Math 320 HW 4

Due: Feb. 13, 2024 at 11:59 PM

1)

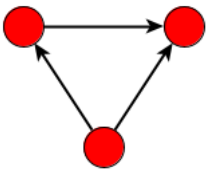
Create a Cayley digraph for each of the following groups. What is the minimum number of generators needed to create the group?

1. D_3
2. S_3

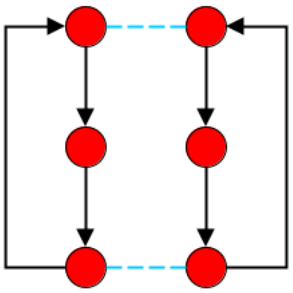
2)

The digraphs below are not valid Cayley Digraphs. Explain why each one is not.

a)



b)



3)

Rewrite the following cycles as a product of 2-cycles and as a product of disjoint cycles.

1. $(631)\,(1247)\,(5)$
2. $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 3 & 1 & 2 & 4 \end{bmatrix}$

5)

1. Draw a shape with D_5 symmetry
2. Draw a shape with C_6 symmetry

6)

For subgroups $H, K \leq G$, show $H \cap K$ is a subgroup of G. Is $H \cup K$ a subgroup of G? If so, prove it. Otherwise, provide a counterexample.

7)

The center $Z(G)$ of group G is defined as follows:

$$Z(G) = \{a \in G | ax = xa \ \forall x \in G\}$$

Show $Z(G)$ forms a subgroup of G.