Math 320 HW 5

Due: Feb. 20, 2024 at 11:59 PM

1)

a)

Let $G = S_8$. Show $H = \{(1), (154)(326)(78), (154)(326), (145)(362), (145)(362)(78), (78)\}$ is a subgroup of G.

b)

Find the orbits of $1, 2, 3, 4, 5, 6, 7, 8$ in H.

c)

Find the stabilizers of $1, 2, 3, 4, 5, 6, 7, 8$ in H.

2)

Write the Multiplication Table (Cayley Table) for each of the following groups. Use these tables to write the permutation group corresponding to this multiplication table.

a)

D_4

b)

$V_4 = \langle a, b | a^2 = b^2 = e, ab = ba \rangle$

3)

Let H be a subset of S_5, such that $H = \{\alpha \in S_5 | \alpha(1) = 1, \alpha(3) = 3\}$.

a)

Show H is a subgroup of S_5.

b)

How many elements are in H?

c)

What proportion of the elements of S_5 are in H? (Hint, there are $n!$ possible permutations on n objects).

4)

Suppose α, β are both permutations on n elements. Prove that exactly one of α, β is odd if $\alpha\beta$ is odd.

5)

A standard soccer ball (football if you prefer) has 12 regular pentagonal faces, 20 regular hexagonal faces, 60 vertices, and 90 edges. Mathematically, we call this shape a truncated icosahedron. A rotating diagram and more information about this polyhedron is provided on the Wikipedia page here: https://en.wikipedia.org/wiki/Truncated_icosahedron. We will count the number of symmetries of the truncated icosahedron.

a)

Number each of the pentagonal faces. Pick a pentagonal face. How many rotations of the soccer ball fix this pentagonal face?

b)

Under the set of rotations of the soccer ball, what is the orbit of the pentagon face you chose from **a)**?

c)

Using orbit stabilizer theorem, with the numbers from **a)** and **b)**, determine the number of symmetries of the soccer ball.

d)

Now, number each of the hexagonal faces. Pick a hexagonal face. How many rotations of the soccer ball fix this hexagonal face? (Hint: a rotation of 60^o about a hexagonal face is not a symmetry of the soccer ball)

e)

Under the set of rotations of the soccer ball, what is the orbit of the hexagonal face you chose from **d)**?

f)

Using orbit stabilizer theorem, with the numbers from **d)** and **e)**, determine the number of symmetries of the soccer ball.

Conclusion

Even though we are describing the symmetries of a soccer ball, this same method is used when counting the symmetries of various molecules. It turns out the rotational symmetries of the soccer ball are the same as the rotational symmetries of a Buckyball. More information about Buckyballs can be found here: https://en.wikipedia.org/wiki/Buckminsterfullerene

6)

Pick any polyhedron from the wikipedia page below (besides the platonic solids, as we have discussed these in class). State the polyhedron, and calculate the number of rotational symmetries using orbit-stabilizer theorem.

https://en.wikipedia.org/wiki/List_of_uniform_polyhedra