A*pex: Efficient Approximate Multi-Objective Search on Graphs

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1 Background

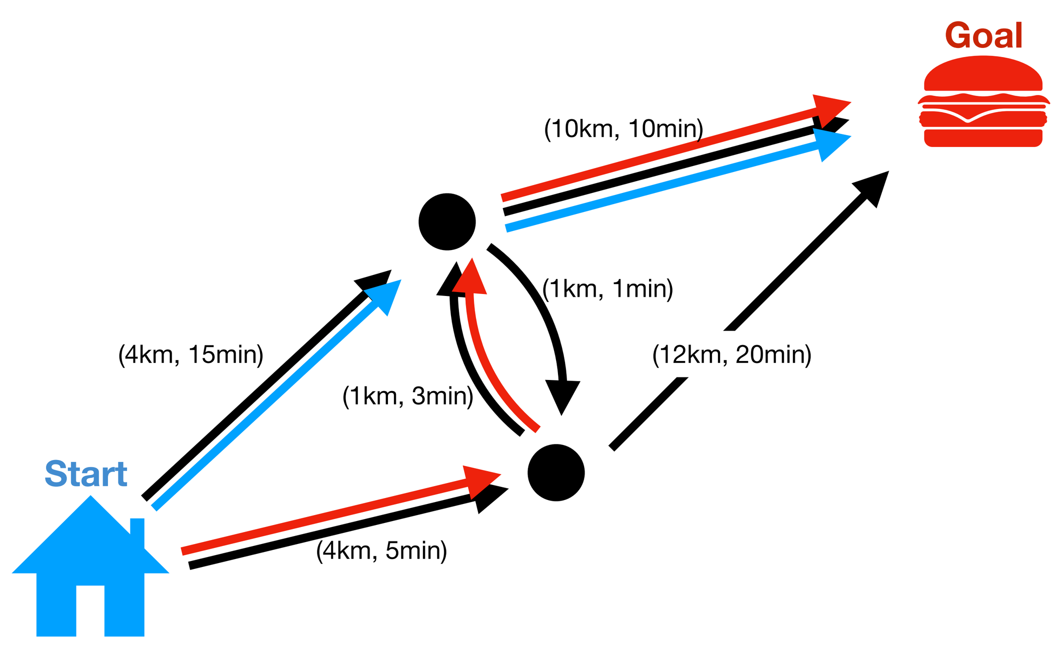


Figure: An example search instance. Blue and red arrows show the two paths in the Pareto-optimal frontier.

Definition (Multi-objective search).

A ***multi-objective search instance*** contains a graph, s start state, and a goal state. A ***cost function*** $\mathbf{c}$ maps an edge in the graph to a cost vector of length N. A ***solution path*** π is a path from the start state to the goal state. The ***cost of path*** π, denoted as $\mathbf{c}(\pi)$, is the accumulated costs of its edges.

Definition (Weak dominance and ε-dominance).

Path π ***weakly dominates*** path π' iff $\mathbf{c}(\pi) \preceq \mathbf{c}(\pi')$. For a non-negative real number ε, path π *ε-dominates* path π' iff $\mathbf{c}(\pi) \preceq (1+\varepsilon)\mathbf{c}(\pi')$.

Definition (Pareto-optimal frontier).

A set of solution paths Π is a ***Pareto-optimal frontier*** iff (1) every solution path for the problem instance is weakly dominated by at least one solution path in Π and (2) solution paths in Π do not weakly dominate each other.

Definition (Approximate Pareto-optimal frontier)

Given an ε-value, a set of solution paths Π_ε is an ***ε-approximate frontier*** iff (1) every solution path for the problem instance is ε-dominated by at least one solution path in Π_ε and (2) solution paths in Π_ε do not weakly dominate each other.

2 Motivation

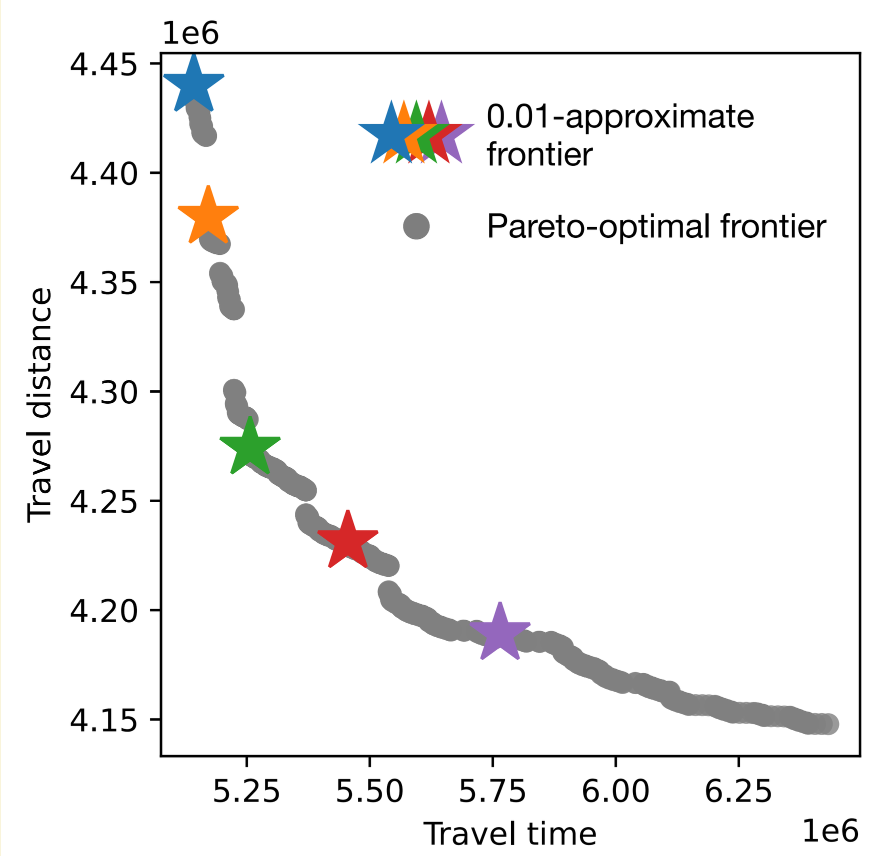
- The Pareto-optimal frontier can be **exponential** in the size of the graph being searched, which makes multi-objective search time-consuming.
- The ε-approximate frontiers are typically much smaller than the Pareto-optimal frontiers even for small ε-values and hence easier to compute.
- The existing algorithm **PP-A*** [1] finds ε-approximate frontiers for bi-objective search instances efficiently. However, it is unclear how to generalize PP-A* to search instances with more than two objectives.

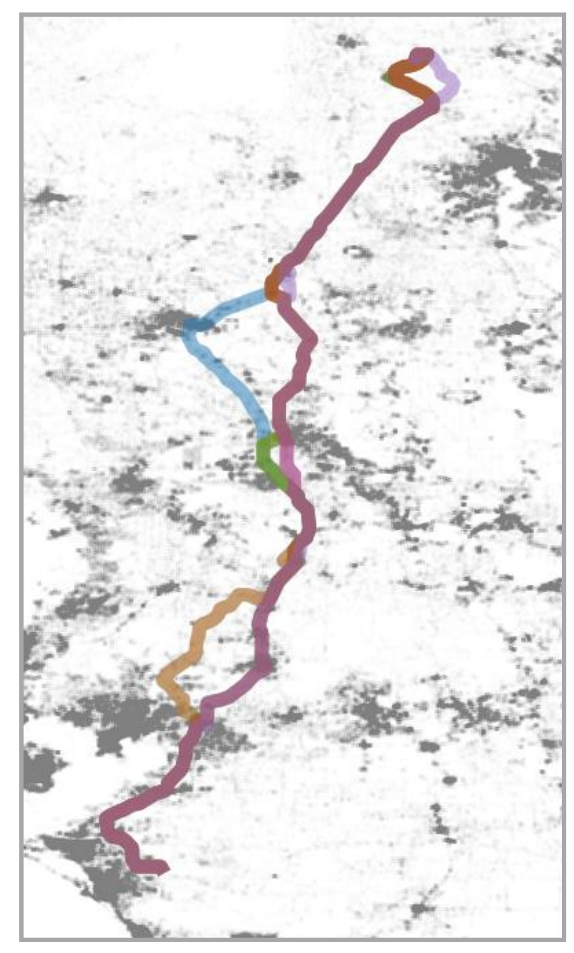
3 Contributions

We proposed an approximate multi-objective search algorithm **A*pex**, which:

- finds an ε-approximate frontier for a user-provided ε-value,
- builds on PP-A* and also makes it **more efficient** for bi-objective search, and
- can solve problem instances with **more than two** objectives while PP-A* cannot.

4 ε-Approximate Frontier: An Example





The left figure shows the Pareto-optimal frontier and an ε (=0.01)-approximate frontier for a road-network problem instance with two objectives. The right figure shows the corresponding paths of the ε-approximate frontier.

In this example, the Pareto-optimal frontier has **many (=2253) solution paths** with many of them being very similar. The ε-approximate frontier contains only five diverse solution paths and takes **a much smaller amount of time (~5%)** to compute.

5 A*pex

In A*pex, each search node is an ***apex-path pair***, which contains a ***representative path*** π and a cost vector $\mathbf{A}$ called ***apex***. An apex-path pair is ***ε-bounded*** iff $\mathbf{c}(\pi) \preceq (1+\varepsilon)\mathbf{A}$.

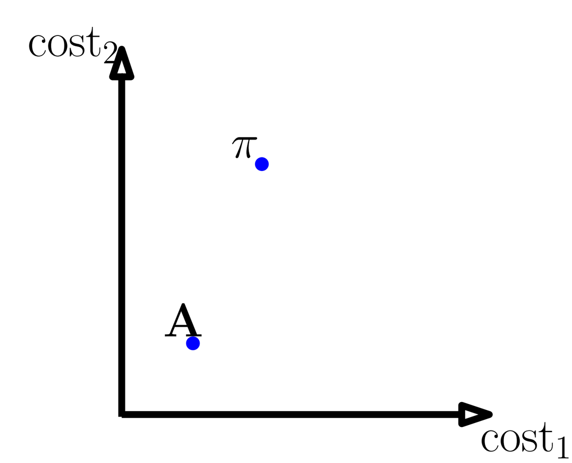


Figure: An apex-path pair

A*pex runs an A*-like search with apex-path pairs as search nodes and merges apex-path pairs whenever the resulted apex-path pairs are ε-bounded. The solution set returned by A*pex contains the representative path of each solution apex-path pair.

Summary: By using ε-bounded apex-path pairs to approximate the Pareto-optimal frontier, A*pex finds an ε-approximate frontier with fewer node expansions than existing algorithms.

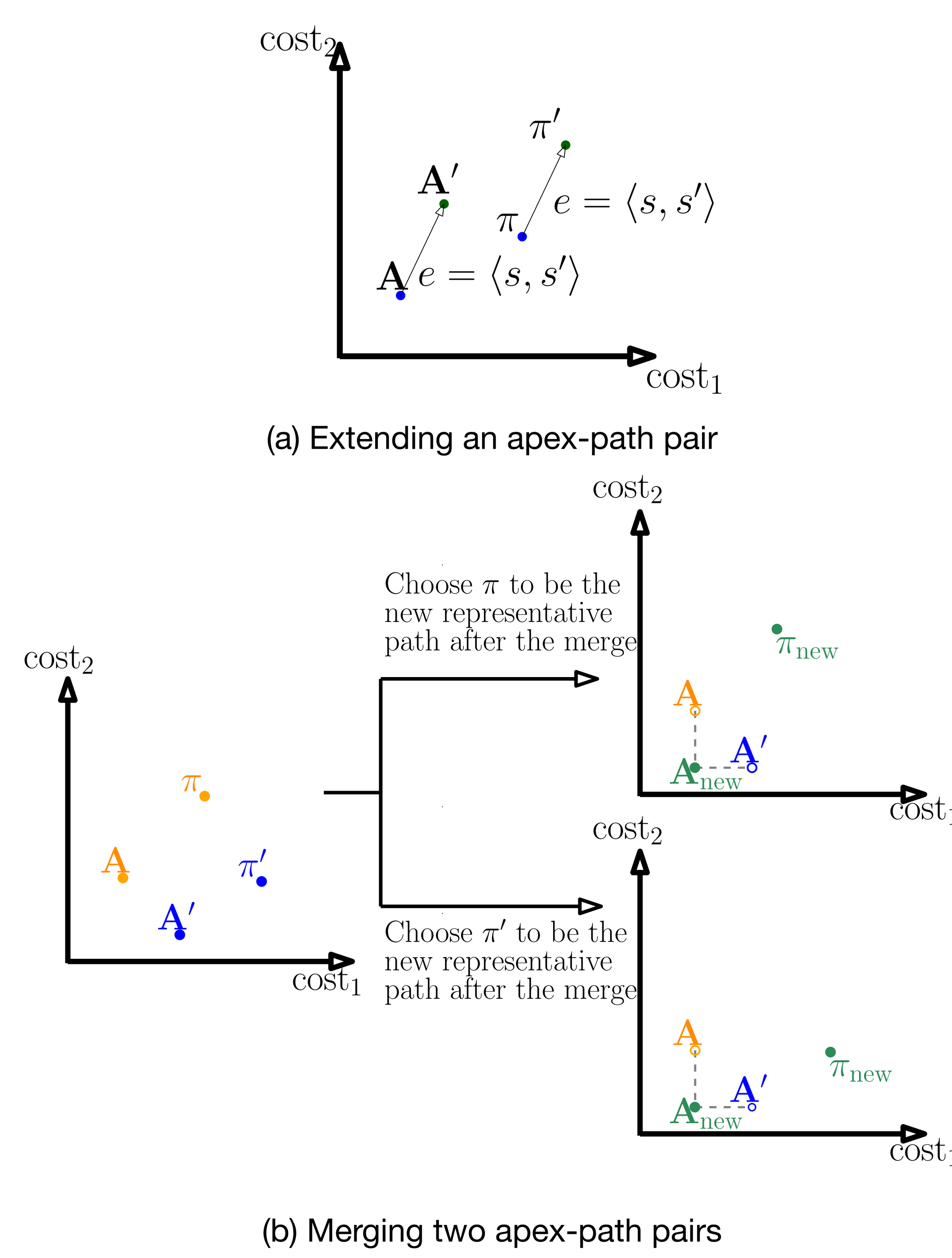
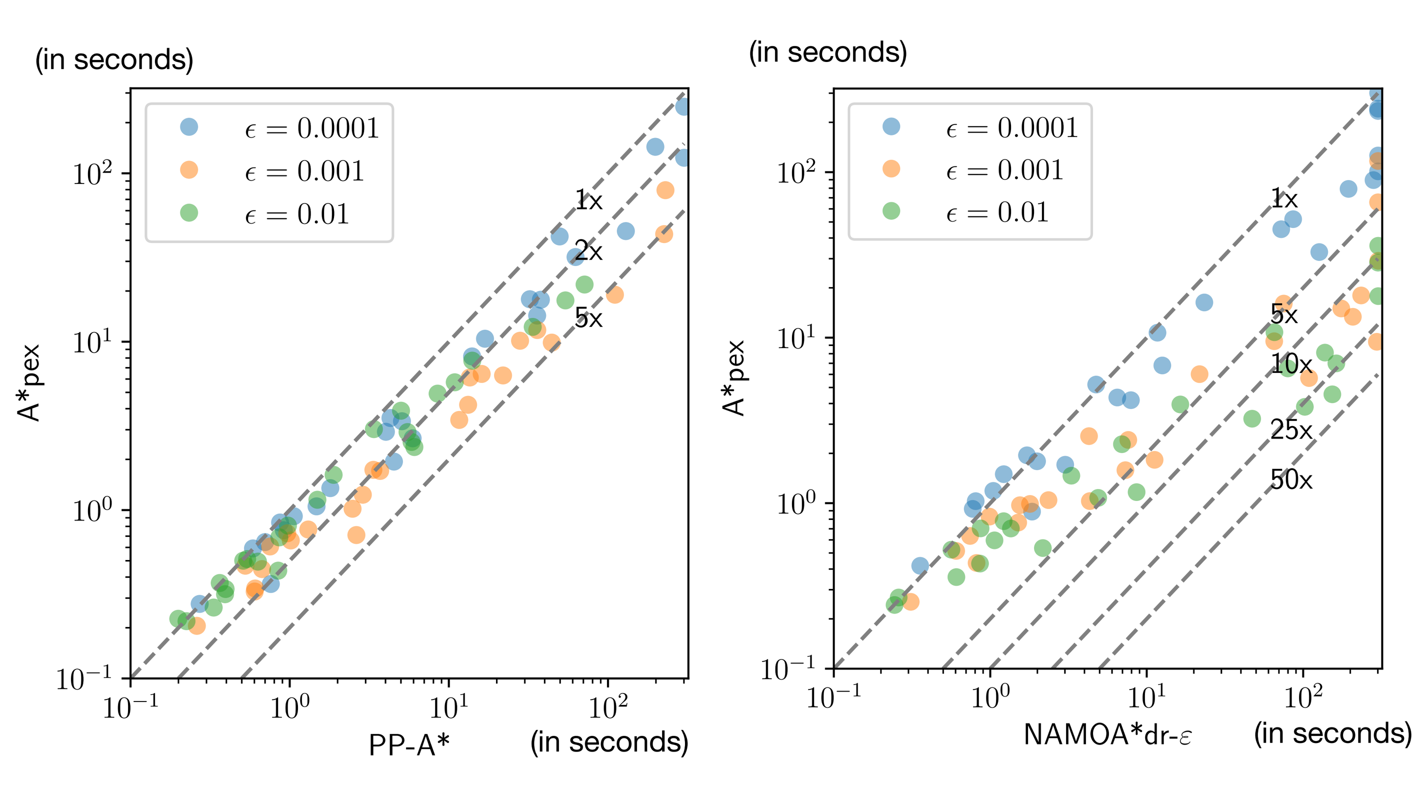


Figure: Operations on apex-path pairs

6 Experimental Results



(a) bi-objective problem instances (b) tri-objective problem instances

The figures show the runtime of A*pex and baseline algorithms for different numbers of objectives, problem instances, and ε-values. The **y-coordinate** of each point is the runtime of A*pex, and the **x-coordinate** of each point is the runtime of the respective baseline algorithm.

Summary: A*pex runs faster than the respective baseline algorithm in most problem instances for both two and three objectives. In some instances, the speedups are more than **5×** and **25×**, respectively.

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[1] Goldin, B.; and Salzman, O. 2021. Approximate Bi-Criteria Search by Efficient Representation of Subsets of the Pareto- Optimal Frontier. In ICAPS, 149–158.