Math 320 Final

April 20, 2023

You will have from the start of class until the end of the class period to complete your exam. Please clearly write each solution on the provided printer paper, and feel free to write any additional scratch work on separate paper. Please clearly number the start of a problem for your scratch work and for your solution.

Computation

Each of these questions will be worth 10 points.

1.

Draw a Cayley Digraph for the following product groups:

a. $C_3 \times C_4$
b. $D_3 \times C_2$

2.

Divide $f(x) = 5x^5 + 3x^2 + 1$ by $g(x) = 2x + 6$ in the polynomial ring $\mathbb{Z}_7[x]$. State your answer in the form $f(x) = q(x)g(x) + r(x)$ where $r(x) = 0$ or $\mathrm{Deg}r(x) < \mathrm{Deg}g(x)$. You may find the following multiplication table helpful:

U(7)						
*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

3.

Let $G = (\mathbb{Z}, +)$, $H = 7\mathbb{Z}$.

a. Find the elements of $G/7\mathbb{Z}$.
b. Find an isomorphism from $G/7\mathbb{Z} \to \mathbb{Z}_7$

4.

Let $G = (\mathbb{C}^*, *)$, the group of non-zero complex numbers under multiplication. Let $f : G \to G$ such that $f(a + bi) = a - bi$.

a. Show f is a group homomorphism.
b. What is the kernel of f?

Proofs

Your lowest scoring response to the next 3 questions will be dropped. Each question is worth 20 points.

5.

a)

State a definition of a subring.

b)

Suppose that R is a ring with unity 1 and a is some element of R such that $a^2 = 1$. Show that the set

$$S = \{ara | r \in R\}$$

is a subring of R.

6.

Let R be a ring. Let $a, b \in R$. Show that the identity

$$(a+b)^2 = a^2 + 2ab + b^2$$

holds for all $a, b \in R$ if and only if R is a commutative ring.

7.

a)

State a definition of an ideal.

b)

Let $R = \{a + bi | a, b \in \mathbb{Z}\}$ be the set of Gaussian integers and let $S = \{a + bi | a, b \in 2\mathbb{Z}\}$ be a subset of R. Show that S is an ideal of R.