Math 320 Midterm

February 16, 2023

You will have from the start of class until the end of the class period to complete your exam. Please clearly write each solution on the provided printer paper, and feel free to write any additional scratch work on separate paper. Please clearly number the start of a problem for your scratch work and for your solution.

Computation

Each of these questions will be worth 10 points.

1.

Draw a shape with the symmetry group below:
a. C_5 but **not** D_5
b. D_5

2.

Let $\alpha = (123)(578)$ and $\beta = (12)(53)(68)$ be permutations in cycle notation. Compute the products below. Please write your solution as a product of disjoint cycles.
a. $\alpha\beta$
b. $\beta\alpha$

3.

Consider the group $G = U(7)$. (The group of integers $1 \leq n \leq 6$ under multiplication). Find all of the subgroups of G (Hint: G is cyclic).

4.

Show $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that $f(x, y) = (x + y, x - y)$ is a bijection.

Proofs

Your lowest scoring response to the next 3 questions will be dropped. Each question is worth 20 points.

5.

a)

State the definition of a subgroup.

b)

Let G be a group. The centralizer of $a \in G$ is defined as

$$C(a) = \{b \in G | ab = ba\}$$

Show $C(a)$ is a subgroup of G.

6.

Let G be a group, $a, b \in G$, and let $n \geq 2$. Show $(ab)^n = a^n b^n$ if and only if G is abelian.

7.

a)

State the definition of an equivalence relation.

b)

Recall groups G, H are isomorphic if there exists a bijection $f : G \to H$ such that $f(ab) = f(a) f(b)$ for all $a, b \in G$. Show isomorphism forms an equivalence relation on the "set" of all groups.