

Domain-Independent Heuristics in Probabilistic Planning – Dissertation Abstract

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Abstract

It has been almost two decades since MDP heuristic search algorithms have been developed. These algorithms guarantee to find an optimal policy for the initial state for several optimization objectives without necessarily expanding the entire state space, if provided with a heuristic that provides optimistic state value estimates. While a large and diverse set of such domain-independent heuristic families is available in classical planning, the same cannot be said about probabilistic planning. So far, except for the particular case of occupation measure heuristics for (constrained) Stochastic Shortest Path Problems, most of the attempts at constructing heuristics pursue the very simple approach of using a classical heuristic on the all-outcomes determinization of the planning problem, in which the probabilistic effect of an action can be chosen at will. Because this approach is agnostic to the uncertainty in the underlying problem, these heuristics are often not very informative. In this thesis, we will investigate heuristics for probabilistic planning which are formulated on the underlying probabilistic model directly instead of delegating to a classical heuristic on the determinization. To this end, we mainly focus on abstraction heuristics, in particular Pattern Database heuristics and Merge-and-Shrink heuristics.

Introduction

AI planning is a long-standing discipline in artificial intelligence which deals with the automatic deduction of strategies for autonomous agents. In classical planning, the simplest form of planning, a single agent acts inside a fully observable and deterministic environment. Probabilistic Planning relaxes these assumptions to allow stochastic environments, where an action leads to one of multiple possible outcomes, each having an associated probability that is known a priori. In this thesis, we will focus on the fully observable case, where problems are commonly modelled as a Markov Decision Process (MDP). In this setting, the behaviour of the agent is typically specified by a function from states to actions, called a *policy*.

There exist various optimization criteria which are considered in this setting to specify the desired behaviour of the agent. In this thesis, we focus on two settings in particular. In the *MaxProb* setting, we want to find a policy that maximizes the probability to reach a set of goal states when starting in the initial state of the problem. On the other hand,

Stochastic Shortest Path Problems (SSPs) associate each action application with a real-valued cost. The objective of the agent is to reach a set of goal states while minimizing the expected accumulated cost to do so.

There exist a plethora of algorithms to solve MDPs both optimally and approximately for these settings. Heuristic search algorithms for SSPs (e.g. Hansen and Zilberstein (2001), Bonet and Geffner (2003), Trevizan et al. (2017)) have the potential to prevent the exhaustive generation of the whole state space of the problem. These algorithms require an *admissible* heuristic to ensure optimality, i.e. a function which underestimates the real minimal expected cost-to-goal of a state. These algorithms can also be extended to MaxProb (Kolobov et al. 2011), where they require an upper-bounding heuristic instead.

Although substantial effort has been invested into the development of MDP heuristic search algorithms themselves, research regarding admissible heuristics which can be supplied to these algorithms is, to this day, rather sparse. Most published work on this topic casts the problem back to a classical planning problem by utilizing the all-outcomes determinization (Yoon, Fern, and Givan 2007), in which the agent can simply choose the probabilistic outcome of an action. With the help of this transformation, any classical heuristic can be used to guide the search by delegating to the determinization.

While the determinization-based approach enables the use of a large arsenal of classical planning heuristics, these heuristics are often not very informative, since the uncertainty in the problem is completely ignored. So far, the only class of admissible heuristics which do not completely ignore the uncertainty in the planning task are occupation measure heuristics (Trevizan, Thiébaux, and Haslum 2017) for (constrained) SSPs. These LP-based heuristics can be seen as extensions of operator counting heuristics (Pommerening et al. 2014) to probabilistic planning. Trevizan, Thiébaux, and Haslum demonstrate two instances of occupation measure heuristics: The projection occupation measure heuristic h^{pom} and the regrouped operator-counting heuristic h^{roc} . In their empirical analysis, both heuristics clearly outperform several determinization-based heuristics. As of today, these heuristics are still considered state-of-the-art.

In this thesis, we develop novel domain-independent heuristics for probabilistic planning which are not agnos-

tic to the uncertainty of the problem. To this end, we extend abstraction heuristics from classical planning to probabilistic planning and explore various types of heuristics which belong to this family, including Pattern Database heuristics (Korf 1997; Haslum et al. 2007; Pommerening, Röger, and Helmert 2013), and Merge-and-Shrink heuristics (Helmert, Haslum, and Hoffmann 2007; Nissim, Hoffmann, and Helmert 2011; Helmert et al. 2014). We compare our new framework both theoretically and empirically with the previously mentioned frameworks and prove interesting dominance properties.

Preliminaries

We consider probabilistic planning problems with full observability in the context of different optimization objectives. This thesis abstract focuses primarily on the MaxProb objective, which prioritizes goal probability maximization, as well as Stochastic Shortest Path Problems (SSPs, Bertsekas and Tsitsiklis (1991)). To represent both settings in a uniform manner, the underlying probabilistic model will be kept separate from the considered optimization objective.

MDPs and Optimization Objectives As the baseline model, we define a Markov Decision Process (MDP) as a 4-tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{T}, s_{\mathcal{I}} \rangle$. \mathcal{S} is the finite and non-empty set of *states*, \mathcal{A} is a finite and non-empty set of *actions*, $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$ is the *transition probability function* and $s_{\mathcal{I}} \in \mathcal{S}$ is the *initial state* of the problem. For any state action pair $\langle s, a \rangle \in \mathcal{S} \times \mathcal{A}$, either $\sum_{t \in \mathcal{S}} \mathcal{T}(s, a, t) = 1$ (a is enabled in s) or $\mathcal{T}(s, a, t) = 0$ for any successor state t (a is disabled in s). The set of actions enabled in s is denoted $\mathcal{A}(s)$. We assume that $\mathcal{A}(s) \neq \emptyset$ for all states. We can easily introduce artificial self-loops to achieve this. Finally, a policy is a mapping $\pi : \mathcal{S} \rightarrow \mathcal{A}$ with $\pi(s) \in \mathcal{A}(s)$ for every state $s \in \mathcal{S}$.

The MaxProb optimization objective is specified by a (non-empty) set of goal states $\mathcal{S}_{\mathcal{G}} \subseteq \mathcal{S}$. The semantics of a policy in presence of this optimization objective is given by the policy state value function $\mathcal{V}_{\text{MP}}^{\pi} : \mathcal{S} \rightarrow [0, 1]$, where $\mathcal{V}_{\text{MP}}^{\pi}(s)$ represents the probability to reach the goal when starting in the state s and following policy π . It can be defined as the (pointwise) *least* solution of the equation system

$$\mathcal{V}_{\text{MP}}^{\pi}(s) = \begin{cases} 1 & s \in \mathcal{S}_{\mathcal{G}} \\ \sum_{t \in \mathcal{S}} \mathcal{T}(s, a, t) \mathcal{V}_{\text{MP}}^{\pi}(t) & s \notin \mathcal{S}_{\mathcal{G}} \end{cases}$$

The optimal state value function $\mathcal{V}_{\text{MP}}^*$ gives the maximal achievable goal probability for a state under any policy and is defined as $\mathcal{V}_{\text{MP}}^*(s) := \max_{\pi} \mathcal{V}_{\text{MP}}^{\pi}(s)$. A policy π^* is optimal if $\mathcal{V}_{\text{MP}}^{\pi^*} = \mathcal{V}_{\text{MP}}^*$. For MaxProb, an optimal policy always exists. Moreover, we say that π is an s -proper policy, if $\mathcal{V}_{\text{MP}}^{\pi}(s) = 1$. If π is s -proper for all s , then π is proper.

The optimization objective considered for SSPs is given by a set of goal states $\mathcal{S}_{\mathcal{G}} \subseteq \mathcal{S}$ and an action cost function $c : \mathcal{A} \rightarrow \mathbb{R}$. This objective makes two additional assumptions: (i) There exists a proper policy and (ii) Every

improper policy eventually accumulates infinite cost¹. The policy state value function $\mathcal{V}_{\text{SSP}}^{\pi} : \mathcal{S} \rightarrow \mathbb{R}$ is only defined for proper policies π for this objective. $\mathcal{V}_{\text{SSP}}^{\pi}(s)$ gives the expected accumulated cost until the goal is reached when starting in state s and acting according to π . It is the unique solution of the equation system

$$\mathcal{V}_{\text{SSP}}^{\pi}(s) = \begin{cases} 0 & s \in \mathcal{S}_{\mathcal{G}} \\ c(\pi(s)) + \sum_{t \in \mathcal{S}} \mathcal{T}(s, a, t) \mathcal{V}_{\text{SSP}}^{\pi}(t) & s \notin \mathcal{S}_{\mathcal{G}} \end{cases}$$

The optimal state value function $\mathcal{V}_{\text{SSP}}^*$ is defined by $\mathcal{V}_{\text{SSP}}^*(s) := \min_{\pi \text{ proper}} \mathcal{V}_{\text{SSP}}^{\pi}(s)$. Analogously, a policy π^* is optimal if $\mathcal{V}_{\text{SSP}}^{\pi^*} = \mathcal{V}_{\text{SSP}}^*$ and always exists.

Probabilistic SAS⁺ Tasks As the underlying planning problem representation, we assume probabilistic SAS⁺ tasks (Trevizan, Thiébaux, and Haslum 2017), except that we factor out the cost function. A probabilistic SAS⁺ task is a tuple $\langle \mathcal{V}, \mathcal{A}, s_{\mathcal{I}}, \mathcal{G} \rangle$. \mathcal{V} denotes the *variables*, where each v is associated with a finite domain \mathcal{D}_v of at least two values. A *partial state* is a partial variable assignment $s : \mathcal{V} \mapsto \bigcup_{v \in \mathcal{V}} \mathcal{D}_v \cup \{\perp\}$ with $s(v) \in \mathcal{D}_v \cup \{\perp\}$. If $s(v) = \perp$, we say s is undefined on v . We denote by $\mathcal{V}(s)$ the set of all variables for which s is defined. s is a *state* if $\mathcal{V}(s) = \mathcal{V}$. The set of states of a probabilistic SAS⁺ task Π is denoted $\mathcal{S}(\Pi)$. For a set of variables $P \subseteq \mathcal{V}$ and partial state s , we denote by $s[P]$ the *projection of s onto P* and define the set $S[P] := \{s[v] \mid s \in \mathcal{S}\}$. We say s *subsumes* t , written $t \subseteq s$, if $s(v) = t(v)$ for all $v \in \mathcal{V}(s)$. The *application* of partial state e onto partial state s is defined by $\text{appl}(s, e)(v) = e(v)$ if e is defined on v , and $s(v)$ otherwise. The *initial state* $s_{\mathcal{I}}$ is a state. The *goal* \mathcal{G} is a partial state. \mathcal{A} is the set of *actions*. An action a specifies its *precondition* $\text{pre}(a)$, and a probability distribution Pr_a over effects, where an effect is a partial state. The possible effects of a are denoted $\text{Eff}(a) := \{e \mid \text{Pr}_a(e) > 0\}$.

A probabilistic SAS⁺ task $\Pi = \langle \mathcal{V}, \mathcal{A}, s_{\mathcal{I}}, \mathcal{G} \rangle$ induces the MDP $\langle \mathcal{S}(\Pi), \mathcal{A}, \mathcal{T}, s_{\mathcal{I}} \rangle$ where $\mathcal{T}(s, a, t)$ is defined as 0 if $\text{pre}(a) \not\subseteq s$ and by

$$\mathcal{T}(s, a, t) := \sum_{\substack{e \in \text{Eff}(a) \text{ s.t.} \\ \text{appl}(s, e) = t}} \text{Pr}_a(e)$$

otherwise. The set of goal states for the MaxProb and SSP objective is given by $\mathcal{S}_{\mathcal{G}} = \{s \mid s \subseteq \mathcal{G}\}$.

Heuristics A heuristic h is an estimator for the optimal state value $\mathcal{V}_{\text{MP}}^*(s)$ or $\mathcal{V}_{\text{SSP}}^*(s)$ of a state. For MaxProb, a heuristic is *admissible* if $h(s) \geq \mathcal{V}_{\text{MP}}^*(s)$, whereas it is admissible in the SSP setting if $h(s) \leq \mathcal{V}_{\text{SSP}}^*(s)$. For SSPs, a heuristic is *consistent* if the equation

$$h(s) \leq c(a) + \sum_{t \in \mathcal{S}} \mathcal{T}(s, a, t) h(t)$$

is satisfied for every $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$, and *goal-aware* if $h(s) = 0$ for goal states $s \in \mathcal{S}_{\mathcal{G}}$. These properties are

¹Less restrictive SSP definitions exist (Kolobov et al. 2011; Guillot and Stauffer 2020), but we focus on this traditional definition for simplicity.

convenient because a heuristic that is both consistent and goal-aware is admissible. Also, some SSP heuristics search algorithms like iLAO* (Hansen and Zilberstein 2001) can be optimized for consistent, goal-aware heuristics.

Lastly, a finite family of heuristics $(h_i)_{i \in I}$ (where I is some index set) is *additive* if $\sum_{i \in I} h_i(s) \leq \mathcal{V}_{\text{SSP}}^*(s)$ and *multiplicative* if $\prod_{i \in I} h_i(s) \geq \mathcal{V}_{\text{MP}}^*(s)$.

Abstraction Heuristics

In classical planning, abstraction heuristics are a fairly versatile family of heuristics that has been studied extensively in various forms, for example through Pattern Databases (e.g. Korf (1997); Haslum et al. (2007); Pommerening, Röger, and Helmert (2013)), Cartesian Abstraction (Seipp and Helmert 2013) and Merge-and-Shrink Abstraction (e.g. Helmert, Haslum, and Hoffmann (2007); Nissim, Hoffmann, and Helmert (2011); Helmert et al. (2014)). In classical planning, an abstraction is typically specified by a surjective abstraction mapping $\alpha : \mathcal{S} \rightarrow \mathcal{S}^\alpha$, which associates each state with a corresponding abstract state $\alpha(s) \in \mathcal{S}^\alpha$. For a labelled transition system (LTS), the deterministic planning model usually assumed in classical planning, an abstraction induces an abstract LTS which overapproximates the behaviour of the original LTS. This abstract LTS can then be solved to obtain an admissible heuristic for the original problem. To do the same with respect to an MDP, a definition for the abstract MDP induced by an abstraction needs to be proposed.

Probabilistic Projection Heuristics

In recent work (Klöbner et al. 2021), we propose a definition for the specific case of *projections*. A projection only considers a subset of variables (a "pattern") $P \subseteq \mathcal{V}$ of the task. It is induced by the abstraction mapping $s \mapsto s[P]$. The abstract MDP for a projection with respect to P is defined as $\langle S[P], A, \mathcal{T}_P, s_{\mathcal{I}}[P] \rangle$, where the transition probability $\mathcal{T}_P(s, a, t)$ is defined as 0 if $\text{pre}(a)[P] \not\subseteq \sigma$, otherwise

$$\mathcal{T}_V(\sigma, a, \tau) = \sum_{\substack{e \in \text{Eff}(a) \text{ s.t.} \\ \text{appl}(\sigma, e[P]) = \tau}} \text{Pr}_a(e)$$

As it turns out, the probabilistic projection heuristic $h^P(s) := \mathcal{V}_{\text{MP}}^*(s[P])$ is an admissible heuristic for MaxProb, and the analogous heuristic $h^P(s) := \mathcal{V}_{\text{SSP}}^*(s[P])$ is even consistent and goal-aware for SSPs, when the abstract set of goal states for both objectives is defined as $\mathcal{S}_{\mathcal{G}}[P]$. Most importantly, these heuristics dominate the respective determinization-based projection heuristic when applied on the same pattern.

Pattern Database Heuristics

In classical planning, Pattern Database heuristics are a family of abstraction heuristics that use several projections in unison to achieve a more accurate heuristic. Given a collection of patterns $C \subseteq \mathcal{P}(\mathcal{V})$, the corresponding pattern database heuristic is constructed by precomputing a lookup table of heuristic values for each individual projection heuristic h^P for $P \in C$. When an estimate for a state s

is requested, these individual projection heuristics can then be combined by performing the necessary table lookups and taking the highest estimate: $h_C(s) = \max_{P \in C} h^P(s)$. An even better approach is to employ additivity constraints to find subcollections $D \subseteq C$ such that the heuristics $(h^P)_{P \in D}$ become additive (Haslum et al. 2007). Max'ing over these subcollections then yields an even stronger heuristic, called the *canonical PDB heuristic* $h_C^{\text{can}}(s)$.

We published two papers (Klöbner et al. 2021; Klöbner and Hoffmann 2021) in which we transfer these concepts to probabilistic planning and construct pattern database heuristics which exploit a collection of probabilistic projections instead. In particular, we show that the well-known additivity constraints considered by Haslum et al. can be adapted and used in a straightforward manner to obtain additivity constraints for SSPs and even multiplicativity constraints for MaxProb.

In more detail, we say that an action *affects* a variable v if there is a possible effect $e \in \text{Eff}(a)$ with $\perp \neq e(v) \neq \text{pre}(a)(v)$. An action a affects a pattern P if a variable $v \in P$ is affected. We show that, for a collection of patterns C , if every action only affects at most one pattern $P \in C$, the projection heuristics $(h^P)_{P \in C}$ are additive for the SSP objective, and multiplicative for the MaxProb objective.

This observation leads to a direct generalization of $h_C^{\text{can}}(s)$ for both MaxProb and SSPs. We show that construction of this heuristic is analogous to the construction in classical planning: Finding the maximal additive subcollections of C can still be reduced to finding the maximal cliques in the graph where nodes are the pattern $P \in C$ and two patterns are connected if their projections are additive, which is easy to check for only two patterns. Our empirical evaluation shows a substantial improvement over the determinization-based canonical PDB heuristics.

In very recent work (Klöbner et al. 2022b), we also deal with the question of how to construct a useful initial pattern collection C in the first place when the problem is no longer deterministic. We consider and extend two approaches that have been studied in classical planning: Pattern construction via Counter-example guided abstraction refinement (CEGAR, Rovner, Sievers, and Helmert (2019)) and pattern construction as a search problem solved using hillclimbing (Haslum et al. 2007). We reformulate both of these frameworks to operate on MDPs, as opposed to using the classical algorithm variants on the determinization. Compared to classical pattern construction techniques on the determinization, both algorithms have a significant advantage in particular problem domains. However, there also exist many domains in which we observe no benefit over determinization-based pattern construction, so these algorithms can by no means be seen as a universal answer to this research question. We might therefore revisit this topic in the future.

Merge-and-Shrink Heuristics

Merge-and-Shrink abstractions are a considerably more flexible type of abstractions in classical planning, which are in theory able to represent any abstraction. In a nutshell, the merge-and-shrink framework operates on a factored rep-

representation consisting of a set of small LTS $\Theta^1, \dots, \Theta^n$ called *factors*. These factors implicitly represent the LTS that is their synchronous product. If $\Theta^i = \langle \mathcal{S}^i, \mathcal{A}, \mathcal{T}^i, s_{\mathcal{T}}^i \rangle$, the synchronous product $\bigotimes_{i=1}^n \Theta^i$ is the LTS $\langle \mathcal{S}^1 \times \dots \times \mathcal{S}^n, \mathcal{A}, \mathcal{T}^\otimes, \langle s_{\mathcal{T}}^1, \dots, s_{\mathcal{T}}^n \rangle \rangle$ where the transition relation \mathcal{T}^\otimes contains the transition $\langle \langle s_1, \dots, s_n \rangle, a, \langle t_1, \dots, t_n \rangle \rangle$ if and only if $\langle s_i, a, t_i \rangle \in \mathcal{T}^i$ for all $i \in [1, n]$.

At the beginning of the merge-and-shrink algorithm, the set of factors is the set of atomic projections (to a single variable). The procedure is iterative. In its simplest form, the algorithm has two choices to make in each iteration:

1. Merge: Select two factors Θ_1 and Θ_2 and replace them by their synchronous product $\Theta_1 \otimes \Theta_2$.
2. Shrink: Select a factor and apply an abstraction on top of it. Replace the old factor with the abstraction.

The procedure stops when there is only one factor left.

The algorithms has several important properties. To formulate them, we need additional terminology. We say that an abstraction α *depends* on variable $v \in \mathcal{V}$ if there are states s and t with $\alpha(s) \neq \alpha(t)$ and $s(w) = t(w)$ for all $w \in \mathcal{V} \setminus \{v\}$. Two abstractions α, β are *orthogonal*, if the sets of variables they depend on are disjoint. Now, if Θ_1, Θ_2 each represent an abstraction α_1, α_2 of the original state space and these abstractions are orthogonal, then $\Theta_1 \otimes \Theta_2$ represents the abstraction $(\alpha_1 \otimes \alpha_2)(s) := \langle \alpha_1(s), \alpha_2(s) \rangle$. It follows easily that, if no shrinking is applied at all, the original state space is reconstructed (up to isomorphy) from all factors, so the set of atomic projection yields a representation of the original LTS. Secondly, if the factor Θ represents an abstraction α , and we shrink Θ using abstraction β , then the new abstract LTS represents the abstraction $(\beta \circ \alpha)(s) = \beta(\alpha(s))$. Due to these properties, the merge-and-shrink framework always returns an abstraction of the state space.

As of yet, it remains an open question whether or how the merge-and-shrink framework can be formulated for probabilistic planning tasks. A reasonable proposal of a merge-and-shrink framework for probabilistic planning should reproduce the above properties. A straightforward approach would model each factor as an MDP and initialize the merge-and shrink algorithms with all atomic (probabilistic) projections. The problem with modelling factors as MDPs is that the merge operation must be defined on MDPs, but the effect probability distributions \Pr_a are abstracted away in this model. To see the problem, consider for example a planning task with variables $v, w \in \{0, 1\}$ and a single action a with the following effects:

$$\begin{aligned} \Pr_a(\{v \mapsto 1\}) &= \frac{1}{4} \\ \Pr_a(\{v \mapsto 1, w \mapsto 1\}) &= \frac{1}{2} \\ \Pr_a(\{w \mapsto 1\}) &= \frac{1}{4} \end{aligned}$$

Unfortunately, this planning task has exactly the same projections onto v and w as the planning task where the effect

probabilities are changed to

$$\begin{aligned} \Pr_a(\{\}) &= \frac{1}{16} \\ \Pr_a(\{v \mapsto 1\}) &= \frac{3}{16} \\ \Pr_a(\{w \mapsto 1\}) &= \frac{3}{16} \\ \Pr_a(\{v \mapsto 1, w \mapsto 1\}) &= \frac{9}{16} \end{aligned}$$

Consequently, we cannot define a merging operation that reconstructs the original MDP of the planning task from the atomic projections, as the necessary information is already lost. Therefore, the planning model used for the factors must remember the individual effects of an action.

Regarding shrinking strategies, previous work on this subject in classical planning deals with variants of bisimulation (Nissim, Hoffmann, and Helmert 2011; Katz, Hoffmann, and Helmert 2012). Therefore, a natural candidate for shrinking strategies in probabilistic planning would be probabilistic bisimulation (Larsen and Skou 1991) as well as possibly relaxed variations of this concept. Alternatively, traditional shrinking strategies are still applicable by considering the determinization of a factor.

Other Contributions and Research Ideas

Although the thesis mainly concentrates on abstraction heuristics, everything else related to domain-independent heuristic construction for MaxProb and SSPs falls into the broader scope of the thesis. In a recent publication (Klößner et al. 2022a), we propose a theory of cost partitioning (Katz and Domshlak 2010) for SSPs. We found out that Trevizan, Thiébaux, and Haslum’s projection occupation measure heuristic h^{pom} essentially computes an optimal cost-partitioning over atomic projections. This has major implications, as it means that optimal cost partitioning over PDB heuristics is theoretically superior to h^{pom} . An experimental evaluation that analyzes the performance of different cost-partitioning techniques, and also yielding an answer to the question: “Can cost-partitioned PDB heuristics outperform occupation measure heuristics in practice?” is an obvious candidate for future work.

Conclusion

Abstraction heuristics for MaxProb and SSPs are promising candidates to extend the landscape of admissible heuristics for these settings and enable more effective use of MDP heuristic search algorithms. So far, we focused in particular on Pattern Database heuristics, for which we observe a clear advantage over determinization-based heuristics. When combined with cost-partitioning, these heuristics even have the potential to outperform occupation measure heuristics, which are the most powerful heuristics for SSPs at present. In future work, we aim to transfer the Merge-and-Shrink framework to probabilistic planning and take a detailed look at various cost-partitioning techniques for SSPs.

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