Efficient Computation and Informative Estimation of h⁺ by Integer and Linear Programming

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Introduction

- Delete-free planning: actions add but do not delete propositions
- The optimal cost of the solution: h^+
- Computing h^+ is NP-complete
- h^+ is hard to approximate

Causal Partial Functions

1. if f(p) = a then a adds p
2. if f(p) = a then for every precondition q of a, either q ∈ I or f is defined for q
3. for every p ∈ G, either p ∈ I or f is defined for p.

Causal Relaxed Plan Representations

- For any causal partial function f, we construct a digraph G_f that includes (q,p) iff for some a, f(p) = a and q is a precondition of a.
- f is a causal relaxed plan representation iff G_f is acyclic.
- cost(f) is the total cost of all actions to which some atomic proposition is mapped by f.

Equivalence

- For every STRIPS planning problem Π there exists a causal relaxed plan representation f for Π such that cost(f) = h+(Π)
- Let Π be a STRIPS planning problem and f be a causal relaxed plan representation for Π. Then cost(f) is an upper bound of h+(Π)

IP Model - Objective Function

$$minimize \quad \sum_{a\in A} f_a \cdot cost(a)$$

IP Model - Causal Partial Functions

$$f_a \in \{0,1\} \qquad f_p \in \{0,1\} \qquad f_{p,a} \in \{0,1\}$$

$$\forall p \in P, \quad f_p = \sum_{p\in add(a)} f_{p,a}$$

$$\forall p,q \in P, \quad \left(\sum_{q\in pre(a),p\in add(a)} f_{p,a} \right) \leq f_q$$

$$\forall p \in G, \quad f_p = 1$$

$$\forall a \in A, p \in add(a) \quad f_{p,a} \leq f_a$$

Cycle Prevention – Time Labels

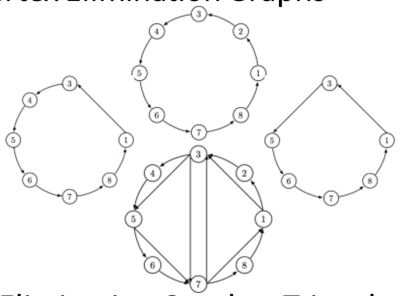
$$t_i \in \{1,\ldots,|P|\}$$

$$\forall a \in A, p_i \in pre(a), p_j \in add(a),$$

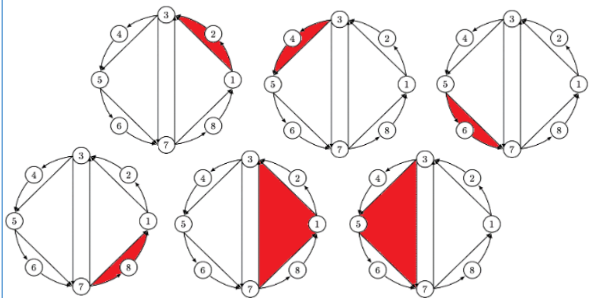
$$t_i - t_j + 1 \leq |P|(1 - f_{p_j,a})$$

IP-TL(∏), h_{TL}(∏,O)

Vertex Elimination Graphs



Vertex Elimination Graphs - Triangles



Cycle Prevention – Vertex Elimination

$$e_{i,j} \in \{0,1\}$$

$$\forall a \in A, p_i \in pre(a), p_j \in add(a) \quad f_{p_j,a} \leq e_{i,j}$$

$$\forall (p_i,p_j) \in E^*_\Pi, \quad e_{i,j} + e_{j,i} \leq 1$$

$$\forall (p_i,p_j,p_k) \in \Delta, \quad e_{i,j} + e_{j,k} - 1 \leq e_{i,k}$$

Main idea: if G_M is cyclic then G_M* has a cycle of length 2

IP-VE(∏,O)

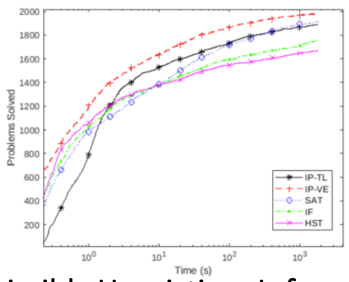
LP-relaxation: h_{VE}(∏,O)

Theoretical Results

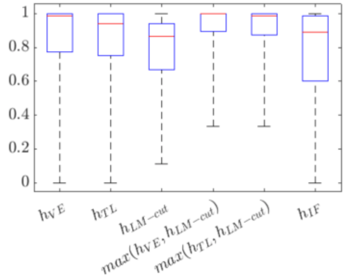
Let Π = (P, A, I, G, cost) be a STRIPS planning problem, and O be any order on members of P.

- **Theorem 1.** If *f* is a causal relaxed plan representation for Π with cost *c*, then IP-VE(Π, O) has a feasible solution with objective value *c*.
- **Theorem 2.** If IP-VE(Π, O) has a feasible solution with objective value *c*, then there exists a causal relaxed plan representation for Π with cost at most *c*.
- **Theorem 3.** h_{VE}(∏,O) ≥ h_{TL}(∏)

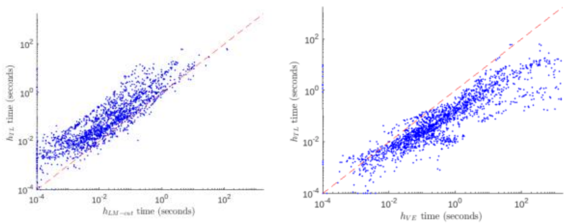
Exact Methods - Coverage



Admissible Heuristics - Informativeness



Admissible Heuristics - Efficiency



Admissible Heuristics – A* Coverage