Lecture 4

DFS and complexity

#Initialization Step | Space

List	Nodes	= [S, 1, 2, ..., n]	n
List	Next	= [ ]	n
List	pred	= [∞, ∞, ..., ∞]	n
List	visited	= [ ]	n

current +1

+1

Length +1

#Visiting S

visited.append(S) 1

For all neighbors v of S: +1

Next.append(v) +n*(2)

pred[v] = S

pred[S] = 0

$2n + 2$

Time Complexity

$DFS = O(n^2)$

Space Complexity

$DFS = O(n)$

While Next is nonempty: $n*(1+1+n*2)$

$(2n+2n^2)$

current=Next.pop[-1] +1

visited.append(current) +1

For all neighbors v of current: $n*(2)$

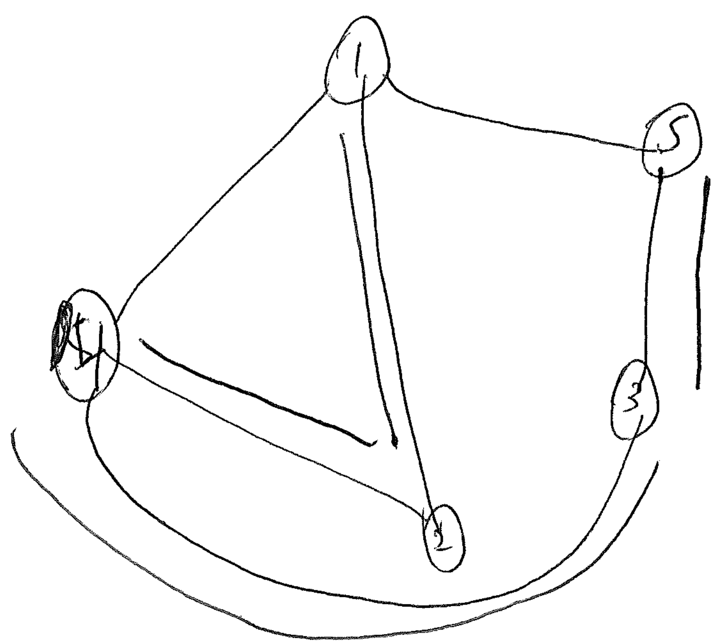
if v is not in visited:

~~visited.append(v)~~

pred[v] = current +1

Next.append(v) +1

return pred



Nodes = [S, 1, 2, 3, 4]

Next = []

pred = [∞, ∞, ∞, ∞, ∞]

visited = []

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visited = [S]

Next = [1, 3]

pred = [0, S, ∞, S, ∞]

Current = 3

Next = [1]

visited = [S, 3]

pred = [0, S, ∞, S, 3]

Next = [1, 4]

Current = 4

Next = [1]

visited = [S, 3, 4]

pred = [0, 4, 4, S, 3]

Next = [1, 2]

current = 2

next = [1]

visited = [5, 3, 4, 2]

pred = [5, 2, 4, 5, 3]

current = 1

next = [ ]

visited = [5, 3, 4, 2, 1]

Complexity

Let $f(n), g(n)$ be functions with positive integer inputs.

We say $g(n) = O(f(n))$ if $\exists\, C > 0$ and a positive integer N s.t.

$$|f(n)| \le C|g(n)| \quad \forall n > N$$

Ex. 1 Let $f(n) = n$

Let $g(n) = n^2 - 4$

Note: $N = 4$, $f(4) = 4$, $g(4) = 12$

So,

$$n \le 1(n^2 - 4) \quad \forall n > 4$$

$$\Rightarrow \quad n = O(n^2 - 4)$$

(Also $n = O(n)$ $n = O(n^2)$, $n = O(n!)$)

Ex2. Let $f(n) = n-1$. Let $g(n) = n-3$

$$n-1 \overset{?}{\leq} n-3$$

$$n-1 \nleq n-3$$

Try changing C!

$$C = 2$$

$$n-1 \leq 2(n-3)$$

$$n-1 \leq 2n-6$$

$$n+5 \leq 2n$$

Suppose $n \geq 100$

Then
$$105 \leq 200$$

and $n+5$ grows slower than $2n$

$$\Rightarrow \quad n-1 = O(n-3)$$

Suppose $f(n) = \sum_{i=0}^{a} c_i n^i$ and suppose $g(n) = \sum_{i=0}^{b} d_i n^i$

Then $|f(n)| \leq C|g(n)|$

$$\Rightarrow \frac{|f(n)|}{|g(n)|} \leq C$$

If $f(n) = O(g(n))$

$$\Rightarrow \lim_{n \to \infty} \left| \frac{f(n)}{g(n)} \right| \leq C$$

$$\Rightarrow \deg(f(n)) = a \leq b = \deg(g(n))$$

So if $f(n), g(n)$ are polynomials

$f(n) = O(g(n))$ if

$\underline{\deg(f(n)) \leq \deg(g(n))}$

ex 3. consider $f(n) = 112n^2 + 3n^3 + 2n^4$

Then $f(n) = O(n^4)$

$f(n) = O(n^5)$

$f(n) \neq O(n^3)$

we also use O to absorb terms

e.g.

$f(n) = \underline{112n^2 + 3n^3} + 2n^4 = \underline{2n^4} + O(n^3)$

suppose

$$f(n) = \sum_{i=1}^{32} (-1)^i (3n)^i = (3n)^{32} + O(n^{31})$$

We have to be careful for some piecewise functions.

ex. $f(n) = \begin{cases} n^2 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$

$f(n) = O(n^2)$

as both the even and odd parts are $O(n^2)$. You always take the worst case scenario.

If your function f has multiple inputs, i.e. $f(n,m)$. then we say

$$f(n,m) = O(g(n,m))$$

if $\exists c > 0$, and integers $N, M > 0$

s.t.

$$|f(n,m)| \leq c\,|g(n,m)|$$

ex. $f(n,m) = n^2 + nm + m^2 \leq n^2 + 2nm + m^2$

$= (n + m)^2$

$$f(n,m) = O(n^2 + nm + m^2)$$

$$f(n,m) = O((n+m)^2)$$

$$f(n,m) = O(n^2 + m^2)$$

Note $nm \leq n^2$ or $nm \leq m^2$

Time and space complexity is measured using O-notation.