

Math 320 Midterm

February 20, 2024

You will have from the start of class until the end of the class period to complete your exam. Please clearly write each solution on the provided printer paper, and feel free to write any additional scratch work on separate paper. Please clearly number the start of a problem for your scratch work and for your solution. **Proof Questions are on the back side of this paper.**

Computation

Each of these questions will be worth 10 points.

1.

Draw a Cayley digraph for D_4 .

2.

Let $\alpha = (143)(23)$ and $\beta = (34)(234)(12)$ be permutations in cycle notation. Compute the products below. Please write your solution as a product of disjoint cycles.

a) $\alpha\beta$

b) $\beta\alpha$

3.

Make a multiplication table for the group $U(5)$.

4.

Show $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $f(x, y) = (2x + y, y - 3)$ is a bijection.

Proofs

Your lowest scoring response to the next 3 questions will be dropped. Each question is worth 20 points.

5.

a) State the definition of an abelian group.

b) Show the set $S = \mathbb{R} \setminus \{-1\}$ forms an abelian group under the binary operation given below:

$$a \diamond b = a + b + ab$$

6.

a) State the definition of antisymmetric relation.

b) Let $|$ be the divisibility relation. That is $a|b$ if a divides b for positive integers a, b . Show this is an antisymmetric relation.

7.

a) State the definition of an isomorphism.

b) Let $\phi : G \rightarrow G$ be an isomorphism. Show

$$\text{Fix}(\phi) = \{a \in G \mid \phi(a) = a\}$$

is a subgroup of G .

8.

Extra Credit (3 points): What do you like about the class so far? Is there anything you wish was done differently? (All responses are valid will count for extra credit, so if you would prefer not to answer, just write “No comment”).