Model and Analysis of Charybdis' Whirlpool in Homer's *The Odyssey*

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Abstract

The pupose of this endeavor is to apply the laws of fluid mechanics to a fictional phenomenon in order to test the validity of the author's description and hopefully gain a greater appreciation for the scene in question. This is done through the use of a computer model which takes into account several situational details gathered from the book and applies the general solutions of fluid dynamics. An equation is given which maps the approximate size and shape of such a whirlpool, and this model could then be interpreted to give an idea of the effect it would have on a boat passing by.

1 Introduction

Homer's ancient Greek epic *The Odyssey* tells the story of a hero's travel from war through all kinds of trials and hardships to return to his family and home. This fictional tale contains many fantastical devices that create intriguing perils, such as witchcraft, gods and goddesses, and mythical monsters. In one of the iconic scenes of the story, the hero Odysseus and his crew must guide their boat between two great monsters, Scilla and Charybdis. The former picks off and eats crew members who get too close. The latter lives underwater and is said to cause whirlpools by sucking in large volumes of water and then spewing them out. The whirlpool is said to draw nearby ships inward, where they are torn to pieces by the violent water.

The story of Charybdis is an intriguing one from a physicist's perspective, because although it is caused by a clearly mythical creature, the effect itself is a very real scientific phenomenon. Whirlpools can arise naturally from a number of different scenarios, one of which being a strong suction force inside a body of water. Other factors come into play, but in the end it is clear that such a whirlpool can be modelled scientifically. This model can then be used to show us just what a perfect whirlpool of such immense size would look like.

2 Constructing the Model

2.1 Setting the Parameters

Before modelling a whirlpool, it is necessary to know as much as we can about the body of water in which we will envision it. A number of inferences can be made from Homer's poem, specifically on pages 273-275 of the Rober Fagles translation.[1] The area is described as a narrow pass, about an "arrow's shot" accross. This is estimated to be about 200 meters. The nearby land is described as shear cliffs that are impossible to climb, so we will assume vertical walls on either side of our body of water. It is reasonable to assume some current through such a narrow strait. The Strait of Messina, to which some scholars attribute this very scene in the Greek myth, has an average water velocity of roughly .1 m/s.[2] However, we should not assume the velocity is uniform. Due to viscous effects, the velocity near the shores is lower than the velocity in the middle, and the velocity profile can be roughly modeled with a basic parabola shape (see next section). As for the depth of water, it is mentioned that there are coral reefs, and coral reefs usually appear about 50 meters below the surface. So, we will set our water depth at 50m.

Now, it is implied that the whirlpool takes up most of the strait, and that there is a narrow way around it which would put the boat in the range of the other monster. If we give Charybdis' whirlpool three-fourths of the strait, and leave the other quarter as the range of Scilla, then the center of our whirlpool should be placed 75 meters away from the shore (and hopefully its influence will reach 75 meters to either side, covering three-fourths of the entire strait).

2.2 Approximating Angular Momentum

An important thing to note about whirlpools is that they don't create any circulation in a fluid. Rather, they are the result of the conservation of angular momentum in the presence of an already-existing circular current. A whirlpool can form in any instance where there is a net angular momentum present in the fluid, and when the fluid is being actively drawn toward a point where a circulation is present (that is, anywhere where the curl of velocity is nonzero).[3] Since our whirlpool is located off-of-center of the the strait, and the velocity profile of the water is nonlinear, we see that there actually is a value of curl at the location of the whirlpool. This concept is explained by Rodney Cole in his basic description of rotational flows.[4] Figure 1 shows the parabolic velocity profile of the water as we look at different points accross the strait. The imbalance of water velocity on either side will provide the initial angular momentum needed for the whirlpool to form.

2.3 Calculating the Whirlpool's Features

Our goal now is to quantify the velocity imbalance on either side of the whirlpool's axis and get a value for the net angular momentum at each point. The classical definition of angular momentum is

$$\vec{L} = m(\vec{r}x\vec{v}) \tag{1}$$

However, it is surely difficult to say what mass to use, since we are dealing with indiscrete volumes of water. Later calculations show that we can actually solve this for angular momentum over volume ($\vec{L}/V$) instead. Dividing both sides of Equation 1, we get

$$\frac{\vec{L}}{V} = \rho(\vec{r}x\vec{v}) \tag{2}$$

Where ρ is the density of ocean water (1020 kg/m^3). After solving this figure by taking the integral of all velocities to the left of the whirlpool's axis, and then all to the right, and then finding the difference between angular momenta, we get a value of $1.4214e6\frac{kg}{ms}$ Given this net angular momentum p[er volume about the location of our source, we can use the principle of conservation of angular momentum to see how the water behaves when it is drawn inward toward our source. We can solve for v by rearranging equation 2 as

$$v = \frac{L}{\rho V r}$$

This relates the water velocity directly to its xy distance from the whirlpool's axis. Note that this only accounts for velocity confined to the xy-plane. It shows us the obvious: that as r decreases, v increases inversely. This also assumes the system has existed for long enough, such that water from all accross the strait (carrying *all* of the angular momentum present) is actively being drawn toward the whirlpool's axis.

Now that we know how the velocity changes as it gets nearer to to whirlpool's axis, we can apply what we know about fluid dynamics via the Bernoulli equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = const \tag{3}$$

where P is the absolute pressure of the water at a given location, g is the gravi-ational constant, and h is the height of the water above the ground. This gives us the relationship between a fluid's velocity, its pressure, and the gravitaitonal potential energy it can occupy. We can rearrange this to get:

$$h = -\frac{P}{\rho g} - \frac{-v^2}{2g} + const$$

Now, if we plug in atmospheric pressure for the absolute pressure P, we can solve for the height of the water (above the floor) at which the water's pressure equals atmospheric pressure (that is, the surface of the water). The constant in this case turns out to be the original height of the water in the absence of a whirlpool (so we set it as 50). So, the final model of the surface height of the water is given in Figure 2.

3 Conclusion

The computer model above effectively simulates the shape and size of Charybdis' whirlpool as described in *The Odyssey.* It applies the general laws of momentum conservation and the Bernoulli Principle in such a way as to effectively model such a complex system. Further work would probably be best spent on analyzing the suction force needed to cause such a disturbance in the water, and possibly finding a way to model the whirlpool with a less powerful central attractive force. Also, the influence of viscosity on the flow of the water was very roughly estimated, and this model would benefit from a more in-depth look. Finally, this model can and should be used to determine just how easily a boat could get sucked in past the "point of no return", and I intend to include that analysis in my final paper.

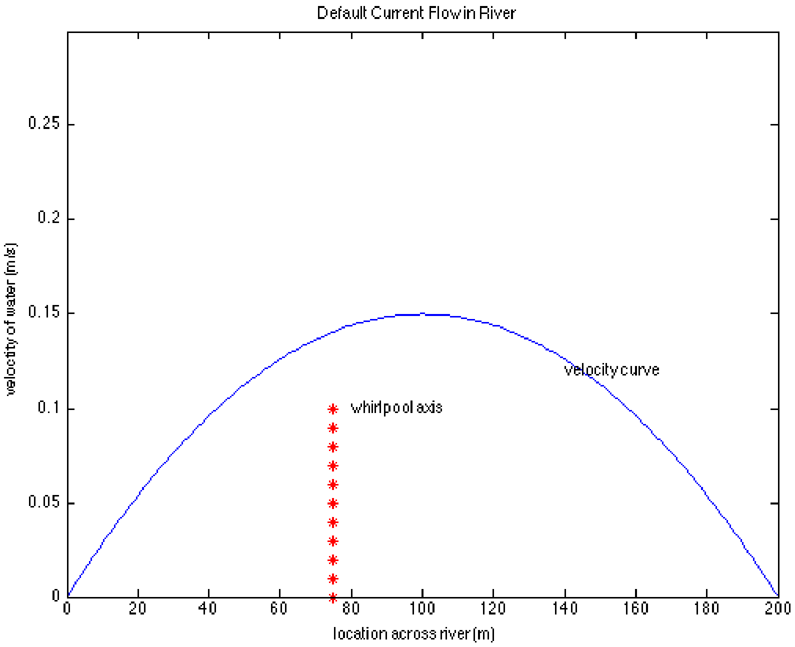

Figure 1: The velocity value of the water, mapped accross the strait.

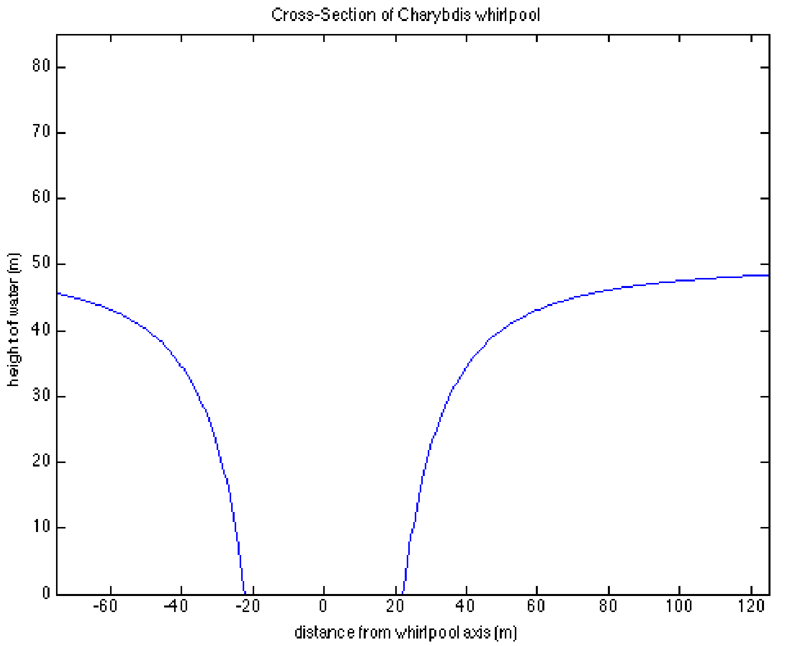

Figure 2: The final model of whirlpool fluid radius in relation to surface height, viewed according to the boundaries of the strait.

References

[1] Homer. *The Odyssey.* Penguin Books Ltd, London, England, 1997.

[2] Global Ocean Associates. Strait of messina. *An Atlas of Oceanic Internal Solitary Waves*, pages 199–206, 2004.

[3] Robert R. Long. Sources and sinks at the axis of a rotating liquid. *The Quarterly Journal of Mechanics and Applied Mathematics*, 9:385–393, 1955.

[4] Dr. Rodney Cole. Rotational flows: Circulation and turbulence, 2012.