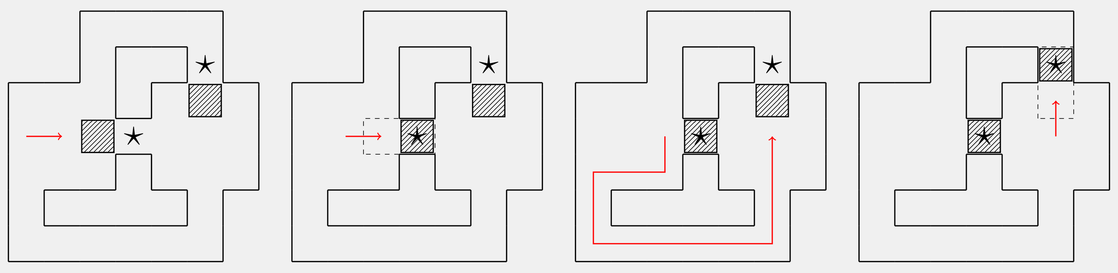
Admissible Heuristics for Multi-Objective Planning

Florian Geißer Patrik Haslum Sylvie Thiébaux Felipe Trevizan

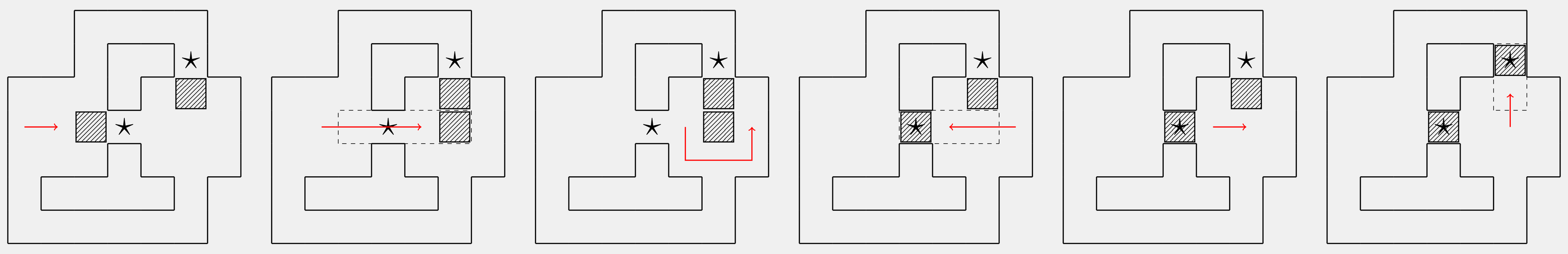
Australian National University

Multi-Objective (MO) Planning

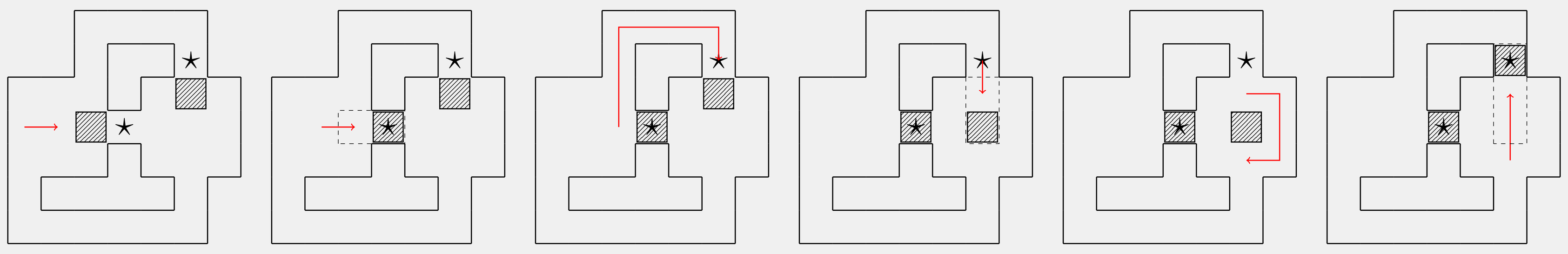
- Multi-objective planning/optimisation problems have k (> 1) metrics.
  - No a priori specified trade-off
  - Plan cost is a vector of k values.
- There may not be a single optimal cost, but a set of incomparable non-dominated cost vectors.
  - We will call this a Pareto (cover) set.
- $\vec{v}$ dominates $\vec{w}$ ($\vec{v} \prec \vec{w}$) if $\vec{v}$ is at least as good as $\vec{w}$ on every metric, and strictly better on at least one.



(2 pushes, 16 moves total)

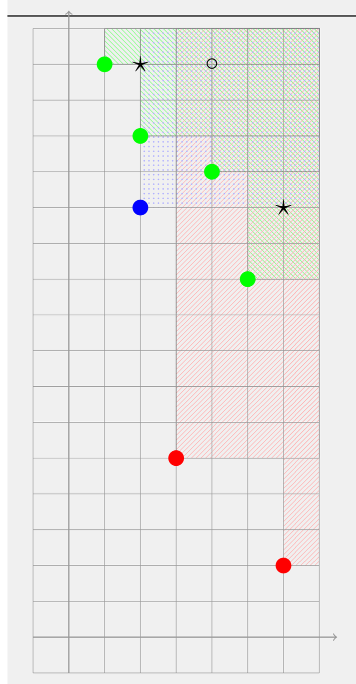


(6 pushes, 12 moves total)



(4 pushes, 16 moves total)

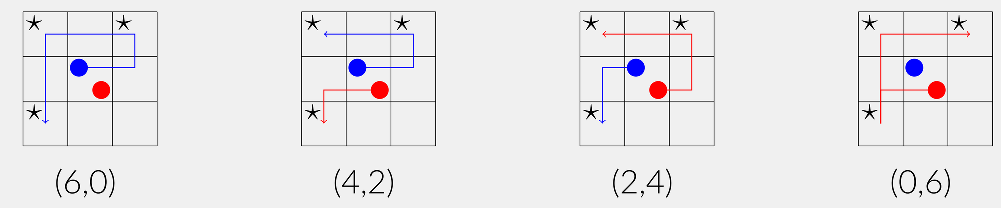
MO Heuristic Search



- There exist MO heuristic search algorithms that compute a Pareto cover set.
  - e.g., NAMOA* and its refinements.
- An MO heuristic value of a state is a *set of cost vectors* (like the Pareto set).
- Such a heuristic is admissible iff, for every state, every non-dominated cost vector of a plan from there is dominated by or equal to some cost vector in the heuristic set.
- $H^{\star} = \{(2,16),(6,12)\}$
- $H = \{(1,16),(2,14),(4,13),(5,10)\}$
- $H = \{(3,5),(6,2)\}$ (not admissible).
- $H = \{(2,12)\}$

Ideal Point Heuristic

- The ideal point heuristic is obtained by applying a classical (single-objective) heuristic for each metric *in isolation*, and combining their values into a single vector.
- General scheme, but weak – fails to account for any necessary trade-off between objectives.



$H^{h^{\star}}_{\text{ideal}} = \{(0,0)\}$ $H^{\max}_{MO} = \{(2,0),(0,2)\}$

Contribution

We derive informative MO heuristics from the same principles as classical planning heuristics

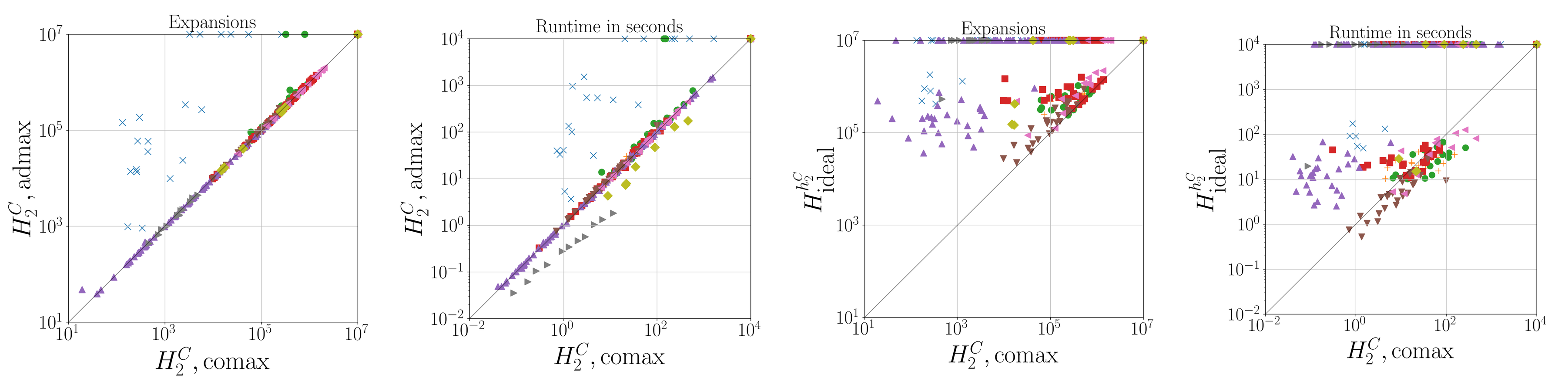
- Canonical abstraction heuristics
- Critical path heuristics
- (I)LP-based operator counting heuristics

We present methods of combining single-objective heuristics into MO heuristics that are more informed than the ideal point

- Not yet quite as general as we would like
- Not yet practical

MO Abstraction Heuristics

- Compute a Pareto cover set for each abstract state.
- Selecting (PDB) abstractions: Simple methods (e.g., enumeration) can be applied as-are.
- Canonical heuristic combining abstractions by using MO maximum and (admissible) sum.



MO Cost Partitioning

- $\vec{c}_1, \dots, \vec{c}_n$ such that $\vec{c}_1(a) + \dots + \vec{c}_n(a) \le \vec{c}(a)$ (vector sum) for each action a.
- Sum of cost vector sets is component-wise:

$$A + \dots + Z = \{\ (\vec{a}^1 + \dots + \vec{z}^1, \dots, \vec{a}^k + \dots + \vec{z}^k) \mid \vec{a} \in A, \vec{b} \in B\}$$

- Sum of cost partitioned heuristics is admissible.
- Evaluated only with 0/1 partitionings for disjoint abstractions.
- How to adapt more advanced cost partitionings is an open question.

MO Maximum – Candidate Definitions

- No unique maximum of sets of cost vectors.
- Component-wise maximum

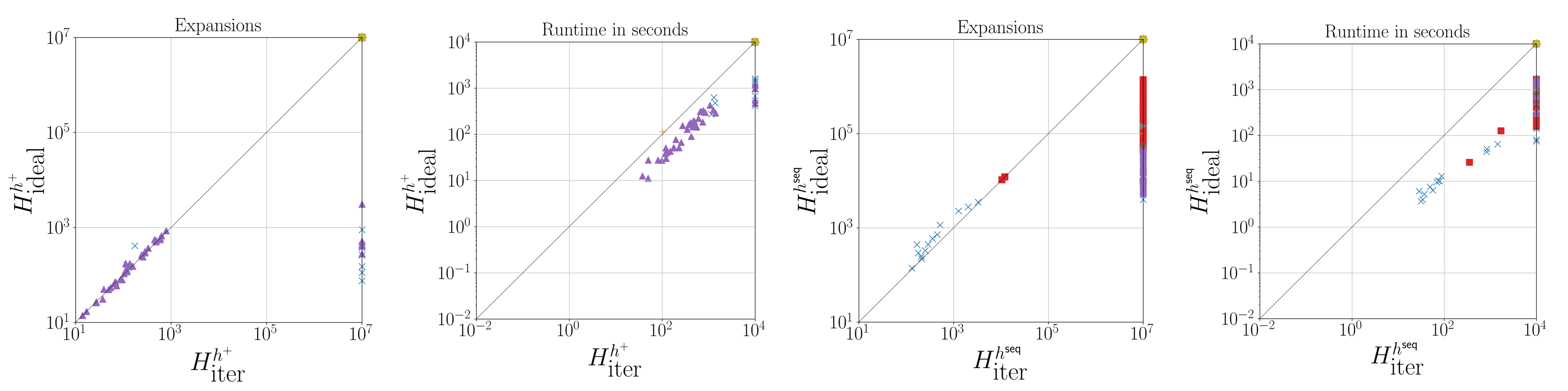
$$\mathrm{comax}(A,B) = \{\ (\max(\vec{a}^1, \vec{b}^1), \dots, \max(\vec{a}^k, \vec{b}^k)) \mid \vec{a} \in A, \vec{b} \in B\}$$

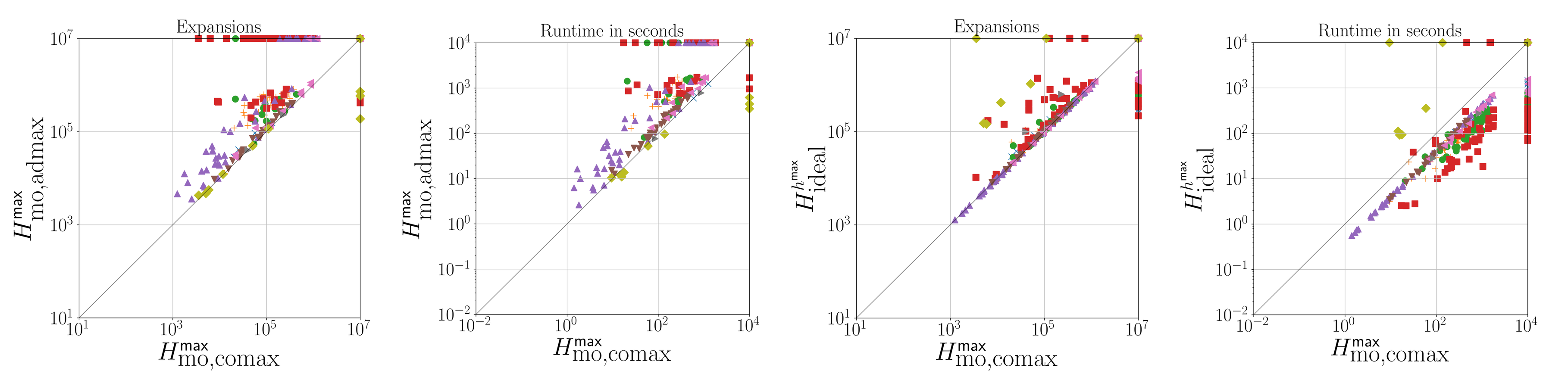
- Anti-dominance maximum

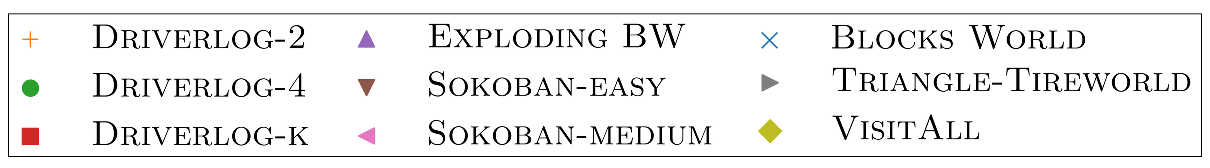
$$\mathrm{admax}(A,B) = \{\vec{a} \in \mathrm{ND}(A) \mid \forall \vec{b} \in \mathrm{ND}(B) : \vec{a} \not\prec \vec{b}\} \cup \{\vec{b} \in \mathrm{ND}(B) \mid \forall \vec{a} \in \mathrm{ND}(A) : \vec{b} \not\prec \vec{a}\}$$

MO Maximum – Candidate Definitions

- Both have some natural properties:
  - associative and commutative;
  - if $A \prec B$, then $\mathrm{comax}(A,B) = \mathrm{admax}(A,B) = B$.
- Both preserve heuristic admissibility.
- comax preserves consistency, admax does not.
- comax has worst-case quadratic size ($|\mathrm{comax}(A,B)| \le |A| \times |B|$).
- admax has worst-case linear size ($|\mathrm{admax}(A,B)| \le |A| + |B|$).
- Experimentally, using comax (in abstraction heuristics and in $H^{\max}_{MO}$) is nearly always better.







momax operator	blind	Critical Path: $H^{h^{\max}}_{\text{ideal}}$	$H^{\max}_{\text{mo}}$ adm.	$H^{\max}_{\text{mo}}$ com.	H^2_{mo} com.	Abstractions: $H^{h^C_2}_{\text{ideal}}$	H^C_2 adm.	H^C_2 com.	$H^{h^C_3}_{\text{ideal}}$	H^C_3 com.	Operator Counting: $H^{h^+}_{\text{ideal}}$	$H^{h^+}_{\text{iter}}$	$H^{h^{\text{seq}}}_{\text{ideal}}$	$H^{h^{\text{seq}}}_{\text{iter}}$	$H^{h^+_{\text{lp}}}_{\text{ideal}}$	$H^{h^+_{\text{lp}}}_{\text{iter}}$	$H^{h^+_{\text{rel}}}_{\text{ideal}}$	$H^{h^{\text{seq}}_{\text{lp}}}_{\text{ideal}}$
Driverlog-2 (52)	22	44	26	42	6	22	40	40	22	**49**	1	1	10	–	1	1	8	37
Driverlog-4 (68)	18	**46**	12	37	2	18	29	31	18	43	–	–	4	–	1	1	1	17
Driverlog-k (111)	22	46	14	39	1	22	43	43	22	**66**	–	–	37	2	10	1	25	43
Sokoban-E (26)	25	**26**	**26**	**26**	–	**26**	**26**	**26**	**26**	25	–	–	2	–	–	–	–	10
Sokoban-M (28)	–	**18**	11	13	–	12	**18**	**18**	9	10	–	–	–	–	–	–	–	2
Exploding BW (107)	13	46	33	46	24	26	**79**	**79**	26	70	39	34	14	–	38	12	31	25
Blocks World (20)	6	10	5	5	–	6	14	**20**	6	18	10	3	19	13	6	1	14	**20**
T-Tireworld (11)	1	2	2	2	1	1	**11**	**11**	1	**11**	–	–	1	–	–	–	–	1
VisitAll (13)	3	4	**9**	6	6	3	7	7	3	6	–	–	–	–	–	–	–	3
Sum (436)	110	242	138	216	40	136	267	275	133	**298**	50	38	87	15	56	16	79	158

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