Math 320 HW 1

Due: Jan. 16, 2023 at Midnight

1)

a)

Given the sets below, perform the following operations:

$$A = \{0, 1, 2\}, B = \{1, 2, 3\}, C = \{0, 2, 4, 5\}$$

1. $A \cap B \cap C$

2. $(A \cup B) - (B \cup C)$

3. $(B \cap A) \cup (A \cap C)$

4. $B \times C$

5. $(P(A) \cap P(B)) \cup (\emptyset \cap P(C))$

b)

Given the sets below, determine if the claims are true or false. Justify your response

$$X = \{0, 1, 2, 3, 4\}, Y = \{1, 3, 5, 7\}, Z = \{x | x \in \mathbb{R} \text{ and } x > 0\}$$

1. $(X \cup \emptyset) \subseteq Z$

2. $X \cap Y \neq \emptyset$

3. $|X \cap Z| = 4$

4. $Y \cap Z \subseteq Z$

5. $Y \subseteq Z$

6. $(Z \cap \emptyset) \supseteq Y$

7. $3 \in X \cap Y \cap Z$

2)

Prove the following claims for arbitrary sets A, B, C:

1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3)

Let n be a positive integer, and let A_i be an arbitrary subset of X for each $i \in \mathbb{Z}$. Prove DeMorgan's laws:

1. $X - \cap_{i=1}^{n} A_i = \cup_{i=1}^{n} (X - A_i)$

2. $X - \cup_{i=1}^{n} A_i = \cap_{i=1}^{n} (X - A_i)$

4)

Prove or provide a counter example the following claims:

1. The union of two sets is commutative

2. The union of two sets is associative

3. The intersection of two sets is commutative

4. The intersection of two sets is associative

5. The Cartesian product of two sets is commutative

6. The Cartesian product of two sets is associative