Math 320 HW 7

March 4, 2024

1)

a)

Compute all the elements of $G = C_2 \times D_3$ and $H = C_2 \times C_6$.

b)

Make a Cayley digraph for G and H.

c)

How many generators do you need to generate G and H?

d)

Are G and H isomorphic?

2)

Determine which of the functions $\phi : G \to H$ below are homomorphisms. If ϕ is a homomorphism, prove it. If ϕ isn't a homomorphism, explain why.

a)

$G = (\mathbb{C}^*, *)$, $H = (\mathbb{R}^+, *)$, $\phi(z) = |z|$ where $\mathbb{C}^*$ is the set of non-zero complex numbers.

b)

$G = (\mathbb{C}, *)$, $H = (\mathbb{R}, +)$, $\phi(z) = |z|$

c)

$G = \left(\mathbb{R}^3, +\right)$, $H = \left(\mathbb{R}^2, +\right)$, $\phi(\vec{x}) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix} \vec{x}$

3)

For each of the functions in **3)** that are homomorphisms, find $\mathrm{Ker}(\phi)$.

4)

Suppose $\phi : G \to H$ is a homomorphism that is onto. Show that ϕ is an isomorphism if and only if $\mathrm{Ker}(\phi) = \{e_H\}$.

5)

Suppose group G is generated by the elements of $X = \{g_1, g_2, \ldots, g_n\}$ and H is generated by the elements of $Y = \{h_1, h_2, \ldots, h_m\}$. Show that $G \times H$ is generated by the elements of $\{(g_i, e_h), (e_g, h_j) | 1 \leq i \leq n, 1 \leq j \leq m\}$ and where e_g, e_h are the identity elements of G and H respectively.

6)

Consider the projection map $\mathrm{proj}_k : \times_{i=1}^{n} G_i \to G_k$ given by

$$\mathrm{proj}_k\left((g_1, g_2, \ldots, g_n)\right) = g_k$$

This mapping will output the coordinate in a given ordered tuple corresponding to the kth group. Show proj_k is a homomorphism and find its kernel.