Math 320 HW 2

Due: Jan. 23, 2023 at Midnight

1)

For the following functions, determine if the function is onto, one-to-one, both, or neither. Provide an argument for your reasoning.

1. $f : \mathbb{R}^+ \to \mathbb{R}$, $f(x) = x^2$

2. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

3. $f : \mathbb{R}^+ \to \mathbb{R}^+$, $f(x) = x^2$

4. $f : \mathbb{R}^4 \to \mathbb{R}^4, f\left(\begin{array}{c} a \\ b \\ c \\ d \end{array} \right) = \left(\begin{array}{c} c \\ a \\ d \\ b \end{array} \right)$

2)

Let $f : A \to B$ and $A_0, A_1 \subseteq A$. Show the following:

1. $f(A_0) \subseteq f(A)$

2. $f(A_0 \cup A_1) = f(A_0) \cup f(A_1)$

3. $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$ in general and $f(A_0 \cap A_1) = f(A_0) \cap f(A_1)$ if f is one-to-one (injective).

3)

Determine if the following relations are equivalence relations, anti-symmetric relations, asymmetric relations, or neither. Provide an explanation for each claim.

1. $R = \subseteq$

2. $R = \subsetneq$

3. $x, y \in \{\text{triangles in } \mathbb{R}^2\}$, $xRy \iff x, y$ are similar triangles

4)

Determine if each of the given functions are binary operations. If a given function is a binary operation, determine if the binary operation is associative and commutative.

1. $a - b$ for positive integers a, b

2. $a * b$ for rational a, b

3. AB for invertible $n \times n$ matrices A, B

4. $\frac{a}{b}$ for rational a, b

5. $|a| \, |b|$ for complex numbers a, b

5)

Let X be a set. How many binary operations $f : X \times X \to X$ are there if:

1. $|X| = 2$
2. $|X| = 3$
3. $|X| = 4$