Option 2 Example

A Method for Simple Card Counting

When playing any number of card games (such as bridge, gin rummy, and pinochle), it is often helpful to remember which cards have been played in a given deal and which cards remain. For small number of cards, this can be done with pure memory. For large numbers of cards or for games with multiple decks of cards, this can be quite difficult to do by memorization alone. We can use some concepts from abstract algebra to help us!

The method here will allow us to determine the last card remaining in a standard deck of 52 cards. In fact, this method can be extended to find the last card remaining in any number of decks of 52 cards shuffled together. The core idea comes from the great gin rummy player, Stu Ungar, who famously used a similar trick to determine the last card remaining in 6 decks of cards (this is a total of 312 cards!). We will assign each card a value from 0 to 51. Each time we see a card of a given type, we will add its value to our running count. As each card appears exactly once in a deck of cards, then we know the total value of all of the cards of the deck. We can subtract the count from the total value of the deck of cards to find the remaining value of the cards left to be played (and once we get to the last card, this will tell us exactly what card we have yet to see). The sum of all the numbers between 0 and 51 can be found by using the well known result.

$$\sum_{i=k}^{n} i = \frac{(n+k)(n-k+1)}{2}$$

Thus

$$\sum_{i=0}^{51} i = \frac{(51+0)(51-0+1)}{2} = \frac{(51)(52)}{2} = 1326$$

So the entire deck of cards has a value of 1326. We can multiply this value by the number of decks to get the total value of all decks to be counted.

Before I present my list of values, I will introduce the first simplification I added to Ungar's method. Rather than keeping each value as a number in base 10, I will keep each card value as a number in base 13. This will make for easier memorization of card values and easier calculations later on. The one's place will store a digit corresponding to the card value, as shown in the table below.

Value	Base 13 One's Digit
10	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
J	J
Q	Q
K	K

As we do not have a single digit character for 10,11, and 12 in base 10, we will use J to represent a 10 (in base 10), Q for an 11 (base 10) and K for a 12 (base 10). Otherwise, every number card is stored by the value in its ones place (10 is stored as a 0). In the thirteen's place, we will store a digit corresponding to the suit of the card at hand. As there are four suits, we will use the digits 0-3, using the following table to assign values:

Suit	Base 13 Thirteen's Digit
♣	0
♦	1
♥	2
♠	3

These suits were chosen in order alphabetically for convenience. Let's do a few examples of converting cards to their corresponding values:

Card	Base 13 Value
3♥	23
7♦	17
K♠	3K
10♣	0

Note that we wrote the 10♣ as a 0, since a 0 in the thirteen's place is redundant. This is analogous to how we don't write a 7 as 07 in base 10. This yields a new number to remember for the total value of a single deck of cards. As

$$1326 = 7 * 169 + 11 * 13 + 0 = 7Q0$$

Although addition base 13 takes some getting used to, we will have an easier total value to memorize for this deck of cards. If we are willing to use a second modification, we can make this process even nicer. If we only care about the last card remaining in the deck, we could instead add all of the card values modulo 52. This simplification works as the integers mod 52 form a group under addition. This gives us the property of unique inverses (which is key to our count working). If we take the total value of the deck, and evaluate it mod 52, we get

$$1326 = 52 * 25 + 26 \equiv_{52} 26$$

Or if we work in base 13:

$$7Q0 = 40 * 1K + 20 \equiv_{40} 20$$

Here, we use 40 as 40 base 13 is the same as 52 base 10. This means converting these values mod 40 (base 13) only requires us to subtract 4 from the thirteen's place. This is the real power of working base 13. Instead of having to subtract 52 repeatedly (base 10), we get to subtract a multiple of 10 (base 13) instead! As each deck has a total value of 20, we can take the total count and subtract it from 20 mod 40 (base 13) to get the value of the remaining card. A proof is shown below. Let x be the value of the remaining card and let c be the count of the deck excluding the remaining card. Then

$$x + c \equiv_{40} 20$$

$$x + c + (-c) \equiv_{40} 20 + (-c)$$

$$x + 0 = x \equiv_{40} 20 - c$$

Now, let's do a few examples of adding and subtracting mod 40 (base 13).

$$2J + 12 = 3K$$

$$34 + 13 = 47 = 40 + 7 \equiv_{40} 7$$

$$1K + Q = 2J$$

$$20 + 7 = 27$$

$$7 - 20 = -13 = -40 + 2J \equiv_{40} 2J$$

Finally, there is one more simplification that can be made. Rather than subtracting the count from 20, we can start the count at a value of 20 if we have a single deck of cards. Then, the value of the unknown card can be taken by finding the negative of the updated count. Consider the following:

$$x + (c + 20) \equiv_{40} 20$$

$$x + c + 20 + (-20) \equiv_{40} 20 + (-20)$$

$$x + c + (20 - 20) \equiv_{40} (20 - 20)$$

$$x + c \equiv_{40} 0$$

$$x \equiv_{40} - c$$

Finally, if we have multiple decks of cards, how can we modify this method? If the total count of one deck is 20 mod 40 (base 13), then we can get the total count of n decks of cards as

$$20n \equiv_{40} \begin{cases} 0 \text{ if } n \text{ is even} \\ 20 \text{ if } n \text{ is odd} \end{cases}$$

So if we have an even number of decks, we will start our count at 0. If we have an odd number of decks, we will start our count at 20. Using group theory, we have managed to take the problem of memorizing multiple decks of cards (which is very difficult) to a problem involving repeated addition (which can be solved with a bit of practice).