Neural Network Action Policy Verification via Predicate Abstraction

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State Space Representation

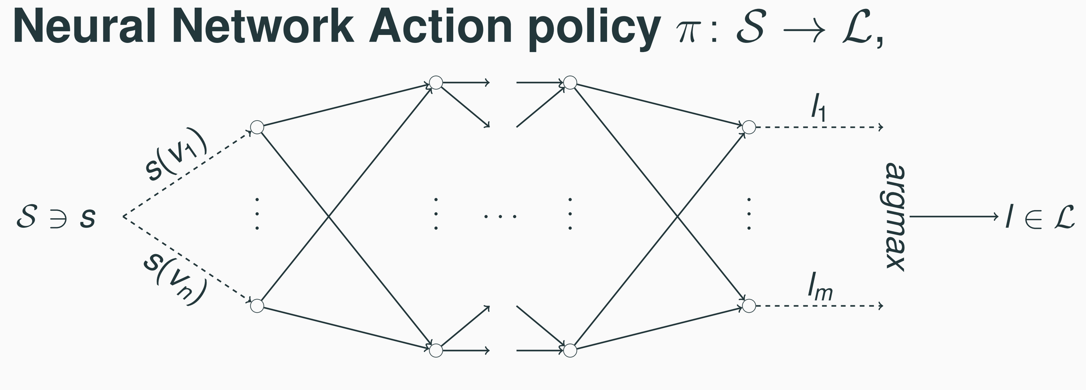
- **State variables** $\mathcal{V}$ with a bounded-integer domain.
- **Linear integer expressions** *Exp* over $\mathcal{V}$, $d_1 \cdot v_1 + \cdots + d_r \cdot v_r + c$ with $d_1, \ldots, d_r, c \in \mathbb{Z}$ and $v_1, \ldots, v_r \in \mathcal{V}$.
- **Linear integer constraints** and conjunctions thereof C, $e_1 \bowtie e_2$ with $e_1, e_2 \in Exp$ and $\bowtie \in \{\leq, =, \geq\}$.
- Labeled **operators** $\mathcal{O}$ of the form (g, l, u), with **action label** $l \in \mathcal{L}$, guard $g \in C$ and (partial) update $u \colon \mathcal{V} \to Exp$.

(Non-deterministic) state space LTS $\Theta = \langle \mathcal{S}, \mathcal{L}, \mathcal{T} \rangle$:

- **States** $\mathcal{S}$: complete state variable assignments over $\mathcal{V}$.
- **Transition** $(s, l, s') \in \mathcal{T}$ iff $s \models g$ (also: $s \models o$) and $s' = s[u(s)]$ (also: $s' = s[\![o]\!]$) for some operator $o = (g, l, u)$ in $\mathcal{O}$.
- State-dependent effects: (g_1, l, u_1) and (g_2, l, u_2) with $s_1 \models g_1$ but $s_2 \not\models g_1$ and $s_2 \models g_2$ but $s_1 \not\models g_2$.
- Action outcome non-determinism: $(s, l, s_1) \in \mathcal{T}$ induced by (g_1, l, u_1) and $(s, l, s_2) \in \mathcal{T}$ induced by (g_2, l, u_2).

Neural Network Action Policy

- **Neural Network Action policy** $\pi \colon \mathcal{S} \to \mathcal{L}$,



ReLU activation : $\max(x, 0)$

- **Policy restriction** $\Theta^\pi = \langle \mathcal{S}, \mathcal{L}, \mathcal{T}^\pi \rangle$ with $\mathcal{T}^\pi = \{(s, l, s') \in \mathcal{T} \mid \pi(s) = l\}$.
- **Safety property** $\rho = (\phi_0, \phi_U)$ with **start condition** $\phi_0 \in C$ and **unsafety condition** $\phi_U \in C$. π is **unsafe** iff there exist states $s_0 \models \phi_0$, $s_U \models \phi_U$ such that s_U is reachable from s_0 in Θ^π.

Policy Predicate Abstraction

- **Idea**: Predicate Abstraction (e.g., Graf and Saïdi (1997)) under π.
- Set of **predicates** $\mathcal{P} \subseteq C$.
- **Abstraction** of concrete state $s \in \mathcal{S}$: $s|_\mathcal{P} \in \mathcal{P} \to \{0, 1\}, p \mapsto p(s)$.
- **Concretization** of abstract state $s_\mathcal{P} \in \mathcal{P} \to \{0, 1\}$: $[s_\mathcal{P}] = \{s' \in \mathcal{S} \mid s'|_\mathcal{P} = s_\mathcal{P}\}$.
- The **policy predicate abstraction** of Θ^π over $\mathcal{P}$ is the LTS $\Theta^\pi_\mathcal{P} = \langle \mathcal{S}_\mathcal{P}, \mathcal{L}, \mathcal{T}^\pi_\mathcal{P} \rangle$, where $\mathcal{S}_\mathcal{P} = \mathcal{P} \to \{0, 1\}$ and $\mathcal{T}^\pi_\mathcal{P} = \{(s|_\mathcal{P}, l, s'|_\mathcal{P}) \mid (s, l, s') \in \mathcal{T}^\pi\}$ (transition preservation).

Motivation: Policy safety verification via (over-approximating) reachability analysis in $\Theta^\pi_\mathcal{P}$.

Transition problem of $\Theta^\pi_\mathcal{P}$:
$(s_\mathcal{P}, l, s'_\mathcal{P}) \in \mathcal{T}^\pi_\mathcal{P}$ iff for some operator $o = (g, l, u)$:
$\exists s \in [s_\mathcal{P}] \colon s \models o \wedge s[\![o]\!] \in [s'_\mathcal{P}] \wedge \pi(s) = l$.

(Without π) routinely encoded in SMT (e.g., *Z3* de Moura and Bjørner (2008)), **but** (under π) expensive due to non-linear NN activation.

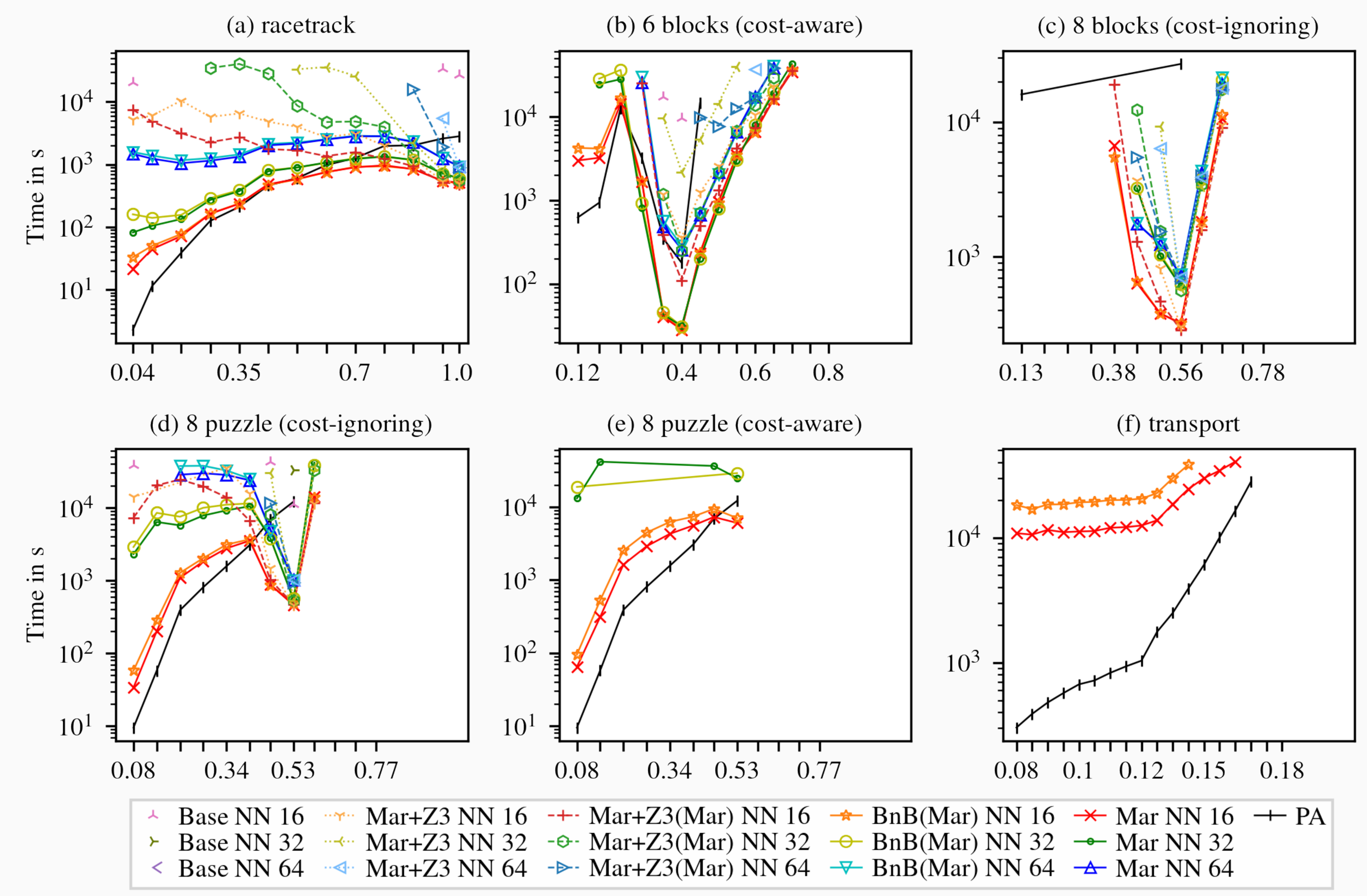
Algorithmic Enhancements: Relaxation + NN Analysis

- Tests on **necessary conditions**: label selection ($l \in \pi(s_\mathcal{P})$), operator applicability ($s_\mathcal{P} \models o$), label selection + operator applicability ($\exists s \in [s_\mathcal{P}] \colon \pi(s) = l \wedge s \models o$), and the non-policy-restricted transition ($\exists s \in [s_\mathcal{P}] \colon s \models o \wedge s[\![o]\!] \in [s'_\mathcal{P}]$). If unsat, one can skip all corresponding transition tests. Also: **non-policy-restricted** tests ($\cancel{\pi(s) = l}$) are much cheaper.
- **Continuously-relax** $\mathcal{V}$ to over-approximate the transition problem, plug in existing SMT solvers tailored to NN analysis (e.g., *Marabou* [Katz et al. (2019)]), branch & bound (over $\mathcal{V}$) to solve the exact transition problem.
- **Fixing activation cases** of ReLU towards SMT encoding: if $x \leq 0$ then $ReLU(x) = 0$, if $x \geq 0$ then $ReLU(x) = x$. Here: Extract bounds derived by *Marabou* to solve the exact transition problem.

Experiments

(on planning benchmarks modeled in JANI [Budde et al. (2017)])

- Compute $\Theta^\pi_\mathcal{P}$ reachable from ϕ_0.
- Scaling $|\mathcal{P}|$ as part of problem input (x-axis) for NN policies of different sizes (neurons per hidden layer).
- SMT via *Z3* [de Moura and Bjørner (2008)] & *Marabou* [Katz et al. (2019)].



$\rightarrow$ Algorithmic enhancements are required for practicality.

- Bounded Model Checking (BMC) competitor:

Benchmark \ NN	$L_U^{\min}$ 16|32|64	$t_U^{\min}$ 16|32|64	$L_{checked}^{\max}$ 16|32|64
Racetrack	3|3|3	36.9|40.1|316.5	12|11|7
6 Blocks (cost-awa)	-	-	6|5|4
8 Blocks (cost-ign)	-	-	5|5|4
8-puzzle (cost-ign)	2| - | -	72.8| - | -	7|3|0
8-puzzle (cost-awa)	-	-	3|3|0
Transport	1|1| -	57.0|20548.0| -	2|1|0

$\rightarrow$ PPA outperforms BMC competitor (& explicit enumeration).

References

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