

Math 320 HW 6

February 26, 2024

1)

Let $n\mathbb{Z} = \{0, \pm n, \pm 2n, \dots\}$ for some $n \in \mathbb{Z}$ such that $n > 0$ and let G be the group of integers under addition.

a)

Show $n\mathbb{Z} \leq G$

b)

Show $n\mathbb{Z} \approx G$ by finding an explicit isomorphism.

2)

Find the automorphism group of \mathbb{Z}_{12} . (Hint: Recall that isomorphisms map generators to generators).

3)

Find the inner automorphism group of D_4 .

4)

Let $G = (\mathbb{Q}^+, *)$ be the group of positive rational numbers under multiplication. Let $\phi : G \rightarrow G$ be given as $\phi(x) = x^3$. Determine if ϕ is an automorphism. If it is, prove this. If it isn't explain why not.

5)

Recall the center of a group G is $Z(G) = \{x \in G \mid ax = xa \ \forall a \in G\}$. Let $\phi : G \rightarrow H$ be an isomorphism. Show

$$\phi(Z(G)) = Z(H)$$

6)

Let G be a group. Show $|\text{Inn}(G)| = 1$ if and only if G is abelian.