume vertical walls on either side of our body of water. It is reasonable to assume some current through such a narrow strait. The Strait of Messina, to which some scholars attribute this very scene in the Greek myth, has an average current of roughly .1 m/s.[2] However, we should not assume the velocity is uniform. Due to viscous effects, the velocity near the shores is almost always lower than the velocity in the middle of a stream, and the velocity profile can be roughly modeled with a basic parabola shape (see next section).[3] As for the depth of water, it is mentioned that there are coral reefs in this strait, and coral reefs often appear about 50 meters below the surface of the water.[4] So, we will set our water depth at 50m.

Now, it is implied that the whirlpool takes up most of the strait, and that there is a narrow way around it which would put the boat in the range of the other monster. Based on this description, it is reasonable to give Charybdis' whirlpool three-fourths of the strait, and leave the other 50 meter section as the range of Scilla. So, the center of our whirlpool should be placed 75 meters away from the shore, and we will analyze its influence to 75 meters away in all directions (covering three fourths of the way across strait). It is unreasonable to consider the whirlpool's influence beyond this length, because beyond 75 meters lies the nearer cliff wall, and this obstruction would easily distort a whirlpool formation. See Figure 1 for an illustration.

2.2 The Source of Angular Momentum

An important thing to note about whirlpools is that they do not create any circular current in a fluid. Rather, they are the result of the conservation of angular momentum in the presence of an already-existing circular current. A whirlpool can form in any instance where the fluid is being actively drawn toward a point about which there is a net angular momentum.[5] Since our whirlpool is located off-of-center of the the strait, and the velocity profile of the water is non-uniform, we see that there actually is a value of curl in the current at the location of the whirlpool, and thus a net angular momentum of the body of water about the axis of the whirlpool. This concept is explained by Rodney Cole in

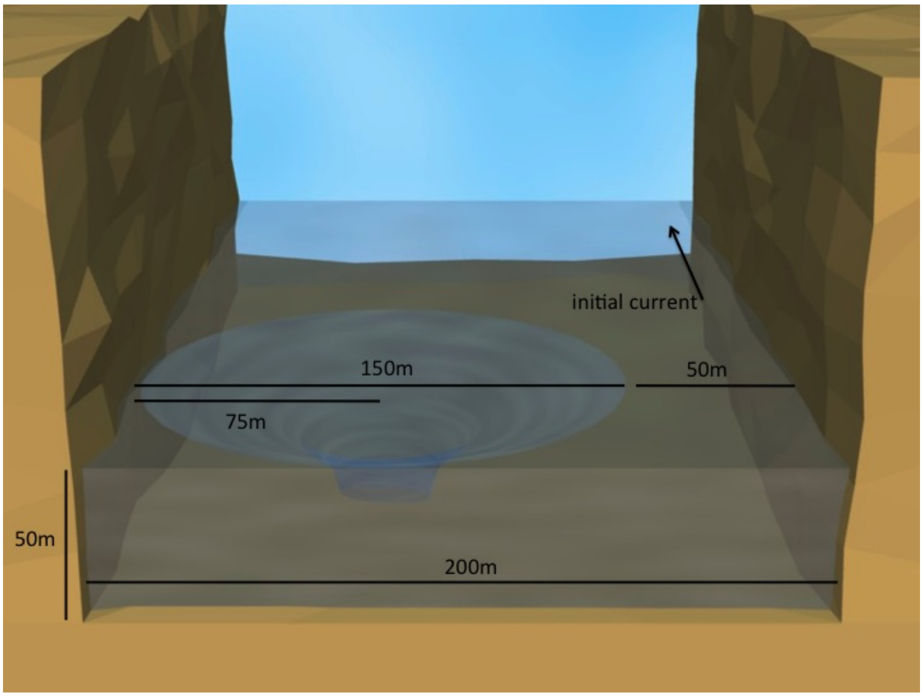

Figure 1: A basic diagram of the whirlpool's position in the strait.

his basic description of rotational flows.[6] Figure 2 shows the parabolic velocity profile of the water as we look (from above) at different points accross the strait. Consistent with its real-world analog, the average current is about $.1m/s$ (there is no need for precision on this value).[2] The imbalance of water velocity on either side of the whirlpool will provide the initial angular momentum needed for the whirlpool to form.

2.3 The Radial Velocity

Having established the condition that makes a whirlpool possible, we will put angular momentum on hold for the time being. The first piece of data we will find is the water's radial-direction motion. As I mentioned, this particular whirlpool is caused by a large suction, a *sink*, located at the bottom of the ocean. Whether there is actually a suction force or just the force of gravity causing water to fall into a vacant space (the body of our monster) is irrelevant. What is imporant is the volumetric flow of the water. I define Q as the volumetric flow rate, in m^3/s of water. Next, we make an important assumption that lessens the complexity of this problem: we assume that the inward flow of water is purely horizontal and is uniform at every height: that is, the flow of water at all depths stays locked in its original horizontal plane and goes toward the whirlpool's axis, *not* directly toward the sink. The rationale for this is that the motion of most of the water will be "filling in" for the water that got sucked downward, and since this is the ocean, we have an effectively infinite supply of water at all depths. See Figure 3 for an illustration of this assumption.

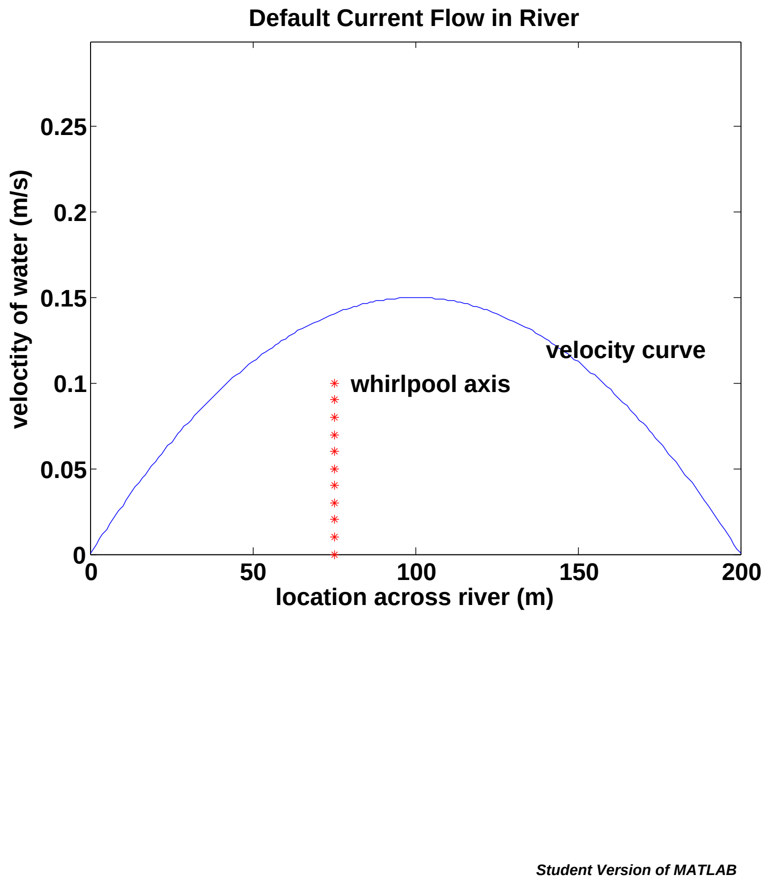

Figure 2: The velocity value of the water, mapped accross the strait.

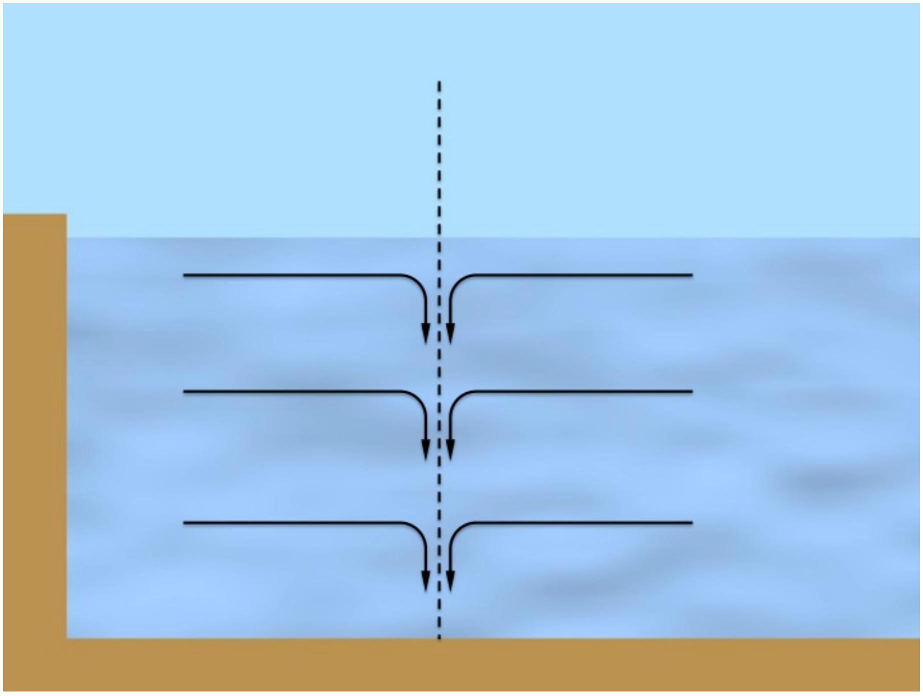

Figure 3: The suction-induced velocity of the water, viewed horizontally. The assumption we make on our model is that the curved parts of these velocity arrows are inifitely “tight”, so that we can deal with only horizontal motion.

Based on this assumption, we can divide Q by the total depth of water ($h = 50m$) to get an "area flow" rate of the cross-sectional area of water, effective at every depth:

$$\frac{dA}{dt} = \frac{Q}{h}$$

In our simplified system, we can say that, in a given *horizontal* plane, this expression is the area of water that "disappears" into the whirlpool's center axis over a given period of time. We can think of concentric circular areas of water, continuously shrinking. The next step is to use this shrinking over time to find how the water's distance from the whirlpool axis, r, changes in time. Really, we want to find $\frac{dr}{dt}$ in terms of r. Since:

$$A = \pi r^2$$

we can differentiate both sides to get:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Plugging in and rearranging, we get:

$$v_r(r) = \frac{dr}{dt} = \frac{Q}{2h\pi r} \tag{1}$$

2.4 Findng the Net Angular Momentum

Our goal now is to quantify the velocity imbalance on either side of the whirlpool's axis and get a value for the net angular momentum of the body of water. I am counting the "body of water" to be the cylinder-shaped volume centered at the whirlpool's axis, with a height of $50m$ and a radius of $75m$. Now, in order to calculate net angular momentum induced by the non-uniform velocity profile, we start with the classical definition of angular momentum:

$$\vec{L} = m(\vec{r} \times \vec{v}) \tag{2}$$

Since v is in the same direction at all locations, we can simplify $|\vec{v} \times \vec{r}|$ to $vcos(\theta)r$. To account for the mass at every point, we will have to integrate the density of water ρ over all volume:

$$m = \rho \int dV$$

and we can take the dimension h out since it is constant:

$$m = \rho h \int dA$$

and since we are using polar coordinates, this becomes:

$$m = \rho h \int_0^{2\pi} \int_0^R r \, dr \, d\theta$$

Putting it all together:

$$L = \rho h \int_0^{2\pi} \int_0^R v \, cos(\theta) \, r^2 \, dr \, d\theta \tag{3}$$

2.5 The Angular Velocity

Given this net angular momentum about the location of our source, our next step is to find how, when volumes of water are drawn inward, velocity in the theta direction increases due to conservation of angular momentum. We know two important things about this: 1) Eventually, the system will be a regular vortex, whose angular velocity is proportional to $1/r$. This statement is derived from the basic angular momentum equation (Equation 2) and is a given feature of it. As r decreases, v_θ must go up proportionally. 2) When our body of water is a vortex, it will still have the same angular momentum as it originally had. The inner volumes of water will be rotating faster and have more angular momentum, and the outer volumes of water will actually have less angular momentum than originally, because they are being pulled inward from all directions evenly, rather than just flowing by with uneven velocity.

The critical figure that relates our original body of water to the swirling vortex pattern is called circulation. It is measured as the line integral of density times velocity around a closed loop:

$$\Gamma = \oint \rho \, \vec{v} \cdot dl \tag{4}$$

One phenomenon of naturally-occuring fluid vortices is that the circulation along any path that encloses the center of the vortex is constant.[6] That is, every vortex has a single, characteristic value of circulation around its center, no matter what path is taken. To find the circulation of our vortex, we will use the helpful relation:[6]

$$\Gamma = 2\,\pi \, \frac{dL}{dV}$$

We can use the total angular momentum and total volume of water to find our average circulation in the body of water. Once we have the circulation, we can modify Equation 4 for a circular path loop centered at the whirlpool axis:

$$\Gamma = \int_0^{2\pi} \rho \, v_\theta(r) \, r \, d\theta$$

This relation is necessarily true at every radius in our whirlpool system. Since the loop integral is evaluated at a constant radius, we can take $v_\theta(r)$ out of the integral and rearrange and simplify everything down to:

$$v_\theta(r) = \frac{\Gamma}{2\pi \, \rho \, r} = \frac{L}{\rho \, V \, r} \tag{5}$$

We now have an expression for the velocity in the theta direction in terms of the radius from the whirlpool's axis. We have taken the angular momentum induced by the non-uniform (but same direction) fluid velocity, used it to find the circulation, and applied that circulation to the fluid as if it were a vortex, which is what inevitably comes about when an inward force is introduced to a body of circulating water. We can now accurately determine the angular velocities at each radius.

2.6 The Change in Height

Now that we have both the r and θ directional components of velocity in terms of r, we can find the equation for total velocity, $v_{tot}(r)$. We can then evaluate how the height of the surface, h_s, changes at different points. To bring it all together:

$$v_{tot}(r) = \sqrt{v_r^2 + v_\theta^2} = \sqrt{\left(\frac{L}{\rho V r}\right)^2 + \left(\frac{Q}{2h\pi r}\right)^2} \tag{6}$$

The Bernoulli Equation gives us what we need to relate the surface height to the radius:

$$P + \frac{1}{2}\rho v^2 + \rho g h_s = const \tag{7}$$

where P is the absolute pressure of the water at a given location, v is $v_{tot}(r)$ that we just found, g is the graviational acceleration, and h_s is the height of the surface above the ground. This gives us the relationship between a fluid's velocity, its pressure, and the gravitaitonal potential energy it can occupy. We can rearrange this to get:

$$h_s = -\frac{P}{\rho g} - \frac{v^2}{2g} + const \tag{8}$$

3 Construction of The Model

Having acquired all of the general equations that will be of use, all that is left to do is plug in our initial parameters and solve each equation, eventually getting the specific relationship between h_s and r. I used MATLAB to work out most of the math, starting with an array of r values going from 0 to 75, and generating corresponding arrays of v_r, v_θ, v_{tot} and finally h_s.

When it came to calculating $v_r(r)$, I gave Q a value of $10m^3/s$, an amount I deemed fitting for a fairly giant creature. Taking the other values into account, the specific solution was:

$$v_r(r) = \frac{.0318}{r}$$

and MATLAB generated an array of values for v_r.

The calculation of $\vec{L}$ was done separately. Given that $R = 75m$, $\rho = 1020kg/m^3$, $h = 50m$, and writing out our velocity profile equation of Figure 2 in terms of r and θ, I calculated (from Equation 2):

$$|L| = 9.5 \times 10^8 \frac{kg\, m^2}{s}$$

The right-hand side of the whirlpool had the greater integral of velocities than the left, and since velocity is pointing away from us in our frame (see Figure 1), we can say the vector $\vec{L}$ at the point of the whirlpool points upward.

For the calculation of $v_\theta(r)$, the value of L was entered in, as well as the dimensions of the body of water (for calculation of volume). The density of sea water, ρ, is $1020kg/m^3$. The specific solution was:

$$v_\theta(r) = \frac{1.078}{r}$$

and MATLAB generated an array of values for v_θ.

Then, Equation 6, the equation for v_{tot}, was evaluated using the arrays of v_r and v_θ. An array of values for v_{tot} was generated.

Next, the model was ready to solve equation 8 to return an array of values for the surface height of the water corresponding to values of r. If we plug in atmospheric pressure ($101,325\,kPa$) for the absolute pressure P, we will get the height (above the ocean floor) at which the water's pressure equals atmospheric pressure (that is, the surface of the water). The constant in Equation 8 determines the original height of the water in the absence of a whirlpool, so I set it such that said height was 50. The model returned the array of h_s values, and these were plotted against the array of r values.

4 Model Presentation and Analysis

The final model of the dimensions of the whirlpool is presented in Figure 4. Here, we can see that, rather disappointingly, the severe part of the whirlpool only has a diameter of roughly 4 meters. The velocity of the water was not high enough to create the pressure differential needed to create a giant gaping hole in the ocean, as the book described. My first thought was that perhaps I should set the volumetric flow, Q, to a higher value. This would increase the values of v_r. However, any reasonable changes in Q had very little effect on the total velocity. And indeed, v_r had a much lower effect than v_θ on the value of v_{tot}. So, my next question was, what factors could make v_θ larger? According to Equation 5, the answer is the $\frac{L}{V}$ ratio. In the modeled body of water, the angular momentum present was too weak, and the volume contained was too large. The most reasonable way to alter this would be to change the original velocity profile equation of Figure 2. I could multiply the entire equation by 5, making the average current roughly $.5m/s$, and this is still fairly common for a narrow strait on occasion.[2] This would increase L, and therefore $v_\theta(r)$ by a factor of 5. A revised model of h_s vs r was generated (Figure 5).

This new model is slightly improved. The whirlpool causes a significant bend in the surface over a diameter of about 10 meters. That is much more likely to pose a danger to a passing ship, but it is still not so big as to be nearly impossible to avoid. It is safe to say that in order for a whirlpool as huge as the one in *The Odyssey* to exist, the pre-existing fluid currents would have to be truly severe, and directed in such a way as to maximize the angular momentum about the point of suction.

One final item of uncertainty in my model was the interpretation of an "arrow's shot" as the width of the strait. This expression could describe a whole range of distances, depending on if it was intended to mean "average range for an arrow shot," "maximum range of accuracy for an arrow shot," or "farthest possible range of an arrow shot." I used the latter, the largest value, but if the strait was meant to be narrower, then it would not take such a large whirlpool to pose a hazard to a ship. Additionally, higher currents are more probable in narrower straits, *and* the volume of the total system would be lower, so $\frac{L}{V}$ would increase quickly.

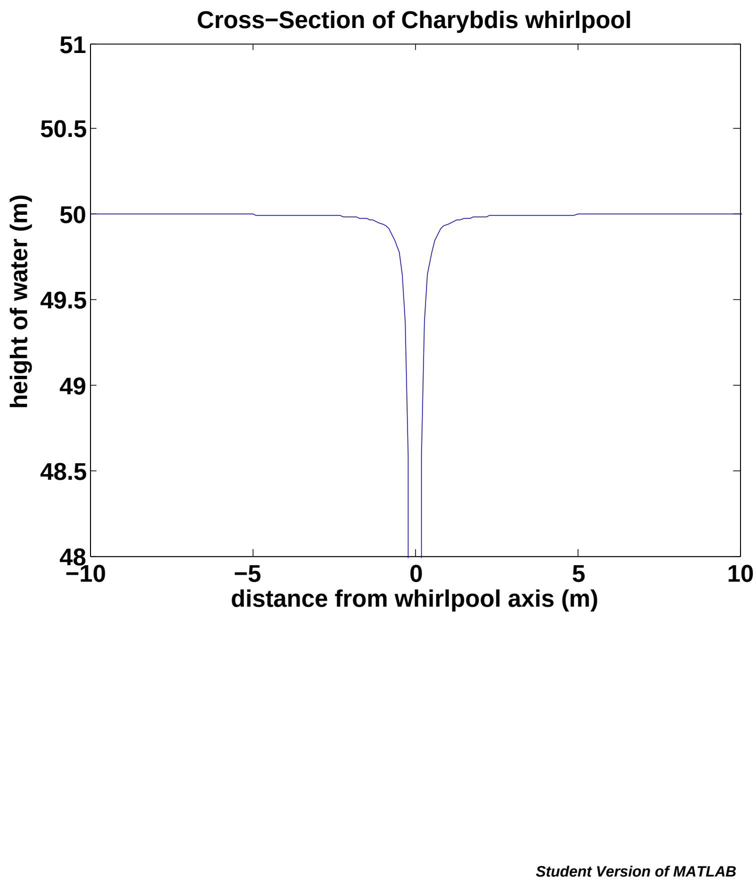

Figure 4: The final model of whirlpool surface height in relation to radius. The plot of h_s vs r was reflected and copied onto the negative-radius side as well, so that a complete 2 dimensional cross-section could be viewed.

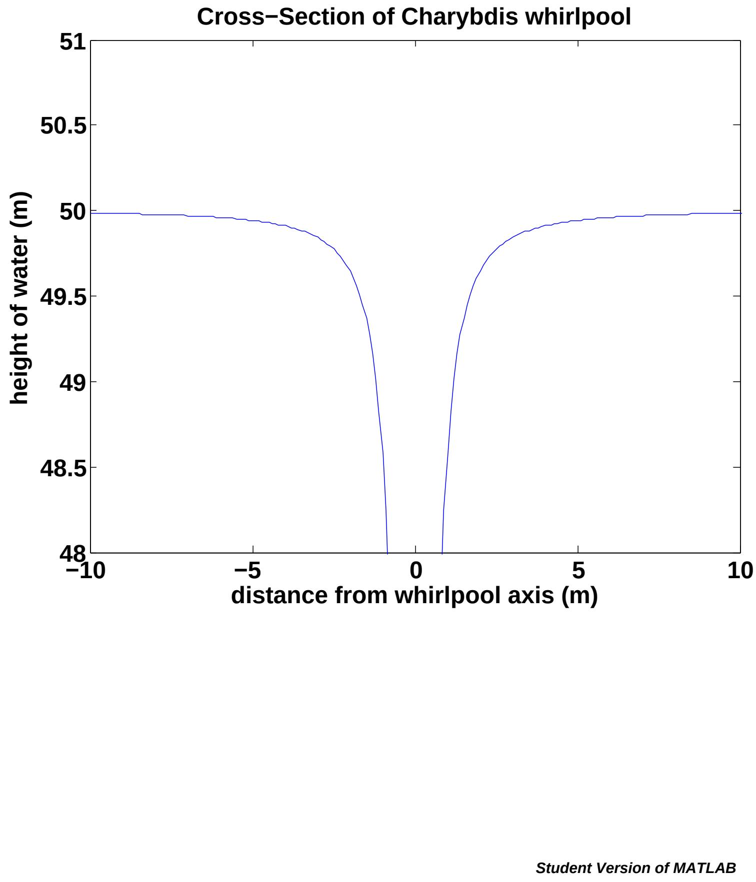

Figure 5: A revised model of whirlpool surface height in relation to radius. The net angular momentum was increased 5 times the original.

5 Conclusion

The computer model above effectively simulated the shape and size of a whirlpool, and I tried my best to enter parameters that were consistent with Charybdis' whirlpool as described in *The Odyssey.* The model applies the general laws of momentum conservation and the Bernoulli Principle, as well as some important integration relations and known properties of fluid vortices, in such a way as to effectively model such a complex system.

While the initial model delivered a much smaller whirlpool than I had inferrred from the description in the book, there was some room for adjustment. Specifically, the original current profile of the water, which set the initial angular momentum, could be intensified to an extent. It also may be the case that the strait was meant to be much narrower than I interpreted, in which case the idea of a whirlpool as a nearly-unavoidable sailing hazard is perhaps reasonable. Further work would probably be well spent applying the model to this case. I could see just how narrow the strait would have to be, and how severe the water currents, in order to make the whirlpool a true hazard. Of course, there would come a point where the narrowness of the passage and the intensity of the current would pose a hazard all by themselves, and this was not mentioned in the text. So, it may simply be the case that Charybdis' whirlpool is as mythical as the monster herself.

References

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